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CHARACTERISTICS AND LIMITATIONS OF THE SERIES TRANSFORMER

BY

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AND

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ENGINEERING EXPERIMENT STATION

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CHARACTERISTICS AND LIMITATIONS OF THE SERIES TRANSFORMER

BY

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CHARACTERISTICS AND LIMITATIONS OF THE SERIES TRANSFORMER

I. Introduction

1. *Uses of the Series Transformer.*—High potential distribution and the large currents carried by feeders have made the use of series transformers, or so-called "current transformers," imperative. Where high voltages are used it would be a source of considerable danger to bring the potential of the distribution system to the switchboard and controlling apparatus. The use of the series transformer, in connection with the potential transformer, makes it possible to meter the power and control such system without handling voltages which are dangerous to life. Furthermore, if live potentials were brought to the switchboard, the proper insulation of instruments and controlling apparatus would be accomplished only with considerable difficulty and expense. Again in the present day system of distribution, even where the voltage may be so low as to overrule the above objection, the feeders leading from the station may carry such large currents as to make the use of series transformers a decided convenience. Obviously it is much cheaper and easier to run small wires to instruments requiring but a low value of current than to bring large feeders to a board on which are mounted cumbersome instruments of sufficient capacity to carry the full live current. The demands of convenience and safety, then, have placed the series transformer in its present position of importance.

In general, the function of the series transformer is to furnish a current in a circuit not metallically connected with the main circuit, which current shall bear a definite constant ratio to the main current, and shall be 180° out of phase with it. The uses to which this secondary current is put may be divided into two classes: (1) to give indications in metering instruments, and (2) to furnish power for the operation of regulators, time-limit relays, and like controlling apparatus. But with the series transformer the ideal is never completely realized, and consequently the above function is but imperfectly performed.

2. *Scope of Bulletin.*—It is the purpose of this bulletin to study the imperfections of the series transformer, to determine how and to what extent certain constants influence its operation, and to deduce certain general characteristics. The first part of the bulletin will be devoted to a discussion of the fundamental principles of the series transformer and the representation by vector diagrams of its operation. A deduction of current relations by the method of complex quantities, and

a discussion of conclusions that may be derived therefrom will follow. The derivation of current relations by the use of instantaneous current values will then be given. In this last named part particular stress will be laid upon the application of the current transformer for the purpose of recording transient phenomena. A comparison of the results obtained for stable condition by the two methods and a general summary will be given in conclusion.

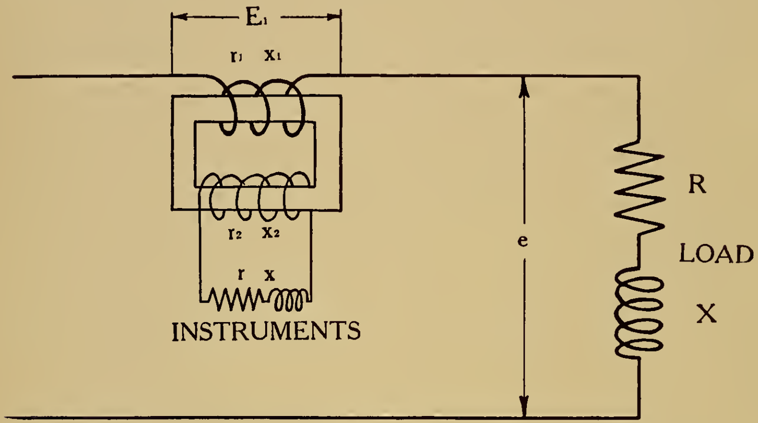

FIG. 1. SYMBOLS USED FOR CONSTANTS OF CIRCUIT

II. CHARACTERISTICS OF THE SERIES TRANSFORMER

3. *General Features of the Series Transformer.*—In construction the series transformer does not greatly differ from the ordinary power potential transformer, inasmuch as both consist essentially of two electrical circuits interlinked by a magnetic circuit. In the potential transformer for furnishing power there is, of course, a variation of primary current with secondary current; in fact, the primary current is a function of the secondary current. In the current transformer, however, the effect of the secondary current upon the primary current is so very small as to be negligible. The primary current is determined by the constants of the main circuit, and is taken to be independent of variations in secondary load and current. Herein then, lies the basic element of distinction between the power potential transformer and the current transformer. This dependence of the primary current upon the constants of the main circuit only, and its independence of variations in secondary current, is the starting point for the theory of the current transformer which follows. Naturally, this difference in operation between the two types of transformers makes a difference in their design, and just how and why some of the design constants differ will be pointed

out later after a development of the theory has made their bearing on the operation of the transformer clearer.

There are certain facts about the series transformer which are generally known among men who have anything at all to do with it. It is known that the series transformer consists of two coils metallically separate, wound on a laminated iron core; that the coil connected in the line generally has fewer turns than the secondary coil across which the instruments are connected, and that the ratio of the turns is about in inverse proportion to the ratio of the currents. It is known that it is dangerous to leave the secondary open-circuited because if a current flows in the primary, high voltages will be induced in the open-circuited secondary and there will be high heating of the iron core which may cause disaster. It is also known that slight variations in secondary load (or resistance and reactance) do not change the transformation ratio to a very great extent, and that the ratio is nearly constant for all values of primary current within a given range. It is also known that when used in connection with wattmeters annoyance is often caused by a certain angle of phase displacement from the ideal position. These very nearly constant factors, however, such as current ratio and phase angle, are very often subject to great variations, and in the following discussion it is proposed to investigate the relations between these factors and the constants of the transformer and load; to determine how and to what extent they are influenced by these constants.

4. *Vector Diagrams for the Series Transformer.*—At the outset it is thought advantageous to study the vector diagram representation of the current transformer in order to gain familiarity with its operation. Fig. 1 is given to indicate the constants represented by the different symbols. Fig. 2 is a simple vector diagram of a series transformer having no core loss. This would be the case with a transformer having an air core. The voltage e across the load is taken as the reference vector, and the primary current I' lags behind it an angle θ dependent upon the power-factor of the load. $\left(\tan\theta = \dfrac{X}{R}\right)$ In phase with I' is ϕ' the total primary flux, or the flux which could be set up in the existing magnetic circuit by the primary ampere-turns if no counteracting force were present. A part of this flux, ϕ_L', however, is set up in a path not interlinked by the secondary coil, that is, it does not thread the secondary coil, and hence has no effect on the secondary circuit.*

*The leakage flux of the primary, it is readily seen, acts only as a reactance in the primary circuit and does not affect the ratio of the currents in the transformer. Since, however, the primary leakage reactance is an inherent constant of the transformer, it is thought well to introduce it here for the completeness of the discussion.

This flux is, of course, in phase with the primary current. Since it is established in a path external to the secondary coil, it is very properly called leakage flux, and if subtracted from the total primary flux, gives as a resultant what might be called the useful primary flux, in phase with the primary current, and represented in the diagram by the vector ϕ'-ϕ_L'. Now in order that a current may flow in a secondary circuit having impedance, there must be induced in the secondary an e.m.f., and to accomplish this there must exist in the core a *real* flux leading the induced e.m.f. by 90°. This flux, indicated in the diagram by ϕ_M is the only flux *really existing* in the core and is the resultant of the useful primary flux and the useful secondary flux $\phi'' - \phi_L''$, as indicated in the diagram. ϕ'' is the total secondary flux, and ϕ_L'' is the secondary leakage flux, corresponding to ϕ' and ϕ_L', for the primary. I'' is, of course, in phase with ϕ''. The induced secondary voltage is indicated in the diagram by E''. The angle α between E'' and I'' is dependent upon the resistance and reactance of the secondary circuit ($\tan \alpha = \dfrac{x_2 + x}{r_2 + r}$),

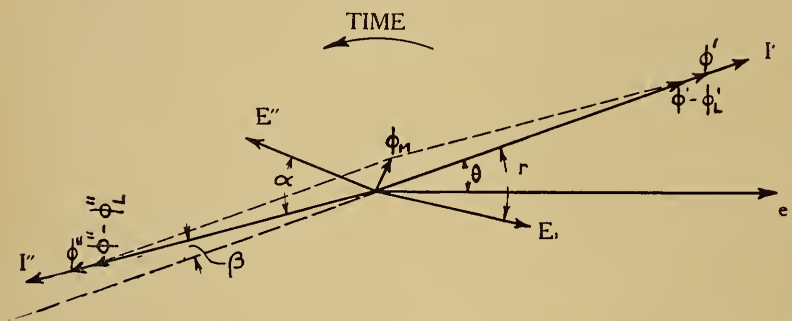

FIG. 2. VECTOR DIAGRAM FOR FLUX IN TRANSFORMER WITH ZERO CORE LOSS

where x_2 is the secondary *leakage* reactance.) β is known as the angle of phase displacement of the secondary current, and is the angle by which the secondary current leads its ideal position of 180° out of phase with the primary current. The function which gives β in terms of other constants of the transformer will be derived later. The variations in the positions and magnitudes of the vectors with the constants of the transformer are not readily seen from this diagram, but there are certain things which might be expected from an examination of the diagram. For instance, it might be expected that a decrease in secondary reactance or an increase in secondary resistance, either of which has the effect of decreasing α, would as a consequence increase β. It will be seen later that such a variation actually takes place.

It is oftentimes convenient to draw the vector diagram using currents in place of fluxes, as is done in Fig. 3. If the ratio of turns is other than 1 to 1, it will be necessary to consider the magnitude of the current vectors given in ampere turns, or what amounts to the same thing, if the primary current vector is drawn to scale the secondary current should be multiplied by the ratio of the number of secondary turns to primary turns to give the magnitude of the secondary current vector. The resulting current vectors will then be in terms of primary current directly, and it will not be necessary to divide by the number of turns as would be the case if the vectors were scaled off as ampere-turns.

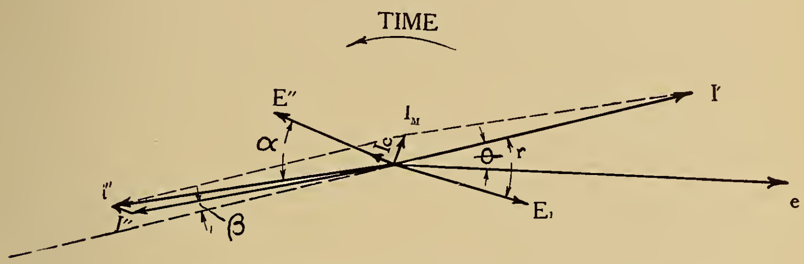

FIG. 3. VECTOR DIAGRAM FOR CURRENTS IN TRANSFORMER WITH ZERO CORE LOSS

Fig. 3 is a vector diagram of a current transformer with an iron core, having consequently some core loss. It is very similar to the diagram in Fig. 2 with the exception that the flux vectors are plotted as currents, and the core loss component I_c is taken into consideration. Core loss represents power dissipated in a secondary circuit, and requires in phase components of current and electromotive force. Since the dissipation of energy takes place in a secondary circuit and we have already drawn a vector for secondary e.m.f. the current vector I_c in phase with E'' will give the required representation of energy dissipation. This current I is consumed in the core, and hence is not available in the useful secondary circuit. Subtracting I_c from i'' gives the true secondary current I'' as represented in the diagram. The angles $\propto$ and β correspond to the angles $\propto$ and β of Fig. 2. In both Fig. 1 and Fig. 2, E_1 is the electromotive force across the primary and γ the angle that it makes with the primary current. From Fig. 3 the effect of core loss is readily seen. It decreases the secondary current I'', and with an inductive secondary circuit it decreases also the phase angle β. Since it is impossible to construct a transformer which has not at least the secondary reactance due

to leakage flux, it must follow that in a commercial transformer core loss has both of the above effects. If these were its only effects it might be thought, ignoring efficiency, that core loss is desirable. But it will be seen later that hysteresis loss is accompanied by a variation in core reluctance that is extremely undesirable so far as regulation is concerned.

5. *Formulas for Current Ratio and Phase Angle—Use of Complex Quantities.*—With these vector diagrams in mind we may proceed with a mathematical discussion of the current transformer by the use of complex quantities. In the following derivation it will be considered for simplicity, that permeability of the iron is constant. The error thus introduced is not so great as might at first be expected, due to the fact that the iron in a current transformer is generally worked at a very low density on the straighter portion of the saturation curve. However, the assumption of constant permeability still makes it possible to discuss from the resulting equations the effects of variation in permeability. If the permeability is taken constant the hysteresis loop vanishes, and the only remaining core loss is the eddy current loss. With sufficiently careful lamination and at the low value of flux density used, this loss may be very small. Indeed in a commercial current transformer the core losses are very small. Neglecting then for the present all core losses, the following derivation will correspond to the vector diagram of Fig. 2 and will be rigid for a transformer with an air core. Let a sine wave of e.m.f. e instantaneous value $= e\sqrt{2} \sin \omega t$, be impressed upon the circuit as indicated in Fig. 1. Then since the effect of the secondary current upon the primary current is neglected

$$\dot{I}' = \frac{e}{R - jX} = i + ji_1 \text{ *} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

where $R—jX$ is the load impedance.

where $$\dot{i} = \frac{eR}{R^2 + X^2} \quad \text{and} \quad \dot{i}_1 = \frac{eX}{R^2 + X^2}$$

The flux set up by the primary current or the total primary flux is proportional to the primary current and in phase with it or

$$\dot{\Phi}' = K_1 N_1 \dot{I}' \times 10^8 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where $K_1 = \dfrac{.4\pi}{\rho_1} \times 10^{-8}$; ($\rho_1$ = combined reluctance of iron circuit

*In all these following equations involving complex quantity, time is considered the independent variable and is represented by counter clock-wise rotation. The inductive impedance is $R - jX$ and thus there is a lagging current $\dot{I} = i + ji_1$.

and leakage path), and N_1 is the number of primary turns. Of Φ' a certain part known as leakage flux does not thread the secondary, but produces a reactive e. m. f. in the primary, which is given by the following expression:

$$e_r' = j\,\frac{2\pi f N_1 \Phi'_L}{10^8} \qquad \text{.................... (3)}$$

It should perhaps be pointed out that the value given above for electromotive force, current, and flux, as well as those to follow, are all effective values.

Equation (3) gives an expression for reactive primary e.m.f. in terms of leakage flux. But the primary leakage reactance x_1 also gives the value of this e.m.f., as follows:

$$e_r' = j\, I' x_1 \qquad \text{........................ (4)}$$

Combining (3) and (4) we obtain

$$\frac{2\pi f N_1 \Phi_L'}{10^8} = I' x_1$$

or

$$\Phi_L' = \frac{I' x_1}{2\pi f N_1}\,10^8 = \frac{I' x_1}{\omega N_1}\,10^8 \qquad \text{.............. (5)}$$

An expression for the magnetizing flux Φ_M may be obtained by starting with the induced secondary e.m.f. This e.m.f. must be sufficient to overcome all of the impedance of the secondary circuit including the leakage x_2. Hence, we may write

$$E'' = I''\, Z'' \qquad \text{........................ (6)}$$

But the induced secondary e.m.f. is dependent upon the magnetizing flux in the following relation:

$$E'' = j\,\frac{2\pi f N_2 \Phi_M}{10^8} \qquad \text{.................... (7)}$$

Combining (6) and (7) an expression for magnetizing flux in terms of secondary current and impedance is obtained as follows:

$$j\,\frac{2\,\pi f\,N_2\,\underset{\cdot}{\Phi}_M}{10^8} = \underset{\cdot}{I}''\,\underset{\cdot}{Z}''$$

Whence,

$$\underset{\cdot}{\Phi}_M = -\,j\,\frac{\underset{\cdot}{I}''\;\underset{\cdot}{Z}''}{\omega\,N_2}\,10^8 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The total secondary flux is proportional to the secondary current and in phase with it, or

$$\underset{\cdot}{\Phi}'' = K_2\,N_2\,\underset{\cdot}{I}'' \times 10^8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where $K_2 = \dfrac{4\pi}{\rho_2} \times 10^{-8}$; ($\rho_2$ = combined reluctance of iron circuit and leakage path).

In a manner similar to that for obtaining equation (5) for the primary leakage flux, the following expression for the secondary leakage flux is obtained,

$$\underset{\cdot}{\Phi}_L'' = \frac{\underset{\cdot}{I}''\,x_2}{\omega\,N_2}\,10^8 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

If we subtract from the total primary flux the primary leakage flux we obtain what may be called the useful primary flux. Similarly the difference between the total secondary flux and the secondary leakage flux gives the useful secondary flux. The sum of these two useful fluxes then gives the magnetizing flux, or expressed in the form of an equation,

$$\underset{\cdot}{\Phi}' - \underset{\cdot}{\Phi}_L' + \underset{\cdot}{\Phi}'' - \underset{\cdot}{\Phi}_L'' = \underset{\cdot}{\Phi}_M \ldots\ldots\ldots\ldots\ldots (11)$$

which may be written

$$\underset{\cdot}{\Phi}' - \underset{\cdot}{\Phi}_L' + \Phi'' = \underset{\cdot}{\Phi}_L'' - \underset{\cdot}{\Phi}_M = 0 \;\ldots\ldots\ldots\ldots (12)$$

Substituting in (12) the expressions for the various fluxes given by equations (2), (5), (9), (10), and (8), and dividing through by 10^8, we obtain

$$K_1\,N_1\,\underset{\cdot}{I}' - \frac{\underset{\cdot}{I}'\;x_1}{\omega\,N_1} + K_2\;N_2\;\underset{\cdot}{I}'' - \frac{I''\,x_2}{\omega\,N_2} + j\,\frac{\underset{\cdot}{I}''\,\underset{\cdot}{Z}''}{\omega\,N_2} = 0 \,..(13)$$

or $$\left(K_1 N_1 - \frac{x_1}{\omega N_1}\right) \dot{I'} + \left(K_2 N_2 - \frac{x_2}{\omega N_2} + j\frac{\dot{Z''}}{\omega N_2}\right) \dot{I''} = 0 \ldots (13')$$

Whence, $$\dot{I''} = - \frac{(K_1 N_1^2 \omega - x_1)\, \omega N_2}{\omega N_1 (K_2 N_2^2 \omega - x_2 + j \dot{Z''})} \dot{I'} \ldots\ldots\ldots (14)$$

But $\dot{Z''} = (r_2 + r) - j(x_2 + x)$

And $j\dot{Z''} = (x_2 + x) + j(r_2 + r) \ldots\ldots\ldots (15)$

Substituting (15) in (14),

$$\dot{I''} = - \frac{(K_1 N_1^2 \omega - x_1) N_2}{N_1 [K_2 N_2^2 \omega + x + j(r_2 + r)]} \dot{I'} \ldots\ldots (16)$$

which might better be written

$$\dot{I''} = - \frac{N_2}{N_1} \frac{(K_1 N_1^2 \omega - x_1)}{(K_2 N_2^2 \omega + x)^2 + (r_2 + r)^2} [K_2 N_2^2 \omega + x - j(r_2 + r)] \dot{I'} \quad (17)$$

The ratio of secondary current to primary current then becomes

$$\frac{I''}{I'} = - \frac{N_2}{N_1} \frac{(K_1 N_1^2 \omega - x_1)}{\sqrt{(K_2 N_2^2 \omega + x)^2 + (r_2 + r)^2}} \ldots\ldots (18)$$

Equation (18) may be reduced as follows: K_1 and K_2 represent the primary and secondary magnetic conductances respectively. If $F = \frac{0.4\pi}{\rho} \times 10^{-8}$ represents the conductance of the iron path, and k_1 and k_2 represent the magnetic conductances of primary and secondary leakage paths respectively, then for K_1 and K_2 in equation (18) may be substituted:

$$K_1 = F + k_1$$

$$K_2 = F + k_2$$

$$\frac{I''}{I'} = - \frac{N_2}{N_1} \frac{(F N_1^2 \omega + k_1 N_1^2 \omega - x_1)}{\sqrt{(F N_2^2 \omega + k_2 N_2^2 \omega + x)^2 + (r_2 + r)^2}}$$

$$k_1 N_1^2 \omega = x_1$$

$$k_2 N_2^2 \omega = x_2$$

$$\frac{I''}{I'} = - \frac{N_2}{N} \frac{F N_1^2 \omega}{\sqrt{(F N_2^2 \omega + x_2 + x)^2 + (r_2 + r)^2}} \ldots . (19)$$

which shows the dependence of current ratio upon the primary leakage reactance. To show more clearly the relation between the currents and the number of turns equation (19) may be reduced to the form:

$$\frac{I''}{I'} = - \frac{N_1}{N_2} \frac{F \omega}{\sqrt{\left(F \omega + \frac{x_2 + x}{N_2^2}\right)^2 + \left(\frac{r_2 + r}{N_2}\right)^2}} \ldots . (20)$$

from which it is seen that with no secondary resistance or reactance the ratio of currents is inversely as the ratio of the number of turns. Furthermore, the lower the reluctance of the iron circuit, and consequently the greater the value of F, the higher the frequency, and the greater the number of secondary turns, the more nearly does the transformation ratio equal the ratio of the number of turns.

Equation (20) is useful in showing certain relations, but equation (18) may be reduced in another way which yields results of interest. From equation (2)

$$\Phi' = K_1 N_1 I' \times 10^8$$

and $\quad N_1 \Phi' = K_1 N_1^2 I' \times 10^8$

But $\quad N_1 \Phi' = L_1 I' \times 10^8$

Whence $\quad K_1 N_1^2 = L_1$

and $\quad K_1 N_1^2 \omega = L_1 \omega = X_1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (21)$

Where X_1 is the total inductive reactance of the primary coil.

Similarly, $\quad K_2 N_2^2 \omega = X_2 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (22)$

If the primary leakage reactance x_1 be subtracted from the total primary reactance X_1, and the difference multiplied by the ratio of the secondary turns to the primary turns the result will be the mutual inductive reactance X_M or

$$\frac{N_2}{N_1}(X_1 - x_1) = X_M \qquad (23)$$

Now substituting (21), (22) and (23), in (18), the following expression is obtained,

$$\frac{I''}{I'} = -\frac{X_M}{\sqrt{(X_2 + x)^2 + (r_2 + r)_2}} \qquad (24)$$

From (24) it is seen that with zero resistance of the secondary coil and load, the ratio of transformation is as the ratio of the mutual inductive reactance to the total reactance of the secondary circuit.

From equation (20) it is seen that the effect of increasing the secondary resistance or reactance is to decrease the secondary current; that the effect of increasing the frequency or increasing F, which means decreasing the magnetic reluctance, is to increase the secondary current. Equation (20) further shows that a large number of secondary turns makes variations in frequency, permeability, and secondary resistance and reactance less effective in influencing the transformation ratio, and hence points to the desirability of a large number of secondary turns.

Referring again to equation (17) it is seen that the phase angle between the secondary current and the projected primary current is

$$\beta = \text{arc tan}\,\frac{r_2 + r}{K_2 N_2^2\,\omega + x} = \text{arc tan}\,\frac{r_2 + r}{FN_2^2\,\omega + x_2 + x} \qquad (25)$$

or $$\beta = \text{arc tan}\,\frac{r_2 + r}{X_2 + x} \qquad (25')$$

Stating equation (25′) in words the tangent of the phase angles is directly proportional to the total secondary resistance, and inversely proportional to the total secondary reactance.

If it is desired to study the current ratio from the design constants of a transformer, equation (18) will be found the more convenient; if it is desired to study this ratio from data taken experimentally, equation (24) may perhaps be used more conveniently. The equations for phase angles are so simple that there would be no trouble in using either one or the other.

As an example a transformer having the following constants may be considered:

$$r_1 = .0058 \text{ ohms.}$$

$$x_1 = .058 \text{ ohms at } 60\sim$$

$$K = .000015$$

$$F = .00001425$$

$$N_1 = 14$$

$$N_2 = 133$$

$$r_2 = 1 \text{ ohm}$$

$$x_1 = K N_1^2 \omega = 1.11 \text{ ohms at } 60\sim$$

$$x_2 = K N_2^2 \omega = 100 \text{ ohms at } 60\sim$$

$$\frac{N_2}{N_1} = 9.5$$

$$X_M = \frac{N_2}{N_1}(X_1 - x_1) = 10 \text{ ohms at } 60\sim$$

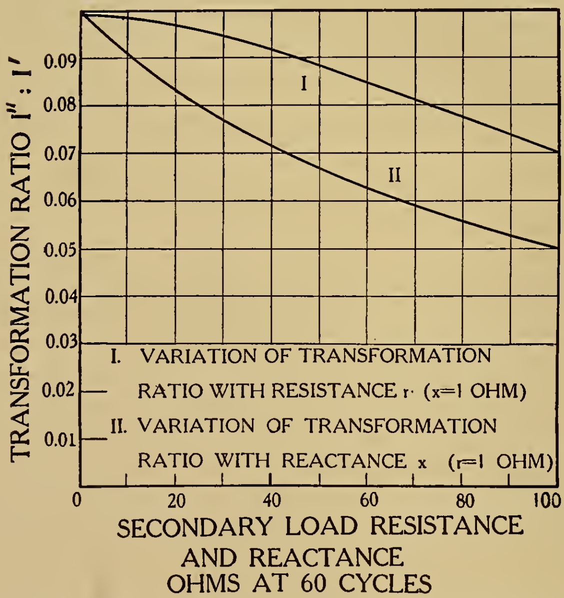

FIG. 4. EFFECT OF SECONDARY LOAD RESISTANCE AND REACTANCE ON TRANSFORMATION RATIO

Taking the secondary load reactance equal to 1 ohm, we may study the effect of secondary load resistance upon the transformation ratio. Fig. 4 Curve I. shows the relation between the transformation ratio and secondary load resistance. It is seen that for reasonable values of resistance (say up to 5 ohms) the ratio is but little affected. Beyond 10 ohms, however, an increase in the resistance results in quite an appreciable decrease in the value of secondary current, and the rate of decrease becomes greater with increased resistance. If the secondary load resistance be taken as 1 ohm, the effect of secondary load reactance upon the transformation ratio may be studied. Fig. 4 Curve II. shows the variation of the transformation ratio with secondary load reactance. It is seen that the ratio is very appreciably decreased by increasing the reactance and that the ratio of decrease is greatest for low values of reactance. Comparing then secondary resistance and reactance as regards their effect upon the transformation ratio, it may be said that for reasonable values the effect of increasing the former is to decrease but negligibly the ratio, whereas an increase in the latter results in a very considerable decrease of the ratio.

Taking the secondary load reactance equal to 1 ohm at 60 cycles, and choosing three values of secondary load resistance, viz., 1 ohm, 10 ohms, and 50 ohms, three curves were plotted, one for each value of resistance, showing the effect of varying the frequency upon the transformation ratio. These curves are given in Fig. 5. An inspection of these curves shows that for high values of frequency the effect of variation in frequency upon the transformation ratio becomes negligible, and also that the lower the secondary resistance, the lower the frequency at which the ratio becomes practically constant. With 1 ohm secondary load resistance it is seen that the ratio is practically constant above 25 cycles. With reasonably low secondary resistance then it may be concluded that the effect of frequency variation, within commercial limits, upon the transformation ratio is quite small. It should not be forgotten, however, that below a certain critical frequency for a given value of secondary resistance decreasing the frequency will result in a very rapid decrease of the transformation ratio.

The effect of secondary load resistance on the phase angle β will be considered next. Taking the secondary load reactance x equal to 1, the curve shown in Fig. 6, was plotted showing the variation of phase angle with the secondary load resistance. It is seen that the increase in phase angle is practically proportional to the increase in resistance for reasonable values of r. By taking the secondary load resistance equal to unity, the variation of phase angle with secondary load reactance may be studied. Fig. 7 shows the relation between these two quantities. It is

noted that increasing the secondary reactance decreases the phase angle, the ratio of decrease being almost constant for reasonable values of x. The effect of x in decreasing the phase angle, however, is not nearly as great as the effect of r in increasing it. As shown by the curves, a given change in x changes the phase angle by less than 2% of that caused by an equivalent change in r.

By taking secondary load resistance equal to 1 ohm, and secondary load reactance equal to 1 ohm at 60 cycles, Fig. 8 was drawn showing the effect of frequency on the magnitude of the phase angle. This curve shows that the phase angle is very sensitive to changes in frequency within the range of commercial frequencies, the phase angle at 25 cycles being more than twice as great as at 60 cycles and over five times as great as at 133 cycles.

6. *Formulas for Current Ratio and Phase Angle of Series Transformers with Iron Cores.*—Throughout the above discussion zero core loss and constant magnetic reluctance have been assumed. In other words the above discussion applies rigidly to a transformer with an air core. The commercial series transformer, however, has an iron core. As was pointed out previously, due to the low flux density in the core and careful lamination, the core loss in such a transformer is very small; so small in fact, as to be of no consequence in affecting the validity of the foregoing discussion. In using an iron core, however, there is an effect more objectionable than the hysteretic core loss, viz., the accompanying variation in core reluctance.

An examination of equation (19) shows that with given constants and frequency, the ratio of secondary current to primary current is constant, independent of the value of primary current. This is true of an air core transformer where the permeability of the core, and consequently the factor F, are constant. But in a transformer with an iron core the permeability and consequently the factor F varies with the flux density. Since the flux density varies with the primary current, it must follow that the factor F is a function of the primary current, and consequently it is not to be expected that the ratio of secondary current to primary current will be constant for all values of primary current in a transformer with an iron core. In order to determine the nature of the variation in F and the consequent variation in the current ratio, it will be necessary to consider that portion of the saturation curve around which the transformer operates. In Fig. 9 Curve I. is given an assumed saturation curve. The scales of the co-ordinates are not given because they are not important for our purpose. The essential thing that concerns us is the shape of the saturation curve.

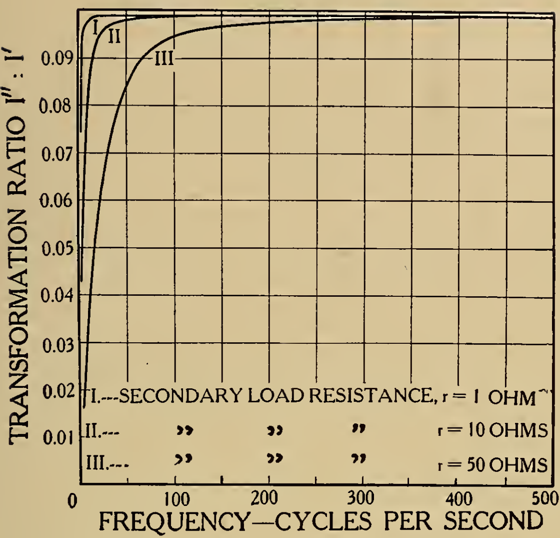

FIG. 5. EFFECT OF FREQUENCY ON TRANSFORMATION RATIO

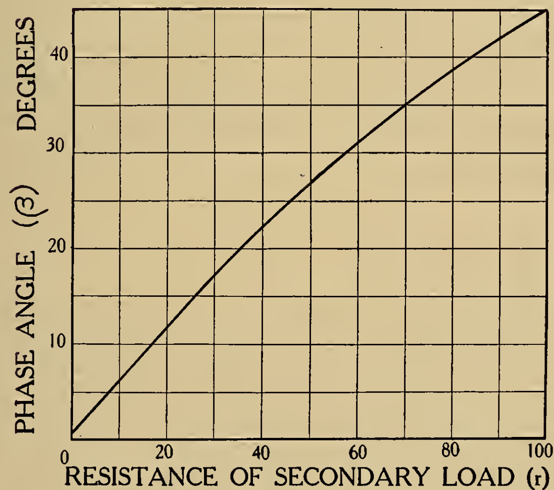

FIG. 6. EFFECT OF SECONDARY LOAD RESISTANCE OF PHASE ANGLE

It will be remembered that the factor F is given by the expression:

$$F = \frac{.4\pi}{\rho} \times 10^{-8}$$

where ρ is the reluctance of the iron circuit. Then we may write

$$F = \frac{.4\pi\, a\, \mu}{l} \times 10^{-8} \qquad (26)$$

where a is the average cross section and l the length of the iron circuit. Again for a given transformer, this may be written

$$F = \text{Constant} \times \mu \qquad (27)$$

From the saturation curve a curve may be derived whose ordinates are proportional to the permeability μ by dividing the ordinates of the saturation curve by corresponding abscissas, and plotting the quotients as ordinates of the new curve. From (27) the ordinates of this new curve will also be proportional to F, and by properly choosing the scale it will be possible to so plot it that the value of F corresponding to any point on the saturation curve may be directly read from it. It will be seen how this may be done.

The point of operation on the saturation curve depends upon the voltage that must be induced in the secondary to send the secondary current through the secondary impedance. This voltage is of course directly proportional to the secondary current, and to the impedance of the secondary circuit. For a given frequency the flux is proportional to the voltage. If the normal secondary load then is known and the full load secondary current, the flux for this condition may be computed from the relation.

$$\Phi_M = \frac{I'' Z''}{\omega N_2} 10^8 \qquad (28)$$

which is obtained from equation (8) by dropping the complex quantity symbols. This establishes a point on the saturation curve, as denoted by A in Fig. 4. From the saturation curve the value of permeability μ at this point may be found, and knowing the dimension of the iron circuit F may be calculated from equation (26). This value of F is then laid off to scale as an ordinate through point A, and the other ordinates are plotted proportionally, as explained above from equation (27). This gives Curve II of Fig. 9. As an example, let the normal secondary load for the transformer under consideration be $r = 1$ ohm and $x = 1$ ohm, and let the value given for F apply to the point of normal full load

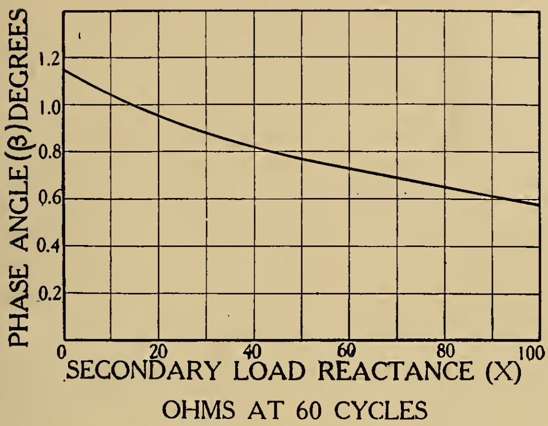

Fig. 7. Effect of Secondary Load Reactance on Phase Angle

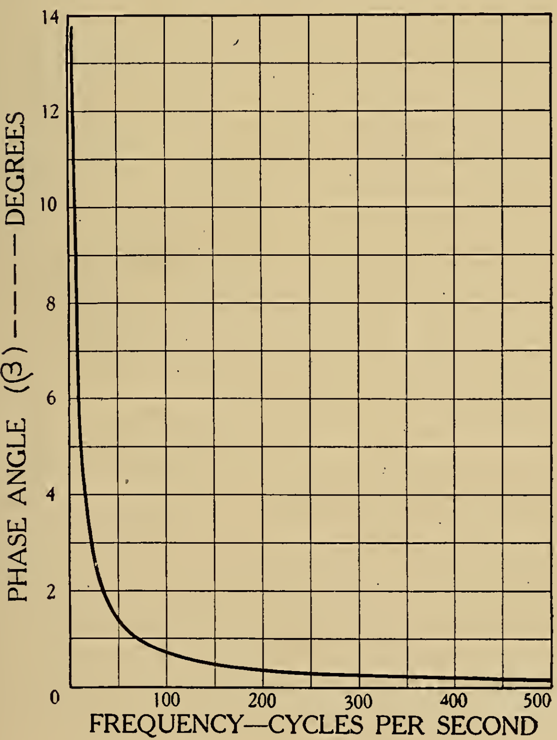

Fig. 8. Effect of Frequency on Phase Angle

operation indicated by A in Fig. 9. Curve II of Fig. 9 gives the values F at other points on the saturation curve. Let the problem of plotting a curve showing the variation of current ratio with primary current be considered.

It is known that with constant secondary load the variations in secondary current and induced voltage are proportional, and that therefore, the flux is proportional to the secondary current. For any percentage of full load secondary current then the corresponding point on the saturation curve may be found from A, Fig. 9 by direct proportion. Going up along the ordinate from the point thus found to Curve II, the corresponding value of F is obtained. This value of F is substituted in equation (19), and the ratio $\frac{I''}{I'}$ calculated. From this ratio and the value of secondary current the corresponding primary current in per cent of full load primary current may be determined, and thus a point on the required curve established. This is done for a sufficient number of values of secondary current to enable a smooth curve to be drawn through the points obtained. Table 1 gives a convenient tabulation for these computations, and Fig. 10 gives the result. It is seen that above 60% of full load primary current the ratio is very nearly constant, and that below 40% it falls off very rapidly. The variation in current ratio with primary current may be more clearly shown by plotting as ordinates the difference between the ratio for a given value of primary current and the ratio for full load primary current expressed in percent of full load primary current. In Fig. 11 such a curve, corresponding to Fig. 10, is shown by a full line. For the purpose of general comparison with the theoretical curve, there is also shown in Fig. 11, by a dotted line, a curve obtained experimentally. No unusual precision was used in obtaining the data for this curve, the currents being measured simply by two ammeters. The very close similarity of the two curves is apparent. It is to be expected that the magnitude of the ordinates, and even the shape of such curves from different transformers will differ even more than these do, inasmuch as they depend not only upon the point of operation on the saturation curve, but also upon the shape of the saturation curve, especially upon the prominence of the first bend, indicated by B in Fig. 9. Since there is very considerable variation in saturation curves for different qualities of iron, close agreement between curves derived from them cannot be expected. The curves in Fig. 10 and Fig. 11, however, point very truly to the general way in which transformation ratio and primary current vary.

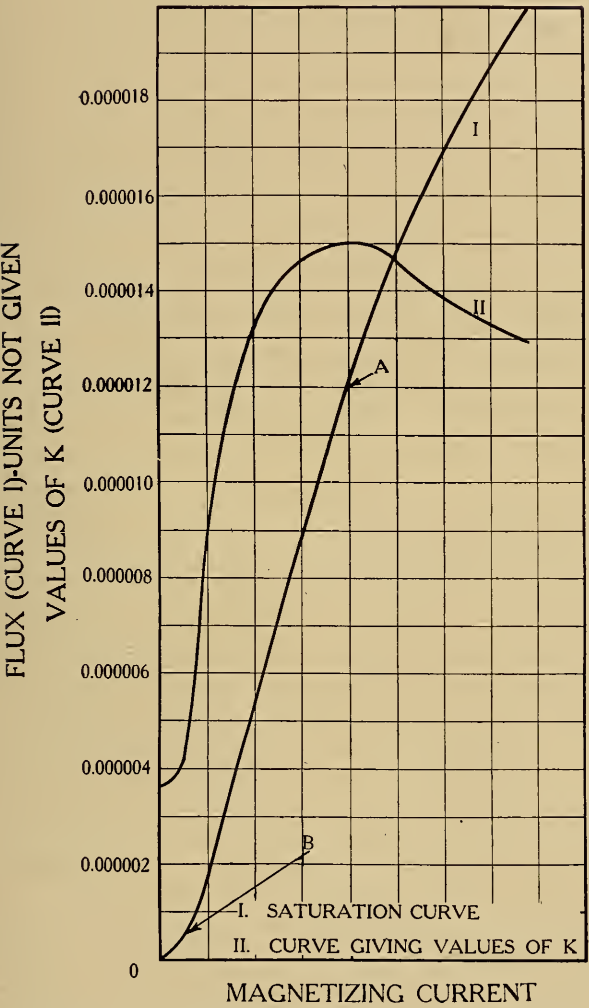

FIG. 9. SATURATION CURVE AND CURVE GIVING VALUES OF K

With values of F already determined for various values of primary current, it is an easy matter to substitute these in equation (25), and find the corresponding phase angles. Fig. 12 shows the variation of the phase angle β with the primary current. It is noted that above 60% full load primary current the variation in phase angle in small, but below 40% it increases quite rapidly.

Curves similar to those shown in Fig. 10, Fig. 11, and Fig. 12 may be determined for any other secondary load by making use of the fact that the ordinate of point A Fig. 9 is directly proportional to the secondary impedance. Knowing the new values of secondary resistance and reactance, and having established the new point A, the method of determining the new curves is exactly the same as that given above.

7. *Discussion of Curves Plotted from Formulas.*—The curves which have been plotted, showing the variations of transformation ratio and phase angle with various constants of the transformer are largely self-explanatory. Perhaps the most general observation that can be made from them is that the factor which least affects the transformation ratio is likely to greatly affect the phase angle, and *vice versa.* Thus from Curve I. Fig. 4 it is seen that for reasonable values the effect of secondary resistance upon the transformation ratio is practically negligible, whereas Fig. 6 shows that its effect upon the phase angle is very great. Again the effect of secondary reactance upon the ratio is very considerable as shown by Curve II. Fig. 4 whereas its effect upon the phase angle is quite small, as shown by Fig. 7. The curves in Fig. 5 show that the effect of changes in frequency becomes less as the secondary resistance is decreased. This then together with the fact that the phase angle increases almost in direct proportion to the secondary resistance points to the desirability of a low value of secondary resistance. On the other hand, the secondary reactance, neglecting its effect upon the point of operation on the saturation curve, is not a matter of such great consequence, for increasing it decreases but little the phase angle, and the *effect* of changes in its value upon the transformation ratio is practically *constant* for all reasonable values of reactance. The effect of changing either secondary resistance or reactance is to change the point of operation on the saturation curve and, as a consequence, to change the shapes of the curves in Fig. 10, Fig. 11, and Fig. 12. The desirability of using iron having as nearly constant permeability at low densities as possible is made evident by Fig. 10, Fig. 11, and Fig. 12. These figures also indicate that the reliable working range lies above 50 or 60 per cent full load current.

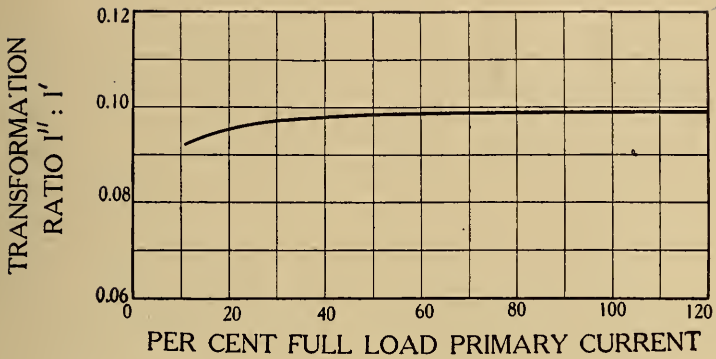

FIG. 10. VARIATION OF TRANSFORMATION RATIO WITH STRENGTH OF PRIMARY CURRENT

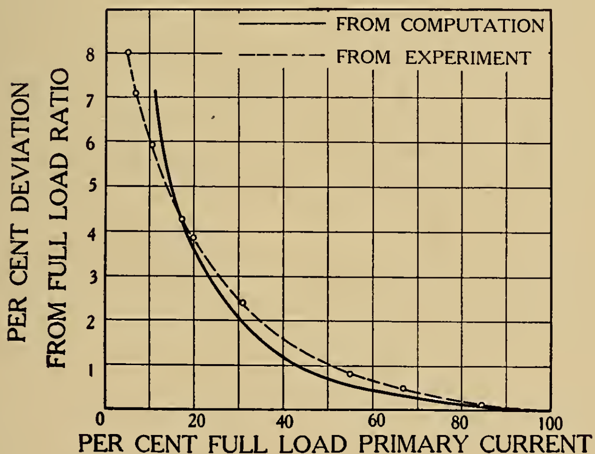

FIG. 11. DEVIATION FROM NORMAL TRANSFORMATION RATIO WITH STRENGTH OF PRIMARY CURRENT

III. THEORY OF THE SERIES TRANSFORMER BASED ON INSTANTANEOUS CURRENT VALUES

8. *Symmetrical Currents in Series Transformer with Air Core.*—The development of the theory of the current transformer based on instantaneous current values will be next considered. This method of treatment is especially valuable when the application of the series transformer to circuits carrying transient currents is studied. The following treatment is largely the outgrowth of a series of tests and calculations on the short-circuit currents of alternators. A number of oscillograph records of short-circuit current were taken, and the instantaneous values compared with those calculated from the constants of the alternator. The agreement was close in tests made in the laboratory where the oscillograph was direct connected to the system; but very considerable discrepancies were found in some tests made outside of the laboratory, where the oscillograph was connected in the secondary circuit of a series transformer. Since the theory of the short circuit currents of alternators would not explain the peculiarites in the latter curves, a study of the action of series transformers on unsymmetrical or transient currents was undertaken in an attempt to explain the phenomena. The results of this investigation show that the series transformer (especially with iron core) is unreliable for recording transient or unsymmetrical currents.

While there is really only one type of current transformer in practical use, which type has an iron magnetic circuit, it is of interest to consider the action of such a transformer without iron. The air core type will be discussed first, since its operation can be expressed by a mathematical equation, and thus clear insight into the whole problem can readily be given.

As has already been explained, a series transformer is a mutual inductance where the effect of the secondary circuit upon the primary current is so small as to be negligible. Since the secondary circuit is closed upon itself, the differential equation of the circuit becomes,

$$X_2 \frac{di''}{d\theta} + r_2 i'' + X_M \frac{di'}{d\theta} = e'' = 0 \quad \ldots\ldots\ldots\ldots (29)$$

Where r_2, X_2, and i'' are the resistance, reactance, and current in the secondary circuit; X_M is the mutual inductive reactance between the primary and secondary circuits; and i' is the primary current. As stated above i' is not affected by i'' and therefore $\frac{di'}{d\theta}$ is not a function of i'', and the single differential equation is sufficient for the solution of the problem.

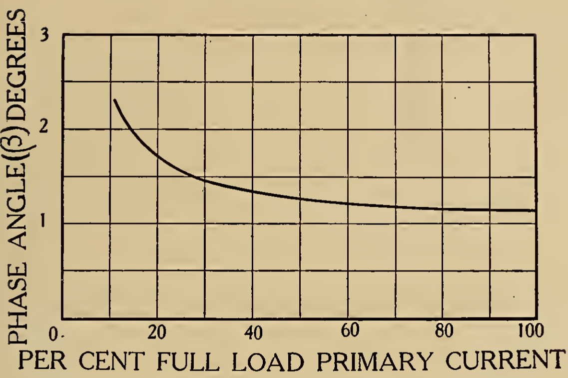

FIG. 12. VARIATION IN PHASE ANGLE WITH STRENGTH OF PRIMARY CURRENT

If the primary current i' contains a transient term, as the starting alternating current in an inductive circuit,

$$i' = A \left[\sin (\theta - s) - \sin (\theta_1 - s) \varepsilon^{-a\theta'} \right] \quad \ldots\ldots\ldots (30)$$

Where A = the maximum value of voltage across the primary divided by the primary impedance $= \frac{E'}{Z'}$, $a = \frac{r_1}{x_1}$, $s = \text{arc tan } \frac{I}{a}$, θ_1 is the angular displacement of the voltage from its zero position when the circuit is closed, and θ' is the time in electrical radians counted from the time of closing the circuit, or $\theta' = (\theta - \theta_1)$. Differentiating equation (30)

$$\frac{di'}{d\theta} = A \left[\cos (\theta - s) + a \sin (\theta - s) \varepsilon^{-a\theta'} \right] \ldots\ldots (31)$$

Equation (31) substituted in equation (29) gives

$$X_2 \frac{di''}{d\theta} + r_2 i'' = - X_M A \left[\cos (\theta - s) + a \sin (\theta_1 - s) \varepsilon^{-a\theta'} \right] (32)$$

Equation (32) is easily written in the familiar form

$$\frac{di''}{d\theta} + bi'' = - \frac{X_M A}{X_z} \left[\cos (\theta - s) + a \sin (\theta_1 - s) \varepsilon^{-a\theta'} \right] (33)$$

Where the factor $\frac{r_2}{X_2}$ is replaced by b

The solution of equation (33) is

$$i'' = -A\frac{X_M}{X_2}\varepsilon^{-b\theta}\left\{\int \varepsilon^{b\theta}\left[\cos(\theta - s) + a\sin(\theta_1 - s)\,\varepsilon^{-a\theta'}\right]d\theta + C'\right\}$$

$$= -A\frac{X_M}{X_2}\left\{\frac{\sin(\theta - s + \beta)}{\sqrt{b^2+1}} + \frac{a}{(b-a)}\sin(\theta_1 - s)\,\varepsilon^{-a\theta'} + C'\varepsilon^{-b\theta}\right\}$$

$$= -A\frac{X_M}{X_2}\left\{\frac{\sin(\theta - s + \beta)}{\sqrt{b^2+1}} + \frac{a}{(b-a)}\sin(\theta_1 - s)\,\varepsilon^{-a\theta'} + C\varepsilon^{-b\theta'}\right\} \quad (34)$$

Where $\beta = \text{arc tan } b$

When $\theta = \theta_1$, that is when $\theta' = 0$,

$i' = 0$ and $i'' = 0$

Therefore, solving equation (34) for C

$$C = -\left[\frac{\sin(\theta_1 - s + \beta)}{\sqrt{b^2+1}} + \frac{a}{(b-a)}\sin(\theta_1 - s)\right] \ldots\ldots (35)$$

$$= -\frac{b}{(b-a)}\sqrt{\frac{(1+a^2)}{(1+b^2)}}\sin(\theta_1 - s + \rho) \ldots\ldots\ldots (36)$$

Where $\rho = \text{arc tan }\dfrac{b-a}{1+ab}$ ....(36′)

Knowing the instantaneous values of i' and i'' and the ratio of the secondary to primary turns $\dfrac{N_2}{N_1}$, the instantaneous value of magnetizing current i is given by the following equation,

$$i = i' + \frac{N_2}{N_1}i'' \ldots\ldots\ldots\ldots\ldots (37)$$

Substituting equation (34) and (30) in (37)

$$i = A\left\{\sin(\theta - s) - \sin(\theta_1 - s)\,\varepsilon^{-a\theta'} - \right.$$

$$\frac{X_M\ N_2}{X_2\ N_1}\left[\frac{\sin\,(\theta - s + \beta)}{\sqrt{b^2+1}} + \frac{a}{b-a}\sin\,(\theta_1 - s)\ \varepsilon^{-a\theta'} + C\varepsilon^{-b\theta'}\right]\Bigg\} \quad (38)$$

Representing $\dfrac{X_M\ N_2}{X_2\ N_1}$ by the constant K and replacing and combining the trigonometric functions into one, the equation becomes

$$i = A\left\{\sqrt{\left(1 - \frac{2k - k^2}{b^2+1}\right)}\sin\,(\theta - s - u) - \right.$$

$$\left.\left(1 + \frac{ka}{b-a}\right)\sin\,(\theta_1 - s)\ \varepsilon^{-a\theta'} - k\ C\ \varepsilon^{-b\theta'}\right\} \quad \ldots\ldots\ldots \quad (39)$$

Where $u = \text{arc tan}\ \dfrac{kb}{(b^2+1-k)}$, which is the angle of lag of the stable magnetizing current behind the primary current.

Example 1. As an example of the application of this method the transformer for which data have already been given may be considered, with the additional data

$$A = 1$$

$$\theta_1 = 175°$$

The following data, previously given, will be needed,

$$a = \frac{r_1}{x_1} = 0.1$$

$$b = \frac{r_2}{x_2} = 0.01$$

$$X_M = 10$$

$$X_2 = 100$$

$$\frac{N_2}{N_1} = 9.5$$

$$s = 85° \text{ (calculated)}$$

Substituting these constants in equation (30)

$$i' = \sin(\theta - 85^\circ) - \varepsilon^{-.1\theta'} \quad \ldots\ldots\ldots\ldots\ldots (A')$$

ρ from equation (36′) is

$$\rho = \text{arc tan}\,\frac{b-a}{1+ab} = \text{arc tan}\,(-.09) = -5^\circ$$

The value of C from equation (36) becomes

$$C = .111$$

$$\beta = \text{arc tan}\, b = 35 \text{ minutes}$$

Hence, from equation (34)

$$i'' = -0.1\left[\sin(\theta - 84.4^\circ) - 1.11\,\varepsilon^{-.1\theta'} + .111\,\varepsilon^{-.01\theta'}\right] \ldots (B')$$

$$u = \text{arc tan}\,\frac{kb}{b^2+1-k} = 10^\circ 45'$$

That is, the stable condition of the magnetizing current lags 10° 45′ behind the primary current.

i from equation (39) becomes

$$i = \left[.05\sin(\theta - 95.75^\circ) + .055\,\varepsilon^{-.1\theta} - .1055\,\varepsilon^{-.01\theta}\right] \ldots (C')$$

In Fig. 13 equations (A′), (B′), and (C′) are plotted with θ as abscissas and i', i'', and i as ordinates. The ordinates derived from equation (B′) are multiplied by the transformation ratio $\frac{X_2}{X_M} = 10$ and turned 180° so as to be superimposed upon the primary current for better comparison. The curves thus plotted show clearly the error in secondary current due to the air core transformer with a 5% magnetizing current.

9. *Unsymmetrical Currents in Series Transformer with Air Core.*—As a second problem a series transformer traversed by an unsymmetrical primary current might be considered. If the current lies entirely above the zero line, the equation would be

$$i' = A\left[\sin\theta + 1 - C\,\varepsilon^{-a\theta'}\right] \ldots\ldots\ldots\ldots (40)$$

Where $C = \sin\theta_1 + 1$

Differentiating (40) and substituting in equation (29)

$$\frac{di''}{d\theta} + bi'' = -A\,\frac{X_M}{X_2}\left(\cos\theta + C\,a\,\varepsilon^{-a\theta'}\right) \ldots\ldots (41)$$

And the solution of equation (41) is

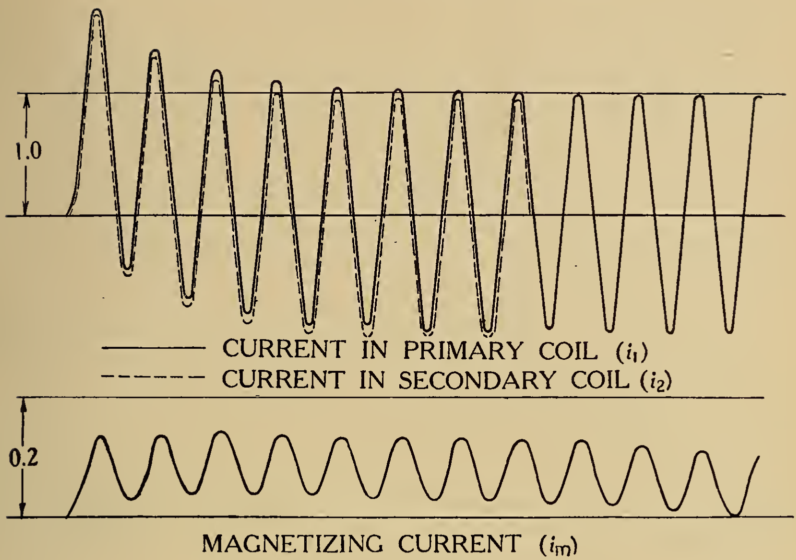

FIG. 13. CURRENT DIAGRAM FOR EXAMPLE 1

$$i'' = -A\frac{X_M}{X_2}\left[\frac{\sin(\theta+\beta)}{\sqrt{b^2+1}} + \frac{aC}{(b-a)}\,\varepsilon^{-a\theta'} + C'\,\varepsilon^{-b\theta'}\right] \quad (42)$$

Where $\beta =$ arc tan b, and since when $\theta = \theta_1$, $\theta' = 0$ and $i'' = 0$

$$C' = -\left[\frac{\sin(\theta_1+\beta)}{\sqrt{b^2+1}} + \frac{aC}{(b-a)}\right]$$

If the time constant $\frac{1}{a}$ of the primary circuit is exceedingly small, then $\varepsilon^{-a\theta'}$ will practically be zero in a very short time; (a) will be large in comparison to (b) and equation (42) becomes, after an infinitesimal time.

$$i'' = -A\frac{X_M}{X_2}\left\{\frac{\sin(\theta+\beta)}{\sqrt{b^2+1}} - \left[\frac{\sin(\theta_1+\beta)}{\sqrt{b^2+1}} - \sin\theta_1 - 1\right]\varepsilon^{-b\theta'}\right\} \quad (43)$$

And equation (40) becomes

$$i' = A\,[\,\sin\theta + 1\,] \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Substituting equations (44) and (43) in (37)

$$i = A\left[\sin\theta + 1 - \frac{k}{\sqrt{b^2+1}}\sin(\theta+\beta) + C\varepsilon^{-b\theta'}\right] \quad (45)$$

Where

$$C = k\left[\frac{\sin(\theta_1+\beta)}{\sqrt{b^2+1}} - \sin\theta_1 - 1\right] \ldots\ldots\ldots (46)$$

Equation (45) may be further reduced to the form

$$i = A\left[\sqrt{1-\frac{(2k-k^2)}{(b^2+1)}}\sin(\theta - s') + 1 + C\varepsilon^{-b\theta'}\right] \quad (47)$$

Where $s' = \text{arc tan}\,\dfrac{kb}{(b^2+1-k)}$

Example 2. Assume that the same transformer is used as in Example 1, but that the constant (a) is very large and the primary wave of the form,

$$i' = \sin\theta + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A'')$$

Then from equation (43)

$$i'' = -.1\,[\sin(\theta + 35') + \varepsilon^{-.01\theta'}] \ldots\ldots\ldots (B'')$$

And from (47)

$$i = .05\sin(\theta - 10.75^\circ) - .95\,\varepsilon^{-.01\theta'} + 1 \ldots\ldots\ldots (C'')$$

10. *Effect of Constants of Transformer on Instantaneous Current Values.*—The curves on Fig. 14, plotted from equations A″, B″, and C″ show clearly how the current in the secondary of the transformer gradually becomes symmetrical in reference to the zero line, while the magnetizing current creeps up to a line which is symmetrical in respect to the primary current.

The reactance of the secondary, X_2, as has previously been indicated, contains two factors, X'_2 and x_2, of which X'_2 represents that portion of the flux that interlinks with the primary coil, and x_2 represents that portion of the flux that does not interlink with the primary coil, and is known as leakage reactance. X'_2 and X_M hold a definite relation to each other, which ratio, is that of the secondary turns N_2 to the primary turns N_1. The transformation ratio of the transformer for zero secondary impedance is that ratio of the mutual inductive reactance X_M to the total secondary self inductive reactance X_2, which ratio can only be considered equal to the ratio of the respective turns in so far as x_2 can be neglected in comparison to X'_2. The resistance in the secondary cir-

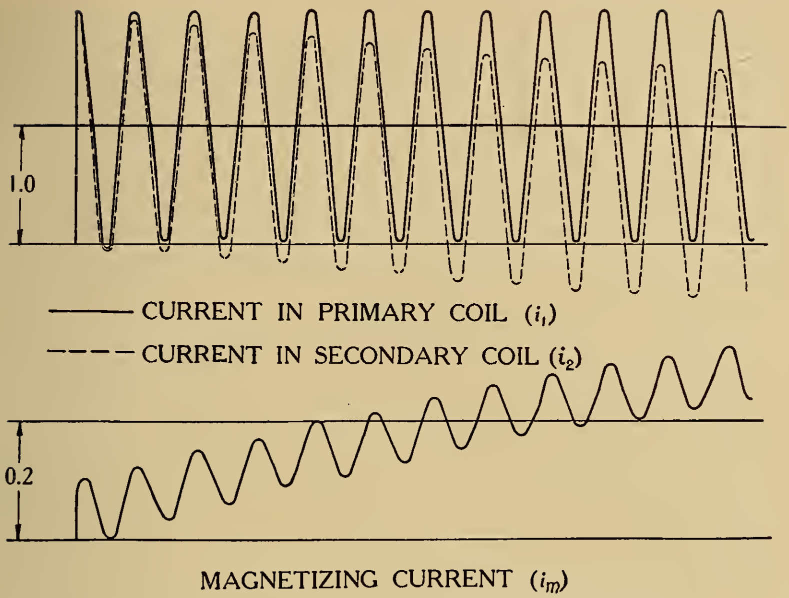

FIG. 14. CURRENT DIAGRAM FOR EXAMPLE 2

cuit has little effect on the transformation ratio, but does produce a displacement in phase in the stable condition, and introduces large errors in the transient term, since b is directly proportional to r_2. The smaller the factor b, the more closely the secondary current follows the primary current and therefore, for observing transient currents, the secondary leakage reactance improves the accuracy; while resistance, though small, introduces large errors. The value of x_2 is limited only in so far as its effect upon the primary current can be neglected. As increase of x_2 does increase the magnetizing current, but this increase is in phase with the primary current and therefore no displacement results, and the transformation ratio is only diminished. Therefore, a transformer designed to be used for transient phenomena should have an exceedingly large secondary reactance and a very small secondary resistance.

Under stable conditions, equation (34) becomes

$$i'' = -A \frac{X_M}{X_2} \frac{\sin(\theta - s + \beta)}{\sqrt{b^2 + 1}}$$

$$= -\frac{A X_M}{\sqrt{X_2^2 + r_2^2}} \sin(\theta - s + \beta) \dots\dots\dots\dots (48)$$

Since X'_2 is large in comparison to either r_2 *or* x_2, it is evident from the above equation that an increase of r_2 has little effect on the transformation ratio, while an increase in x_2 has an appreciable effect. Also, since β is small (arc tan $\dfrac{r_2}{X'_2 + x_2}$) the angle of displacement β is directly proportional to r_2, while increasing x_2 decreases the displacement. In other words, the magnetizing current required to force the secondary current through the resistance r_2 lags ninety degrees behind that portion of the primary current that is transferred to the secondary, while the magnetizing current required to force the secondary through the reactance x_2 is in phase with the said current.

11. *Series Transformer with Iron Core and Negligible Secondary Leakage Reactance.*—This gradual increase, or creeping of the magnetizing current of the series transformer with unsymmetrical currents, has a much greater and more disastrous effect in the case of an ordinary transformer with an iron core. In the case of the air core transformer the magnetizing current was a direct function of the primary current in all stable conditions; but this, as has already been shown, does not hold true in the case of the transformer with an iron core. The mutual inductive reactance X_M and the corresponding self-inductive reactance X'_2 depend, in exactly the same way, upon the reluctance of the core circuit, and consequently their ratio is a constant, with a value the same as that of the ratio of the number of primary and secondary turns. The leakage reactance x_2 is more or less independent of the permeability of the core, and consequently the transformation ratio $\dfrac{X_M}{X'_2 + x_2}$ is not a constant but depends upon the magnitude of the magnetizing current. Furthermore as X'_2 decreases it necessarily approaches the value of r_2 and the error due to the increasing factor b becomes very large.

It is unfortunate that the saturation curve of the iron cannot be represented by a mathematical equation and therefore the only solution of the problem is the tedious step by step method. Although this method obviously is not absolutely correct, it does give a close approximation. Consider first the problem where the secondary leakage reactance x_2 is negligible. From the terminal conditions and constants of the circuit, the primary current at any instant may be determined, as by equation (30). From equation (37),

$$i' - \frac{N_2}{N_1} i'' = i \qquad \qquad (49)$$

where the quantities are expressed arithmetically instead of algebraically.

The nature of what follows makes it more convenient to express the relation in this manner. Since the secondary leakage reactance is negligible, the change of flux need give only the e.m.f. necessary to force the secondary current through the resistance r_2, or

$$D \frac{d\,\phi}{d\,\theta} = i'' r_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

where D is a proportionality constant. Equation (50) may be written

$$i = \frac{D}{r_2} \frac{d\,\phi}{d\,\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

Substituting (51) in (49)

$$i' - \frac{N_2}{N_1} \frac{D}{r_2} \frac{d\,\phi}{d\,\theta} = i \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

Changing from differential to difference, that is, replacing as approximation d by $\triangle$ gives

$$i' - \frac{N}{N_1} \frac{D}{r_2} \frac{\triangle\,\phi}{\triangle\,\theta} = i \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

Taking increments of 10° for $\triangle\ \theta$ gives

$$i' - \frac{N_2}{N_1} \frac{D}{r_2}\, 5.73\ \triangle\,\phi = i$$

Whence
$$\triangle\phi = \frac{N_1}{N_2} \frac{r_2}{D}\, .175\ [i' - i] \ldots\ldots\ldots\ldots (54)$$

If the remanent magnetism in the core of the transformer at the time the circuit is made be denoted by ϕ_0, the flux ϕ at any instant is given by the expression,

$$\phi = \phi_o + \Sigma \triangle\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Substituting (54) in (55),

$$\phi = \phi_o + \Sigma \frac{N_1}{N_2} \frac{r_2}{D}\, .175\ [i' - i] \ldots\ldots\ldots\ldots (56)$$

For convenience let r_2 have such a value that $.175\ \dfrac{r_2}{D}$ becomes equal to unity. Then (56) becomes

$$\phi = \phi_o + \Sigma \frac{N_1}{N_2} [i' - i] \ldots\ldots\ldots\ldots (57)$$

As indicated above, the primary current may be determined from the conditions of the main circuit. Assuming a value for ϕ_0, we may take as a first approximation

$$\phi' = \phi_0 + \Sigma \frac{N_1}{N_2} i \dots\dots\dots\dots\dots\dots \quad (58)$$

From the saturation curve a value of i corresponding to ϕ' is obtained, and substituted in equation (57). From (57) then a second approximate value of ϕ is obtained, which from the saturation curve gives a very close second approximate value of i. The following example may illustrate more fully the method of procedure.

Example 3. Assume the ratio of secondary to primary turns to be 10; the per cent of primary magnetizing current to be 1.2%; the primary current as given by equation (A′); a residual magnetism in the core of .5 units; and the saturation curve given in Fig. 15 to be that of the primary coil. The tabulation shown in Table 3 is a convenient form. Table 2 is a convenient tabulation for determining values of i'.

Column 2 of Table 3 is obtained from Table 2. The value of $\triangle\, \phi'$ in column 3 is equal to $\frac{N_1}{N_2}$ times the primary current i', and ϕ in column 4 is the sum of $\triangle\, \phi'$ in column 3 and the value of ϕ in column 8 of the preceding line. (i) in column 5 is found from the saturation curve for the value of flux corresponding to 4. From (i'—i) of column 6 the values of $\triangle\, \phi$ in column 7 are obtained and added to 8 of the preceding line. This gives the value of flux from which the final value of magnetizing current i is determined. From column 10, i'' is obtained. In Fig. 16 are plotted curves for the above case similar to those in Fig. 12. The much more disastrous effect of the magnetizing current in a transformer with an iron core is readily seen from a comparison of these two sets of curves.

12. *Series Transformer with Iron Core and Appreciable Secondary Leakage Reactance.*—If the reactance of the secondary must be considered the change of flux must give an e.m.f. sufficient to overcome the reactance drop as well as the resistance drop of the secondary, or

$$D \frac{d\phi}{d\theta} = r_2\, i'' + x_2 \frac{di''}{d\theta} \dots\dots\dots\dots\dots\dots \quad (59)$$

Example 4. Take the data of Example 3, but consider secondary leakage reactance. Changing from differentials to differences, and taking increments of $\theta = 10°$, we obtain

$$5\ 73\ D\triangle\phi = r_2 i'' + x_2\, 5.73\ \triangle i'' \dots\dots\dots\dots \quad (60)$$

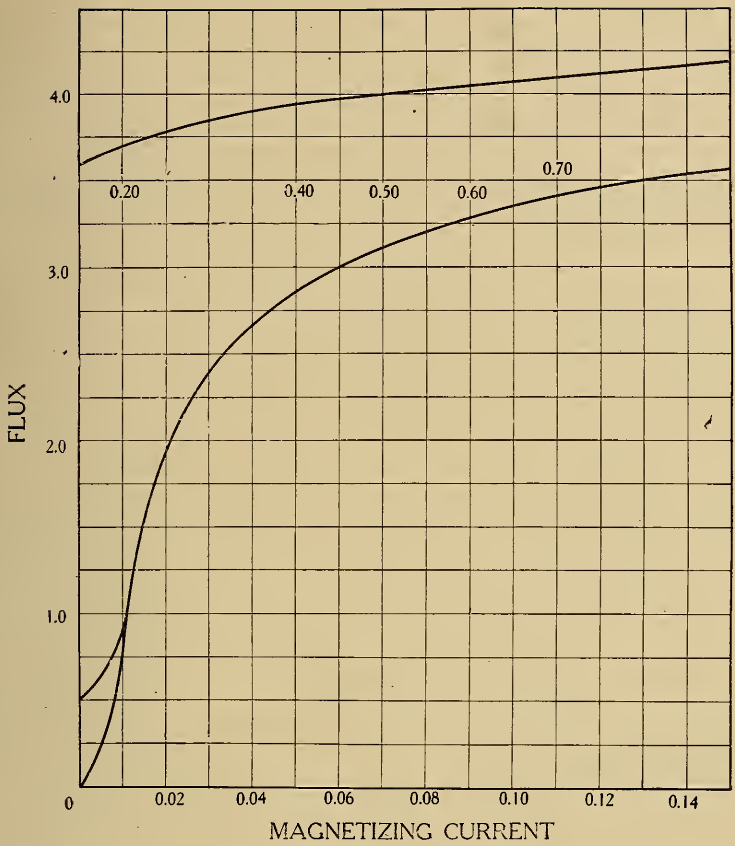

FIG. 15. SATURATION CURVE FOR EXAMPLE 3

And $$i'' = \frac{5.73}{r_2}\left[D\triangle\phi - x_2\triangle i''\right] \qquad (61)$$

Substituting (61) in (49)

$$i' - \frac{N_2}{N_1}\frac{5.73}{r_2}\left[D\triangle\phi - x_2\triangle i''\right] = i$$

Which reduces to

$$\triangle\phi = \frac{.175 r_2}{D}\frac{N_1}{N_2}\left[i' - i\right] + \frac{x_2}{D}\triangle i'' \qquad (62)$$

Again, let r_2 have such a value that $.175 \frac{r_2}{D}$ becomes equal to unity. Then $D = .175\, r_2$, and equation (62) becomes

$$\triangle \phi = \frac{N_1}{N_2} [i' - i] + 5.73 \frac{x_2}{r_2} \triangle i'' \ldots\ldots\ldots\ldots (63)$$

Taking the secondary reactance equal to the secondary resistance, equation (63) becomes

$$\triangle \phi = \frac{N_1}{N_2} [i' - i] + 5.73 \triangle i'' \ldots\ldots\ldots\ldots (64)$$

But from equation (49) $\triangle i'' = \frac{N_1}{N_2} [\triangle i' - \triangle i] \ldots\ldots\ldots\ldots (65)$

Substituting (65) in (64) it becomes

$$\triangle \phi = \frac{N_1}{N_2} \left\{ [i' - i] + 5.73 [\triangle i' - \triangle i] \right\} \ldots\ldots (66)$$

Now substituting equation (66) in equation (55), an expression for the flux is obtained

$$\phi = \phi_o + \Sigma \frac{N_1}{N_2} \left\{ [i' - i] + 5.73 [\triangle i' - \triangle i] \right\}$$

$$\phi = \phi_o + \Sigma \frac{N_1}{N_2} \left\{ (i + 5.73 \triangle i') - (i + 5.73 \triangle i) \right\} \ldots (67)$$

As a first approximation, the parenthesis containing i may be omitted, and equation (67) becomes

$$\phi = \phi_o + \Sigma \frac{N_1}{N_2} (i' + 5.73 \triangle i') \ldots\ldots\ldots\ldots (68)$$

With this value of ϕ' an approximate value i is obtained from the saturation curve. By this value of i the parenthesis of equation (67) is then filled in, and a second approximate value of ϕ obtained, whence by means of the saturation curve, a second very close approximate value of i is obtained.

Table 4 is a convenient form for tabulation. Column 7 of Table 4 gives ϕ' as obtained by the first approximation, and column 8 gives the corresponding magnetizing current. Column 12 gives $\triangle \phi$ as obtained when using i, and column 13 gives the second approximation of ϕ. Column 14 gives the corresponding value of magnetizing current, and from 15 the value of secondary current may be obtained. ϕ' in column 7 is obtained by adding $\triangle \phi'$ of column 6 to ϕ of the previous line in column 13. In Fig. 17 are plotted curves from the above case similar to those shown in Fig. 13 and in Fig. 14.

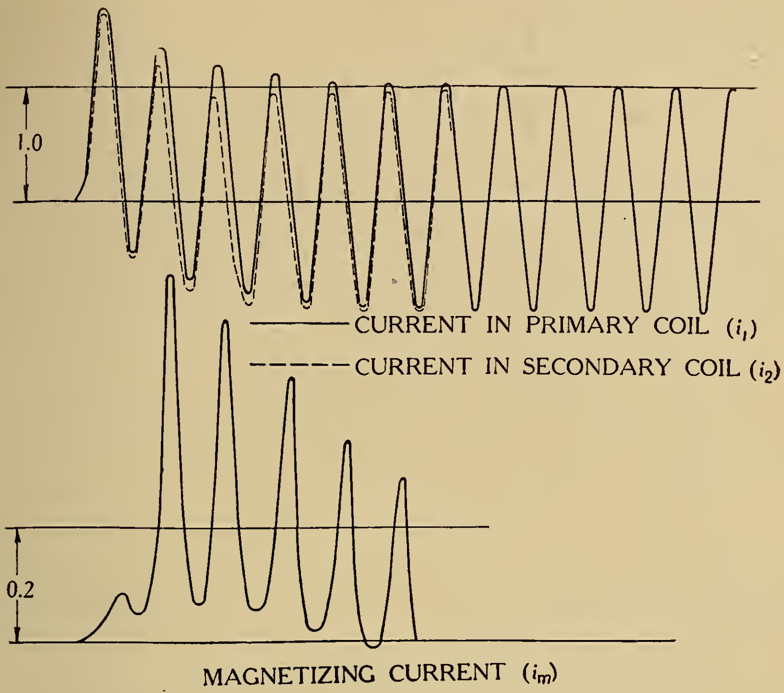

FIG. 16. CURRENT DIAGRAM FOR EXAMPLE 3

Fig. 16 and Fig. 17 (corresponding to examples 3 and 4), show clearly the errors introduced by the use of the series transformer with an iron core in recording transient phenomena. The "steadying" effect of secondary reactance is made evident by a comparison of the two curves.

The oscillograph record, shown in Fig. 18, is that of the current through the secondary of the series transformer together with the current through the primary, which is the starting current of an inductive circuit with an electrical time constant $(\frac{x}{r})$. of 10. These experimental curves are very similar to those as calculated from Examples 3 and 4 although the constants are somewhat different.

13. *Comparison of Methods of Computation for Series Transformers.*—The agreement between the two methods of solution, namely, the "complex quantity method" and the "differential equation method" may be pointed out briefly as follows. If transient terms be dropped, and the maximum value of equation (34) be divided by the maximum value of

equation (30) this will be the same as the quotient of the effective values, or the transformation ratio will be

$$\frac{i''}{i'} = -\frac{X_M}{X_2\sqrt{b^2+1}} = -\frac{X_M}{\sqrt{r_2^2+X_2^2}} \ldots\ldots \quad (69)$$

It will be remembered that secondary load resistance and reactance were taken equal to zero in the second method. If therefore, r and x in equation (24) be replaced by zero, the agreement with equation (69) is evident. Again a comparison of equations (34) and (30) shows that the angle of phase difference between the two currents i'' and i' under stable conditions is β and

$$\beta = \text{arc tan } b = \text{arc tan } \frac{r_2}{X_2}$$

If r and x in equation (25′) be replaced by zero, the agreement in phase angle as obtained by the two methods is evident.

IV. CONCLUSIONS

The conclusions that may be drawn from the preceding discussion in regard to the behavior of current transformers, are of a qualitative rather than a quantitative nature. The examples chosen, however, have been such as to give a fair and reasonable representation of magnitude, but the main purpose has been to derive and explain the general characteristics of the current transformer, and to point out some of its limitations.

To enumerate again in detail all the results of this investigation is thought unnecessary. An examination of the figures will show most of them. In conclusion a few of the most general and important results are given:

1. The transformation ratio and phase angle of a series transformer having a core of constant permeabilty (as air) are constant under given conditions for all values of primary current; but this is not so with a transformer having an iron core. With an iron-cored series transformer the form of variation depends upon the shape of the saturation curve, and upon the range over which the transformer operates. The range over which the *permeability* remains most nearly constant is the range over which the *ratio* remains most nearly constant. In a transformer of constant core reluctance the so-called magnetizing current is proportional to the primary current, and its phase position is constant.

2. The introduction of resistance in the secondary circuit of a series transformer has the effect of increasing the phase angle, and this increase in phase angle is practically proportional to the secondary resistance for

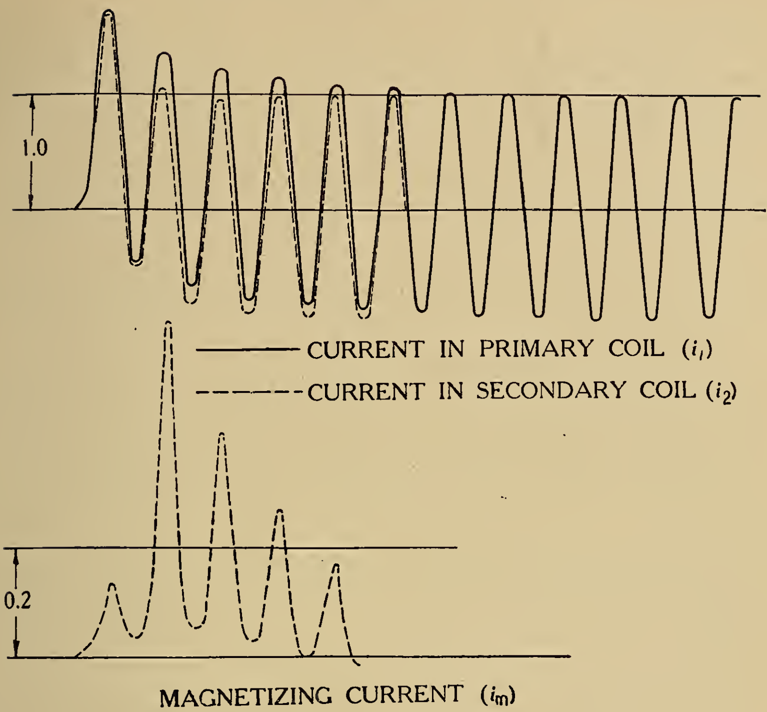

FIG. 17. CURRENT DIAGRAM FOR EXAMPLE 3—SECONDARY REACTANCE CONSIDERED.

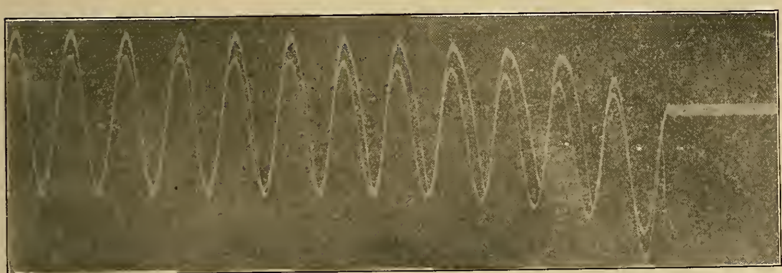

FIG. 18. OSCILLOGRAPH RECORD OF PRIMARY AND SECONDARY CURRENTS IN SERIES TRANSFORMER

reasonable values. Increasing the secondary resistance decreases but slightly the transformation ratio. Hence it may be said that in general the introduction of secondary resistance is very objectionable when the transformer supplies current for a wattmeter, but is not seriously objectionable when the transformer supplies current for an ammeter.

3. The effect of secondary reactance, and the equivalent effect of magnetic leakage is to reduce the phase angle slightly, and the transformation ratio very considerably.

4. The phase angle increases with decreased permeability, and consequently in a transformer with an iron core the phase angle increases as the line current decreases.

5. The effect of changes in frequency within a range of 10 cycles is not generally serious. It should be pointed out, however, that in addition to the effects of frequency shown in the curves, in a transformer with an iron core, decreasing the frequency raises the point of operation on the saturation curve, and hence increases the core loss and alters the form of variation of transformation ratio and phase angle with primary current.

6. The desirability of a high number of turns was pointed out. With a reasonably high number of turns and a not excessive value of secondary resistance the effect of frequency over a considerable range is negligible.

7. The effect of core loss is to decrease the secondary current, this effect being lessened by inductive secondary load. Increased core loss decreases the phase angles, and this effect is increased by inductive secondary load.

8. In an iron-core series transformer the value of flux density should be low (say $B = 2{,}000$ at full load). This means a *low* value of magnetizing current. To this end excessive secondary impedance should be avoided, as increased impedance requires an increase in flux in practically direct proportion to the impedance. Since the effect of magnetic leakage is equivalent to the effect of secondary reactance, the transformer should be designed with a view to minimum magnetic leakage. This requires a well closed iron circuit.

9. For recording instantaneous values of current in transient or unsymmetrical systems, the commercial series transformer with an iron core is quite inadequate, and cannot be relied upon.

10. If necessity demands the use of a series transformer in recording transient or unsymmetrical currents, an air-core transformer, designed to have a very small secondary resistance and a large secondary reactance will be found to give results nearer to those desired than can be obtained with an iron core transformer.

TABLE I.

NOMINAL AND ACTUAL CURRENT RATIOS

Secondary Current—per cent of Full Load	100	70	50	30	20	10
F	0.00001425	0.00001375	0.00001288	0.00001100	0.00000934	0.00000650
$F N_1^2 \omega$	1.052	1.017	0.951	0.812	0.690	0.480
$A = \frac{N_2}{N_1} F N_1^2 \omega$	10.00	9.67	9.04	7.71	6.55	4.56
$F N_2^2 \omega$	95.0	91.75	86.0	73.4	62.3	43.3
$F N_2^2 \omega + x_2 + x$	101.0	97.75	92.0	79.4	68.3	49.3
$(F N_2^2 \omega + x_2 + x)^2$	10200	9550	8460	6300	4660	2430
$(r_2 + r)^2$	4	4	4	4	4	4
$(F N_2^2 \omega + x_2 + x)^2 + (r_2 + r)^2$	10204	9554	8464	6304	4664	2434
$B = \sqrt{(F N_2^2 \omega + x_2 + x)^2 + (r_2 + r)^2}$	101.0	97.8	92.0	79.4	68.3	49.3
$\frac{I''}{I'} = -\frac{A}{B}$	0.0990	0.0988	0.0983	0.0971	0.0958	0.0925
Primary Current—per cent of Full Load	100	70.2	50.4	30.6	20.6	10.7
Deviation of $\frac{I''}{I'}$ *from Full Load Value*	0	0.0002	0.0007	0.0019	0.0032	0.0065
Error in Nominal Value of $\frac{I''}{I'}$ *— per cent*	0	0.202	0.707	1.920	3.230	6.570

TABLE 2

DETERMINATION OF VALUES OF i'

1	2	3	4	5	6	7	8	9	10
θ' degrees	θ' radians	(θ_1-s) degrees	$a\,\theta'$	$e^{-a\theta'}$	$\sin(\theta_1-s)\ e^{-a\theta'}$	θ	$(\theta-s)$	$\sin(\theta-s)$	i'
0	.0	90	0	1	1.000	175	90	1.000	0
10	.175	"	.0175	.983	.983	185	100	.985	—.002
20	.349	"	.0349	.965	.965	195	110	.940	—.025
30	.524	"	.0524	.950	.950	205	120	.866	—.084
40	.699	"	.0699	.935	.935	215	130	.766	—.169
50	.873	"	.0873	.92	.92	225	140	.643	—.277
60	1.048	"	.1048	.90	.90	235	150	.500	—.400
70	1.223	"	.1223	.885	.885	245	160	.342	—.543
80	1.398	"	.1398	.87	.87	255	170	.174	—.696
90	1.572	"	.1572	.855	.855	265	180	0	—.855
100	1.747	"	.1747	.84	.84	275	190	—.174	—1.014
110	1.922	"	.1922	.825	.825	285	200	—.342	—1.167
120	2.095	"	.2095	.81	.81	295	210	—.500	—1.310
130	2.27	"	.227	.795	.795	305	220	—.643	—1.438
140	2.44	"	.244	.785	.785	315	230	—.766	—1.551
150	2.62	"	.262	.77	.77	325	240	—.866	—1.636
160	2.79	"	.279	.76	.76	335	250	—.940	—1.700
170	2.97	"	.297	.745	.745	345	260	—.985	—1.730
180	3.14	"	.314	.73	.73	355	270	—1.000	—1.730
190	3.32	"	.332	.715	.715	365	280	—.985	—1.700
200	3.49	"	.349	.705	.705	375	290	—.940	—1.645
210	3.67	"	.367	.69	.69	385	300	—.866	—1.556
220	3.84	"	.384	.68	.68	395	310	—.766	—1.446
230	4.02	"	.402	.67	.67	405	320	—.643	—1.313
240	4.19	"	.419	.66	.66	415	330	—.500	—1.160
250	4.37	"	.437	.65	.65	425	340	—.342	—.992
260	4.54	"	.454	.635	.635	435	350	—.174	—.809
270	4.72	"	.472	.625	.625	445	360	0	—.625
280	4.89	"	.489	.615	.615	455	370	.174	—.441
290	5.07	"	.507	.605	.605	465	380	.342	—.263
300	5.24	"	.524	.59	.59	475	390	.500	—.090
310	5.42	"	.542	.58	.58	485	400	.643	.063
320	5.59	"	.559	.57	.57	495	410	.766	.196
330	5.76	"	.576	.56	.56	505	420	.866	.306
340	5.94	"	.594	.55	.55	515	430	.940	.390
350	6.11	"	.611	.54	.54	525	440	.985	.445
360	6.28	"	.628	.535	.535	535	450	1.000	.465

TABLE 3

SOLUTION OF EXAMPLE 3

1	2	3	4	5	6	7	8	9	10
θ'	i'	$\Delta\phi'=\frac{N_1}{N_2}i'$	ϕ'	i	$(i'-i)$	$\Delta\phi=\frac{N_1}{N_2}(i'-i)$	ϕ	i	$(i'-i)=\frac{N_2}{N_1}i''$
0	0	0	.500	0	0	0	.500	0	0
10	0	0	.500	0	0	0	.500	0	0
20	.025	.002	.502	0	.025	.002	.502	0	.025
30	.084	.008	.510	.001	.083	.008	.510	.001	.083
40	.169	.017	.527	.002	.167	.017	.527	.002	.167
50	.277	.028	.555	.003	.274	.027	.554	.003	.274
60	.400	.040	.595	.004	.396	.040	.594	.004	.396
70	.543	.054	.649	.006	.537	.054	.648	.006	.537
80	.696	.070	.718	.007	.689	.069	.717	.007	.689
90	.855	.085	.803	.008	.847	.085	.802	.008	.847
100	1.014	.101	.903	.010	1.004	.100	.902	.010	1.004
110	1.167	.117	1.019	.011	1.156	.116	1.018	.011	1.156
120	1.310	.131	1.149	.012	1.298	.130	1.148	.012	1.298
130	1.438	.144	1.291	.013	1.425	.143	1.290	.013	1.425
140	1.551	.155	1.445	.014	1.537	.154	1.444	.014	1.537
150	1.636	.164	1.607	.016	1.620	.162	1.606	.016	1.620
160	1.700	.170	1.776	.018	1.682	.168	1.774	.018	1.682
170	1.730	.173	1.947	.020	1.710	.171	1.945	.020	1.710
180	1.730	.173	2.118	.023	1.707	.171	2.116	.023	1.707
190	1.700	.170	2.286	.026	1.674	.167	2.283	.026	1.674
200	1.645	.164	2.448	.031	1.614	.161	2.444	.031	1.614
210	1.556	.156	2.600	.036	1.520	.152	2.596	.036	1.520
220	1.446	.145	2.741	.043	1.403	.140	2.737	.043	1.403
230	1.313	.131	2.868	.050	1.263	.126	2.863	.050	1.263
240	1.160	.116	2.979	.059	1.101	.110	2.973	.058	1.102
250	.992	.099	3.072	.067	.925	.093	3.066	.066	.926
260	.809	.081	3.147	.074	.735	.073	3.139	.073	.736
270	.625	.062	3.202	.080	.545	.055	3.194	.079	.546
280	.441	.044	3.238	.085	.356	.036	3.229	.084	.357
290	.263	.026	3.256	.086	.177	.018	3.247	.085	.178
300	.090	.009	3.256	.086	.004	.000	3.247	.085	.005
310	—.063	—.006	3.241	.084	—.147	—.015	3.233	.083	—.146
320	—.196	—.020	3.213	.081	—.277	—.028	3.205	.080	—.276
330	—.306	—.031	3.174	.077	—.383	—.038	3.167	.076	—.382
340	—.390	—.039	3.128	.072	—.462	—.046	3.120	.071	—.461
350	—.445	—.044	3.076	.067	—.512	—.051	3.069	.066	—.511
360	—.465	—.046	3.023	.062	—.527	—.053	3.017	.061	—.526

TABLE 4

SOLUTION OF EXAMPLE 4

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
θ'	i'	$\Delta i'$	$5.73 \Delta i'$	$[i' + 5.73 \Delta i']$	$\Delta \phi'$	ϕ'	i	Δi	$5.73 \Delta i$	$[i + 5.73 \Delta i]$	$\Delta \phi$	ϕ	i	$(i' - i) = \frac{N_2}{N_1} i''$
0	0	0	0	0	0	.500	0	0	0	0.	0	.500	0	0
10	0	0	0	0	0	.500	0	0	0	0	0	.500	0	0
20	.025	.025	.143	.168	.017	.517	.001	.001	.006	.007	.016	.516	.001	.024
30	.084	.059	.338	.422	.042	.558	.003	.002	.011	.014	.041	.557	.003	.081
40	.169	.085	.487	.656	.066	.622	.005	.002	.011	.016	.064	.621	.005	.164
50	.277	.108	.619	.896	.090	.710	.007	.002	.011	.018	.088	.709	.007	.270
60	.400	.123	.705	1.105	.110	.819	.009	.002	.011	.020	.108	.817	.009	.391
70	.543	.143	.820	1.363	.136	.953	.010	.001	.006	.016	.135	.952	.010	.533
80	.696	.153	.877	1.573	.157	1.109	.012	.002	.011	.023	.155	1.107	.012	.684
90	.855	.161	.923	1.778	.178	1.285	.013	.001	.006	.019	.176	1.283	.013	.842
100	1.014	.159	.911	1.925	.192	1.475	.015	.002	.011	.026	.190	1.473	.015	.999
110	1.167	.153	.877	2.044	.204	1.677	.017	.002	.011	.028	.202	1.674	.017	1.150
120	1.310	.143	.820	2.130	.213	1.887	.019	.002	.011	.030	.210	1.884	.019	1.291
130	1.438	.128	.733	2.171	.217	2.101	.022	.003	.017	.039	.213	2.097	.022	1.416
140	1.551	.113	.647	2.198	.220	2.317	.027	.005	.029	.056	.214	2.312	.027	1.524
150	1.636	.085	.487	2.123	.212	2.524	.033	.006	.034	.067	.206	2.517	.033	1.603
160	1.700	.064	.366	2.066	.207	2.724	.042	.009	.052	.094	.197	2.714	.042	1.658
170	1.730	.030	.172	1.902	.190	2.905	.053	.011	.063	.116	.179	2.893	.052	1.678
180	1.730	0	0	1.730	.173	3.066	.066	.013	.074	.140	.159	3.052	.065	1.665
190	1.700	—.030	—.172	1.528	.153	3.205	.080	.014	.080	.160	.137	3.189	.078	1.622
200	1.645	—.055	—.315	1.330	.133	3.322	.095	.015	.086	.181	.115	3.304	.093	1.552

TABLE 4.—*Continued*

SOLUTION OF EXAMPLE 4

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
θ'	i'	$\Delta i'$	$5.73 \Delta i'$	$[i' + 5.73 \Delta i']$	$\Delta \phi'$	ϕ'	i	Δi	$5.73 \Delta i$	$i + 5.73 \Delta i]$	$\Delta \phi$	ϕ	$\imath$	$(i'-i) = \frac{N_2}{N_1} \imath''$
210	1.556	—.089	—.510	1.046	.105	3.408	.110	.015	.086	.196	.085	3.389	.106	1.450
220	1.446	—.110	—.630	.816	.082	3.470	.122	.012	.069	.191	.062	3.451	.118	1.328
230	1.313	—.133	—.762	.551	.055	3.506	.129	.007	.040	.169	.038	3.489	.126	1.187
240	1.160	—.153	—.877	.283	.028	3.518	.132	.003	.017	.149	.013	3.503	.129	1.031
250	.992	—.168	—.963	.029	.003	3.506	.129	—.003	—.017	.112	—.008	3.495	.127	.865
260	.809	—.183	—1.048	—.239	—.024	3.471	.121	—.008	—.046	.075	—.031	3.463	.120	.689
270	.625	—.184	—1.054	—.429	—.043	3.420	.112	—.009	—.052	.060	—.049	3.414	.111	.514
280	.441	—.184	—1.054	—.613	—.061	3.353	.101	—.011	—.063	.038	—.065	3.349	.100	.341
290	.263	—.178	—1.020	—.757	—.076	3.273	.089	—.012	—.069	.020	—.078	3.271	.088	.175
300	.090	—.173	—.992	—.902	—.090	3.181	.077	—.012	—.069	.008	—.091	3.180	.077	.013
310	—.063	—.153	—.877	—.940	—.094	3.086	.068	—.009	—.052	.016	—.096	3.085	.068	—.131
320	—.196	—.133	—.762	—.958	—.096	2.989	.060	—.008	—.046	.014	—.097	2.988	.060	—.256
330	—.306	—.110	—.630	—.936	—.094	2.894	.052	—.008	—.046	.006	—.094	2.893	.052	—.358
340	—.390	—.084	—.481	—.871	—.087	2.806	.047	—.005	—.029	.018	—.089	2.805	.047	—.437
350	—.445	—.055	—.315	—.760	—.076	2.729	.042	—.005	—.029	.013	—.077	2.727	.042	—.487
360	—.465	—.020	—.114	—.579	—.058	2.669	.039	—.003	—.017	.022	—.060	2.667	.039	—.504

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THE ELECTRON THEORY OF MAGNETISM

BY

ELMER H. WILLIAMS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 62 NOVEMBER, 1912

THE ELECTRON THEORY OF MAGNETISM

BY ELMER H. WILLIAMS, ASSOCIATE IN PHYSICS

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ELECTRON THEORY OF MAGNETISM

I. Essential Features of the Electron Theory of Magnetism

1. *Introduction.*— During the last decade the development of the subject of magnetism has made rapid strides. Not only have the older theories and methods been extended by improved facilities, but new theories have been advanced which are intended to correlate the great mass of data and facts, and thus enable our present knowledge to be extended along new lines. Among the new theories which have been advanced by various authors, the *electron theory of magnetism* is one of the most important and interesting. This theory seems to account for magnetic phenomena in a very direct way. We have only to assume that the molecular currents of Ampère, which form the elementary magnets, are revolving electrons in order to express Ampère's theory of magnetism in terms of the electron theory. However, a closer study of their orbits, due to Voigt and J. J. Thomson, showed that these currents cannot account sufficiently for the phenomena of diamagnetic and paramagnetic bodies. It was only on the basis of the researches of Curie that Langevin was able to give a more satisfactory theory of diamagnetism and paramagnetism.

The theory, worked out by Langevin for paramagnetic gases only, was later extended by Weiss to ferromagnetic substances. Weiss introduced a new notion into the theory of magnetism, viz., that of an intrinsic or molecular magnetic field by means of which he could account in a very beautiful way for the magnetic properties of the crystal pyrrhotite and many of the magnetic properties of iron, nickel, and cobalt. He has also contributed most essentially to our experimental knowledge of the ferromagnetic phenomena.

One of the co-workers of Weiss in the fundamental investigations on pyrrhotite was J. Kunz, who also contributed to the theory of magnetism by determining the elementary magnetic moment and the charge of the electron from purely magnetic phenomena. In his lectures on the electron theory given at the University of Illinois, Kunz gave an account of the present theory of magnetism and of the experimental and theoretical work of Weiss. The author of this bulletin has used these lectures as a basis, drawing in addition, from the works of the various authors who have made further experimental advances.

Weiss recently advanced a new theory in which the magnetism of a substance appears to be made up of magnetons just as a negative electrical charge is an aggregation of electrons. This theory, if confirmed by further experimental evidence, represents a new fundamental step in the development of our knowledge of the material universe. The experimental evidence, however, seems hardly strong enough to warrant a detailed discussion of it in this bulletin.

The present theory of magnetism as developed by Langevin and Weiss is open to certain objections and fails to explain a considerable number of magnetic phenomena. It gives for instance no connection

between the elastic and magnetic phenomena of ferromagnetic substances. It is possible that even the foundation of the present theory will undergo changes, but the experimental facts which have been considered in developing the present form of the theory will still be of value. It seems advisable, therefore, in order to give to this bulletin a more permanent value, to lay considerable stress on the experimental methods applied by Weiss and his followers and to give also the results obtained since the time when the classical book on magnetism was written by Ewing.

The first part of the present bulletin contains, therefore, the essential features of the electron theory of magnetism, the second and the third parts give an account of the properties of ferromagnetic crystals, while the fourth part gives further experimental evidence in favor of the electron theory of magnetism together with an account of some of the phenomena for which the theory in its present form fails to give a satisfactory explanation.

2. *Kinds of Magnetism.*— Bodies are divided, from the point of view of their magnetic properties, into three distinct groups: ferromagnetic, paramagnetic, and diamagnetic.

Under ferromagnetic substances are classed those substances of which the intensity of magnetization at saturation is of the same order of magnitude as that of iron. They are iron, nickel, cobalt, magnetite, pyrrhotite, and the Heusler alloys (which consist of copper, manganese and aluminum).

Paramagnetic substances are those which, while they become magnetized in the direction of the field, do so very feebly. Among paramagnetic bodies are found oxygen, nitrogen dioxide, palladium, platinum, manganese, and the salts of various metals.

Diamagnetic bodies, which include the greater number of all simple and compound bodies, have properties very different from those of either ferromagnetic or paramagnetic bodies. When placed in a magnetic field, they become slightly magnetized in a direction opposite to the direction of the field.

Some bodies, such as iron, when heated, show a gradual transition from the ferromagnetic to the paramagnetic state or vice versa, but as yet no body, with the exception of tin, has been found which, by change of physical conditions, will pass from the diamagnetic to the paramagnetic state.

Within the last decade a large amount of work has been done on the ferromagnetic substances, magnetite, hematite, and pyrrhotite, which are found in nature in crystals of such size and shape as will permit of a study of their magnetic properties. However, it is not possible to obtain comprehensive results with magnetic crystals merely by adapting to them the methods that have been applied with success to isotropic substances.

In the case of isotropic substances, the intensity of magnetization has always the same direction as the field, and, as in all directions the behavior is the same, it is sufficient to apply to the substance a magnetic field of any direction and to determine the intensity of magnetization

corresponding to each of its values. It is true that it is necessary to take into account the different secondary phenomena, hysteresis, retentivity, etc., which influence considerably the character of the principal phenomena.

For allotropic substances, it is necessary to consider, besides the magnitude of the field and of the intensity of magnetization, their directions, which, in general, are different for these two quantities. In place of a function of one variable there will be a system of three functions of three variables if one represents the field and the magnetization by their components. As the secondary phenomena are as complicated as in isotropic substances, it is easy to see that the complete investigation of the magnetic properties of a crystal constitutes a very difficult problem.

3. *General Properties of Electrons.*—The electron theory of magnetism supposes that the atom is made up of positive and negative electricity, the latter always occurring as exceedingly small particles called electrons, and that these electrons, whenever they occur, are always of the same size and always carry the same quantity of electricity. It is this peculiar way in which the negative electricity occurs both in the atom and when free from matter that gives to the theory its name. An electron is then, an "atom" of electricity, or the smallest amount of electricity which can be isolated. These electrons are given out by all bodies at a sufficiently high temperature accompanying the phenomena of radiation. These electrical particles leave the metals and other substances under the action of visible and invisible light, Roentgen rays, radium rays, etc. They appear in most of the radioactive processes and in chemical reactions. Thus when a metal is oxidized it emits electrons. Whatever be the source of the electron, its electrical charge has always been found to be from 4.65×10^{-10} to 4.69×10^{-10} absolute electrostatic units. The mass of the electron is about 1,800 times smaller than the mass of the atom of hydrogen, which is the lightest chemical element and which has a mass of about 1.61×10^{-24} grams. The mass of the electron is not ordinary chemical or ponderable matter, but apparent or electromagnetic mass and is due to the electromagnetic field which surrounds the electron in motion. The radius of the electron has been found to be 1.8×10^{-13} cm. To illustrate the size of an electron as compared with an atom, imagine a hydrogen atom increased in volume to that of a large cathedral, the electron being increased proportionally. Then the volume of the electron would be that of a fly flying about in the vast space. In spite of this minute size of the electron, or rather because of this minute size, the actions of the elementary charge are surprisingly great. Thus the electrical field on the surface of the electron is 1.4×10^{15}, or 10^{12} times stronger than any which we are able to produce by artificial means.

An electric current in a metal consists of electrons in motion, while a current through electrolytic solutions and through gases at ordinary and reduced pressures consists of positive and negative ions. As the pressure in a discharge tube becomes smaller, the electrical current is car-

ried more and more by free electrons or cathode rays which, by their impact on a solid obstacle, give rise to Roentgen rays.

The electron theory aims to explain all the phenomena of light, electricity, and magnetism, and in many cases it is the only theory that is able to explain the great variety of physical phenomena. The electron forms a part of each atom of the universe and it plays an important rôle in the chemical theories of matter. It is probable that the forces of affinity in the chemical reactions can be reduced to electrical forces between the electron and the positive charge of the atom. Thus chemical phenomena are drawn into the circle of the electron theory. Even mechanics, the oldest branch of exact natural science, is affected by the discovery that the mass of the electron depends on its velocity, so that Newton's equations of dynamics, the basis of the physical science, have to be slightly changed*. Finally, in the radioactive transformations, in which one element is transformed into another element, the electron plays an essential rôle.

An electron in motion is surrounded by a magnetic field. When an electron moves in a closed orbit, it is accompanied by a permanent magnetic field identical with that of an elementary magnet. Ampère considered the elementary magnets of iron as due to electrical currents flowing in closed molecular orbits without resistance. If we replace these currents of Ampère's theory by electrons moving in closed orbits, we have the fundamental idea of the electron theory of magnetism.

4. *Electromagnetic Force Due to an Electron in Motion.*— Rowland's experiments show that a moving electron is surrounded by a magnetic field. Consider a small element dl, Fig. 1. of the conductor carrying the current i. Let m be the magnetic pole, and ϕ the angle between the direction of the current and the radius r. Then the electromagnetic force produced at m by this element is

$$dK = \frac{midl \sin \phi}{r^2}$$

Suppose

$$m = 1$$

then

$$dK = \frac{idl \sin \phi}{r^2} \quad \text{(1)}$$

Suppose now that the current i is that due to an electron moving with a velocity v, where v is not greater than one third that of the velocity of light. If the electron moves through the distance dl in the time dt, we will have

$$dl = vdt \quad \text{(2)}$$

Also

$$i = \frac{e}{dt} \quad \text{(3)}$$

* For all engineering purposes and for the motion of the heavenly bodies this change is too small to be considered. It is only when the velocity of the mass approaches in magnitude the velocity of light that the effect is appreciable.

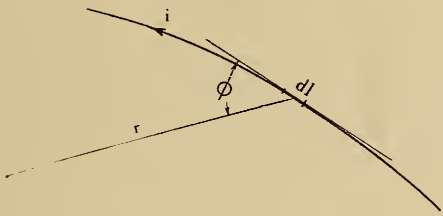

FIG. 1

Substituting (2) and (3) in (1) we get

$$dK = \frac{ev \sin \phi}{r^2} \quad \text{.......................} (4)$$

If we consider a sphere of radius r, Fig. 2, with an electron at the center moving with a velocity v, the magnetic force at the point m will be, from equation (4)

$$dK = \frac{ev \sin\phi}{r^2}$$

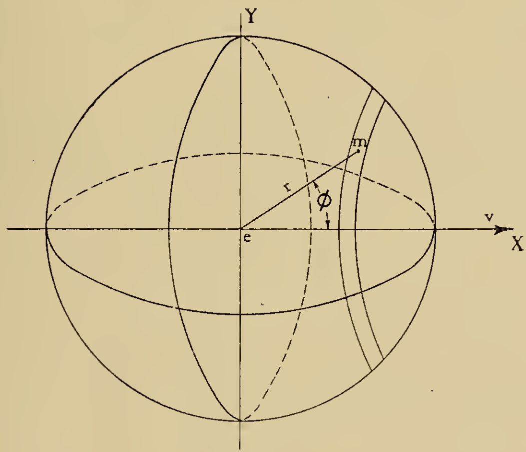

FIG. 2

This is the same for all points on the surface of the sphere where ϕ is the same. Therefore there is a circle around the sphere where the magnetic force is constant. The direction of the magnetic force is at right angles to the motion of e, or in a plane perpendicular to the x-axis.

5. *Diamagnetism.*— As defined before, a diamagnetic body is one which when placed in a magnetic field becomes slightly magnetized in a direction opposite to that of paramagnetic substances. Thus, a cylindrical diamagnetic body will set itself perpendicular to a magnetic field.

Consider in any atom an electron of mass m and charge e, Fig. 3. moving with a velocity v, in an orbit of radius r the plane of which is perpendicular to a magnetic field of intensity H. In the absence of the magnetic field, the centrifugal force on the electron is opposed by elastic forces, which we will suppose to be directed toward the center of the orbit and to be proportional to its radius. Then

$$\frac{mv^2}{r}=fr \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where f is the force of attraction toward the center, when

$$r=1\ cm.$$

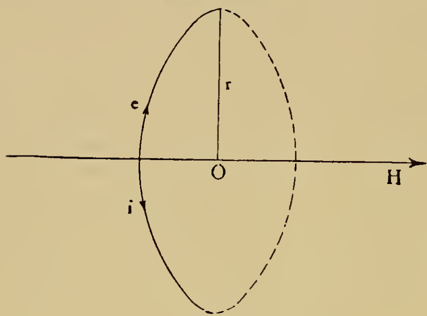

FIG. 3

Now apply the external magnetic field, and the electron is subject to a force at right angles to the field and to the direction of its motion, that is, along the radius of its orbit. The magnitude of this force is found as above by applying the fundamental law of electromagnetism. As before we have

$$dK=\frac{mi\ dl \sin\phi}{r^2}$$

$$=\frac{m}{r^2}idt\frac{dl}{dt}\sin\phi$$

$$=Hev\sin\phi$$

If the angle $\phi = 90°$, the force which the magnetic field exerts on the electron moving through the distance dl is

$$dK=Hev$$

Applying Ampère's rule we see that if the electron is negatively charged the force is directed outward along the radius. Since the electromagnetic forces acting on the electron are perpendicular to the direction of its motion, the magnitude of its velocity v is unchanged by the action of these forces.

Denoting the period of the new orbit produced when the field H is acting by T' and its radius by r', we have

$$\frac{mv^2}{r'}=fr'-Hev$$

or

$$\frac{mv^2}{r'^2}=f-\frac{Hev}{r'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

(6) − (5) gives
$$\frac{mv^2}{r'^2} - \frac{mv^2}{r^2} = -\frac{Hev}{r'} \quad \text{(7)}$$
$$T = \frac{2\pi r}{v}$$
or
$$T^2 = \frac{4\pi^2 r^2}{v^2}$$
Therefore
$$\frac{v^2}{r^2} = \frac{4\pi^2}{T^2} \quad \text{(8)}$$
Also
$$T' = \frac{2\pi r'}{v}$$
or
$$T'^2 = \frac{4\pi^2 r'^2}{v^2}$$
Therefore
$$\frac{v^2}{r'^2} = \frac{4\pi^2}{T'^2} \quad \text{(9)}$$
Substituting (8) and (9) in (7)
$$\frac{4\pi^2 m}{T'^2} - \frac{4\pi^2 m}{T^2} = -\frac{2\pi He}{T'}$$
whence
$$\frac{1}{T'^2} - \frac{1}{T^2} = -\frac{He}{2\pi m T'}$$
or
$$\frac{T^2 - T'^2}{T^2 T'^2} = -\frac{He}{2\pi m T'}$$
This may be written
$$\frac{(T+T')(T-T')}{T^2 T'^2} = -\frac{He}{2\pi m T'} \quad \text{(10)}$$
Now the diamagnetic phenomena are very small, therefore we can without appreciable error put
$$T + T' = 2T$$
and
$$TT' = T^2$$
Substituting these values in (10) we have,
$$T - T' = -\frac{HeT^2}{4\pi m} \quad \text{(11)}$$

In order to calculate the intensity of the induced magnetization, let us replace the revolving electron by an equivalent current flowing in a circuit coincident with its orbit. The strength i of the equivalent current is given by
$$i = \frac{e}{T}$$
Now the magnetic moment of a circuit of area A carrying a current i is given by
$$M_1 = Ai$$

In the case of an undisturbed electron revolving in an atom

$$i=\frac{e}{T}$$

and
$$A=\pi r^2$$

where r is the radius of the orbit.

Therefore the moment of the equivalent elementary magnet is

$$M_1=\frac{e}{T}\pi r^2$$

When the magnetic field H is applied this becomes

$$M_1'=\frac{e}{T}\pi r'^2$$

Let

$$M_1-M_1'=\triangle M_1$$

be the induced magnetic moment for one revolving electron.
Therefore

$$\triangle M_1=e\pi\left(\frac{r^2}{T}-\frac{r'^2}{T'}\right).$$

If we have N electrons revolving in unit volume, the induced magnetic moment per unit volume is

$$\triangle M=N\triangle M_1=Ne\pi\left(\frac{r^2}{T}-\frac{r'^2}{T'}\right)\quad\text{(12)}$$

From (8)

$$\frac{r^2}{T}=\frac{v^2T}{4\pi^2}\quad\text{(13)}$$

From (9)

$$\frac{r'^2}{T'}=\frac{v^2T'}{4\pi^2}\quad\text{(14)}$$

Substitute (13) and (14) in (12) and

$$\triangle M=\frac{Ne\pi v^2}{4\pi^2}(T-T')\quad\text{(15)}$$

From (11)

$$T-T'=-\frac{HeT^2}{4\pi m}$$

Therefore (15) becomes

$$\triangle M=-\frac{Ne^2v^2HT^2}{(4\pi)^2m}\quad\text{(16)}$$

Substitute (13) in (16) and there is obtained

$$\triangle M=-\frac{Ne^2Hr^2}{4m}$$
$$=-\frac{HNe^2r^2}{4m}$$

This is the induced magnetism per unit volume due to an external magnetic field H. Therefore $\triangle M$ is proportional to H, or

$$\triangle M=-kH,$$

where
$$k=\frac{Ne^2r^2}{4m}$$

is defined to be the diamagnetic susceptibility.

All the quantities on the right hand side of this equation are essentially positive, hence ΔM is negative and the body is diamagnetic whatever the sign of the electronic charge e. Thus all substances possess the diamagnetic property according to the above theory. Some substances are also paramagnetic, that is, one phenomen on is superimposed on the other.

In the above calculations it has been assumed that all the electronic orbits are so arranged that their axes are in the direction of the magnetic intensity of the inducing field and their planes perpendicular to their direction. It would be more accurate to assume that the axes are distributed in all directions. However, the change introduced by this assumption would consist only in multiplying the right hand side of the last equation by a proper fraction whose value is not very different from unity.

Multiplying numerator and denominator of the above expression for k by m, the mass of an electron, then

$$k=\frac{Nme^2r^2}{4m^2}$$

$$=\frac{\rho}{4}\left(\frac{e}{m}\right)^2r^2 \qquad (17)$$

where ρ is the density of the electrons, or the mass of the electrons per unit volume. It is seen that the effect produced contains as one factor, the square of the ratio $\frac{e}{m}$, and since this ratio is at least a thousand times greater for the negative electrons than for the positive corpuscles it is the former, which are present in all substances, that play the essential rôle in the production of diamagnetism.

In the case of water, of which the diamagnetic constant is $.8\times10^{-6}$, the density ρ of the negative electrons, which constitute only a part of the molecule, is less than unity and probably greater than 1/2000, the ratio of the mass of a negative electron to that of the atom of hydrogen. The ratio e/m is known and is

$$\frac{e}{m}=1.8\times10^7$$

absolute electromagnetic units.

Substituting the above values in equation (17) we get, using 1 for ρ,

$$r<10^{-8}\ cm.$$

Substituting 1/2000 for ρ we get

$$r>2\times10^{-10}\ cm.$$

Therefore

$$2\times10^{-10}<r<10^{-8}.$$

Experimental determinations indicate that r actually lies within these limits.

As the electronic orbits are considered to belong to the interior of the atom, which is not affected by temperature, we should expect that the diamagnetic susceptibility does not depend upon temperature. Curie's results indicate that, in general, this is the case, although du Bois and Honda found a large number of exceptions. Also the position

of the lines of the spectrum, which are due to the revolution of electrons inside the atom, is almost entirely independent of the temperature.

Equation (16) above may be interpreted simply as follows. The equation

$$\triangle M = -\frac{HNe^2r^2}{4m}$$

expresses the increase in the magnetic moment per unit volume, which contains N electrons. For one electron the increase in the magnetic moment is

$$\triangle M_1 = -\frac{He^2r^2}{4m}$$
$$= -\frac{He^2r^2\pi}{4m\pi}$$

This may be written

$$\triangle M_1 = -\frac{He^2A}{4\pi m} = -\frac{e^2}{4\pi m}HA \dots\dots\dots\dots\dots (18)$$

if the electron describes a circular orbit whose area is πr^2. The change of the magnetic moment of the electronic orbit is determined by the flux of magnetic induction HA which is produced by the external field passing through the orbit.

This is exactly the result that is obtained by merely applying to the electronic orbits the elementary laws of induction for the elementary circuits. Let us suppose that the resistance of the orbit is zero, and the self-inductance equal to $\triangle L$. If i is the current, the equation for the induced e. m. f. gives

$$\frac{d(\triangle Li)}{dt} = -\frac{d(\phi)}{dt} = -\frac{d(HA)}{dt}$$

whence

$$\triangle Li = -HA \dots\dots\dots\dots\dots\dots\dots\dots (19)$$

if H is zero in the beginning.

$$\triangle M_1 = \triangle Ai = -\frac{e^2}{4\pi m}HA$$

Substituting the value of HA given by (19),

$$\triangle Ai = \frac{e^2}{4\pi m}\triangle Li$$

It is sufficient to take

$$Ai = \frac{e^2}{4\pi m}Li$$

Whence since

$$A = \pi r^2$$
$$L = \frac{4\pi^2 mr^2}{e^2}$$

Thus the apparent self-induction is proportional to the mass of the electron and to the square of its radius, and inversely proportional to the square of the charge. This self-induction will identify itself with a real self-induction corresponding to the creation of a magnetic field by the electronic orbit, only if the inertia of the electron is wholly of electromagnetic origin.

If we assume that the self-induction of the current is due to the electromagnetic inertia of the electron,

$$\tfrac{1}{2}Li^2 = \tfrac{1}{2}mv^2$$

where

$$i = \frac{e}{T},$$

and

$$v = \frac{2\pi r}{T}$$

Whence

$$L\frac{e^2}{T^2} = \frac{m\,4\pi^2 r^2}{T^2}$$

or

$$L = \frac{4\pi^2 m r^2}{e^2}$$

which corresponds to the value of L found above.

The diamagnetic modification corresponds to a slight change in the magnetic moment of the original circuit. We have seen that this magnetic moment has the value

$$Mi = Ai = A\frac{e}{T}.$$

This magnetic moment, owing to the variation of the magnetic field, undergoes a change

$$\triangle M_1 = -\frac{He^2r^2}{4m}$$

The relative variation of the magnetic moment is

$$\frac{\triangle M_1}{M_1} = -\frac{HTe}{4\pi m}$$

which in the case of all diamagnetic bodies is very small. Now $\frac{e}{m}$ is of the order of 10^7 for negative electrons and still less for positive particles. T must necessarily have a value larger than that of the period of light which is of the order of 10^{-15}, for if the time of vibration were so small, the magnet would be a spontaneous source of light and permanent magnetism would be impossible. Let us assume that T is of the order 10^{-12}, which corresponds to the longest wave length that has been isolated in the spectrum of mercury vapor. Then in order to make $\frac{\triangle M_1}{M_1}$ approach unity, H needs to be of the order of only 10^6. Now we are able to produce fields of the order of 10^5, which, on the above assumption, would cause a change in the magnetic moment of the diamagnetic substance of one tenth of that of its original value.

6. *Magnetic Energy.*— In order to calculate the energy required to produce the diamagnetic modification, let us assume an electronic orbit which is without motion during the establishment of the magnetic field H. The magnetic force is perpendicular to the plane of the orbit and therefore produces no work. If there is a displacement dl along the

orbit (Fig. 4) and the electric force is E, the work done is

$$dW_1 = eEdl \cos \alpha$$
$$= e\,(E_x dx + E_y dy + E_z dz)$$

or

$$W_1 = e\int_C (E_x dx + E_y dy + E_z dz).$$

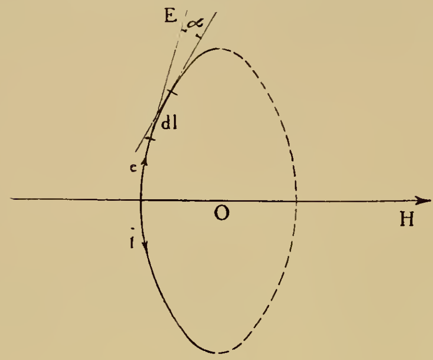

FIG. 4

During the time T of one revolution of the electron, which is of the order of 10^{-15} seconds, a time extremely short in comparison with the time necessary for the establishment of the field by the creation of currents or the displacement of magnets, E_x, E_y, E_z will not change appreciably and the work W may be calculated by an application of Stokes's Theorem.

$$W_1 = e\int_C (E_x dx + E_y dy + E_z dz) = e\iint_A \left[\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\cos nx + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\cos ny + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\cos nz\right] dA \quad \text{.........}(20)$$

where n is the normal to the surface.

Introduce Maxwell's equations,

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial H_x}{\partial t} &= -\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) \\ \frac{1}{c}\frac{\partial H_y}{\partial t} &= -\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) \\ \frac{1}{c}\frac{\partial H_z}{\partial t} &= -\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \end{aligned}\right\} \quad \text{................}(21)$$

c, the ratio between electrical energy expressed in electrostatic and electromagnetic units, is already included in E_x, E_y, and E_z of equation (20), therefore it should be omitted from equation (21) before substitution. Substituting (21) in (20), we have

$$W_1 = -e\iint_A \left[\frac{\partial H_x}{\partial t}\cos nx + \frac{\partial H_y}{\partial t}\cos ny + \frac{\partial H_z}{\partial t}\cos nz\right] dA$$

Now

$$H_x\cos nx + H_y\cos ny + H_z\cos nz = H$$

Therefore

$$W_1 = -e\iint\limits_A \frac{dH}{dt}\,dA$$

$$= -e\frac{dH}{dt}A$$

since $\frac{dH}{dt}$ is the same for all points of the area A.

The work done by the increasing magnetic field per unit of time is

$$W_1 = -\frac{e}{T}\,\frac{dH}{dt}A$$

$$= -\,iA\,\frac{dH}{dt}$$

Therefore

$$W_1 = -\,M_1\frac{dH}{dt}$$

The work done in the time dt is

$$dW_1 = \;-\,M_1\,dH$$

whence

$$W_1 = -\int_0^H M_1 dH \dots\dots\dots\dots\dots\dots\dots (22)$$

where M_1 is the magnetic moment of the electronic current in the direction dH. This work, done during the establishment of the magnetic field H by the current, or by the displacement of a magnet, is transformed into kinetic or potential energy of the electron which produces the electronic current. It represents the potential energy between the revolving electron and the magnetic field H. This transformation of energy accompanies the production of the diamagnetic state.

If the initial magnetic moment of the revolving electron is equal to M_o, the external field being zero, the magnetic moment under the action of the field will be

$$M_o + \triangle M_1 = M_o - \frac{e^2r^2H}{4m}$$

From (22)

$$W_1 = -\int_0^H \left(M_o - \frac{e^2r^2H}{4m}\right)dH = -\,M_oH + \frac{e^2r^2H^2}{8m}$$

If the molecule has N orbits, the resultant initial moment M will be

$$M = M_oN$$

whence

$$W = -\,MH + \frac{e^2r^2H^2N}{8m}$$

If the body is purely diamagnetic, M is zero and we have simply

$$W = \frac{e^2r^2H^2}{8m}\,N$$

This is the energy brought into play in purely diamagnetic phenomena. It is always present even in the case where the body is paramagnetic, but is small in comparison with the energy of the latter.

7. *Paramagnetism.*— We have seen that in all cases the creation of an exterior magnetic field modifies the electronic orbits by polarizing diamagnetically, all the molecules. This phenomenon is manifested only in the case where the resultant moment of the electronic orbits is zero, when the matter is diamagnetic in the ordinary sense of the word.

If the resultant moment is not zero, upon the diamagnetic phenomena is superimposed another phenomenon due to the orientation of the molecular magnets by the external field. The substance is then paramagnetic if the mutual action between molecular magnets is negligible, as in the case of gases and of solutions, and ferromagnetic in the case where the mutual actions play the essential rôle. As soon as the paramagnetism appears it is, as a rule, enormous in comparison with the diamagnetism and therefore completely conceals it. This explains the absence of continuity between paramagnetism and diamagnetism; paramagnetism may not exist; but if it exists, it hides completely the diamagnetism.

Therefore, substances whose atoms have their electrons in revolution in such a way that their effects are additive, are paramagnetic. The atoms of such substances may be looked upon as elementary magnets. The energy of such an elementary magnet may be represented by

$$W = -MH \cos \alpha$$

where α is the angle that the magnet makes with the magnetic field H.

If a magnetic field acts on a paramagnetic substance in a gaseous state and if the molecules have no thermal agitation, they will rearrange themselves in a direction parallel to the magnetic field. By virtue of this rearrangement they will, like a falling body, lose their original potential energy and acquire kinetic energy.

8. *Curie's Rule.*— Let us consider a paramagnetic body in the gaseous state, such as oxygen, whose molecules have a magnetic moment M. The molecules of such a body will turn, when under the influence of a uniform magnetic field H, in such a way as to place their magnetic axes parallel to the field. Let us calculate the magnetic moment per unit volume after the rearrangement has taken place.

If the magnetic moment makes an angle α with the direction of the uniform field H, then the molecule possesses a potential energy equal to

$$-MH \cos \alpha$$

The increase of this potential energy is derived from the kinetic energy of rotation of the molecules in the same way in which the potential energy of gravitation of the molecules of a gas is derived from the kinetic energy of translation when it is rising. The resultant inequalities in the distribution of kinetic energy between the orientations and the degrees of freedom of the molecules, rotation and translation, are not compatible with thermal equilibrium. A rearrangement takes place at the instant of the collisions during which the magnetic polarity appears and the energy

$$-HdM$$

of thermal agitation turns into potential energy of magnetization.

If the molecules have no relative potential energy of orientations, as in the case of gases and liquids, it is necessary, in order to maintain the medium at a constant temperature during the process of magnetization, to introduce into the system the energy $-HdM$, in the form of heat. In the case of a solid, where the molecules have a potential energy of orientation, this heat will be introduced only in case a cycle has been performed.

In the former case, we may apply the principles of thermodynamics in order to find the law experimentally established by Curie, since the process is perfectly reversible. The magnetic moment M for a given mass of the substance will be a function of the magnetic field H, and of the absolute temperature T. During a reversible change dH, dT, one can take out of the system a quantity of heat of which the portion which depends on H is

$$\begin{aligned} dQ &= HdM \\ &= H\left(\frac{\partial M}{\partial H}dH + \frac{\partial M}{\partial T}dT\right). \end{aligned}$$

In a reversible process

$$dA = \frac{dQ}{T}$$

Therefore

$$dA = \frac{H}{T}\frac{\partial M}{\partial T}dT + \frac{H}{T}\frac{\partial M}{\partial H}dH.$$

According to the second law of thermodynamics this must be an exact differential, therefore

$$\frac{\partial}{H\partial}\left(\frac{H}{T}\frac{\partial M}{\partial T}\right) = \frac{\partial}{\partial T}\left(\frac{H}{T}\frac{\partial M}{\partial H}\right)$$

or

$$\frac{1}{T}\frac{\partial M}{\partial T} + \frac{H}{T}\frac{\partial^2 M}{\partial T\partial H} = \frac{H}{T}\frac{\partial^2 M}{\partial H\partial T} - \frac{H}{T}\frac{2\partial M}{\partial H}$$

whence

$$\frac{1}{T}\frac{\partial M}{\partial T} = -\frac{H}{T^2}\frac{\partial M}{\partial H}.$$

Therefore

$$\frac{\partial M}{\partial T} = -\frac{H}{T}\frac{\partial M}{\partial T}$$

or

$$\frac{\frac{\partial M}{\partial T}}{\frac{\partial M}{\partial H}} = -\frac{H}{T} \qquad (23)$$

The general integral of (23) is

$$M = f(H/T) \qquad (24)$$

In the beginning of the magnetization, where the susceptibility may be considered constant at a given temperature

$$M = kH \qquad (25)$$

Comparing (24) with (25), we see that k, the paramagnetic susceptibility, must vary inversely as the absolute temperature, that is

$$k = A/T$$

This is called Curie's Rule, and the constant A is sometimes called Curie's constant. When this rule was first given out by Curie, it was thought to be general, but since then some substances have been found in which temperature does not affect the diamagnetic susceptibility k, and others in which k actually increases with increase of temperature.

9. *Langevin's Theory.*— The following comparison, which is due to Langevin,* will make clear the theory which precedes. Imagine a gaseous mass contained in a given receptacle Fig. 5, without being subject to the action of gravity. The molecules will distribute themselves in such a manner that the density of the gas will be the same at all points, which is similar to that which takes place in the case of a magnetic gas, such as oxygen, in the absence of an exterior magnetic field when the molecules have their axes distributed uniformly in all directions, Fig. 7.

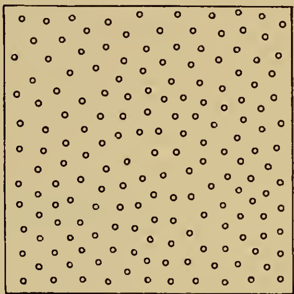

Fig. 5

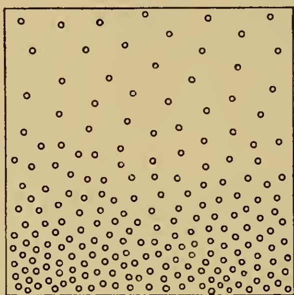

Fig. 6

If the force of gravitation is applied, Fig. 6, the molecules will acquire an acceleration directed toward the base, and, in the absence of mutual collisions, each molecule will have a greater velocity at the bottom than at the top of the vessel. But this inequality of velocity is incompatible with thermal equilibrium, and a rearrangement will take place due to the mutual collisions, after which the distribution which is established is given by the formula of barometric pressure. The center of gravity is lowered, and in order to maintain the gas at the initial temperature, it is necessary to remove from it a quantity of heat equivalent to the product of the mass of the gas by this lowering of the center of gravity, or equivalent to the loss of potential energy. One deduces from a thermodynamic reasoning analogous to that given above, that this lowering of the center of gravity is inversely proportional to the absolute temperature.

After the rearrangement in a mass of gas of uniform temperature, the distribution of the molecules takes place between the various regions in a manner such that the molecules will be more numerous where the

*P. Langevin, Ann. de Chem. et de Phys., Ser. 8, Vol. 5, p. 70, 1908.

potential energy is the least, that is to say, at the lowest points in the case of gravity. Boltzmann has calculated the distribution by generalizing the law of barometric pressure. The ratio of the densities of the gas in two points between which the potential energy varies by W is

$$e^{\frac{W}{RT}}$$

where e is the base of Naperian logarithms, T the absolute temperature of the gas and R the constant of the equation of a perfect gas, a constant such that, according to the kinetic theory, RT represents two thirds of the mean kinetic energy of translation.

The change when the magnetic field H is applied to a paramagnetic gas such as oxygen, Figs. 7 and 8, is the same as in the case of gravity except that here we have a rotation of the axes of the elementary magnets which assume the direction of H. Here too there is a loss of potential energy and a gain of heat, the rise in temperature being due to the thermal agitation of the molecules which produce a certain amount of heat due to their rotation.

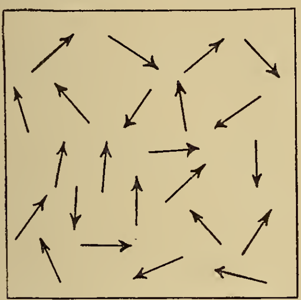

Fig. 7

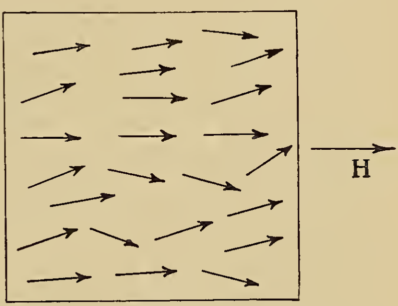

Fig. 8

The distribution of the molecules between the various orientations will be determined by the static equilibrium which will establish itself under the superimposed influence of the potential magnetic energy, $MH \cos \alpha$, and the energy RT of thermal agitation, the molecules being from preference oriented in the direction of least potential energy, that is to say, with their magnetic axes in the direction of the field. If one considers the distribution of the magnetic axes between the various directions, the density per unit of solid angle will vary from one direction to the other proportional to

$$e^{\frac{mH \cos \alpha}{RT}}$$

all directions being equally probable if M or $H=O$. The number of molecules whose axes are directed within the solid angle $d\alpha$, Fig. 9, will be

$$dn = Ke^{\frac{MH \cos \alpha}{RT}} d\omega$$

where the element $d\omega$ is a zone of aperture $d\alpha$ around the direction of the field

$$d\omega = 2\pi \sin \alpha d\alpha$$

α varying from 0 to π.

Therefore

$$dn = Ke^{\frac{MH \cos a}{RT}} 2\pi \sin a da \quad \text{.................} (26)$$

The total number of molecules per unit volume N will be

$$N = 2\pi K \int_0^\pi e^{a \cos a} \sin a da$$

where

$$a = \frac{MH}{RT}$$

Let

$$\cos a = x$$

then

$$dx = -\sin a da$$

and

$$N = 2\pi K \int_{-1}^{+1} e^{ax} dx$$

$$= \frac{2\pi K}{a} \left[e^{ax} \right]_{-1}^{+1}$$

$$= \frac{2\pi K}{a} \left(e^a - e^{-a} \right)$$

Now

$$\frac{e^a - e^{-a}}{2} = \sinh a$$

Therefore

$$N = \frac{4\pi K}{a} \sinh a$$

whence

$$K = \frac{aN}{4\pi \sinh a} \quad \text{.......................} (27)$$

The total magnetic moment of the N molecules is evidently directed parallel to the field and is equal to the sum of the projections of the component moments on this direction. For the unit of volume supposed to contain N molecules, this resultant moment represents the intensity of magnetization I.

$$I = \int M \cos a \, dn.$$

Substitute for dn its value given by equation (26) and

$$I = \int_0^\pi M \cos a K e^{a \cos a} 2\pi \sin a \, da$$

Therefore

$$I = 2\pi MK \int_{-1}^{+1} x e^{ax} dx$$

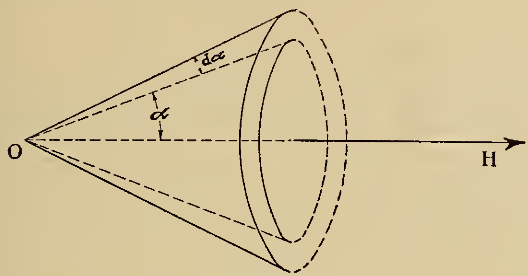

FIG. 9

Now

$$\int_{-1}^{+1} xe^{ax}\,dx = 2\left(\frac{\cosh a}{a} - \frac{\sinh a}{a^2}\right),$$

therefore

$$I = 4\pi MK\left(\frac{\cosh a}{a} - \frac{\sinh a}{a^2}\right)$$

Substitute the value of K given by equation (27) and

$$I = MN\left(\frac{\cosh a}{\sinh a} - \frac{1}{a}\right)$$

For a given number of molecules N, I is therefore a function solely of a, that is, of H/T, in accordance with the results given by thermodynamics.

The expression $\left(\frac{\cosh a}{\sinh a} - \frac{1}{a}\right)$ vanishes with a, which is proportional to H, and tends toward unity when a increases indefinitely, the intensity of magnetization approaching the maximum value $I_m = MH$ which corresponds to saturation, that is, the condition when all the molecular magnets are oriented parallel to the magnetic field.

The curve of magnetization of a magnetic gas at constant temperature ODE, Fig. 10, representing I/I_m as a function of a, that is, as a function of H, would be represented by the expression

$$I/I_m = \frac{\cosh a}{\sinh a} - \frac{1}{a} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

$$= f(H/T).$$

It is evident that the magnetic susceptibility will not be constant and I will be proportional to H only for values small compared with unity.

Developing equation (28) in series we have

$$\frac{I}{I_m} = \frac{\cosh a}{\sinh a} - \frac{1}{a} = \frac{1}{3} - \frac{2}{90}a^3 + \frac{4}{45.42}a^5 \quad\ldots\ldots\ldots\ldots (29)$$

Taking account only of the terms of the first degree with respect to a we find

$$I = \frac{I_m}{3}a = \frac{M^2NH}{3RT} = k'\frac{H}{T} = kH$$

where

$$k=\frac{M^2N}{3RT}$$

that is the paramagnetic susceptibility is inversely proportional to the absolute temperature, which agrees with the rule obtained experimentally by M. Curie.

If all the molecules were oriented parallel to the field, that is if the body were magnetized to the point of saturation, the intensity of magnetization would be,

$$I_m=MN$$

Combining this with the expression above, we get

$$k=\frac{M^2N^2}{3RTN}=\frac{I^2_m}{3p}$$

p being the pressure of the gas at which k is measured.

Curie found for oxygen at the standard pressure 10^6 and at a temperature of 0° $C.$,

$$k=1.43\times10^{-7}$$

per unit of volume.

$$\begin{aligned} I_m{}^2 &= k\times 3p \\ &= 1.43\times10^{-7}\times3\times10^6 \\ &= 0.43 \end{aligned}$$

whence

$$I_m=0.65$$

This would correspond for liquid oxygen, which is at least 500 times more dense than the gas, to a maximum magnetization.

$$I_m=325$$

which is not very much smaller than that of iron. In fact, liquid oxygen possesses such intense magnetic properties that it forms a liquid bridge between the poles of an electromagnet.

From the above, one is able to obtain the order of magnitude of the quantity a under ordinary experimental conditions.

$$a=\frac{MH}{RT}=\frac{MNH}{NRT}=\frac{I_mH}{NRT}$$

But NR is the constant of a perfect gas for the unit of volume which is supposed to contain N molecules. Under the normal conditions for which I_m has been calculated, one will have

$$NRT=p=10^6 c.\ g.\ s.\ units.$$

Therefore

$$a = 0.65\times10^{-6}H.$$

For a field of 10,000 units a will be 0.0065, and one would still be near the origin of the curve of magnetization, Fig. 10, where the curve coincides with the straight line. In order to make $a=1$, the region where the curve commences to leave the straight line, it would be necessary to have fields greater than 10^6, which we are not able to produce.

One sees then, in the ferromagnetic substances, the importance of the mutual actions between molecules which alone makes possible magnetic saturation. For the same exterior fields, magnetic saturation is still far removed in the case of paramagnetic substances where mutual actions are not appreciable.

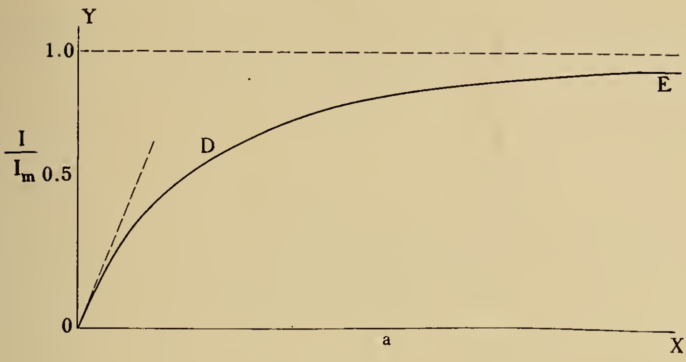

Fig. 10

From this point of view, Curie's comparison of the transition between paramagnetism and ferromagnetism to the transition between gaseous and liquid states where the mutual actions play an essential rôle, is perfectly justifiable. In the pure gaseous state, as in paramagnetism, each molecule reacts individually by its own kinetic energy against the exterior forces of pressure and magnetic field.

10. *Electronic Orbit.*— If one assumes that the magnetic moment M of one oxygen molecule is due to only one electron of charge equal to that of the atom of hydrogen obtained in electrolysis, moving along a circular orbit whose radius is equal to the radius of the molecule of air, 1.5×10^{-8} cm., one can calculate the velocity of the electron along the orbit. We have

$$\begin{aligned} M &= Ai \\ &= A\frac{e}{T} \\ &= \frac{\pi r^2 ev}{2\pi r} \\ &= \frac{rev}{2}. \end{aligned}$$

Also

$$I_m = MN = Ne\frac{rv}{2}.$$

But the product Ne is given by electrolysis, since e is the charge of the atom of hydrogen. Under normal conditions

$$\begin{aligned} Ne &= 1.22 \times 10^{10} \text{ electrostatic units} \\ &= 0.4 \text{ electromagnetic units.} \end{aligned}$$

Therefore

$$I_m = 0.65 = 0.4\,\frac{1.5\times 10^{-8}}{2}\,v$$

whence

$$v = 2 \times 10^{8} \textit{ cm. per sec.}$$

Suppose the revolving negative charge e, Fig. 11, to be attracted toward the center of its orbit by an equal positive charge. Then the centrifugal force equals the centripetal force, that is

$$\frac{mv^2}{r}=\frac{e^2}{r^2}$$

or

$$v^2=\frac{e^2}{mr}$$

where e is measured in electrostatic units.

$$e = 4.65 \times 10^{-10}$$
$$r = 1.5 \times 10^{-8} cm.$$
$$m = 8.77 \times 10^{-28}\ gram.$$

Therefore

$$v^2=\frac{(4.65 \times 10^{-10})^2}{8.77 \times 10^{-28} \times 1.5 \times 10^{-8}}$$
$$= 1.6 \times 10^{16}$$

or
$$v = 1.2 \times 10^8\ cm.\ per\ sec.,$$

which is in accordance with the value found by the above method.

It is a remarkable thing that the magnetic moment of the molecule of oxygen can be due to the revolution of a single electron. The same is probably true in the case of iron of which the maximum magnetization is, as we have seen, of the same order of magnitude as that of oxygen. The other electrons in the atom neutralize each other as in the purely diamagnetic body.

The paramagnetic electron also probably plays a part in chemical actions, the number of electrons of a molecule acting being equal to the valence. Therefore, the paramagnetic properties of an element change with change of molecular combinations, while diamagnetism seems to be an internal and invariable property of the atom.

11. *Molecular Field of Ferromagnetic Substances.*— Weiss Hypothesis: Each magnetic molecule in a ferromagnetic substance is subject to a uniform intrinsic magnetic field NI proportional to the intensity of magnetization I and acting in the same direction. This molecular magnetic field is due to the action of the neighboring molecules. It may be called the internal magnetic field in comparison with the internal pressure of Von der Waal's equation. This field, added to the external field, accounts for the great intensity of magnetization of ferromagnetic bodies by means of the laws of paramagnetic bodies, in the same way as the internal pressure, added to the external pressure, accounts for the great density of the liquids by invoking the compressibility of the gas.

This hypothesis has proved to be in agreement with experimental facts in a large number of cases. Among these cases may be named the properties of pyrrhotite, the heat developed when a substance passes from the paramagnetic to the ferromagnetic state and the law of temperature and intensity of magnetization of magnetite. The properties of hysteresis of iron can be given a theoretical interpretation and from $H_m = NI$, where N is a constant, I the intensity of magnetization and H_m the

intrinsic molecular field, can be calculated the absolute values of the elementary magnets and the fundamental quantities of nature.

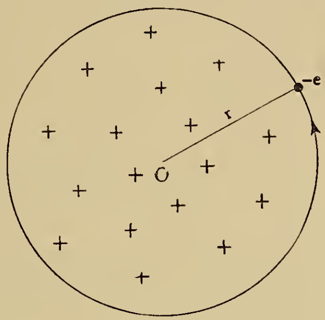

Fig. 11

There are, however, exceptions to this hypothesis, such as the law of approach to saturation and many temperature variations of ferromagnetism. Liquid oxygen does not follow the temperature law nor do iron and nickel at very low temperatures. Furthermore, when iron loses its magnetism we should expect that its volume would increase since the molecules lose their mutual actions and should go farther apart. On the contrary, iron contracts in passing from the ferromagnetic to the paramagnetic state. Colby* has shown that, in the case of nickel, the coefficient of expansion decreases very decidedly as the metal passes through the critical temperature.

II. Experimental Determination of the Magnetic Properties of Crystals

12. *Pyrrhotite.*— Pyrrhotite is one of the most simple of the crystalline magnetic substances. It is composed of sulphur and iron in quantities that are not perfectly constant; it also contains traces of nickel. Various authors have proposed the formulæ Fe_7S_8, Fe_8S_9, etc., all of which can be expressed in the form, n (FeS) + FeS_2. The color of pyrrhotite varies from brown to dark bronze.

Weiss† has shown that, in the case of pyrrhotite, if it be brought into a magnetic field in such a manner that the plane of its hexagonal base is perpendicular to the lines of force, little or no attraction will be exerted on it, while there is a very strong attraction with it in any other position. From this we see that there is one direction in which the magnetization is practically impossible. However, pyrrhotite is strongly magnetized in a plane perpendicular to this direction. This last named plane is called the plane of magnetization. In this plane, which is parallel to the base, there are two directions at right angles to each other in which the magnetization is different: (1) the direction of easy magnetization; (2) the direction of difficult magnetization.

*W. F. Colby, Phys. Rev., Vol. XXX, No. 4, p. 506, 1910.
†Weiss, Jour. de Phys., ser. 3, Vol. 8, p. 542, 1899.

13. *Apparatus and Methods for Determining the Magnetic Properties of Pyrrhotite.*—Weiss,* in his experimental determination of the magnetic properties of pyrrhotite, used two methods: (1) the ballistic method, in which a ballistic galvanometer was used to measure the quantity of electricity induced in a coil surrounding the sample by variations in the magnetic field; (2) a method in which he measured the couple produced on the crystal by the field.

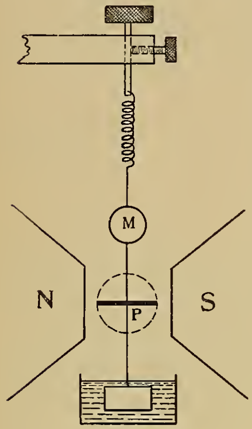

Fig. 12

The latter method is the more sensitive and permits of the examination of much smaller specimens. In this method a disk of pyrrhotite, P, Fig. 12, from 1 cm. to 2 cm. in diameter and about ½ mm. thick, is placed between the poles of a magnet of field strength H, and the magnetic properties of the crystal are studied with the disk in various positions. The intensity of magnetization is not necessarily in the direction of H, in fact, it is in this direction only in one position of the disk, i.e., when the direction in which the disk is most easily magnetized coincides with the direction of H. In general, the intensity of magnetization makes an angle a with H.

14. *Component of Magnetization Perpendicular to the Field.*—Suppose the crystal is turned so that the elementary magnet, with center at O, Fig. 13, makes an angle α with the direction of the field, H.

Let N_e be the moment of the couple acting on the elementary magnet. Then

$$N_e = 2lm\ H \sin \alpha$$
$$= \mu\ H \sin \alpha$$

where $2l$ is the length of the magnet, m its pole strength and μ the magnetic moment of the elementary magnet.

I, the intensity of magnetization, is equal to $n\mu$, where n is the number of elementary magnets per unit volume. Therefore,

$$N = n\mu H \sin \alpha$$
$$= I\ H \sin \alpha$$

where N is the moment of the couple acting on the crystal.

Now, $I \sin \alpha$ equals the component of the intensity of magnetization perpendicular to H. Therefore

$$N = HI_{(p)} = C\alpha'$$

where C is the constant of the apparatus and α' is the deflection of the mirror. If the magnet is mounted in a horizontal plane so that it can be turned about a vertical axis, the perpendicular component of the intensity of magnetization can be studied for various relative positions of the magnet and crystal.

*Weiss, Jour. de Phys., ser. 4, Vol. 4, p. 469, 1905.

Weiss' results show that this is a periodic function which repeats itself every 180°. The curve shown in Fig. 14 represents the ideal curve obtained in this manner. In practice the curve is not smooth but has irregularities.

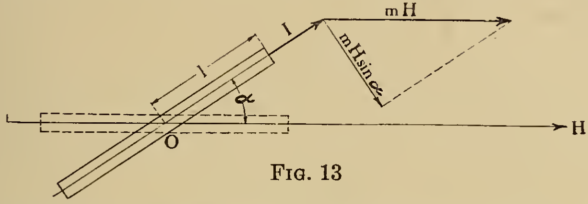

Fig. 13

15. *Component of Magnetization Parallel to the Field.*— If the disk is now placed between the poles of the magnet in a vertical position (see dotted line of Fig. 12) so that the plane of the disk makes an angle of 3° or 4° with the direction of the field H, and so that it can turn about a horizontal axis, it will be in a position to study the component of the intensity of magnetization which is parallel to the field.

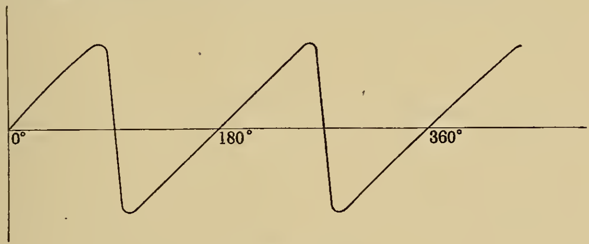

Fig. 14

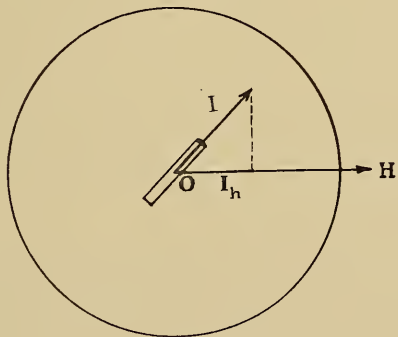

Fig. 15

Resolve the intensity of magnetization into a horizontal and a vertical component (see Fig. 15). Let I_h be the horizontal component. If the disk is placed so that I_h makes an angle of 3° or 4° with the direction of H, Fig. 16, there will be a restoring force tending to turn the disk back into a plane parallel to the field H. This can be measured in a manner similar to that shown in Fig. 12. The restoring force is

$$HI_h \sin \gamma = C\gamma'$$

where γ' is the deflection of the mirror. Therefore, if γ is kept constant the restoring force is proportional to I_h. Now $I_h \cos \gamma$, the component of I_h parallel to the field, is very nearly equal to I_h, since γ is only 3° or 4°. Hence, if we turn the disk through 180°, we obtain the components of the intensity of magnetization I_p, parallel to the field.

The component of the intensity of magnetization parallel to the field may also be studied by the method of induction by use of a solenoidal coil S, with a secondary S' inside. If the pyrrhotite disk d is thrust into the secondary with its plane parallel to the axis of the cylinder as shown in Fig. 17, there is a change of the flux through the secondary and hence a deflection of the ballistic galvanometer. The galvanometer will not be affected by the component of magnetization perpendicular to H, but only by that parallel to H. Therefore, by rotating the disk, one can study the relation between I_p and the angle of rotation of the disk. Weiss, by direct experiment, obtained results which are shown in Fig. 18, in which the abscissas are the azimuths of a constant magnetic field. The ordinates of the upper curve are the components of the magnetization parallel to the field, whereas those of the lower curve are the components perpendicular to the field. The phenomenon repeats itself every 180°.

In order to interpret these curves, Weiss makes use of the following illustration. Suppose a magnetic field is made to turn in the plane of an elliptic plate of soft iron. At the axes the magnetization coincides with the direction of the field. The longer axis will have a maximum, the shorter axis a minimum of magnetization. For all other directions of the field, the magnetization will be more nearly that of the longer axis than of the field. With the continuous rotation of the magnetic field, the magnetization will turn more slowly than the field in the neighborhood of the long axis and more rapidly in the neighborhood of the short axis. If the ellipse is very elongated, the component of magnetization perpendicular to the field will pass almost instantly from a very large negative value to a very large positive value, as represented in Fig. 14, while the direction of the field changes from one side to the other of the short axis. The lower curve of Fig. 18 may evidently be obtained by the addition of three curves, similar to that of Fig. 14, displaced with respect to each other by 60° and 120°.

From the above analogy, Weiss assumes that the complex structure of the crystal of pyrrhotite results from the juxtaposition of elementary crystals of which the magnetic planes are parallel and which possess each a direction of maximum and minimum magnetization at right angles to each other, and that these crystals are associated in the magnetic plane by the angles of 60°, or what amounts to the same thing, 120°.

The amplitudes of the abrupt variations of the component of magnetization perpendicular to the field give the relative importance of the three components of magnetization due to superposition of the crystals. The angles of the upper curve of Fig. 18 correspond to the minima of the magnetization parallel to the field of each of the components.

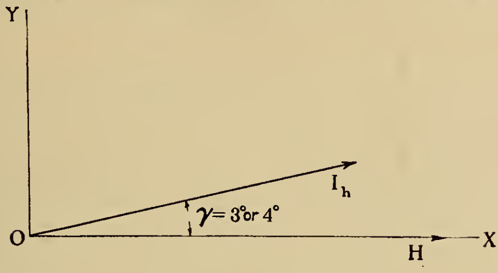

FIG. 16

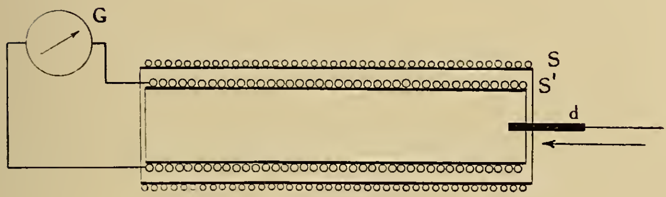

FIG. 17

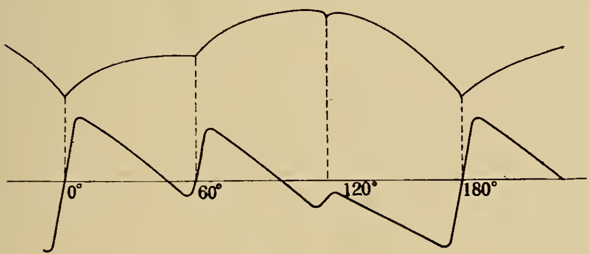

FIG. 18

16. *The Molecular Magnets of Pyrrhotite.*— In order to make a clear representation of the properties of a crystal of pyrrhotite, imagine that it be composed of rows of small needle magnets, equidistant from each other, pointing in the direction of easy magnetization Ox (Fig. 19), the axis Oy being the direction of difficult magnetization. Let the axis of rotation be perpendicular to the magnetic plane. Suppose that these magnets are small in comparison with the distances which separate them and strong enough to exercise on each other a directing action. Imagine moreover, that because of a compensation or of a greater distance, the rows do not affect one another. Left to themselves, the magnets of a row will adopt a position of equilibrium in which the north pole of each magnet will face the south pole of the following magnet. On this assumption, the substance would be saturated in the direction of easy magnetization Ox with little or no external field. Let a field H, making an angle α with the direction of the rows, act on the above system. The magnets will be deviated by an angle ϕ. As soon as the magnets are deviated from the direction Ox, there will be a force tending to restore

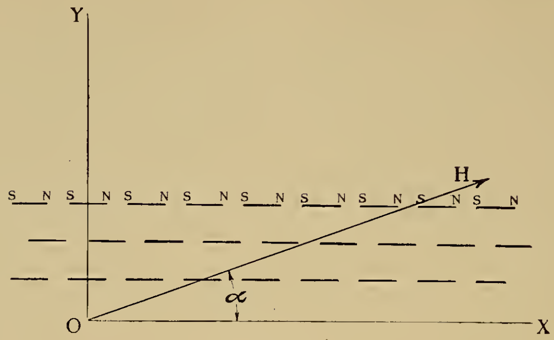

FIG. 19

them to their original position. If there were only two magnets, this force would take the direction indicated by the dotted line (1) Fig. 20, but the combined effect of all the magnets will give the force a direction (2), which is more nearly that of Ox. Let us assume that this force makes an angle ϕ with the direction Ox. The resultant action of the rows then exercises on each small magnet a magnetizing field $A\mu\cos\phi$ in the direction Ox, and in the direction Oy, a demagnetizing field $-B\mu\sin\phi$, where A and B are constants and where $\mu = ml$ is the magnetic moment of an elementary magnet. Let H_x and H_y be the sum of the components of the forces acting in the x and y directions respectively.

$$H_x = H\cos\alpha + A\mu\cos\phi \ldots\ldots\ldots\ldots\ldots\ldots (30)$$
$$H_y = H\sin\alpha - B\mu\sin\phi \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

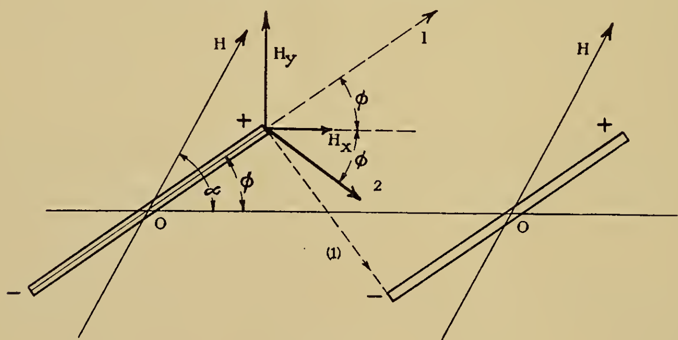

FIG. 20

For the condition of equilibrium

$$H_x \sin\phi = H_y \cos\phi$$

Therefore from (30) and (31)

$$(H\cos\alpha + A\mu\cos\phi)\sin\phi = (H\sin\alpha - B\mu\cos\phi)\cos\phi \ldots\ldots (32)$$

or,

$$H\sin(\alpha - \phi) = (A + B)\mu\sin\phi\cos\phi \ldots\ldots\ldots\ldots (33)$$

Writing

$$(A + B)\,\mu = NI_m$$

where N is a constant and I_m the intensity of magnetization per unit volume,

$$H \sin(\alpha - \phi) = NI_m \sin\phi \cos\phi \quad \ldots\ldots\ldots\ldots (34)$$

Thus with the above assumptions we arrive at an expression which as will be shown later, is the expression for the law of magnetization of pyrrhotite determined experimentally.

17. *Weiss' Law of Magnetization of Pyrrhotite.* — (a) Effect of Alternating Field.— It has been shown that the elementary magnets of pyrrhotite lie in the plane of the base of the crystal. The direction x is called the direction of easy magnetization; the perpendicular direction y, that of the difficult magnetization. If H is in the direction of easy magnetization, saturation takes place from the beginning. If, in this case, the field is reversed in direction the hysteresis curve will run through a cycle which is rectangular in shape (see Fig. 21). When the field has acquired a certain value $\pm H_c$, the elementary magnets become unstable and suddenly all of them swing around into the other position of equilibrium. Experiment gives very nearly a rectangle.

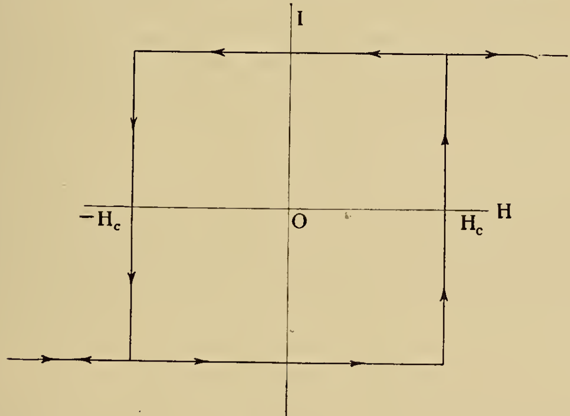

Fig. 21

(b) Effect of Rotating Field.— If a constant field is applied and rotated in the magnetic plane interesting phenomena present themselves. The elementary magnets have a tendency to remain in their original position so that when the field is rotated the direction of the intensity of magnetization turns slower than the field. If a field of 5,000 units acts in the direction Ox Fig. 22, and then rotates toward the direction Oy, the direction of the intensity of magnetization turns round much more slowly, and when H is in the direction Oy, I the intensity of magnetization, will be in the direction OI. Then when the field rotates a little farther, the magnets swing around very quickly so that, in the neighborhood of Oy, the rate of change of the direction of the intensity of magnetization is much greater than that of the field. Whatever the intensity of the field, the vector representing the intensity of mag-

netization falls within a circle C whose radius is equal to I_m, the maximum intensity of magnetization, which is the value obtained when the direction of the field coincides with the direction of easy magnetization. This circle is called the circle of saturation.

When the field is relatively weak, say 1,000 units, the intensity of magnetization vector follows the circle of saturation for only a short distance after which it describes a chord parallel to Ox. When the field has a strength of 7,300 units, or more, we have saturation in every direction of the field and the intensity of magnetization vector follows more closely the circle of saturation C.

Thus in order to turn and maintain the magnets in the direction of difficult magnetization Oy, it is necessary to apply in this direction a field of at least 7,300 units.

While for saturation in the direction Ox little or no field is required, the field necessary to produce saturation in the direction Oy must have a rather high value. Furthermore the intensity of magnetization remains constantly in the plane Oxy, the component of magnetization in the direction Oz perpendicular to this plane being very small in comparison with the magnetization in the characteristic plane. Thus while the demagnetizing field in the direction Oy is 7,300 units, that in the direction Oz is 150,000 units, or about 20 times greater.

Let α and ϕ be the angles which H and I make with Ox (Fig. 23): resolve H into components H_d directed along Oy and H_I parallel to the intensity of magnetization. There is a constant ratio

$$\frac{H_d}{I \sin \phi} = N \qquad (35)$$

between the components H_d of the field and the components of magnetization in the direction Oy. This fact was established experimentally by Weiss, a straight line relation between H_d and $I \sin \phi$ being obtained. The value of the constant N, for the sample that he used, was found to be

$$\frac{H_d}{I \sin \phi} = \frac{7300}{47} = 155.3$$

Everything takes place as if there were acting in the direction Oy, a demagnetizing force due to the structure of the crystal, proportional to the component of the intensity of magnetization in the direction of difficult magnetization Oy, and as if the remaining component of H were parallel to the direction of easy magnetization Ox.

From Fig. 23 we see that

$$H_d = AH = \frac{BH}{\cos \phi} = \frac{H \sin (\alpha - \phi)}{\cos \phi} \qquad (36)$$

Substituting (36) in (35) gives,

$$\frac{\dfrac{H \sin (\alpha - \phi)}{\cos \phi}}{I \sin \phi} = N$$

or

$$H \sin (\alpha - \phi) = NI \sin \phi \cos \phi \qquad (37)$$

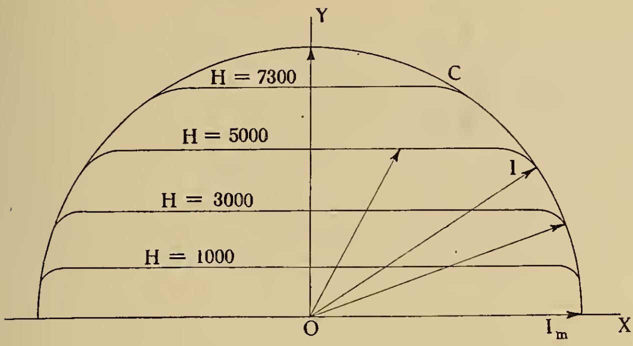

FIG. 22

This is the equation stating the law of magnetization of pyrrhotite, and is the same as that obtained when considering an elliptic plate of soft iron in a rotating field (Equation 34).

If, starting from the direction Ox, a field of 7,300 units or less has been rotated to the position Oy, the expression for the law of magnetization

$$H \sin(\alpha - \phi) = NI \sin\phi \cos\phi$$

becomes

$$H \cos\phi = NI \sin\phi \cos\phi$$

since in this position $\alpha = 90°$.

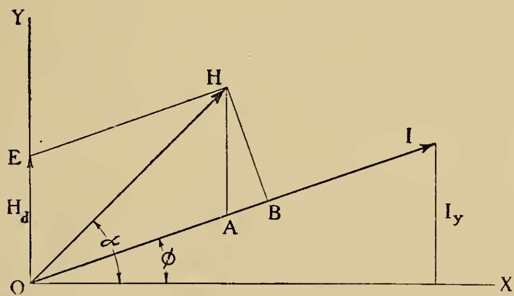

FIG. 23

For saturation this equation becomes

$$H \cos\phi = NI_m \sin\phi \cos\phi$$

therefore

$$\sin\phi = \frac{H}{NI_m}$$

For the same field there is a second position of equilibrium ϕ', Fig. 24, symmetrical to the first with respect to Oy. One might imagine that a coercive force H_c directed along $-Ox$ causes the various rows of the elementary magnets to turn from the first into the second position of equilibrium. This coercive force, in the case of pyrrhotite, is about 15 units whereas the demagnetizing field, or the field necessary to make the elementary magnets stand at right angles to the direction of easy magnetization, is about 7,300 units. The relation between the two quantities has not yet been explained, but it has been suggested that it has to do with the disturbed region of the extremity of the rows.

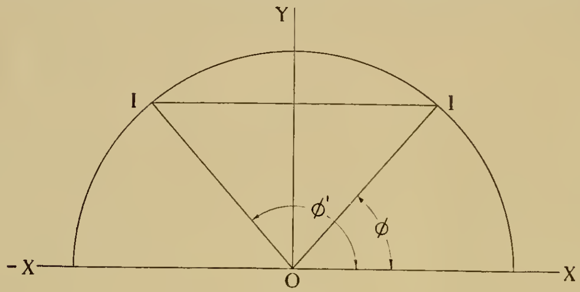

FIG. 24

18. *Magnetization in Different Directions.*— Let us now consider the laws of magnetization in directions other than that of Ox and Oy. As stated above, when a magnetizing field is applied in the direction of easy magnetization, saturation takes place from the beginning. If we plot intensity of magnetization I against field strength H, we will obtain the straight line AB (Fig. 25), parallel to the axis OH; that is, the intensity of magnetization remains the same whatever the value of the magnetizing field. Saturation occurs even without any external field.

Now if we apply a magnetizing force in the direction Oy, we obtain, from the general law of magnetization of pyrrhotite

$$H \cos \phi = NI \sin \phi \cos \phi$$

or

$$H = NI \sin \phi$$
$$= NI_y$$

whence

$$\frac{H}{I_y} = N = \text{constant}$$

This holds for fields below that necessary to produce saturation and is represented by the line OC Fig. 25. For fields equal to or greater than that necessary to produce saturation the curve is the same as for easy magnetization. Therefore the whole curve for the intensity of magnetization when the field is applied in the direction Oy is given by OCB Fig. 25.

We have now to inquire what happens when the direction of the magnetizing field is intermediate between the direction Ox and Oy. We get different expressions for the law of magnetization depending on the way in which I is resolved. For a field of constant direction we may

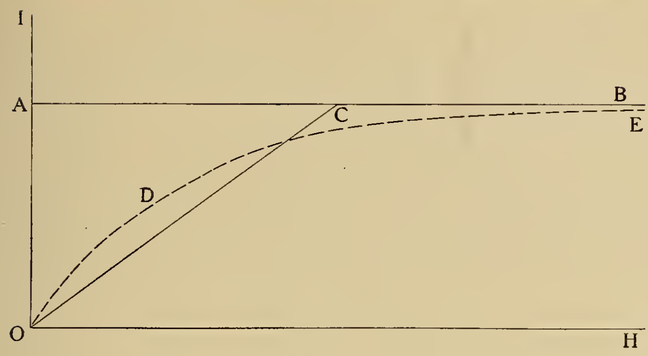

FIG. 25

take the component of the intensity of magnetization in the direction of the field. In this case (from Fig. 26)

$$I_h = I \cos(\alpha - \phi) \qquad (38)$$

In general

$$H \sin(\alpha - \phi) = NI \sin\phi \cos\phi$$

For saturation this equation becomes

$$H \sin(\alpha - \phi) = NI_m \sin\phi \cos\phi \qquad (39)$$

and (38) becomes

$$I_h = I_m \cos(\alpha - \phi) \qquad (40)$$

Eliminating ϕ between (39) and (40) we find

$$H = \frac{N}{2} \sin 2\alpha \frac{I_h}{\sqrt{I^2_m - I^2_h}} - NI_h \cos 2\alpha - \frac{N}{2} \sin 2\alpha \sqrt{I^2_m - I^2_h}$$

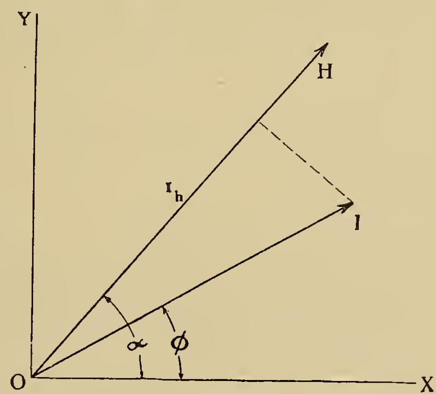

FIG. 26

If I_h approaches I_m, the first term of the right side of the equation approaches ∞; in other words, if H becomes ∞, I_h approaches I_m. Therefore the curve between the intensity of magnetization I_h, and the magnetizing force H approaches the line of saturation AB Fig. 25 asymptotically, and the law of magnetization for directions intermediate

between Ox and Oy is given by the curve ODE Fig. 25. It is possible to explain all the ordinary curves of magnetization by a superposition of properties analogous to those of pyrrhotite in the direction of easy, difficult, and intermediate magnetization.

As has already been shown, the relation

$$H \sin (\alpha - \phi) = NI \sin\phi \cos\phi$$

deduced from the triangle OHE (Fig. 23), is the analytic expression of the law of magnetization. This expression multiplied by I gives the couple, or mechanical moment, exerted by the field on the substance.

$$M = HI \sin (\alpha - \phi) = NI^2 \sin \phi \cos \phi$$
$$= \frac{2NI^2}{2} \sin\phi \cos\phi$$
$$= \frac{NI^2}{2} \sin 2\phi \quad \ldots\ldots\ldots\ldots\ldots (41)$$

This couple is a maximum when

$$I = I_m$$
$$\phi = 45°$$

If we measure M as a function of α, the angle of orientation of the field with respect to the substance, we shall find a couple which becomes constant as soon as the field is strong enough to bring about saturation of the intensity of magnetization after this vector I has described an arc of 45° or more along the circle of saturation. This has been very clearly demonstrated by Weiss* in which he obtains the following curves (Fig. 27) experimentally. In these curves the angles of rotation of the field with respect to the substance are used as abscissas, and the couples, expressed in mm. divisions of the scale are used as ordinates. It will be noticed that the maxima for curves III, IV, and V are approximately the same, notwithstanding the fact that the magnetizing fields in the three cases are widely different. In order to reduce these couples to their absolute values per unit of volume it is necessary to multiply by 950.

From equation (41), the value of the maximum couple is $\frac{NI_m^2}{2}$. Taking $I_m = 47$, one obtains from the mean of the three values of this couple a demagnetizing field,

$$H_d = NI_m = 7300$$

which agrees with experimental determination.

19. *Hysteresis Phenomena.*—(a) Alternating Hysteresis.—We have seen that when a field $H = H_c = 15$ gauss is applied parallel to the direction of easy magnetization of pyrrhotite the elementary magnets turn completely over, yet it takes 7,300 gauss to make them stand at right angles to this direction.

Weiss has shown that, if one considers the direction of easy magnetization in a substance that is infinite and without fractures, the intensity of magnetization would remain constant as the magnetizing field increases and, if the magnetizing force were to describe a cycle, the theoretical hysteresis loop would be a rectangle as indicated by the

* Weiss, Jour. de Phys., ser. 4, Vol. 4, p. 469, 1905.

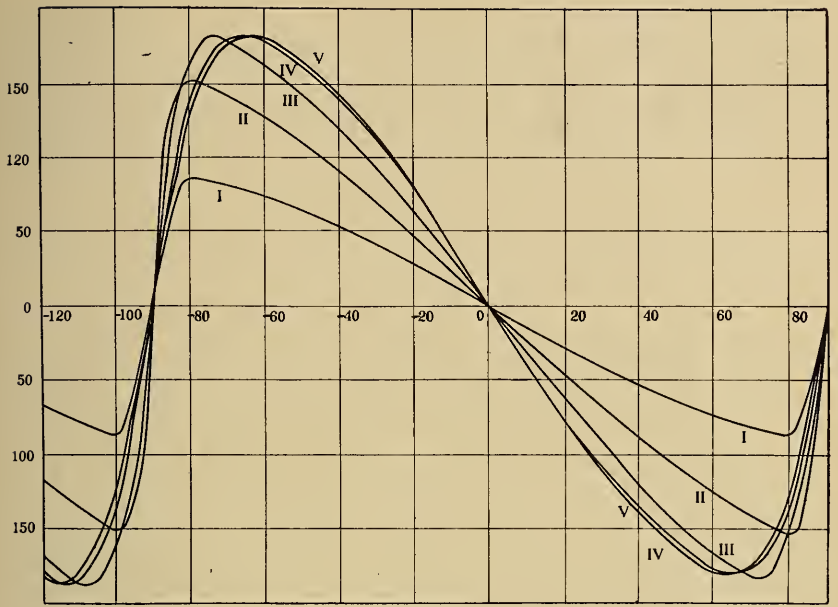

FIG. 27

Curve I. H= 1992 gauss
Curve II. H= 4000 gauss
Curve III. H= 7310 gauss
Curve IV. H=10275 gauss
Curve V. H=11140 gauss

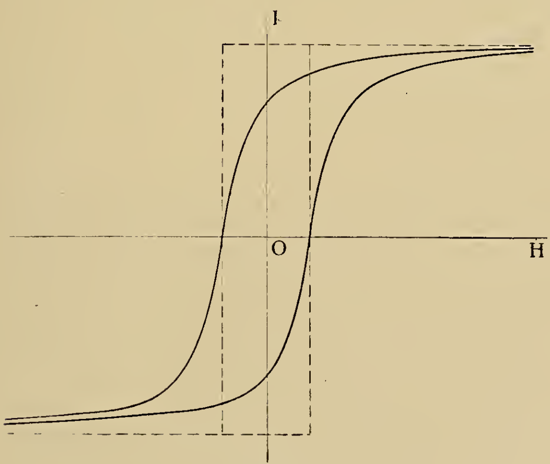

FIG. 28

dotted line in Fig. 28. The more uniform the magnetic material is, the less the experimental curve deviates from the theoretical curve. Weiss found that the distance between the ascending and descending branches of the experimental curve, measured parallel to the axis of abscissas, is approximately constant and equal to 30.8 gauss. Thus in order to move the vector of magnetization along the diameter of easy magnetization it is necessary to overcome a constant coercive field

$$H_c = 15.4 \text{ gauss.}$$

The energy dissipated per cycle in the form of heat in a unit of volume is

$$E_a = 4H_cI_m = 4 \times 15.4 \times 47$$
$$= 2900 \text{ ergs.}$$

(b) Rotating Hysteresis.— Excluding the direction of easy magnetization, the knowledge of magnetization in the magnetic plane has been obtained by causing a field of constant magnitude to turn in this plane. From these experiments one can obtain information on the form of hysteresis that has been called rotating hysteresis as distinguished from alternating hysteresis which has been considered above. If we rotate the field from the direction of easy magnetization OX (Fig. 29) through the angle XOY, the vector of the intensity of magnetization describes the arc AB of the circle of saturation. Then, the field passing the direction of difficult magnetization OY, the vector of the intensity of magnetization describes quickly the chord BFD, after which it describes the arc DE as the field describes the angle YO $(-X)$.

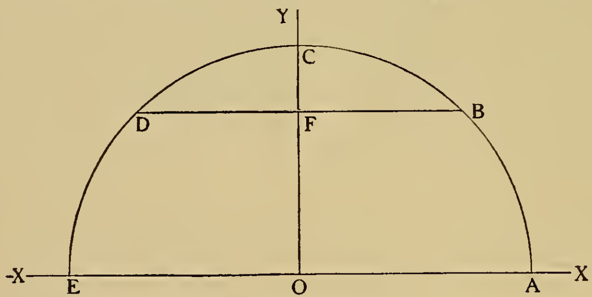

Fig. 29

Hysteresis occurs only in the direction of difficult magnetization OY when the molecules swing from one position of equilibrium into the other. In other words hysteresis accompanies the change of intensity of magnetization as the vector moves along BFD. When the magnetizing field H reaches the value 7,300 gauss the intensity of magnetization I follows the circle of saturation and the rotating hysteresis disappears. It follows from this that the hysteresis area along the circle of saturation is equal to zero. In the curves of Fig. 30, taken from experimental results obtained by Weiss with a sample of pyrrhotite in a field of about 600 gauss, 0 represents the region in the direction of easy magnetization and 90° that in the direction of difficult magnetization. It is seen that the two curves coincide exactly for some distance

in the neighborhood of easy magnetization. In the neighborhood of the direction OY, marked by a, the two curves are distinctly different. The smaller the field with which one works the greater the divergence, but whatever be the field the curve corresponding to rotation in one direction can be superimposed on the return curve by a horizontal displacement.

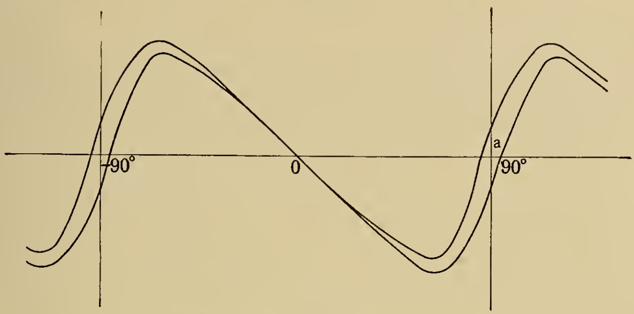

FIG. 30

20. *Energy of Rotating Hysteresis.*—So long as the intensity of magnetization I follows the curve of saturation there is no hysteresis loss. The loss takes place along SBS' Fig. 31. The force necessary to turn the elementary magnet over at S is less than at A. Call this force H'_c. Experiment has shown that

$$\frac{H_c}{I_m}=\frac{H'_c}{I_m-I_y}=\text{Const.}$$

therefore

$$H'_c=C(I_m-I_y).$$

To obtain the value of C let

$$I_y=O$$

then

$$H'_c=CI_m=H_c$$

or

$$C=\frac{H_c}{I_m}.$$

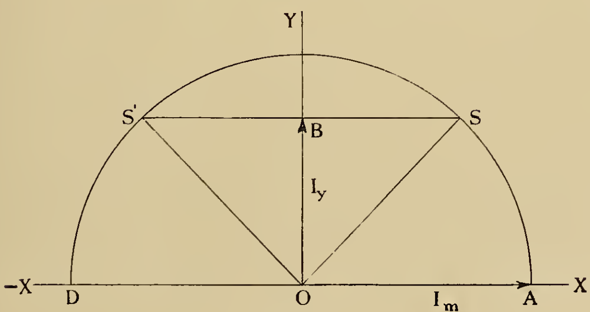

FIG. 31

Therefore the coercive force

$$H'_c=\frac{H_c}{I_m}(I_m-I_y).$$

If

$$I_y=I_m$$
$$H'_c=0$$

and there is no hysteresis loss.

Now the energy dissipated in an alternating hysteresis, that is a hysteresis along AD (Fig. 31), was found above to be equal to $4\,I_mH_c$. The rotating hysteresis loss along SS' is $4\,H'_cBS$.

Now

$$(BS)^2=I^2_m-I^2_y=I^2_m\left(1-\frac{I^2_y}{I^2_m}\right)$$

whence

$$BS=I_m\sqrt{1-\left(\frac{I_y}{I_m}\right)^2}$$

Therefore the energy dissipated in one rotating hysteresis cycle is

$$\begin{aligned}E_r&=4H'_cBS\\&=4H'_cI_m\sqrt{1-\left(\frac{I_y}{I_m}\right)^2}\\&=4\frac{H_c}{I_m}\left(I_m-I_y\right)I_m\sqrt{1-\left(\frac{I_y}{I_m}\right)^2}\\&=4H_cI_m\left(1-\frac{I_y}{I_m}\right)\sqrt{1-\left(\frac{I_y}{I_m}\right)^2}\qquad\qquad(42)\end{aligned}$$

If

$$I_y=0$$

this reduces to the value for alternating hysteresis. Since the demagnetizing force, if the substance is infinite and continuous, is

$$H_d=NI_y$$
$$I_y=\frac{H_d}{N}$$

Substituting this in equation (42) we have

$$E_r=4\,H_cI_m\left(1-\frac{H_d}{NI_m}\right)\sqrt{1-\left(\frac{H_d}{NI_m}\right)^2}$$

The energy E_a, dissipated in alternating hysteresis remains constant, whereas that of rotating hysteresis decreases with increase of I_y. This is shown graphically in Fig. 32 by the full line curve AB. The points (marked +) in the neighborhood of this curve are the results of measurements made by Weiss with a hysteresis-meter. They are subject to the correction $-nI_y$ since in a finite sample as was used in the experiment

$$H_a = (N + n) I_y$$

instead of

$$H_a = N I_y$$

where

$$n = \frac{H_I}{I}$$

in which H_I is the component of magnetization parallel to I.

21. *Magnetic Properties of Hematite.*— The magnetic properties of the crystal hematite are similar in many ways to those of the crystal pyrrhotite except that they are less pronounced. As in pyrrhotite, the elementary magnets of hematite lie in the plane of the base and it is paramagnetic perpendicular to this plane.

The crystal hematite has been studied by J. Kunz* and others. Kunz studied two distinct groups of crystals. The first group consisted of crystalline disks parallel to the base of the crystal. These crystals show a regular magnetic behavior and possess very small values of alternating and rotating hysteresis. The second group consisted of crystals composed of elementary magnets inclined toward each other at an angle of 60°. They possess so large an hysteresis effect that in many instances this overshadows the intensity of magnetization.

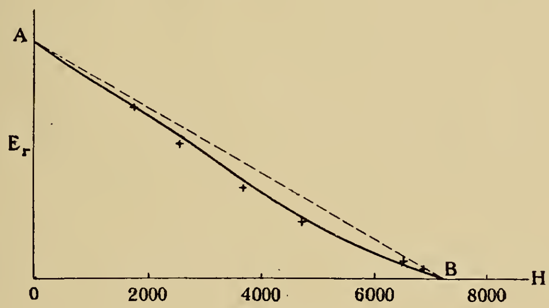

FIG. 32

It was found that the first group was not affected by temperature, that is while the intensity of magnetization decreased with increase of temperature, becoming zero at 645°, yet when the temperature was again decreased the intensity of magnetization returned to its original values. This is not the case with crystals of the second group, for after the crystal has been heated to 650° its intensity of magnetization does not return to its original value on its return to lower temperatures.

The maximum value of the intensity of magnetization in the direction of easy magnetization is reached much more quickly in the case of pyrrhotite than in the case of hematite. In the case of alternating and rotating hysteresis, the effect is very much the same in the two crystals.

*J. Kunz, Archives des Sciences, Vol. XXIII, 1907.

22. *Magnetic Properties of Magnetite.*— Besides the ferromagnetic crystals pyrrhotite and hematite there is a third, magnetite, whose magnetic properties are similar in many ways to those of the other two. It is classified as belonging to the regular system of crystals, but its magnetic properties indicate that it does not belong to this system. The magnetic properties of magnetite have been studied by Curie,* Weiss,† Quittner‡ and others, who found that they were more pronounced than those of hematite but less pronounced than those of pyrrhotite. From about 535° C. the temperature of magnetite transformation to 1375° C. the temperature of fusion of magnetite Curie found that the intensity of magnetization is independent of the field and that it decreases very regularly with the increase of temperature. For a part of this temperature range Curie formulated the following law: "The coefficient of magnetization of magnetite varies inversely as the absolute temperature between 850° C. and 1360° C."

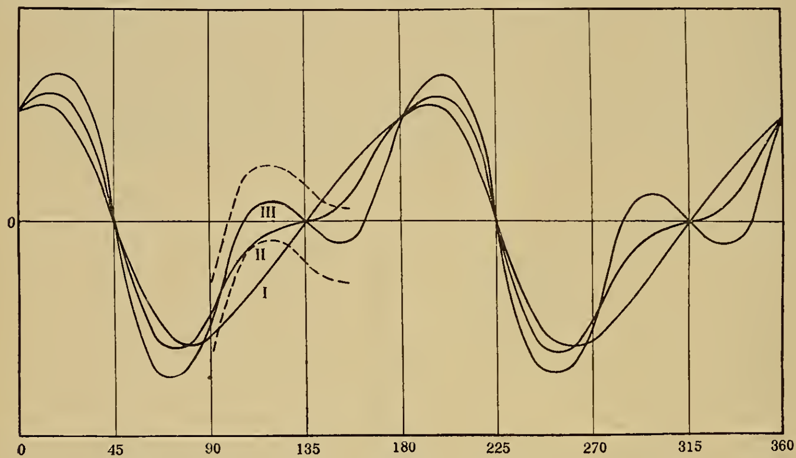

FIG. 33

Weiss and Quittner found that the magnetic properties indicate that the crystals of magnetite do not belong to the regular system. This was shown by taking a plate of magnetite cut parallel to the surface of the cube and placing it in the magnetic field in a horizontal position and then rotating the magnetic field round about it. Plotting their results with the angles of rotation of the magnetic field as abscissas, and as ordinates the deflection of the suspension which is proportional to the intensity of magnetization in a direction perpendicular to the magnetic field, they obtained the curves of Fig. 33. Each of these curves is the mean of the two curves obtained by rotating the field from 0° to 360° and then back to 0°. A portion of the two curves from which curve III is obtained is represented by the dotted lines. The area between the two curves is the hysteresis area of rotation.

* P. Curie, Ann. de Chem., ser. 7, Vol. 5, p. 391, 1895.
† P. Weiss, Archives des Sciences, Vol. XXXI, 1911.
‡ V. Quittner, Archives des Science, Vol. XXVI, 1908.

If the symmetry were that of the cubic system the curve should show a period of 90°, thus forming between 0° and 360° four similar waves. Instead of this the figure shows only two identical waves between 0° and 360° for $H = 57.3$ gauss. The symmetry is therefore not that of the cubic system. Now the geometrical properties of the crystal leucite are such that it is usually classed as an isomeric crystal, yet when it is investigated optically it is found to be isotropic. However, as the temperature is raised, the optical properties also become those of the regular system. It would be interesting to see whether the properties of symmetry of magnetite, from the magnetic point of view, change with change of temperature. If according to its magnetic properties magnetite does not belong to the regular system, the question arises, to which system does it belong from a magnetic point of view?

The curves of Fig. 33 show that the two principal axes situated in the plane of the cubic plate are not equivalent. It remains to be determined whether the third axis, perpendicular to the plane of the plate, is equivalent to either of the other two or whether it behaves differently from the magnetic point of view. In the first case, we would have the symmetry of the quadric system, in the second, that of the othorombic system. Thus if the system is cubic, there should be between 0° and 180° three identical waves; if it is quadratic there should be two similar waves and a third which is different; and finally, for the symmetry of the othorombic system, the three waves should be different, with the possibility that one or even two of these waves may disappear completely. Quittner in his researches found the three waves to be different and that their relative magnitudes depend on the magnitude of the magnetic field. We must conclude therefore, that magnetite, so far as its magnetic properties are concerned, possesses the symmetry of the othorombic system.

In order to see the irregularities in the magnetic behavior of a crystal of magnetite and the dependence of the magnetic properties on the value of the magnetizing field, we need only to observe the curves, Fig. 34, obtained by Quittner with a plate cut from a crystal in such a way that its plane makes equal angles with the three axes. It will be noticed that for a field $H = 167.2$ gauss all three waves are practically equal; for a field $H = 368.1$ gauss there are only two waves which are greatly reduced and displaced, the third being barely visible. If the field is still increased one again finds, for $H = 757$ gauss, three well-defined waves which, however, are displaced by half a wave length.

The theoretical interpretation of these very complicated phenomena is hardly possible in the present state of our knowledge, but the investigators of magnetite have proposed that the crystal is made up of equal parts of three elementary magnets whose magnetic planes are perpendicular to each other, such as would be the case if it were possible to superimpose three plates of pyrrhotite cut parallel to the plane of easy magnetization in such a manner that their planes would be mutually perpendicular to each other.

III. Effect of Temperature upon the Magnetic Properties of Bodies

23. *Method of Investigation Used by Curie.*— Extensive investigations of the effect of temperature upon the magnetic properties of various substances have been made by Hopkinson,* Curie,† and others. Curie's method was to place the body to be tested in a non-uniform magnetic field and measure the resultant force of the magnetic actions by utilizing the torsion of a wire. Let *ABCD* (Fig. 35) represent the horizontal arms of an electromagnet, and let the axes of these two arms form an angle with each other. The body to be investigated is placed at the point O on the line Ox, which is the intersection of the horizontal plane passing through the axis of the arms of the electromagnet and the vertical plane of symmetry. When the electromagnet is excited, a force f of attraction or repulsion acts along Ox. Call H_y the intensity of the magnetic field at O. This field is directed, by reason of symmetry, along Oy perpendicular to Ox. Let I be the specific intensity of magnetization, that is the intensity of magnetization per unit mass, and m the mass of the body, then

$$f = mI\frac{dH_y}{dx}$$

If diamagnetic or paramagnetic bodies are being studied the demagnetizing force arising from the magnetization of the body is insignificant, and if K is used to designate the coefficient of specific magnetization, there is obtained

$$I = KH_y$$

and

$$f = mKH_y\frac{dH_y}{dx}$$

Now K for most diamagnetic and paramagnetic bodies is practically constant and therefore f is proportional to $H_y\frac{dH_y}{dx}$. For greatest sensitiveness the body should be placed at the point on Ox at which this product is maximum. Curie's method was to surround the sample under investigation by a vertical electric furnace so that it could be heated to any desired temperature. The sample itself was mounted on the end of a lever *lm*, which was suspended by a torsion wire *tm*. This lever was connected to another lever *mn* so that any movement of the sample would be greatly magnified at the other end of the system. The whole movable system was suspended in such a manner that any movement of the body, which was very small in the substances investigated by Curie, would be along Ox. With his apparatus, Curie claimed to be able to measure movements of the object to 0.001 mm. As the body was heated to various temperatures, which could be determined by means of a thermocouple, the forces of attraction or repulsion could be determined from the movement of the levers and the constant of the apparatus.

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* Hopkinson, Phil. Trans., p. 443, 1889.

† P. Curie, Ann. de Chem., ser. 7, Vol. 5, p. 289, 1895.

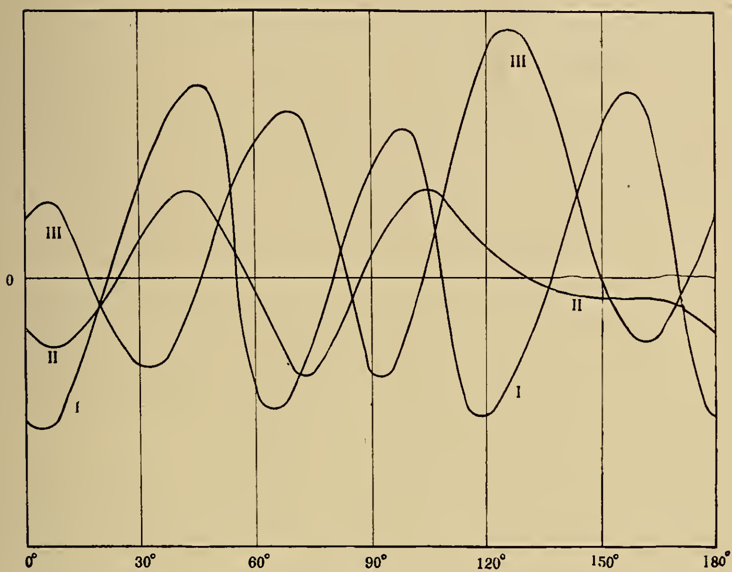


FIG. 34

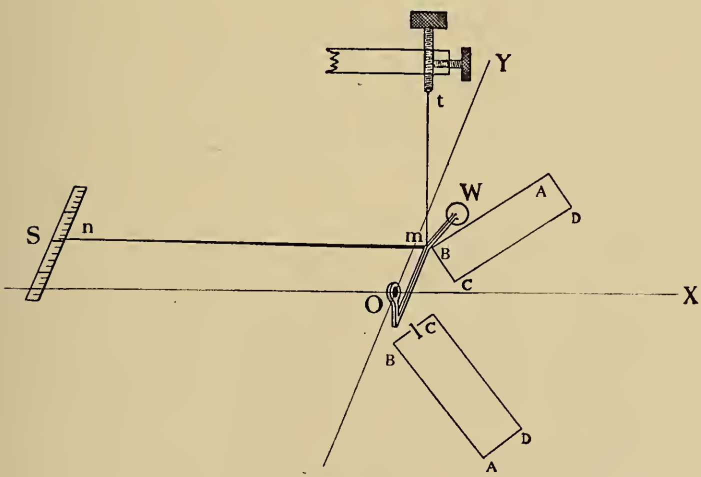


FIG. 35

24. *Results Obtained by Curie.* — After a very extended study, covering a wide range of substances — diamagnetic, paramagnetic, and ferromagnetic — Curie came to the following conclusions:

The coefficient of specific magnetization of diamagnetic bodies is independent of the intensity of the field, and as a general rule independent of the temperature. Antimony and bismuth are exceptions to this rule. The coefficient of magnetization of these bodies diminishes with increase of temperature. For bismuth the law of variation is a linear one. The physical and chemical changes of state often have only a slight effect on the diamagnetic properties. This is true in the case of the fusion of white phosphorus at 44°, and in the various transformations which are undergone when sulphur is heated. However, this is not always the case; the coefficient of magnetization of white phosphorus experiences a considerable diminution when this body is transformed into red phosphorus, antimony deposited by electrolysis in the allotropic state is much less diamagnetic than the ordinary variety, and the coefficient of magnetization of bismuth becomes, by fusion, twenty-five times more weak.

Paramagnetic bodies have also a coefficient of magnetization independent of the intensity of the field, but these bodies behave quite differently from the point of view of the changes produced by the change of temperature. The coefficient of specific magnetization varies simply in inverse ratio to the absolute temperature.

The difference in the effect of temperature on the coefficient of magnetization of magnetic and diamagnetic bodies is very marked, which is in favor of the theories which attribute magnetism and diamagnetism to causes of a different nature.

The properties of ferromagnetic and paramagnetic bodies are, on the contrary, intimately related. When a ferromagnetic body is heated it is transformed gradually and takes the properties of a paramagnetic body. The curve of Fig. 36 represents graphically the relation between temperature and intensity of magnetization I for a sample of iron subject to a magnetizing field of 1,300 units. In the region β, which commences at 760° C and extends to 920° C, the coefficient of specific magnetization obeys exactly an hyperbolic law up to 820°, after which it decreases more rapidly. Between 820° and 920°, at which point the γ state begins, it is probable that a gradual transformation takes place. In the γ state the iron possesses a susceptibility inversely proportional to the absolute temperature, which is characteristic of paramagnetic bodies. At 1280° where the last change of state takes place the coefficient of specific magnetization increases rapidly in the ratio of 2 to 3, after which it seems to take a variation in reverse ratio to the absolute temperature.

25. *Results of du Bois and Honda.* — H. du Bois and K. Honda* extended the investigations of Curie to a large number of elements

* H. du Bois and K. Honda, Konink. Akad. Wetensch., Amsterdam, Proc. 12, pp. 596-602, 1909-1910.

(43 in all) and decided that Curie's conclusions do not admit of such extensive generalizations as have been given to them. They found that of the twenty or more diamagnetic elements examined there are only six which do not vary within the whole temperature range, and that during a change of physical state a discontinuity in the intensity of magnetization frequently occurs. This change may consist of a large or small break in the curve showing the relation between intensity of magnetization and temperature or of a rather sudden change in the shape of the curve.

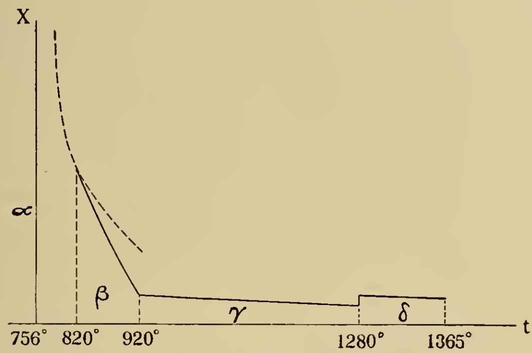


FIG. 36

Of the thermomagnetic examinations of polymorphous transformations made by du Bois and Honda, the most remarkable properties are shown by tin. They found that if diamagnetic grey tin is slowly heated, at 32° the intensity of magnetization, which is negative, suddenly changes (like the density) and at 35° passes through zero. Further heating continuously increases the magnetization until the value for paramagnetic tetragonal tin is reached at about 50°, after which it remains practically constant.

26. *Analogy Between the Manner in Which the Intensity of Magnetization of a Magnetic Body Increases under the Influence of Temperature and the Intensity of the Field, and the Manner in Which the Density of a Fluid Increases under the Influence of Temperature and Pressure.*— There are many analogies between the function $f\ (I, H, T) = O$ as applied to a magnetic body and the function $f\ (D,\ p,\ T) = O$ as applied to a fluid. The intensity of magnetization I corresponds to the density D, the intensity of the field H corresponds to the pressure p, and the absolute temperature T plays the same rôle in the two cases. For a paramagnetic body or a ferromagnetic body at a temperature above that of the transformation point, the relation is found

$$I = A\frac{H}{T}$$

where A is a constant. Similarly for a fluid sufficiently removed from its temperature of liquefaction one has the relation

$$D=\frac{I}{R}\times\frac{p}{T}$$

where $\frac{I}{R}$ is a constant. The law of the constancy of the intensity of magnetization when the field varies, and the inverse law of the absolute temperature for the coefficient of magnetization correspond respectively to the laws of Boyle and of Charles.

The manner in which the intensity of magnetization as a function of the temperature varies in the neighborhood of the temperature of transformation, the field remaining constant, corresponds to the manner in which the density of the fluid as a function of the temperature varies in the neighborhood of the critical temperature, the pressure remaining constant. The analogy between $I=\phi$ (T) and $D=\phi$ (T) corresponding to pressures above the critical pressures is shown graphically in Figs. 37 and 38. Although as Curie has shown the analogy seems to be almost perfect when the field strength in the one case and pressure in the other is kept constant, yet it has not been shown to hold in the case where temperature is kept constant in both phenomena.

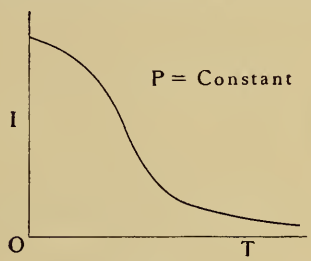


Fig. 37

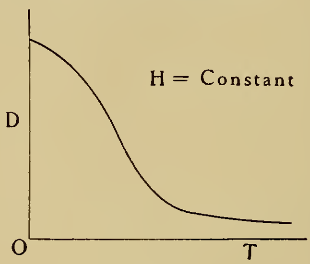


Fig. 38

In the case of magnetization if the temperature is kept constant and the field strength alternated a hysteresis loop is obtained when intensity of magnetization I is plotted against field strength H. That this is true in the case of a fluid when the temperature is kept constant and the relation between density and pressure plotted has not been shown. It is true that in the case of solids the phenomena of lag occurs with variation of density and pressure at constant temperature, and it may be true to a small degree in the case of liquids, but it is hard to conceive of it as being true in the case of gases.

IV. Experimental Evidence in Favor of the Electron Theory of Magnetism

27. *The Molecular Magnetic Field of Pyrrhotite.*—Let us assume that we have two fields, H and H_m, acting in a crystalline substance and that the crystalline structure possesses three rectangular planes of symmetry coincident with the planes of the system of coördinates, and that each component of the molecular field is proportional to the corresponding component of the intensity of magnetization with a coefficient

$N_1 > N_2 > N_3$ for each axis. Then the molecular field H_m has, in general, not the direction of the intensity of magnetization except when the latter is directed along one of the axes. Thus the intensity of magnetization assumes of itself the direction of one of the axes, Ox, Oy, or Oz, Fig. 39, and this orientation will correspond to a stable equilibrium only for the axis Ox for which the coefficient N_1 is the greatest.

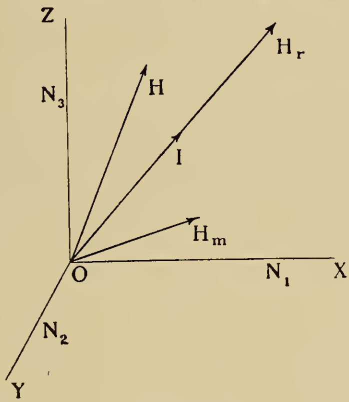


FIG. 39

If an external field H is added to the molecular field the magnetization assumes the direction of the resultant magnetic field. Now

$$H_x + H_{mx} = H_x + N_1 I_x,$$
$$H_y + H_{my} = H_y + N_2 I_y$$
$$H_z + H_{mz} = H_z + N_3 I_z$$

If the resultant H_r coincides with I in one direction,

$$\frac{H_x + N_1 I_x}{I_x} = \frac{H_y + N_2 I_y}{I_y} = \frac{H_z + N_3 I_z}{I_z}$$

and if the magnetization takes place in a plane, say the xy plane,

$$\frac{H_x + N_1 I_x}{I_x} = \frac{H_y + N_2 I_y}{I_y}$$

Let α and ϕ, Fig. 40, be the angles which H and I make with the axis Ox.

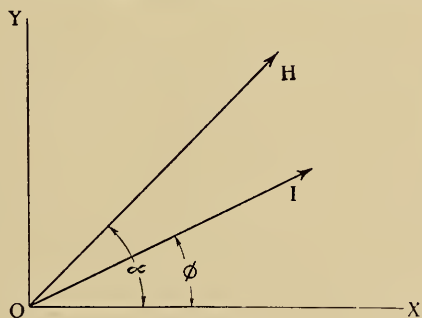


FIG. 40

Then

$$\frac{H \cos \alpha + N_1 I \cos \phi}{I \cos \phi} = \frac{H \sin \alpha + N_2 I \sin \phi}{I \sin \phi}$$

whence

$$IH \cos \alpha \sin \phi + N_1 I^2 \cos \phi \sin \phi = IH \sin \alpha \cos \phi + N_2 I^2 \sin \phi \cos \phi$$

Transposing and simplifying,

$$IH \sin (\alpha - \phi) - (N_1 - N_2) I^2 \sin \phi \cos \phi = 0$$

or

$$H \sin (\alpha - \phi) = (N_1 - N_2) I \sin \phi \cos \phi$$

Put

$$N_1 - N_2 = N$$

and

$$H \sin (\alpha - \phi) = NI \sin \phi \cos \phi$$

This equation expresses the law of magnetization of pyrrhotite, (see equation (37), page 9) as determined experimentally by Weiss. Therefore the above hypothesis accounts for the experimental properties of pyrrhotite in the xy plane. These properties are different from those in the xz plane only in the magnitude of the constant.

The equation

$$HI \sin (\alpha - \phi) = (N_1 - N_2) I^2 \sin \phi \cos \phi$$

states that the couple $HI \sin (\alpha - \phi)$ exerted by the external field on the intensity of magnetization is equal to the couple $(N_1 - N_2) I^2 \sin \phi \cos \phi$, which is due to the structure of the crystal and which would remain if the external field were suppressed. The latter couple tends to bring the elementary magnets back into the direction of easy magnetization. The position of equilibrium corresponding to the orientation of the magnets in the direction Ox is stable only when $N_1 > N_2 > N_3$.

28. *Variation of the Intensity of Magnetization of Magnetite with Temperature.*— It has been shown, page 21, that

$$\frac{I}{I_m} = \frac{\cosh a}{\sinh a} - \frac{1}{a} \quad \ldots\ldots\ldots\ldots (28)$$

where

$$a = \frac{MH}{RT} \quad \ldots\ldots\ldots\ldots (43)$$

In the case of ferromagnetic substances we have, in addition to the external field, an anterior or molecular field H_m which is due to the action of the molecules upon each other and which has been called by Weiss the "intrinsic molecular field." Just as liquids can exist when the external pressure is zero, so ferromagnetic bodies can take a finite intensity of magnetization in the absence of exterior fields. If the interior field acted alone the intensity of magnetization would be proportional to it and we would have

$$H_m = NI$$

and equation (43) would become

$$a = \frac{NMI}{RT}$$

or

$$I = \frac{aRT}{MN} \quad \ldots\ldots\ldots\ldots (44)$$

where N is the factor of proportionality.

This equation is represented in Fig. 41 by the straight line OA while equation (41) is represented by the curved line OBA. The intensity of magnetization being satisfied by equations (41) and (44), the points of intersection of the curve and the straight line give the values of I. One solution of these equations is

$$I = 0$$
$$a = 0$$

from which it follows that

$$H = 0$$

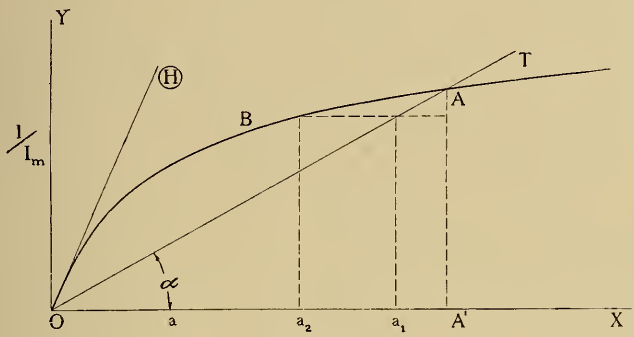


Fig. 41

It can be shown, however, that only the point A represents a state of stable equilibrium of magnetization. For suppose that we were able to decrease the intensity of magnetization I directly. For the same value of I and T, the value of a given by the straight line (namely a_1) is larger than a_2 given by the curve (See Fig. 41). a_1 is the value due to the molecular field which is much stronger than the external field whose action is represented by the curve and by a_2. Now a decrease in the intensity of magnetization means that fewer elementary magnets have their magnetic axes pointing in the same direction. Referring to Fig. 20, it will be seen that if the molecular field or the field due to the action of one elementary magnet on another, is stronger than the external field H, equilibrium will be established only when the magnets have oriented themselves into a position such that the components of the moments of the two fields are the same, that is, when the intensity of magnetization has risen to the point A. On the other hand the point O represents a value at which the intensity of magnetization is zero, that is, where as many elementary magnets point in the direction of easy magnetization as in the opposite direction. A slight mechanical or magnetic disturbance will cause half of the elementary magnets to swing around into the direction of easy magnetization where they remain in stable equilibrium. Thus the point O corresponds to unstable magnetization and the point A to stable magnetization.

As the paramagnetic susceptibility is very small enormous magnetic fields would be required to increase this spontaneous magnetization

which is due to the external field. Assuming the mechanical analogy of Weiss in which the increase in the density of a liquid requires external pressures that are incomparably greater than those by means of which the density of gas is changed, we arrive at the conclusion that for the absolute temperature T, the ordinate $A'A$ represents the saturation value of the intensity of magnetization. According to this a ferromagnetic substance is saturated without the least external field. Though this inference, drawn from the theory, seems in contradiction to the larger number of experimental facts observed, yet it agrees perfectly with the phenomena of magnetization observed in crystals, and especially in those of normal pyrrhotite. This crystal has the very valuable property of a magnetic plane in which all the elementary magnets are situated. In this magnetic plane is a direction in which saturation takes place with very little or no external field, while a field of about 7,300 units is required to bring about saturation in a perpendicular direction. The assumption of a molecular field accounts very satisfactorily for the laws that govern the magnetization of the normal pyrrhotite as a function of the external field. The intensity of magnetization as a function of the temperature is a very complicated phenomenon, varying from one substance to another, and also varying in the same substance with the magnetic field; therefore it appears doubtful whether the most simple hypothesis of the uniform molecular magnetic field will be able to account for all the observed phenomena. In the case of magnetite, however, Weiss has shown that the theoretical curve coincides with the experimental curve between the temperatures — 79° C. and + 587° C. On the assumption that a piece of ordinary iron is composed of small crystals having the property of a magnetic plane, Weiss has also shown that the hysteresis loops of annealed iron can be given a theoretical interpretation.

In order to determine the absolute values of the internal magnetic field we have to examine the magnetic properties of the ferromagnetic substances in the neighborhood of the point where the spontaneous magnetization disappears. Thus iron loses its spontaneous magnetization at the temperature of 756° C. Between this point and 920° C. iron has still a considerable susceptibility, the magnetism, however, appearing only under the combined action of the external and internal fields. In this region we have

$$a=\frac{M(H_e+NI)}{RT} \qquad \text{(45)}$$

or

$$T=\frac{M(H_e+NI)}{aR} \qquad \text{(46)}$$

where M is the resulting magnetic moment of each molecule and H_e the external field. Equation (29), page 21, gives

$$\frac{I}{I_m}=\frac{\cosh a}{\sinh a}-\frac{1}{a}=\frac{1}{3}a-\frac{2}{90}a^3+\frac{4}{45.42}a^5-\dots$$

As long as we consider only the beginning of the curve OB of Fig. 41 whose tangent at the origin corresponds to the temperature $\Theta = 756 +$

$273=1029$, we may consider only the first term on the right-hand side of the last equation. Then

$$I/I_m=a/3 \qquad (47)$$

Up to the temperature Θ we have spontaneous ferromagnetism where the external field H_e is negligible in comparison with NI the internal magnetic field, so that in equation (45), we may write

$$a=\frac{MNI_m a}{3R\Theta}$$

whence

$$\Theta=\frac{MNI_m}{3R} \qquad (48)$$

Dividing (46) by (48)

$$\frac{T}{\Theta}=\frac{3H_e}{aNI_m}+\frac{3I}{aI_m}$$

or from (47)

$$\frac{T}{\Theta}=\frac{3H_e}{a\,NI_m}+1$$

Since

$$I_m=\frac{3}{a}I$$

this equation reduces to

$$\frac{T-\Theta}{\Theta}=\frac{H_e}{NI}$$

or

$$(T-\Theta)\,I=\frac{\Theta H_e}{N} \qquad (49)$$

This equation represents an equilateral hyperbola and allows us to determine the coefficient N. Weiss found for

iron	$N = 3{,}850$	$H_m = 6{,}560{,}000$
nickel	$N = 12{,}700$	$H_m = 6{,}350{,}000$
magnetite	$N = 33{,}200$	$H_m = 14{,}300{,}000$

If the molecular magnets act upon one another with magnetic forces of this enormous amount, the potential energy due to the molecular magnetic field must have very large values.

29. *Specific Heat and Molecular Field of Ferromagnetic Substances.* — The mutual energy of a number of magnets of invariable magnetic moment M is

$$W=-\tfrac{1}{2}\Sigma MH\cos\alpha$$

H being the field in which is placed one of the elementary magnets and which is due to all the other magnets and α being the angle between H and M. When this summation is extended to all the elementary magnets contained in 1 cc. H becomes the molecular field H_m and the intensity of magnetization I is the geometric sum of the magnetic moments M. The energy of magnetization per unit of volume is then

$$W=-\tfrac{1}{2}IH_m.$$

The molecular field is related to the intensity of magnetization I by

$$H_m = NI$$

where N is a constant coefficient. Therefore

$$W = -\tfrac{1}{2} NI^2.$$

Since this energy is negative it is necessary to add energy in order to demagnetize. Thus the intensity of magnetization decreases continuously as the temperature increases from absolute zero to the temperature Θ of the disappearance of spontaneous ferromagnetism. The total quantity of heat absorbed by the magnetic phenomena per unit of mass of the body, between the temperature where the intensity of magnetization is I and the temperature Θ is

$$q_m = \tfrac{1}{2} \frac{NI^2}{JD}$$

where J is the mechanical equivalent of the calorie and D is the density.

The specific heat due to change in the intensity of magnetization is then

$$s_m = \frac{dq_m}{dt} = \frac{1}{2} \frac{N}{JD} \frac{\partial I^2}{\partial t}$$

which must be added to the ordinary specific heat. According to Curie's experimental results I, the intensity of magnetization for iron at ordinary temperatures, is equal to 1700 and

$$N = 3850.$$

Therefore

$$H_m = NI = 6{,}540{,}000.$$

The energy of diamagnetization per unit mass is

$$W = \tfrac{1}{2} NI^2 = 70.6 \times 10^7 \text{ ergs.}$$

At 20° C.

$$q_m = \frac{70.6 \times 10^7}{4.19 \times 10^7} = 16.8 \text{ calories.}$$

The following results for iron were obtained from Curie's experimental data.

t°C.	*I/D C. G. S.*	q_m *cal.*	$s_m = \frac{\Delta q_m}{\Delta t}$	*In the Interval*
20	216.3	16.8		
275	207.5	15.5	.005	20° – 275°
477	189.6	12.9	.013	275° – 477°
601	164.0	9.7	.027	477° – 601°
688	127.	5.8	.045	601° – 688°
720	100.7	3.6	.068	688° – 720°
740	64.	1.5	.108	720° – 740°
744.6	50.1	.9	.136	740° – 744.6°
753	0. (by extrapolation)			

From this we see that at ordinary temperatures the specific heat is altered, due to the magnetic phenomena, by only a small amount, while in the neighborhood of Θ, the temperature at which spontaneous magnetization disappears, the effect amounts to 0.136, or about two fifths of the total value.

The results of Weiss and Beck * show a very interesting relation between the variations of the ordinary specific heat and

$$s_m = \frac{\Delta q_m}{\Delta t}$$

as functions of the temperature. Their results are represented graphically in Fig. 42, curve *A* representing the relation between ordinary specific heat and temperature and curve *B* that between s_m, the specific heat due to magnetization, and temperature. The same close agreement has been found in the ferromagnetic substances, nickel and magnetite.

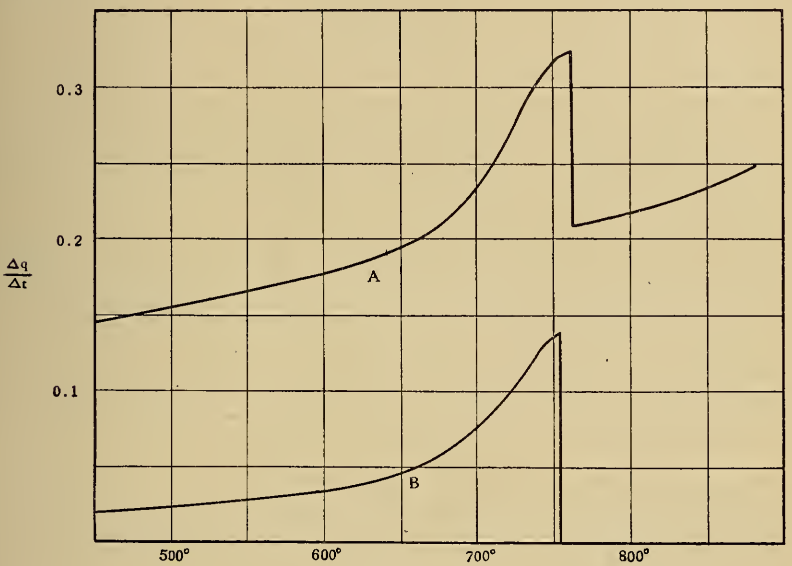


Fig. 42

30. *The Elementary Magnets of Iron, Nickel, and Magnetite.* — The electron theory of ferromagnetism gives us a means of determining the moment *M* of the molecular magnets of those substances whose internal magnetic field has been determined. Various methods may be applied for this purpose, but the one used by J. Kunz† is probably the most simple. In his method Kunz makes use of the equation

$$a = \frac{MNI}{RT} \qquad (50)$$

where *N* is a constant. For the absolute temperature Θ, the temperature at which the spontaneous ferromagnetism disappears, we have the relation

$$I/I_m = a/3 \qquad (51)$$

* Weiss and Beck, Jour. de Phys., ser. 4, Vol. 7, p. 249, 1908.
† J. Kunz, Phys. Rev., Vol. XXX, No. 3, March, 1910.

Substituting (51) in (50), we find

$$a=\frac{MNI_m a}{3\,RT}$$

or

$$M=\frac{3\,R\,\Theta}{NI_m} \qquad (52)$$

where Θ is the particular temperature considered. R is given by the equation of gases

$$p=RN_1T$$

where

$$p=10.1\times 10^6$$
$$T=273$$
$$N_1=2.7\times 10^{19}$$

Therefore

$$R=1.36\times 10^{-16}.$$

I_m, the saturation value of the intensity of magnetization at the absolute zero of temperature, has to be determined from the above theory by means of the value I, the intensity of magnetization for the case of saturation at the temperature $t°$.

In the case of iron, we have from Curie's results, $I = 1700$ for ordinary temperatures and a field strength of 1300 units, and $I_m = 1950$. Taking $N = 3850$, the value found by Weiss and Beck,* and $\Theta = 756 + 273 = 1029$, we find, by substitution in equation (50) $M = 4.445 \times 10^{-20}$ absolute electromagnetic units.

Let N_1 be the number of molecular magnets in unit volume at the absolute zero. Then we have

$$N_1M = I_m = 1950$$

or

$$N_1=4.386\times 10^{22}.$$

If this number N_1 of elementary magnets is at the same time the number of molecules of iron, and if the mass of one molecule of iron is equal to μ, we have

$$N_1\mu=\delta=7.36$$

where δ is the density of iron at the absolute zero.

$$\mu=\delta/N_1=1.792\times 10^{-22}\text{ grams}$$

Let us assume that the molecule of iron consists of two atoms, then it will be 111.8 times heavier than the atom of hydrogen, and the mass of the atom of hydrogen μ_H will be equal to

$$\mu_H=\frac{1.792\times 10^{-22}}{111.8}=1.603\times 10^{-24}\text{ grams}$$

Du Bois and Taylor Jones† found the intensity of magnetization of iron continues to increase up to field strengths of 1500 units. At this value they found the value of I for ordinary temperatures to be 1850. Substituting this value instead of 1700, the value obtained by Curie, we have

$$\mu_H=1.66\times 10^{-24}\text{ grams}$$

* Weiss and Beck, Jour. de Phys., Vol. 7, pp. 249, 1908.

† Du Bois and Taylor Jones, Elektrot. Zeitschr., Vol. 17, p. 544, 1896.

A recent value of this quantity has been determined by Rutherford by means of radioactive phenomena. He found

$$\mu_H = 1.61 \times 10^{-24} \text{ grams}$$

The close check of the values for the mass of one atom of hydrogen deduced from so very different phenomena as those of radioactivity and ferromagnetism by considerations entirely different, is strong evidence in favor of the theories involved. It follows from the above that the molecular magnet of iron contains two atoms.

In the case of magnetite, Fe_3O_4, we find the following numbers:

$$\begin{aligned} N &= 33{,}200 \\ I &= 430 \\ I_m &= 490 \\ \delta &= 5.2 \end{aligned}$$

According to Curie

$$\Theta = 536 + 273 = 809.$$

Using these values we find

$$M = 2.02 \times 10^{-20}.$$

Now

$$N_1 M = I_m = 490$$

therefore

$$N_1 = 2.42 \times 10^{22}$$

If the mass of each molecule is equal to μ we have

$$N_1 \mu = 5.2$$

whence

$$\mu = 2.15 \times 10^{-22}.$$

Now

$$Fe_3O_4 = 231.7$$

therefore

$$\mu_H = \frac{2.15 \times 10^{-22}}{231.7} = 0.93 \times 10^{-24} \text{ grams,}$$

a value that agrees pretty well with that obtained from other sources when we take into consideration the comparatively small amount of work that has been done on magnetite.

According to Weiss

$$\Theta = 589 + 273 = 862,$$

instead of 809 as given above. Substituting this value for Θ we find

$$\mu_H = 0.99 \times 10^{-24} \text{ grams.}$$

In the case of nickel we have the following values:

$$\begin{aligned} N &= 12{,}700 \\ I &= 500 \\ I_m &= 570 \\ \Theta &= 376 + 273 = 649 \\ M &= 3.65 \times 10^{-20} \\ N_1 &= 1.56 \times 10^{22} \\ N_1 \mu &= \delta = 8.8 \\ \mu &= 5.63 \times 10^{-22} \end{aligned}$$

The atomic weight of nickel is 5.87, hence, assuming that each molecular magnet contains two atoms, we find

$$\mu_H = \frac{5.63 \times 10^{-22}}{117.4} = 4.8 \times 10^{-24},$$

a value that is just three times larger than that given by Rutherford. As the degree of accuracy is the same in the case of iron, nickel, and magnetite, the experimental evidence indicates that the molecular magnet of nickel is made up of six atoms, or that the number of degrees of freedom is only one third as great as in the case of iron. In a recent investigation, Stifler* has determined the above quantities for cobalt. He obtained the following values:

$$\begin{aligned} N &= 6{,}180 \\ I &= 1{,}310 \\ I_m &= 1{,}435 \\ \Theta &= 1{,}348 \\ M &= 6.21 \times 10^{-20} \\ N_1 &= 2.31 \times 10^{22} \\ N_1\mu &= \delta = 8.77 \\ \mu &= 3.8 \times 10^{-22} \end{aligned}$$

Since the atomic weight of cobalt is 59, we obtain, on the assumption that each molecular magnet contains two atoms,

$$\mu_H = \frac{3.8 \times 10^{-22}}{118} = 3.22 \times 10^{-24}$$

a value that is just two times larger than Rutherford's value. If we apply the above reasoning we must conclude that the elementary magnet of cobalt is made up of four atoms.

The quantities considered above are given in the following table: n is the number of atoms corresponding to one elementary magnet.

Substance	$I_t = 20°$	I_m	$\Theta°C.$	N	$NI = H_m$	$M \times 10^{20}$	$\mu_H \times 10^4$	n
Fe	1,700	1,950	756	3,850	6,540,000	4.445	1.603	2
Fe_3O_4	430	490	536	33,200	14,300,000	2.02	0.93	
Ni	500	570	376	12,700	6,350,000	3.65	4.8	6
Co	1,421	1,435	1,075	6,180	8,870,000	6.21	3.22	4

As the ratio of the density of nickel and iron, 1.12, is nearly equal to the ratio of the atomic weights, 1.05, of the two metals, the number of molecules per unit volume must be the same for both metals, assuming that each molecule contains two atoms. Since the moment of the molecular magnets of nickel is only about 18 per cent smaller than that of iron, we should expect that the intensity of magnetization of nickel would vary by about this amount from that of iron, while in reality the magnetization of iron is 3.4 times greater than that of nickel. This consideration indicates again, that either the molecular magnet of nickel contains six atoms or that only every third molecule is an elementary magnet.

* W. W. Stifler, Phys. Rev., Vol. XXXIII, No. 4, p. 268, 1911.

This fundamental difference in the molecular magnets of iron and nickel must be taken into account when explaining some of the very interesting differences in the magnetic behavior of the two metals. Thus, the first layer of electrolytically deposited nickel is stronger magnetically than the subsequent layers, while for iron the opposite is true, that is, thin layers of iron are much less magnetic than thicker layers. In addition, a longitudinal compression decreases the magnetization of iron and increases that of nickel. In a recent article the author has shown * that the effect of transverse joints in nickel bars is to increase the magnetic induction rather than decrease it as in the case of iron.

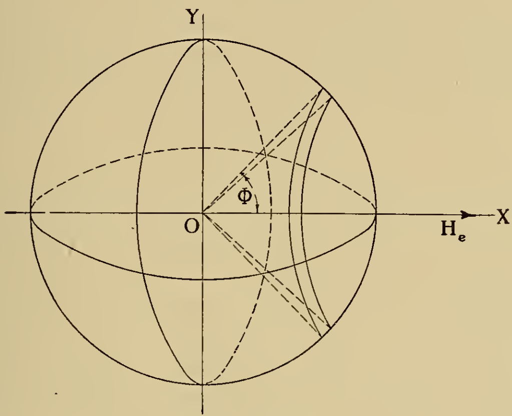


Fig. 43

31. *The Hysteresis Loop of Iron.*— Let us assume that the elementary crystal of iron has properties analogous to those of the crystal of pyrrhotite and that the direction of easy magnetization is distributed uniformly throughout the volume. Working with weak fields let us first consider only the irreversible phenomena. When the substance is in the neutral state, the magnetization vectors of the different elementary crystals will terminate on the surface of a sphere with uniform density. If the field H acting in the direction Ox, Fig. 43, exceeds the coercive field H_c, all the elementary magnets which were originally directed in the negative direction will swing round so that all the intensity of magnetization vectors will be contained in a cone having OH for its axis and of semiangle ϕ which is given by

$$\cos\phi = H_c/H.$$

Each of the elementary magnets that swings around will contribute its moment M_1 to the resulting intensity of magnetization in the direction x. Now the number of vectors ending on the sphere is equal to N, the number of elementary crystals with a plane of magnetization,

* E. H. Williams, Phys. Rev., Vol. XXXIII, No. 1, p. 59, 1911.

and the number ending on the zone subtended by the angle ϕ before the field is applied will be

$$\frac{2\pi r \sin\phi \, d\phi}{4\pi r} N = N/2 \sin\phi \, d\phi$$

The moment, due to these magnets, in the direction Ox is

$$M_x = M_1 \cos\phi \frac{N}{2} \sin\phi \, d\phi.$$

The moment due to all the magnets that swing round into the direction Ox is

$$M = \int_0^\phi \frac{M_1 N}{2} \sin\phi \cos\phi \, d\phi.$$

$$= \frac{M_1 N}{4} \sin^2\phi$$

$$= \frac{I_m}{4} \sin^2\phi$$

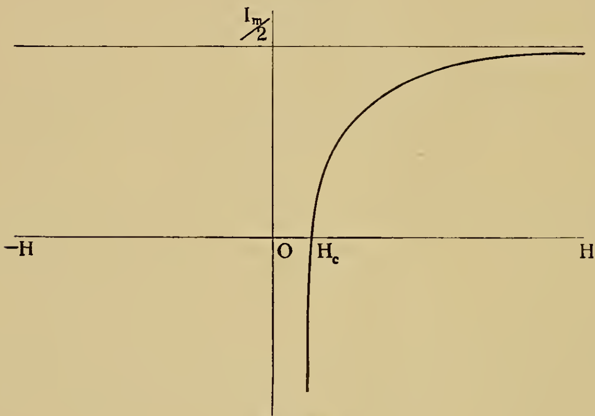


FIG. 44

since the number originally in the positive direction is equal to the number that have been turned around, the resulting moment in the direction Ox will be

$$M = 2 \frac{I_m}{4} \sin^2\phi$$

$$= \frac{I_m}{2} \sin^2\phi$$

Now

$$\cos\phi = H_c/H$$

therefore

$$M = \frac{I_m}{2} [1 - (H_c/H)^2] \qquad (53)$$

where M is the resultant magnetic moment per unit volume, that is, the intensity of magnetization I.

The graphic representation of the equation

$$I = I_m/2\left[1-(H_c/H)^2\right] \qquad (54)$$

is given by Fig. 44. This resembles an hyperbola whose horizontal asymptote is given by $I = \frac{I_m}{2}$ and whose vertical asymptote is given by $H = 0$. If H is equal to H_c the intensity of magnetization $I = 0$ and if $H = \infty$, $I = I_m/2$. But physically this must be the value of saturation I_m therefore we shall write

$$I = I_m\left[1-(H_c/H)^2\right] \qquad (55)$$

If we were to draw the curve corresponding to this equation, we should find a curve of exactly the same character as the previous one except that for a given value of H the ordinate I would be twice as large as before.

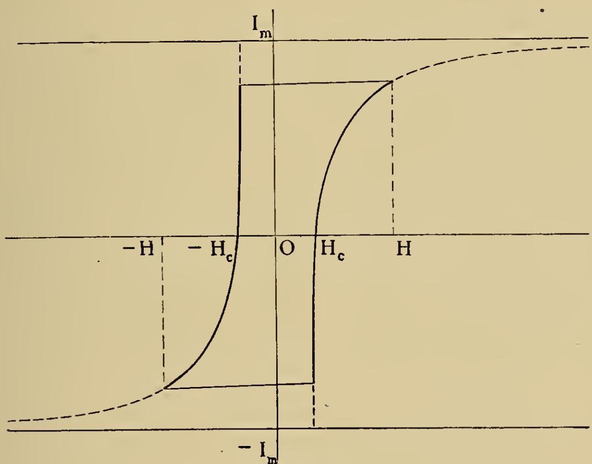


Fig. 45

If one causes the field to oscillate between the value $+H$ and $-$ H, the graphical representation of the curve of equation (55) will be given by Fig. 45. When the field $-H$ is applied, the figurative points will be collected on the negative side of the sphere of Fig. 43. With the sample in this condition let us begin the description of a cycle, varying the field from $-H$ to increasing positive values. The intensity of magnetization will change very little so long as the field is less than $+H_c$. At this point it begins to change very rapidly and will describe a curve similar to the curves considered above. This curve with the portion of the straight line already described will constitute half of a cycle corresponding to a variation of the field from $-H$ to $+$H. The cycle is completed, from symmetry, by returning to the origin.

Equation (55) assumes that the coefficients N_2 and N_3 of Fig. 39 are zero. Now, in the case of iron, this is only approximately true. Making these corrections, the theoretical considerations give cycles which are shown in Fig. 46. The scale has been chosen so as to repro duce as nearly as possible the experimental curves of Fig. 47, which are taken from the results of Ewing.* The similarity of the ascending and descending curves, more particularly the outer ones, is very marked.

The principal differences to be noted between the experimental and theoretical curves are, first, that the upper limits of the cycles for medium fields fall more nearly on the outer cycle in the experimental than in the theoretical curves, and second, that for fields but slightly greater than H_0 the theoretical curves are rectangular in shape while the experimental curves are not.

32. *Exceptions to the Electron Theory.* — While the electron theory is capable of explaining many of the phenomena of magnetism, yet in its present form and present stage of development it is unable to account for a large number of cases.

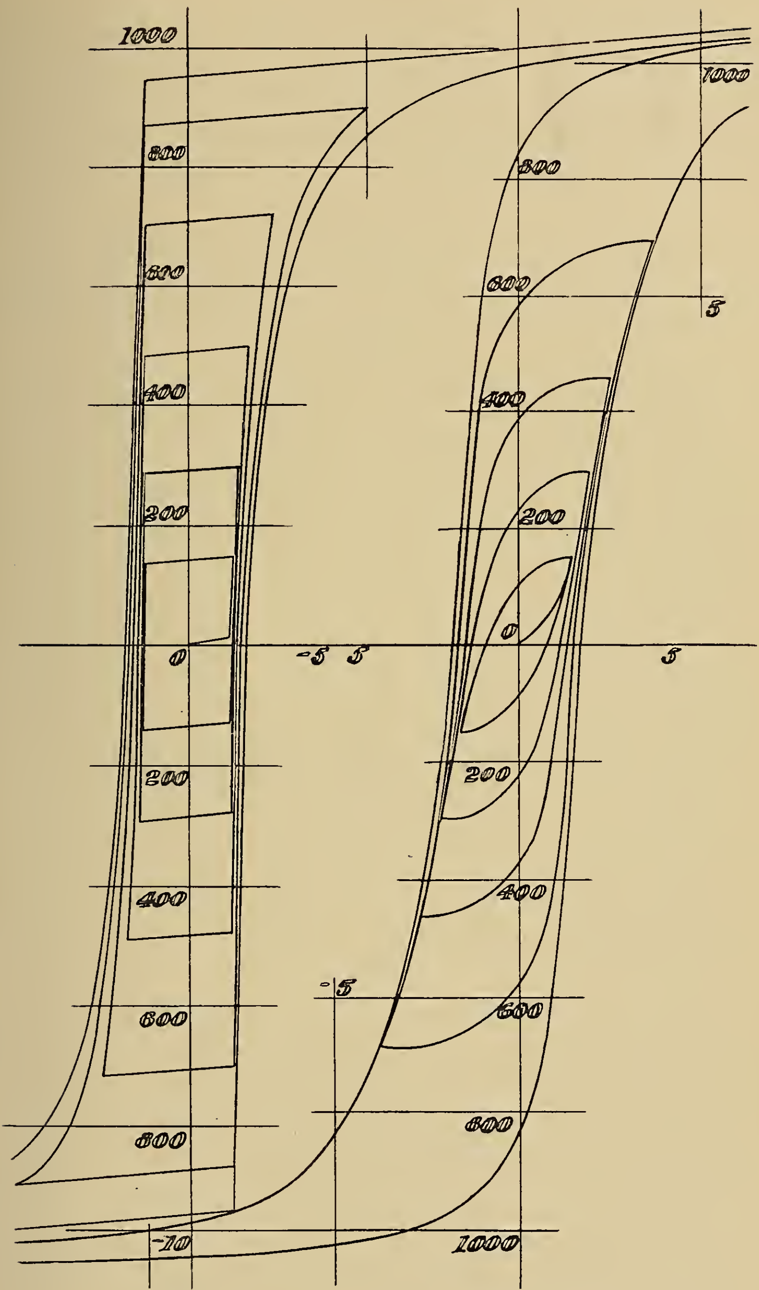
Curie's rules, which are the basis of the present theory, hold rigidly for very few substances. Thus, according to these rules the diamagnetic susceptibility is independent of the temperature. However, there are substances whose diamagnetic susceptibility increases with increase of temperature, while in other substances the opposite is the case. Another of Curie's rules states that, for paramagnetic substances, the susceptibility is inversely proportional to the absolute temperature. While this holds for a very large number of substances, there are cases where the rule fails to represent the facts as determined by experiment. Recently H. Kamerling Onnes and A. Perrier† have shown that for several substances the law does not hold for very low temperatures. Some substances at temperatures below those at which Curie's c/T law is obeyed follow more nearly a $\frac{c}{\sqrt{T}}$ law. None of the salts investigated by the above authors show signs of saturation phenomenon. Pyrrhotite, whose magnetic properties conform to the electron theory very closely up to Θ, the temperature of transformation, is very abnormal above this temperature.

From his experimental results upon a limited number of substances, Curie comes to the conclusion that the paramagnetic susceptibility is independent of the state of aggregation of chemical combination of elements. Now oxygen and boron are paramagnetic, oxygen strongly so, yet the oxide of boron is diamagnetic. Likewise Al_2O_3, M_gO, M_oO, ThO, UrO and other oxides are diamagnetic.

The law of approach of the intensity of magnetization to the constant value of saturation holds only for cobalt and not for iron and nickel.

* Ewing, Magnetic Induction, 3rd. Ed., p. 106, Fig. 50.

† H. Kamerling Onnes and A. Perrier, Konink. Akad. Wetensch., Amsterdam Proc. 14, p. 115, 1911.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200
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-5
600
600
1000

FIG. 46

FIG. 47

The large number of exceptions to the electron theory in its present form requires either that it be abandoned or that the theory be modified to fit more exactly experimental results. The fact that it agrees in such a large number of cases with experiment and that, by its application, the fundamental quantities of nature can be obtained in such close agreement with observation, gives hope that ultimately the present theory will be modified so that it will hold universally.

V. Bibliography.

The following bibliography is intended to give only those works and results which have contributed most toward the advancement of the electron theory of magnetism to its present state of development.

Experimental.

1. P. Weiss. The Magnetization Plane of Pyrrhotite, Jour. de Phys., Ser. 3, Vol. 8, p. 542, 1899.
2. P. Curie. Magnetic Properties of Bodies at Various Temperatures, Ann. de Chem., Ser. 7, Vol. 5, p. 289, 1895.
3. P. Weiss. The Magnetic Properties of Pyrrhotite, Jour. de Phys., Ser. 4, Vol. 4, p. 469, 1905. Jour. de Phys., Ser. 4, Vol. 4, p. 829, 1905.
4. Weiss & Kunz. The Thermal Variation of the Magnetization of Pyrrhotite, Jour. de Phys., Ser. 4, Vol. 4, p. 847, 1905.
5. J. Kunz. The Magnetic Properties of Hematite, Archives des Sciences, Vol. 23, p. 137, 1907.
6. Weiss & Planer. Hysteresis of the Rotating Field, Jour. de Phys., Ser. 4, Vol. 7, p. 5, 1908.
7. Weiss & Beck. Specific Heat and Molecular Field of Ferromagnetic Substances, Jour. de Phys., Ser. 4, Vol. 7, p. 249, 1908.
8. V. Quittner. The Magnetic Properties of Magnetite, Archives des Sciences, Ser. 4, Vol. 26, p. 358, 1908.
9. Du Bois & Honda. The Thermomagnetic Properties of Elements, Konink. Akad. Wetensch., Amsterdam, Proc. 12, p. 596, 1910.
10. Weiss & Foex. The Magnetization of Ferromagnetic Bodies, Archives des Sciences, Ser. 4, Vol. 31, p. 5, 1911. Archives des Sciences, Ser. 4, Vol. 31, p. 89, 1911.
11. W. W. Stifler. The Magnetization of Cobalt as a Function of the Temperature and the Determination of its Intrinsic Magnetic Field, Phys. Rev., Vol. 33, No. 4, p. 268, 1911.
12. P. Weiss. On a New Property of the Magnetic Molecule, Comptes Rendus, Vol. 152, p. 79, p. 187, p. 367, and p. 688, 1911.
13. Weiss & Block. On the Magnetization of Nickel, Cobalt, and the Alloys of Nickel and Cobalt, Comptes Rendus, Vol. 153, p. 941, 1911.

Theoretical.

1. W. Voigt. Electron Hypothesis and Theory of Magnetism, Ann. de Phys., Vol. 9, p. 115, 1902.
2. J. J. Thomson. The Magnetic Properties of Systems of Corpuscles Describing Circular Orbits, Phil. Mag., Ser. 6, Vol. 6, p. 673, 1903.
3. P. Langevin. Magnetization and the Electron Theory, Ann. de Chem. et de Phys., Ser. 8, Vol. 5, p. 70, 1905.
4. E. E. Fournier. The Electron Theory, Ch. VIII (1906 ed.).
5. N. R. Campbell. Modern Electrical Theory, Ch. IV (1907 ed.).
6. P. Weiss. The Hypothesis of the Molecular Field and the Ferromagnetic Property, Jour. de Phys., Ser. 4, Vol. 6, p. 661, 1907.
7. J. Kunz. The Absolute Values of the Moments of the Elementary Magnets of Iron, Nickel and Magnetite, Phys. Rev., Vol. 30, No. 3, p. 359, 1910.
8. P. Weiss. On the Rationality of the Ratios of the Molecular Magnetic Moments and the Magneton, Archives des Sciences, Ser. 4, Vol. 31, p. 401, 1911.

**Bulletin No. 1.* Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

**Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

**Bulletin No. 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

**Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

**Circular No. 3.* Fuel Tests with Illinois Coal. (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904–1907,) by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

**Bulletin No. 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

**Bulletin No. 4.* Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

**Bulletin No. 5.* Resistance of Tubes to Collapse, by Albert P. Carman. 1906. *Fifteen cents.*

**Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906 *Thirty-five cents*

**Bulletin No. 7.* Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *Thirty-five cents.*

**Bulletin No. 8.* Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

**Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

**Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No. 11.* The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

**Bulletin No. 12.* Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No. 13.* An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

**Bulletin No. 14.* Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No. 15.* How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

**Bulletin No. 16.* A study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

**Bulletin No. 17.* The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *Twenty cents.*

**Bulletin No. 18.* The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

**Bulletin No. 19.* Comparative Tests of Carbon, Metallized Carbon, and Tantalum Filament Lamps, by T. H. Amrine. 1908. *Twenty-five cents.*

**Bulletin No. 20.* Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

**Bulletin No. 21.* Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

**Bulletin No. 22.* Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. *1908. Thirty-five cents.*

**Bulletin No. 23.* Voids, Settlement and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

Bulletin No. 24. The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

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ENTROPY-TEMPERATURE AND TRANSMISSION DIAGRAMS FOR AIR

BY

C. R. RICHARDS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 63 JANUARY, 1913

ENTROPY-TEMPERATURE AND TRANSMISSION DIAGRAMS FOR AIR

By C. R. Richards, Professor of Mechanical Engineering

CONTENTS

ENTROPY-TEMPERATURE AND TRANSMISSION DIAGRAMS FOR AIR

I. Introduction

1. *Preliminary.*—In engineering calculations graphical methods not infrequently afford a satisfactory degree of accuracy, or when extreme precision is required, a check on analytical methods which prevents the possibility of arithmetical errors. Among the thermodynamical calculations which need to be frequently made, those dealing with the compression, expansion, and transmission of air and other gases are tedious and liable to error.

It is the purpose of this bulletin to present the theory and use of three graphical charts through whose aid all problems pertaining to compressed air may be quickly solved with a minimum of labor and with a degree of accuracy which is entirely satisfactory in engineering work. As with all graphical and mechanical aids, practice in the use of the charts is necessary to a full realization of their convenience and reliability.

2. *Acknowledgment.*—The original entropy-temperature diagram for air, Fig. 1, was designed by the writer several years ago. Recently, Mr. John A. Dent, Instructor in Mechanical Engineering, simplified the construction of this chart and improved it by the addition of a series of lines representing the curves of $pv^n = C$, as shown by Fig. 2. Fig. 5 is a diagram for the solution of problems on the flow of air in pipes, designed at the writer's suggestion by one of his former students, Mr. Walter J. Wohlenberg, and recomputed and redrawn for presentation here.

Construction of the Entropy-Temperature Diagrams

3. *Values of dQ and dφ.*—As a result of the application of the fundamental principles of thermodynamics, there may be developed three general equations for the change of heat, dQ, for perfect gases. Of these, two are conveniently used in calculating the entropy change, thus

$$dQ = c_v dt + (c_p - c_v)\frac{T}{v}\,dv \qquad (1)$$

$$dQ = c_p dt - (c_p - c_v)\frac{T}{p}\,dp \qquad (2)$$

Since the change of entropy ($d\phi$) is represented by $\frac{dQ}{T}$, the change of entropy for perfect gases is found by dividing equations (1) and (2) by the absolute temperature, T.

Hence $$d\phi = \frac{dQ}{T} = c_v \frac{dt}{T} + (c_p - c_v) \frac{dv}{v} \qquad (3)$$

and $$d\phi = \frac{dQ}{T} = c_p \frac{d}{T} - (c_p - c_v) \frac{dp}{p} \qquad (4)$$

4. *Constant Volume Lines.*—Integrating equation (3) between limits of ϕ and ϕ_1, T and T_1, and v and v_1,

$$\phi - \phi_1 = c_v log_e \frac{T}{T_1} + (c_p - c_v) log_e \frac{v}{v_1} \qquad (5)$$

If the volume is constant during the change of entropy, equation (5) reduces to

$$\phi - \phi_1 = c_v log_e \frac{T}{T_1} \qquad (6)$$

In Fig. 1, equation (6) gives a logarithmitic curve, and in Fig. 2, plotting log T as ordinates, the equation gives a straight line. From equation (4) it is evident that successive constant volume lines are parallel on both diagrams. The various lines may be drawn with any arbitrarily assumed zero of entropy.

5. *Constant Pressure Lines.*—From equation (4) the change of entropy between finite limits, becomes

$$\phi - \phi_1 = c_p \log_e \frac{T}{T_1} - (c_p - c_v) \log_e \frac{p}{p_1} \qquad (7)$$

which, at constant pressure, becomes

$$\phi - \phi_1 = c_p \log_e \frac{T}{T_1} \qquad (8)$$

When plotted, equation (8) becomes a logarithmitic curve in Fig. 1,—whose slope is different from that of the constant volume lines,—and a straight line in Fig. 2.

The proper location of the initially plotted constant pressure line with reference to the previously drawn constant volume lines is determined through the aid of the characteristic equation

$$pv = RT$$

Having drawn the initial constant pressure line, successive lines are parallel thereto as shown by equation (7).

6. *Isothermal and Adiabatic Lines.*—By construction, horizontal lines on both diagrams are lines of constant temperature, or isothermals, and vertical lines are lines of constant entropy, or adiabatics.

7. *pv^n Lines.*—For air the equation of the adiabatic is

$$p_o v_o^k = pv^k = pv^{1.405}$$

and for the isothermal it is

$$p_o v_o = pv$$

In practice expansion or compression lines lie somewhere between these two extremes and the equation of the real curve becomes

$$p_o v_o^n = pv^n$$

where n lies between 1 and 1.405 for air.

Since $\quad pv = RT$

$$pv^n = p_1 v_1^n$$

and $\quad Tv^{n-1} = T_1 v_1^{n-1}$

then $\quad \frac{v}{v_1} = \left(\frac{T_1}{T}\right)^{\frac{1}{n-1}} \quad \ldots\ldots (9)$

Inserting this value of $\frac{v}{v_1}$ in equation (5)

$$\phi - \phi_1 = c_v log_e \frac{T}{T_1} + \frac{c_p - c_v}{n-1} log_e \frac{T}{T_1}$$

$$= c_v \frac{n-k}{n-1} log_e \frac{T}{T_1} \quad \ldots\ldots (10)$$

Assuming an initial condition of pressure and temperature and a value for n, equation (10) may be plotted as a curve in Fig. 1 or a straight line in Fig. 2. A series of such lines with different values of n are shown on Fig. 2. Being more difficult to construct and to use when plotted on Fig. 1, these pv^n lines are omitted on this diagram.

8. *Intrinsic Energy.*—The intrinsic energy of the so-called perfect gases is in the form of sensible heat. Since the intrinsic energy is changed into mechanical energy, during an adiabatic expansion, the total intrinsic energy could be realized during an expansion to infinity. That is, the intrinsic energy in B. t. u. is

$$AE_1 = A\int_{v_1}^{\infty} pdv = A\frac{p_1 v_1}{k-1} = A\frac{RT_1}{k-1} \quad \ldots\ldots (11)$$

where A is the reciprocal of the mechanical equivalent of heat, and is therefore equal to $\frac{1}{778}$. For air, equation (11) becomes

$$AE_1 = 0.16851\ T_1\ B.\ t.\ u. \quad \ldots\ldots (12)$$

or $\quad E_1 = 131.1\ T_1$ foot pounds $\ldots\ldots (13)$

Since for air (or any other perfect gas) the intrinsic energy is thus a function of the absolute temperature, only, its values for the several temperatures have been calculated and indicated on the diagrams. Obviously all curves and figures given are for one pound of air, since the value of R used is based on this unit of weight.

From equations (11) and (12) it is evident that for air and other perfect gases, isothermals are lines of constant intrinsic energy or isodynamics.

9. *Variable Specific Heats.*—In the construction of both entropy-temperature diagrams, the specific heats c_p and c_v are assumed to be constant within the temperature range given. If these specific heats are considered to be variable, and if their values are assumed to be of the form

$$c_p = a + bT$$
$$c_v = a_1 + bT$$

the expressions for the change of entropy become respectively

$$\phi - \phi_1 = a_1 \log_e \frac{T}{T_1} + b(T - T_1) + (c_p - c_v) \log_e \frac{v}{v_1} \dots\dots\dots (14)$$

$$= a \log_e \frac{T}{T_1} + b(T - T_1) - (c_p - c_v) \log_e \frac{p}{p_1} \dots\dots\dots (15)$$

The constant volume, constant pressure, and pv^n lines from the above equations all become curves, whether plotted as in Fig. 1 or Fig. 2. Within the range of temperatures given on the charts, it is probable that the constant and generally used values of c_p and c_v are amply accurate.

III. The Use of the Diagrams

10. *Values of p, v, and T for Isothermal Changes.*—Values of p, v, and T for any condition of one pound of air may be read directly from the diagrams, Fig. 1 or Fig. 2. Since $pv = p_1v_1$ for the isothermal line, the final pressure or volume may be determined by following a constant temperature line through the given initial condition to the given final condition. Thus for example, find the initial and final volumes of one pound of air at 100° Fahr., which, starting at 100 lb. per sq. in. absolute, expands to 14.7 lb. absolute. From the diagram 1, the intersection of the 100 lb. and 100° lines is at 2.08 cu. ft.; and the intersection of the 100° and 14.7 lb. lines is at 14 cu. ft. By calculation the initial volume is 2.07 cu. ft., giving an error of −0.01 cu. ft. or 0.48%; and the final volume is 14.08 cu. ft., giving an error of −0.08 cu. ft. or 0.57%.

11. *Values of p, v, and T for Adiabatic Changes.*—For the adiabatic, the following relations hold:

$$p_1v_1^k = pv^k \dots\dots\dots (16)$$
$$T_1v_1^{k-1} = Tv^{k-1} \dots\dots\dots (17)$$
$$T_1p_1^{\frac{1-k}{k}} = Tp^{\frac{1-k}{k}} \dots\dots\dots (18)$$

These relations are determined graphically by following the vertical adiabatic or isentropic line through the given initial and final conditions. For example, let the initial absolute pressure and volume be respectively 100 lb. per sq. in. and 4 cu. ft., and the final pressure, 14.7 lb.

absolute. From the diagram the 100 lb. and 4 foot lines intersect at 625° Fahr. Following a vertical line through this point, the final temperature is found to be 162° Fahr. and the final volume 15.6 cu. ft. By calculation, the initial temperature is 621° Fahr. showing the error due to the use of the chart to be +4° or 0.37%; the final temperature is 165° Fahr., giving an error of +3° or 0.5%; and the final volume is 15.66 cu. ft., giving an error of +0.06 cu. ft. or 0.38%.

12. *Values of p, v, and T for pv^n Changes.*—If k be replaced by n, the formulas just given hold for changes along the pv^n lines. Fig. 2 permits the graphical relation of the several quantities to be determined in a manner similar to that just described for the adiabatics, except that a line is drawn through the given initial condition, parallel to the line having the desired value of n, and the results read at the final condition.

13. *Heat Changes.*—In general the heat change is expressed by the equation

$$dQ = A(dE + pdv)$$

or

$$Q = A\left(E_1 - E_2 + \int_{v_2}^{v_1} pdv\right) \quad \text{.................... (19)}$$

where Q = the heat added or rejected in $B.\ t.\ u.$

E_1 and E_2 = the initial and final intrinsic energy in foot pounds.

A = the reciprocal of the mechanical equivalent of heat.

$\int_{v_2}^{v_1} pdv$ = the external work in foot pounds done during the change Q.

14. *Graphical Representation of Heat Changes.*—On the diagram, Fig. 1, areas are quantities of heat in $B.\ t.\ u.$ according to the scale of the drawing shown. The total heat change in passing from one condition to another on the diagram, is the area under the line or curve representing the path of the air during the change, to absolute zero. Since the lowest temperature shown is −200° Fahr. (259.5° absolute), the area under the given path to −200° must be increased by the addition of an amount of heat equal to 259.5 times the total change of entropy between the initial and final conditions of the air, to find the value Q.

15. *Values for External Work.*—Since $A(E_1 - E_2)$ can be read directly from the diagram the external work becomes

$$A\int_{v_2}^{v_1} pdv = Q - A(E_1 - E_2) \text{ in } B.\ t.\ u. \quad \text{........................ (20)}$$

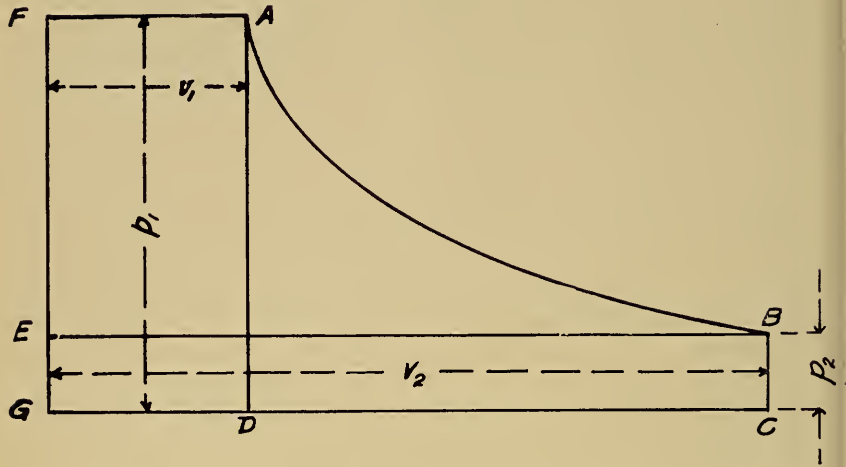

FIG. 3

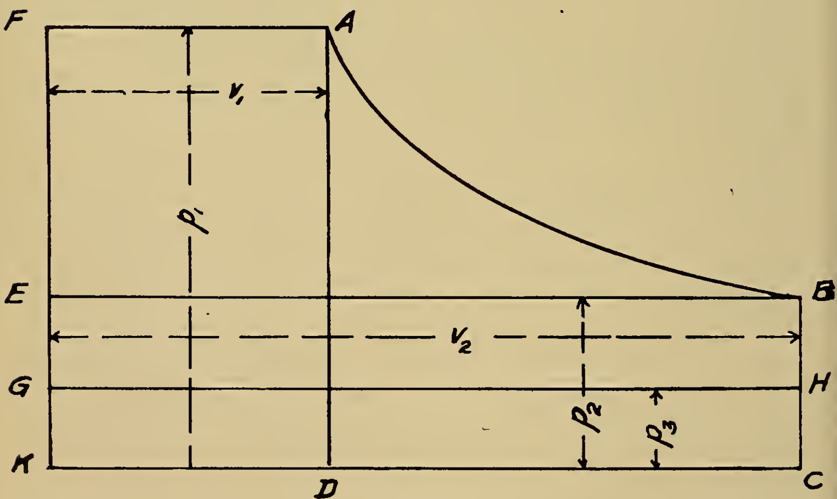

FIG. 4

16. *Expansion or Compression Along Irregular Paths.*—If for any reason air is expanded or compressed along an irregular path, and the changes of pressure and volume are known, the several pressures and volumes (reduced to a basis of one pound weight) may be directly plotted on Fig. 1, and the change in heat determined by mechanical integration of the area under the curve, as described in the preceding article.

The method here presented for finding Q offers no particular advantage, since the work done may be obtained by mechanical integration of the pv diagram, and the change of intrinsic energy, calculated or taken from the diagram, added to the work done to secure the value of Q, as indicated by equation (19).

17. *Isothermal Expansion and Compression.*—The work done in *B. t. u.* during an isothermal change is

$$W = Ap_1v_1 \log_e \frac{v_2}{v_1} = Ap_1v_1 \log_e \frac{p_1}{p_2}$$

$$= ART_1 \log_e \frac{p_1}{p_2} \text{ in } B.\ t.\ u. \qquad (21)$$

In these formulas p_1 and v_1 are the pressure and volume at A in Fig. 3 and 4, and p_2 and v_2 are the pressure and volume at B.

On either Fig. 1 or Fig. 2 the heat change in *B. t. u.* can be obtained for an isothermal expansion or compression by multiplying the absolute temperature (459.5°+ temperature Fahr. on the chart) by the change in entropy between the given initial and final condition of the gas. Inspection of the diagram shows that the change of intrinsic energy is 0 during an isothermal change, hence the heat added or rejected is exactly equal to the external work done, expressed in *B. t. u.*

The work thus obtained is equivalent to that obtained under the curves AB in Fig. 3 and 4, represented by the areas $ABCD$.

The work of a complete cycle of admission, expansion, and exhaust, or of suction, compression, and discharge as shown in Fig. 3 by the area $ABEF$ is exactly equal to the work done under the curve AB (area $ABCD$) since $p_1v_1 = p_2v_2$. In Fig. 4, with incomplete expansion, the area $ABCD$ = area $ABEF$, hence the total work done in *B. t. u.* is equal to area $ABCD$, found from the entropy-temperature diagram, increased by the addition of the area $EBHG$, or $A(p_2 - p_3)v_2$.

It must be remembered that in these formulas pressures are in pounds per square foot and volumes in cubic feet.

To illustrate the application of the diagrams let it be required to find the work done during an expansion of one pound of air from 100 lb. per sq. in. absolute to 14.7 lb. absolute at 100° Fahr. From the diagram the initial and final values for the entropy are 0.2316 and 0.1, or the entropy change is 0.1316. Hence the work in *B. t. u.* is

$$W = 0.1316\ (100+459.5)$$
$$= 73.63$$

By calculation, $W = 73.43$ *B. t. u.*, so the error is $+0.2$ *B. t. u.* or 0.27%

18. *Adiabatic Expansion or Compression.*—The work done during an adiabatic expansion, represented by the areas *ABCD* in Figs. 3 and 4, and expressed in *B. t. u.*, may be found analytically by one of the following formulas:

$$W = \frac{Ap_1v_1}{k-1}\left[1 - \left(\frac{v_1}{v_2}\right)^{k-1}\right] \qquad (22)$$

$$= \frac{Ap_1v_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \qquad (23)$$

$$= \frac{A\ (p_1v_1 - p_2v_2)}{k-1} \qquad (24)$$

$$= AR\ \frac{(T_1 - T_2)}{k-1} \qquad (25)$$

Similarly, the work done during an adiabatic compression, represented by the area *ABCD* in Fig. 3 and expressed in *B. t. u.*, may be found by the use of any one of the following formulas:

$$W = \frac{Ap_2v_2}{k-1}\left[\left(\frac{v_2}{v_1}\right)^{k-1} - 1\right] \qquad (26)$$

$$= \frac{Ap_2v_2}{k-1}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} - 1\right] \qquad (27)$$

$$= \frac{A(p_1v_1 - p_2v_2)}{k-1} \qquad (28)$$

$$= \frac{AR(T_1 - T_2)}{k-1} \qquad (29)$$

Through the aid of the charts, the use of these somewhat troublesome formulas is avoided, since the work in *B. t. u.* is found by determining the difference between the initial and final intrinsic energy along an adiabatic (isentropic) or vertical line through the initial and final states of the gas.

For example, if one pound of air expands from an initial pressure and volume of 100 lb. per sq. in. absolute and 4 cu. ft., respectively, to 14.7 lb. per sq. in. absolute, the change of intrinsic energy (and hence the work) is 78.03 *B. t. u.* Through the aid of equation (23) the work is calculated to be 77.56 *B. t. u.*, giving an error of +0.47 *B. t. u.* or 0.66%.

19. *The Work of the Complete Cycle.*—If the work of a complete cycle of admission, expansion to back pressure, and exhaust; or of suction, compression, and discharge is required for a motor or compressor without clearance as shown by area *ABEF* in Fig. 3, it can be determined analytically as follows:

Area $ABEF$ = Area $ABCD$ + Area $ADGF$ − Area $BCGE$ or, using formulas (24) or (28).

$$Ap_1v_1+\frac{A(p_1v_1-p_2v_2)}{k-1}-Ap_2v_2=\frac{Ak\ \ (p_1v_1-p_2v_2)}{k-1} \quad \ldots\ldots\ldots\ldots (30)$$

That is, the area of the cycle *ABEF* in Fig. 3 is equal to the absolute work done under the curve *AB* multiplied by k, the value of which for air = 1.405. Evidently the change in intrinsic energy multiplied by k gives the net work of the cycle under the conditions here stated.

If a motor does not expand the air to the back pressure line, as in Fig. 4, the area $BHGE = A(P_2-P_3)V_2$ must be added to k times the change of intrinsic energy between A and B to secure the work of the cycle, *ABHGF*.

20. *Expansion or Compression Along a pv^n Line.*—If k be replaced by n in the equations for adiabatic expansion and compression, equations (22) to (30) inclusive, the work done under a pv^n curve is obtained. Taking equation (25) or (29) and replacing k by n the work done becomes, in *B. t. u.*,

$$W=\frac{AR(T_1-T_2)}{n-1}=\frac{k-1}{n-1}\cdot\frac{AR(T_1-T_2)}{k-1} \quad \ldots\ldots\ldots\ldots (31)$$

From this last value for W in equation (31) it is evident that the work done during an adiabatic change between T_1 and T_2 needs only to be multiplied by $\frac{k-1}{n-1}$ to secure the corresponding work done during a pv^n change.

Similarly to the adiabatic cycle, the work of the complete pv^n cycle can be found by multiplying the work done during expansion or compression by n (See Fig. 3).

TABLE 1

Values of n	Values of $\frac{k-1}{n-1}$
1.05	8.10
1.10	4.05
1.15	2.70
1.20	2.025
1.25	1.62
1.28	1.446
1.30	1.35
1.32	1.266
1.35	1.157
1.38	1.066
1.405	1.000

21. *Heat Changes Along a pv^n Line.*—Since there must be a change in the value of Q along a pv^n curve, the heat added or rejected may be determined by the algebraic addition of the change in intrinsic energy between the initial and final conditions and the work done during the change. If the heat change is required without regard to the work done, it may be found as above and reduced to the simplest terms, as follows:

$$Q_{12} = \mp\frac{AE(T_1-T_2)}{k-1} \pm \frac{k-1}{n-1} \cdot \frac{AR(T_1-T_2)}{k-1}$$

$$= \pm\frac{k-n}{n-1} \cdot \frac{AR(T_1-T_2)}{k-1} \qquad \text{(32)}$$

Evidently the heat change is found by multiplying the change of intrinsic energy from the diagram by the factor $\frac{k-n}{n-1}$. Table 2 gives values of this factor.

TABLE 2

Values of n	Values of $\frac{k-n}{n-1}$
1.05	7.10
1.10	3.05
1.15	1.70
1.20	1.025
1.25	0.62
1.28	0.446
1.30	0.35
1.32	0.266
1.35	0.157
1.38	0.066
1.405	0.000

To show the method of handling the pv^n lines on the chart, the following problem is solved:

Given $p_1 = 150$ lb. per sq. in. absolute

$p_2 = 14.7$ lb. per sq. in. absolute

$t_1 = 200°$ Fahr.

to determine the initial and final volumes, the final temperature, the work done during expansion in *B. t. u.*, the heat added in *B. t. u.*, and the work of the complete cycle, all per pound of air, if $pv^{1.2} = C$.

Following a line through the given initial and final conditions, parallel to the $pv^{1.2}$ line it is found from the diagram that $V_1 = 1.613$

cu. ft., and by calculation $V_1=1.625$, an error of -0.012 cu. ft. or 0.7%; from the diagram $V_2=11.48$ cu. ft., and by calculation $V_1=11.25$ cu. ft., an error of $+0.23$ cu. ft. or 2%; from the diagram $T_2=-12°$ Fahr., and by calculation $T_1=11.7°$. The change of intrinsic energy is found to be 35.724 *B. t. u.* from the diagram. From Table 1, for $n=1.2$ the factor for work $=2.025$. Hence $35.724\times 2.025=72.34$ *B. t. u.* is the external work under the expansion curve. By calculation the value is found to be 72.39 *B. t. u.* The work of the cycle is

$$72.34\times 1.2=86.81 \ B.\ t.\ u.$$

while the heat change is

$$+(35.724\times 1.025)=+36.617 \ B.\ t.\ u. \ \text{(See Table 2).}$$

As is evident, the agreement between the values from the chart with those calculated is amply close for all practical purposes.

22. *Flow of Air Through Nozzles.*—It is easily shown that if air flows through a short nozzle the heat energy changed to kinetic energy per pound of air is, in *B. t. u.*,

$$\frac{k}{k-1}.A(p_1v_1-p_2v_2)=A\ \frac{u^2}{2g} \qquad (33)$$

Where $u=$ the velocity of flow in feet per second. The left hand member of the above equation is recognized as the work of the cycle $ABEF$ shown in Fig. 3, the determination of which has been discussed in article 19. Remembering that $A=\frac{1}{778}$ and $2g=64.4$, the value of u can be easily calculated.

It must be remembered that the pressure, p_2, at the point of minimum diameter of the nozzle has a value such that

$$p_2=0.5767\ p_1$$

when the nozzle has a well rounded entrance and the pressure p_1 is more than twice the pressure in the space into which the nozzle is discharging. If the nozzle is properly flared beyond the point of minimum diameter the air will be further expanded with a resultant increase in velocity of flow. The minimum diameter and the relation between p_2 and p_1 determine the actual weight or volume of free air discharged.

The velocity resulting from a given expenditure of heat is plotted graphically on the "Mollier Diagram" accompanying Marks & Davis' "Steam Tables." If, therefore, the heat converted into kinetic energy is found graphically from Figs. 1 or 2 or analytically from equation (33) the graphical relation referred to enables the velocity to be determined without further calculation. In the design of nozzles, these graphical methods save a large amount of time while giving a satisfactory degree of accuracy.

IV. Use of the Charts for Gases Other Than Air

Although the entropy-temperature diagrams here shown are calculated for one pound of air, it is possible to use them for other gases through the aid of corrective factors.

In Table 3 the gases listed each have almost exactly the same value of k, and the values of the constant R vary directly as the relative specific volumes. In consequence, the equation $\frac{pv}{T} = R$ for air must be multiplied by the relative specific volume of another gas. Thus, for example, if the pressure and volume of one pound of hydrogen are the same as on the air diagram, the *absolute* temperature will be 14.46 times as great as the absolute air temperature; or if the pressure and temperature of hydrogen are the same as on the air diagram, the volume will be 14.46 times as great as the indicated volume of air. Further, it is evident that the change of intrinsic energy between two points on the air diagram will be 14.46 times as great for H as for air, since k is the same as for air. Or, stating the case in another way, *the diagrams are exactly accurate for* $\frac{1}{14.46}$ *pounds of hydrogen.* In a similar way the corrective factors may be employed for the other gases given.

Table 3

$k = 1.4+$

Kind of Gas	Relative Density	Relative Specific Volume	Correction Factor
H	1	14.46	14.46
O	16	0.904	0.904
N	14	1.03	1.03
CO	14	1.03	1.03
Air	14.46	1.00	1.00

For gases whose values of k are different from that for air, a correction factor is needed in determining the relations p, v, and T, and another factor for determining the relations between the work done in comparison with air.

The relation between the work done during an adiabatic change between two temperatures for any gas compared with air is

$$\text{Work per pound of gas} = \frac{R'\ (k-1)}{R\ (k'-1)}\ (\text{Work per pound of air})$$

where R' and k' are the values of these constants for the given gas. Table 4 shows the two correction factors for CO_2, CH_4 and C_2H_4.

Mixed gases whose properties are known may be treated in the same manner as for the three gases mentioned. The necessity for using these correction factors renders this application of the entropy-temperature charts of doubtful value.

TABLE 4

Kind of Gas	Value of k	Relative Density $H=1$	Relative Specific Volume	Correction Factor for p, v, and T	Correction Factor for Adiabatic work $\frac{R'(k-1)}{R(k'-1)}$
CH_4	1.267	8	1.808	1.808	2.73
C_2H_4	1.213	14	1.03	1.03	1.96
CO	1.29	22	0.657	0.657	0.919

V. THE AIR FLOW DIAGRAM

The application of the principles of thermodynamics to the flow of air in pipes leads to the following formula connecting the several variables entering into the problem:

$$u_1=\sqrt{\frac{gRTm}{KL}\cdot\frac{p_1^2-p_2^2}{p_1^2}} \qquad (35)$$

where u_1 = the initial velocity of flow in feet per second.

g = the acceleration due to gravity.

T = the absolute temperature, considered constant.

m = the "hydraulic mean depth," or the ratio of the area to the perimeter of the pipe.

K = the coefficient of resistance.

L = the length of the pipe in feet.

p_1 and p_2 = the initial and final pressures in the pipe in pounds per square inch, absolute.

If V = the number of cubic feet of free air at 70° Fahr. and 14.7 pounds pressure flowing per minute.

d = the *actual* diameter of pipe in inches.

$r=\frac{p_2}{p_1}$ = the ratio of the final to the initial pressures.

$m=\frac{d}{4}$ for circular pipes.

then equation (35) reduces to

$$V=3.061\sqrt{\frac{d^5p^2_1\,(1-r^2)}{KL}} \qquad (36)$$

In plotting the chart (Fig. 5) the value of K has been assumed to be

$$K=0.003\left(1+\frac{3.6}{d}\right)$$

and, instead of plotting r direct

$100\left(\frac{p_1-p_2}{p_1}\right)$ has been used as more convenient, since it shows the percentage of pressure drop in the pipe in terms of the initial pressure.

While the formula for V is theoretically correct, its ultimate accuracy depends upon the assumption of a correct value of K, which is not easy considering the paucity of reliable data available.

Should it be desired to use K', another value for K than the one given, the values of V from the chart are to be multiplied by $\sqrt{\frac{K}{K'}}$

Should the discharge from a pipe be required for another temperature than 70° Fahr. (530° absolute) the values of V from the chart are to be multiplied by $\sqrt{\frac{T'}{530}}$, where T' is the absolute temperature desired.

The use of the air flow diagram is simple. If the volume of free air per minute, the length of pipe, the initial pressure, and the permissible drop in pressure are given, proceed as follows:

From the length of pipe in feet, follow a vertical line to its intersection with the volume line; from this point follow a horizontal line to its intersection with the initial pressure line, and from there follow a vertical line until it intersects the horizontal line through the given pressure drop. The position of this last point determines the proper pipe diameter. Any other combination of given data is handled similarly.

This diagram may be adapted to other gases than air (assuming the coefficients of resistance to be the same) by multiplying the volume of air by $\sqrt{\frac{v'}{v}}$ where v^1 and v are respectively the specific volumes of the given gas and of air; or by $\sqrt{\frac{1}{D}}$ where D is the density of the given gas in terms of air.

To illustrate, let $L=2000$ feet, $V=1000$ cu. ft., $p_1=100$ lb., drop $=5\%$. Following a vertical line from $L=2000$ to $V=1000$, a horizontal line to $p=100$, a vertical line to drop $=5\%$, it is found that a 4-inch pipe is the nearest commercial size available, although the drop for this size is about 6%.

Without the aid of the chart, the solution of the problem just given would require considerable time since the pipe diameter can only be obtained from the formula by a series of approximations, and at least three trial solutions would ordinarily be necessary.

VI. Maximum Power Transmitted Through Pipes

The maximum power which may be secured per pound of air is realized during a complete cycle when the expansion is complete to the back pressure line, that is to atmospheric pressure, as shown by Fig. 3.

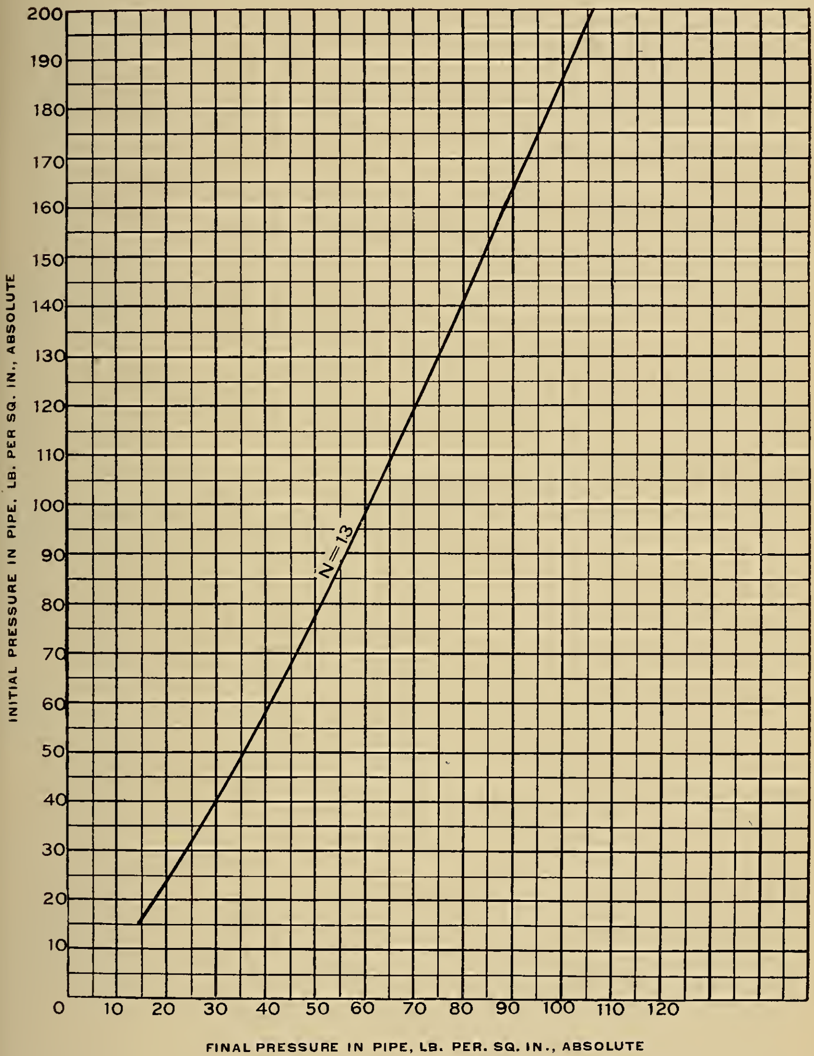

FIG. 6. MAXIMUM POWER TRANSMITTED BY AIR, $N = 1.3$

When air is transmitted through pipes there is a loss of pressure due to friction, but, in general, there will be no loss of temperature; hence the product of the pressure and volume remains constant. Because of this fact, the loss of *power* is less than the loss of *pressure*, assuming that the air may be effectively used at the reduced pressure. Obviously, the power developed per cycle is a function of the value of n for the expansion curve, so its probable value is necessarily known in determining the actual efficiency of transmission.

Let p_1 = the absolute initial pressure in the pipe.
p_2 = the absolute final pressure in the pipe.
p_a = the atmospheric pressure.

The work developed per cycle with no loss of pressure, is, in foot pounds,

$$W_1=\frac{n}{n-1}p_1v_1\left[1-\left(\frac{p_a}{p_1}\right)^{\frac{n-1}{n}}\right] \qquad (37)$$

Where the pressure is reduced to p_2 during the transmission the maximum possible work developed becomes

$$W_2=\frac{n}{n-1}p_1v_1\left[1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}\right[\qquad (38)$$

The efficiency of power transmission is then

$$E_1=\frac{W_2}{W_1}=\frac{1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}}{1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}} \qquad (39)$$

while the efficiency of *pressure transmission* is

$$E_2=\frac{p_2}{p_1} \qquad (40)$$

Equation (38) gives the power theoretically available at the end of a main of given diameter and length if v_1 is replaced by V the volume of air transmitted per minute. Inserting the value of V from equation (36) in equation (38) gives

$$W=3.061\frac{n}{n-1}\left[\frac{d^5p_1^2\,(1-r^2)}{KL}\right]^{\frac{1}{2}}\left[1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}\right]$$

Since $r^2=\left(\frac{p_2}{p_1}\right)^2$

$$W=3.061\frac{n}{n-1}\left[\frac{d^5\,(p_1^2-p_2^2)}{KL}\right]^{\frac{1}{2}}\left[1-\left(\frac{p_a}{p_2}\right)^{\frac{n-1}{n}}\right] \qquad (41)$$

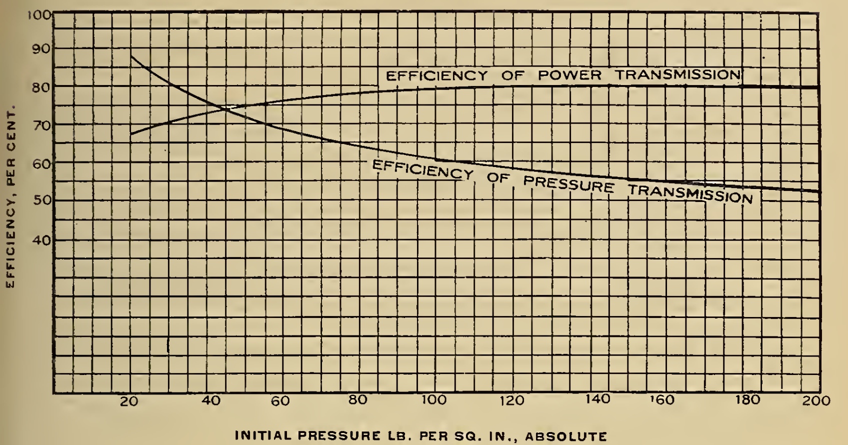

FIG. 7. EFFICIENCY OF POWER TRANSMISSION FOR MAXIMUM POWER, $N=1.3$

Since W and p_2 are the only variables in equation (41), if W be differentiated with respect to p_2 and the result placed equal to 0, it is found that W becomes a maximum when

$$p_1{}^2=\frac{p_2{}^2}{n-1}\left[n\left(\frac{p_2}{p_a}\right)^{\frac{n-1}{n}}-1\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Solving equation (42) for various values of n from 1.2 to 1.4, it is found that the influence of n affects but slightly the relation between p_1 and p_2. Since in practice the real value of n is probably not far from 1.3, this value is used in plotting the curves shown Fig. 6 and 7.

In Fig. 6, values of p_1 are plotted as ordinates and of p_2 as abscissas so the final pressure in the pipe for theoretical maximum power transmitted can be determined directly for initial pressures up to 200 pounds per square inch absolute. Fig. 7 shows the efficiency of *power* transmission, calculated from equation (39), and the efficiency of *pressure* transmission, calculated from equation (40), when the power transmitted is a maximum with the final pressures as shown in Fig. 6 for $n=1.3$. As previously stated other values of n between 1.2 and 1.4 give but little deviation from the results shown in Fig. 6 and 7.

From this discussion it is evident that there is a theoretical maximum carrying power for an air pipe line. Thus with air at 100 pounds initial pressure, the power transmitted is a maximum when the final pressure is 61 pounds absolute. The efficiency of *pressure* transmission is then 61%, while the efficiency of *power* transmission is nearly 79%.

This latter efficiency cannot be secured if the air is used non-expansively or with only partial expansion, nor can the maximum power be attained under such conditions.

While this subject is probably of greater theoretical than practical interest it is of value in determining the minimum size of pipe which should be used for a given service. It is possible in temporary installations that the reduced cost of the pipe, carrying its maximum power, would more than offset the loss in the transmission. Under normal conditions, however, this would not be true, and the pipe line should be proportioned to give not more than 5% to 10% pressure drop.

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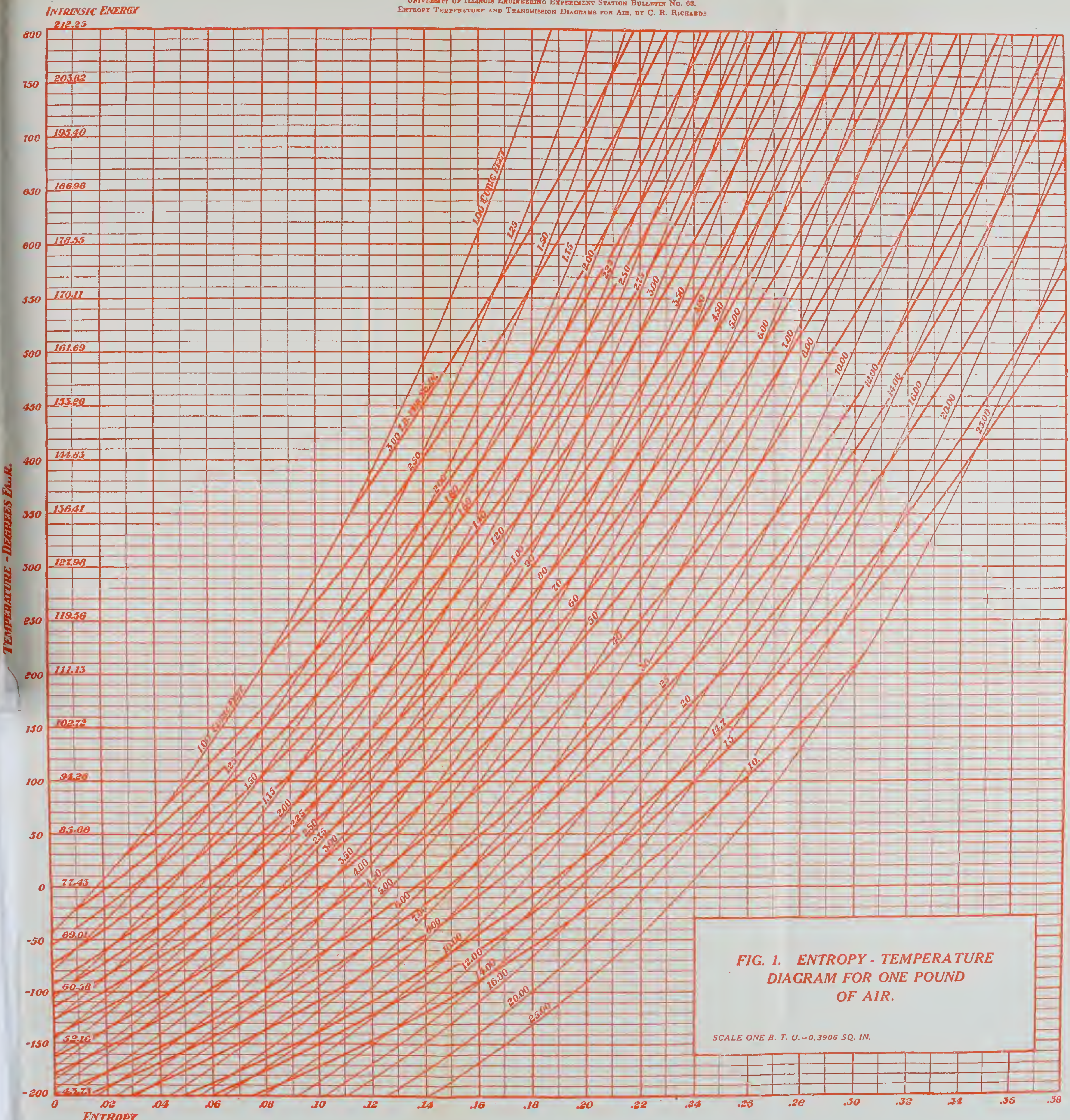

FIG. 1. ENTROPY - TEMPERATURE DIAGRAM FOR ONE POUND OF AIR.

SCALE ONE B. T. U. = 0.3906 SQ. IN.

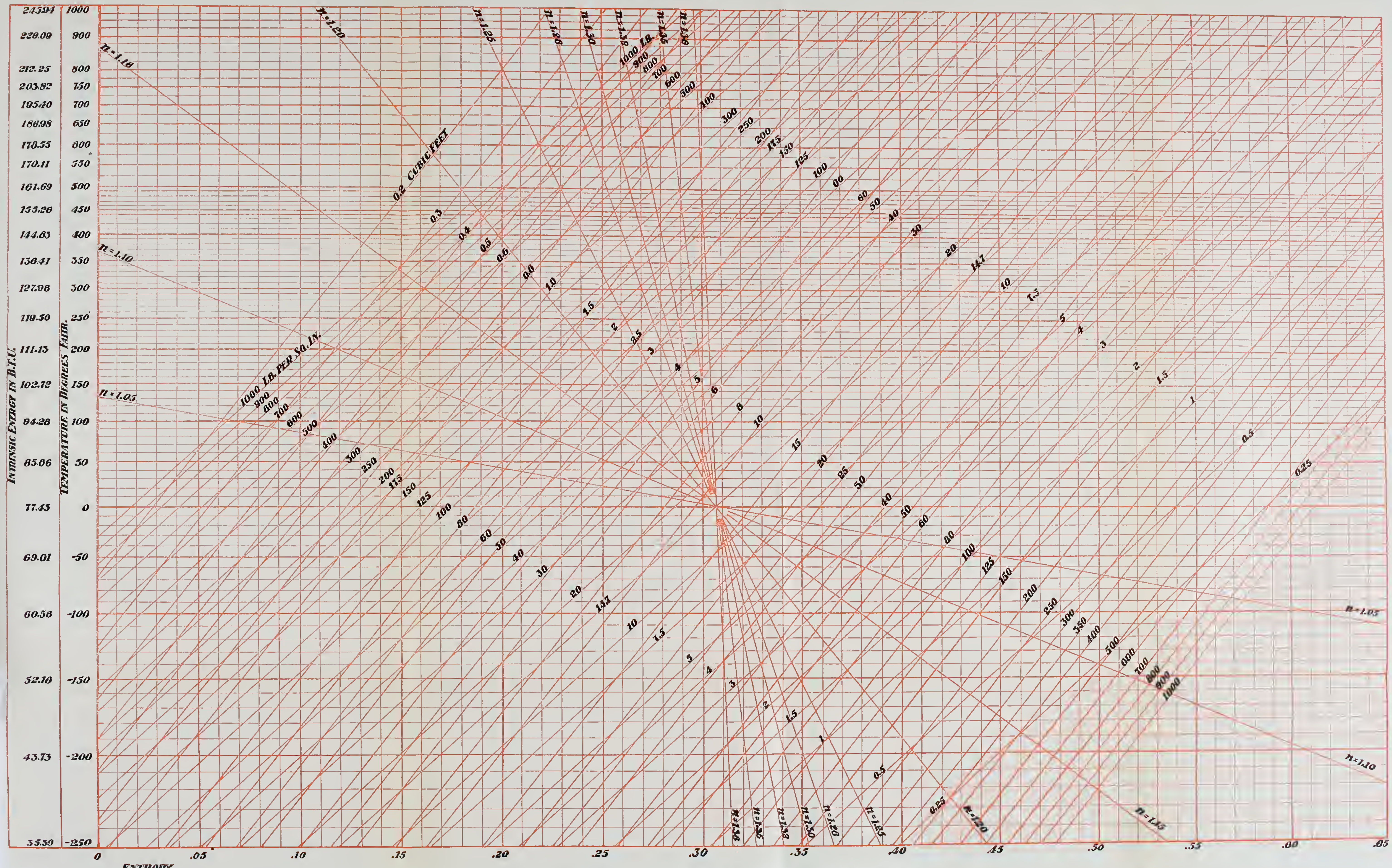

FIG. 2. ENTROPY-LOG. TEMPERATURE DIAGRAM FOR AIR

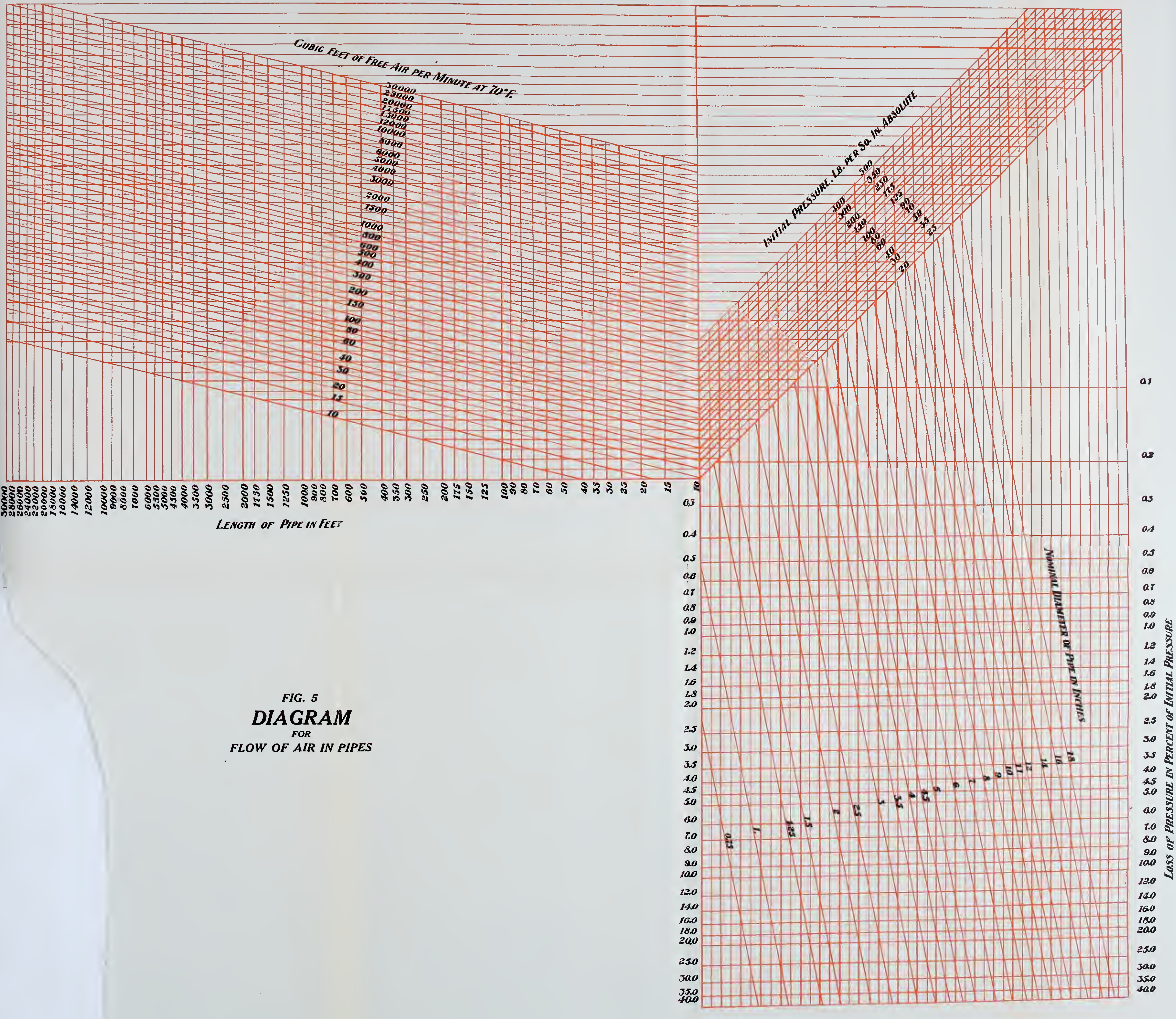

FIG. 5
DIAGRAM
FOR
FLOW OF AIR IN PIPES

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TESTS OF REINFORCED CONCRETE BUILDINGS UNDER LOAD

BY

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AND

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 64 JANUARY, 1913

TESTS OF REINFORCED CONCRETE BUILDINGS UNDER LOAD

By ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS, AND WILLIS A. SLATER, FIRST ASSISTANT IN ENGINEERING EXPERIMENT STATION

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I. Introduction.

1. *Preliminary.*—In the development of the newer types of building construction, the need of further information on the action of the structure in its various parts has been felt. Analysis gives methods of calculation of stresses and laboratory tests give data on the action of individual members; but the truth of the assumptions used in analysis may be questioned, and because of the method of fabrication or the influence of one part on another the action of the structure may not be in exact accord with the conclusions derived from analytical considerations. It is especially important that knowledge on the amount and distribution of deformations and stresses actually developed in structures be extended, and every effort may well be made to determine these stresses by tests of structures themselves. Many load-deflection tests of structures have been made, and such tests are required by city building departments as a condition of acceptance for allowable loading, and these tests have been used by construction companies and engineers to demonstrate the adequacy of various designs. Load-deflection tests are of value in judging of the quality of the workmanship and in giving confidence in the structure, but they throw little light on the stresses developed in the different parts or upon their distribution. The deflections observed in such tests constitute a very inadequate measure of the stresses and may even be misleading in this respect. Slight deflections, which have been taken to indicate low stresses in steel and concrete, may actually be accompanied by high stresses. In the matter of design there has been a divergency of views on the relation between the bending moment at a section at the support and that at the middle of the beam, on the distribution of stresses across a flat slab acting as the flange of a T-beam, on the restraint of girders and beams, and on the stresses developed in the flat slab type of floor construction. It is evident that measurements of the deformation in structures may be expected to greatly assist the settlement of such questions as these.

The measurement of deformation in the various parts of a structure by a field test is a recent development in testing work. It may be expected that in the early stages of the development of such field tests difficulties will be encountered and that experience will bring out the

methods which are most satisfactory and will indicate the precautions which must be observed to insure accurate and trustworthy results. The statement of the requirements for such a test will be of value in making other tests, and the methods of course should be carefully stated with the record of such tests.

2. *Scope of Bulletin.*—This bulletin records the results of three field tests made on reinforced concrete floor systems in which the measurement of deformations or strains in the parts of the structure was an important feature. As these tests comprise the earliest known measurements of this kind made upon reinforced concrete buildings and as the writers have been connected with the development of this method of testing, it has seemed proper to include a discussion of the method of testing—the use of the instruments, the methods of observation, the precautions to be taken, the accuracy of the results and the methods of loading. The bulletin then gives a record of the results of the tests on the floor systems of two buildings of the beam and girder type and of one building of the flat slab type, and contains discussions of the stresses developed and the general phenomena observed.

3. *Acknowledgment.*—The technical part of making the tests was done as the work of the Engineering Experiment Station of the University of Illinois. The first building test in which deformations of steel and concrete were measured was made on the Deere and Webber Building in November, 1910. This test was under the direct supervision of Mr. Arthur R. Lord, then Research Fellow in the Engineering Experiment Station. Mr. Lord is entitled to much credit for his work in directing this test and for the initiative, foresight and care used in developing methods and in making the test. The report of the test on the Deere and Webber Building and the discussion of the results were prepared by Mr. Lord and with his permission are included in this bulletin. Mr. W. A. Slater was in direct supervision of the test of the Wenalden Building and the Turner-Carter Building, and has been intimately connected with the other tests of the kind named in this bulletin, and to him credit is due for many of the methods and details of the testing work and for formulating the provisions and precautions necessary to give accuracy and trustworthiness to the results.

The tests were undertaken as co-operatove work. The tests on the Wenalden Building and the Turner-Carter Building were made in connection with the Committee on Reinforced Concrete and Building Laws of the National Association of Cement Users, and the president and the treasurer of the Association raised the funds to defray expenses of the test. The contractors who erected the buildings also assisted in these

tests. The expense of the Deere and Webber test was borne by the building contractor, to whom especial credit should be given for very active interest and co-operation in initiating a new line of tests.

The tests were conducted by members of the staff of the Engineering Experiment Station of the University of Illinois. These included Messrs. H. F. Moore, W. A. Slater, A. R. Lord, D. A. Abrams, N. E. Ensign and H. F. Gonnerman. The observers on the Deere and Webber Building test were Messrs. Moore, Slater and Lord; on the Wenalden Building test, Messrs. Moore, Slater and Ensign; and on the Turner-Carter Building, Messrs. Moore and Slater. Professor Talbot was in charge of the work. Papers covering much of the ground of this bulletin have been presented before the National Association of Cement Users and published in Vols. VII and VIII of the Proceedings of the Association.

4. *Comment.*—A few words on the basis and limitations of such tests may not be out of place here. It must be borne in mind that the measurements and observations are subject to some uncertainty as compared with certain laboratory tests; they are not exact or precise, and some erratic readings may be expected. The measuring instrument is used under unfavorable conditions. The gauge holes are deep in the concrete and the measurements may be interfered with by dust or other obstructing matter. It is evident that great care and much skill is necessary in making observations. Each test made has shown advances in accuracy and certainty, and further experience ought to show additional progress. Besides, it must be understood that the structure itself is not entirely homogeneous and that all parts of it do not act alike. Further, the structure itself is tied together so closely that stress in one portion may be modified or assisted in an unknown amount by another portion, which may not be thought to affect it. The modulus of elasticity of the concrete in the structure is not easily determined. The load-deformation diagrams may be irregular and imperfect. This all means that care must be taken in the interpretation of results and that some irregularities and uncertainties must be expected. With careful work important information will be brought out, as these tests show, and an accumulation of data on the action of structures, and tests of special features of construction will advance knowledge of structural action and be worth many times the cost of the work.

II. The Testing of Buildings.

5. *Building Tests Made.*—The number of building tests in which deformations have been measured is comparatively small. A list is here given of all known tests on reinforced concrete building floors in which

deformations in the steel and the concrete have been measured. The methods used in all these tests are essentially the same; they have been developed at the University of Illinois Engineering Experiment Station. Fig. 1 shows the range in size of the test areas in the buildings tested.

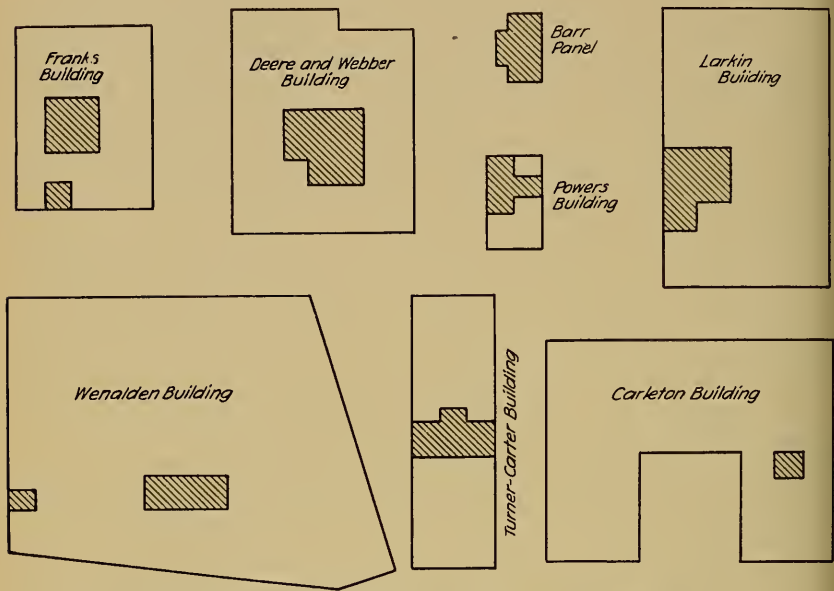

FIG. 1. DRAWING SHOWING RELATIVE SIZES OF TESTS.

Tests have been made on steel structures by the U. S. Bureau of Standards using methods somewhat similar to those here described, but as this bulletin is concerned primarily with the results of tests on reinforced concrete floor systems specific mention of the tests by the Bureau of Standards is omitted.

Test No. 1. Deere and Webber Building, Minneapolis, Minnesota, October and November, 1910. Flat slab floor with four-way reinforcement, built by Leonard Construction Company of Chicago, and tested by co-operation between the contractors and the Engineering Experiment Station of the University of Illinois.

Test No. 2. Wenalden Building, Chicago, Illinois, June and July, 1911. Beam and girder building constructed by Ferro-Concrete Construction Company of Cincinnati, and tests made by co-operation between the National Association of Cement Users, the construction company, and the Engineering Experiment Station of the University of Illinois.

Test No. 3. The Powers Building, Minneapolis, Minnesota, July and

August, 1911. Flat slab floor with two-way reinforcement, built and tested by Corrugated Bar Company of St. Louis.

Test No. 4. Franks Building, Chicago, Illinois, August, 1911. Flat slab floor with four-way reinforcement, built and tested by Leonard Construction Company of Chicago. Prof. W. K. Hatt of Purdue University was employed as consulting engineer for this test.

Test No. 5. Turner-Carter Building, Brooklyn, New York, September, 1911. Beam and girder floor, built by Turner Construction Company of New York; test made by co-operation between National Association of Cement Users, the construction company, and the Engineering Experiment Station of the University of Illinois.

Test No. 6. Carleton Building, St. Louis, Missouri, October, 1911. Flat slab floor with two-way reinforcement, built and tested by Corrugated Bar Company.

Test No. 7. Barr Building, St. Louis, Missouri, December, 1911. Full size test panel (25 x 26 ft. 9 in.); terra-cotta tile used to lighten construction; consists of two-way T-beams with web between tile on tension side and concrete flange above the tile on the compression side. Panel built by the Corrugated Bar Co. to demonstrate efficiency of design proposed for Barr Building in St. Louis; test made by Corrugated Bar Company.

Test No. 8. Ford Motor Building, Detroit, Michigan, February and March, 1912. Flat slab floor, built and tested by the Corrugated Bar Company.

Test No. 9. Larkin Building, Chicago, Illinois, August, 1912. Flat slab floor, built and tested by Leonard Construction Company.

The Deere and Webber Building test was undertaken to learn of the general action of the flat floor slab. The tests on the Wenalden Building and the Turner-Carter Building were made to find the general action of the beam and girder type of construction. The tests made by the Corrugated Bar Company were for their own information but the results of the tests on the Powers Building and on the Barr Building test panel were presented before the Eighth Annual Convention of the National Association of Cement Users. Those of the Carleton Building and the Ford Motor Building were in the nature of investigation of special features of design. The Franks Building test made by the Leonard Construction Company was an investigation to obtain a basis for making provision in the Chicago building code for this form of construction. The Larkin test was the most extensive of those enumerated and was made with the object of furnishing the Leonard Construction Company with additional information for the design of flat slab floors.

6. *Definitions.*—In the following descriptions of tests, many terms will be used for which somewhat arbitrary definitions will need to be made. These definitions are given here:

Gauge Hole: A small hole (0.055 in. is here recommended) drilled into the steel bar or into the plug inserted in the concrete has been termed a gauge hole. It is for the admission of the point of a leg of the extensometer.

Gauge Line: The gauged length connecting a pair of gauge holes is termed a gauge line.

Reading: A reading is a single observation on any gauge line.

Observation: An observation as here used is the average of a number of readings.

Correction: A correction is the amount which if added algebraically to the observation will give the observation which would have been obtained if the instrument had not changed from its reference length.

Series of Observations: A set of observations on all gauge lines or on a selected number of gauge lines at a given load and taken in an established order is termed a series of observations.

Interval: An interval as used here is the time elapsing between consecutive observations, and all intervals in any series are (for lack of more exact information) assumed to be equal. For this purpose the average of two consecutive observations on standard gauge lines is considered a single observation.

Standard Gauge Line: Changes in the temperature of the instrument always occur in the course of a test. Frequently these changes are sufficient to cause an appreciable change in the length of the instrument. These and other small changes (usually unaccounted for) in the length of the instrument will introduce errors into the results unless the necessary corrections are applied to the observations. For the purpose of determining what these corrections should be, it is necessary to have reference to the standard bar may be understood to signify the standard constant as possible. This gauge line is termed a standard gauge line. Usually it is placed on a steel bar separate from the structure, and this has given rise to the term standard bar. In several of the tests, however, the standards have consisted of gauge lines placed in the steel and concrete of the structure remote from the area affected by the load. Standard gauge line is adopted, therefore, as the more general term and any reference to the standard bar may be understood to signify the standard gauge line on a bar separate from the structure.

Reference Length and Reference Observation: In order to determine changes in length of instrument it is necessary to make comparison

of all observations on the standard gauge line. To facilitate this comparison the length of the instrument at the time of some reference observation on the standard may be chosen as a reference length. A comparison of all other readings on the same standard gauge line with the reference observation chosen will show whatever variation there is in the length of the instrument. For convenience the first observation on the standard gauge line has been assumed as the reference observation. The length of the instrument at the time of this observation will then be known as the reference length.

7. *General Outline of Method of Testing.*—After determining what measurements will best give the information desired from the test, the gauge lines are laid off on the surface of the concrete and small holes are cut or drilled in the concrete at a predetermined distance apart in order to expose the steel or allow a metal plug to be inserted, according as the measurement is of steel deformation or concrete deformation. The metal plugs used are securely held in place by embedment in plaster of paris. The gauge holes having been carefully prepared, a set of zero observations is taken on all gauge lines, an increment of the loading material is then applied and a second series of observations on the gauge lines is taken. The difference between the two observations on the same gauge line represents the deformation in that gauge line. It is possible that this apparent deformation may be due partly to temperature changes in the instrument instead of stress changes of the material by reason of applied load. For this reason reference measurements are made on standard unstressed bars made of invar steel which has a very low coefficient of expansion and whose change in length due to change in temperature would therefore be very slight. From these readings on the standard bar, temperature corrections are computed as shown in a later paragraph and applied to the observations in order to determine the actual change in length of the gauge line. Another increment of load is then applied and another series of observations taken.

Floor deflections also have been measured in all of these tests, but they have been considered as of secondary importance. They have been used to throw light on the correctness or incorrectness of the deformation readings and to gain some idea of the general distribution of stresses throughout a floor. Apparently they can be depended upon to show with considerable accuracy the proportional rate of increase of stress, but deflection formulas are so imperfect that measurements of deflections can not be depended upon to give actual values of stresses.

Measurements of dimensions such as span, depth of beams, location of

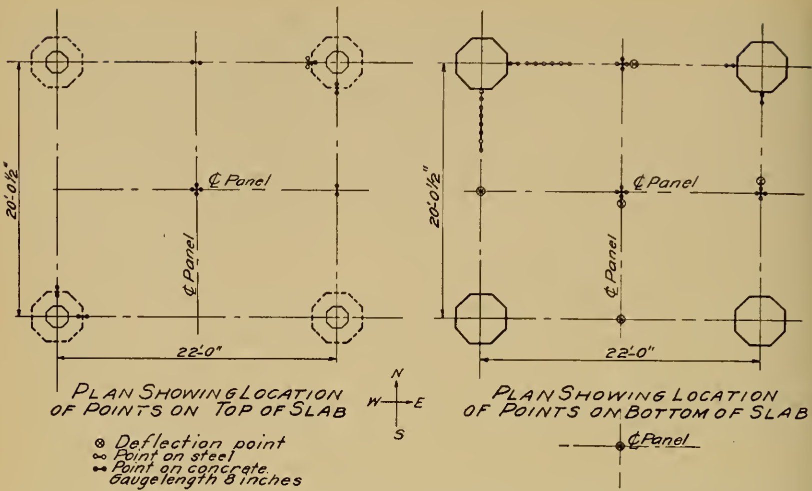

FIG. 2. CARLETON BUILDING TEST; PLAN SHOWING POSITION OF GAUGE LINES.

observation points, weight of loading material, location of cracks and any other measurements which are of value in working up results are carefully taken.

The gauge lines are usually distributed over and under the surface of the floor tested in order to gain an idea of the changes occurring in different parts of the structure. The statements in the preceding paragraphs give in general terms the features of a field test. There are many difficulties to be overcome and many chances for error. The methods of overcoming the difficulties and avoiding the errors will be discussed in the following pages. Most of the statements there made represent the results of experience on building tests.

8. *The Planning of a Test.*—Each test made will involve individual consideration of the choice of area to be loaded, the number and location of gauge lines and deflection points, the number of laborers required, the loading material to be used and its distribution, and the provisions for storage of the loading material near the test area without appreciably affecting the stresses which are to be measured. Other matters will come up for consideration but generally different solutions will be required for each test.

The area to be loaded should be chosen so as to fulfill the following conditions as completely as possible.

(a) It should be so located as to give conditions in the beams, slabs, columns, etc., as severe as will be found anywhere in the building when in use.

(b) It should be free from irregularities of construction.

(c) It should be as free as possible from disturbances by workmen.

(d) It should be as easily accessible to the loading material as possible.

In most cases some limitation is found on part or all of the conditions named. For example, in the test of the Wenalden Building it was impossible to find an area entirely free from irregularities of construction. An industrial track crossed one of the panels chosen, and the floor was thicker immediately under this track than at other places. On the edge of one or two of the panels tested, beams about an inch deeper than the regular beams were located. However, none of the measurements assumed to give typical results were taken in these panels, and it is believed that the stresses in the other panels were not affected appreciably by the irregularities. Again, in the test of the Franks Building it was not possible to choose a lower floor convenient to the loading material. The choice of an upper floor fulfilled one of the conditions mentioned—it gave a much more severe test of the columns than a test on a lower floor would have done. Also, in the test of the Carleton Building at St. Louis the area to be tested was specified by the city building department, and there was no choice as to location, on the part of those making the test.

The number of measurements to be taken will depend upon the nature of the test, the number of observers, and the number of laborers. If the test is a part of a series by which it is expected to gain scientific information which will afford a basis for design, it is likely that it will be made deliberately enough that a large number of measurements may be taken. Such tests were those of the Wenalden Building, the Franks Building, the Turner-Carter Building and the Barr test panel. If, on the other hand, the test has more of a commercial nature or is a utilization of the opportunity offered by the acceptance test to take some measurements which will show actual stresses, or if for any other reason the test is hurried, the number of measurements will necessarily be rather small. Of this class, the tests of the Carleton Building in St. Louis and of the Ford Motor Building in Detroit, Michigan, are good examples. Notice was given the engineers only about one day in advance that a test would be made on the Carleton Building. Permission was obtained from the contractor to expose bars for measurement in various places and to erect the necessary scaffolding. The measurements were made more for the purpose of checking the analysis upon which the design was based than to form in itself a basis of design. Therefore comparatively few observation points were used. It is believed that this test is representative of the type of test which is practicable on a commercial basis, hence

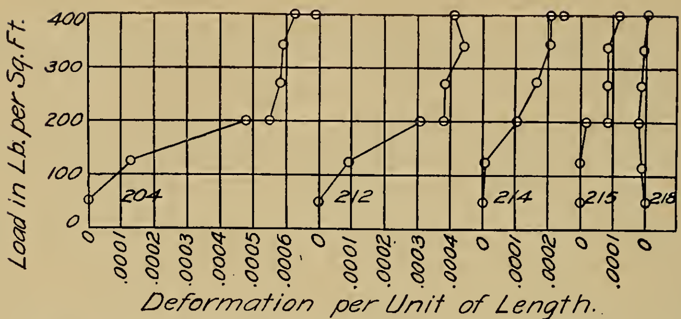

FIG. 3. POWERS BUILDING TEST; LOAD-DEFORMATION DIAGRAMS FOR SERIES OF GAUGE LINES ON REINFORCING BAR.

(by courtesy of the Corrugated Bar Company) a plan is given in Fig. 2 showing the points where measurements were taken.

The principal subjects for investigation in any test will determine the arrangement of observation points. Whatever the subject of study may be, the observation points should be arranged in such a way that a curve of deformations may be plotted against distance, showing a gradual progression from the condition at one part of the structure to the condition at another, for it is found that even under the most careful work there are inconsistencies which will make the results look doubtful if standing by themselves. The points so arranged should be numerous near the place where the measurements of greatest importance are to be taken, so that the results will not depend upon measurements at a single point, or upon the average at portions of the structure supposed to be similarly situated but in different parts of the building where unknown conditions actually may cause a large variation in the phenomena of the test. It will not be possible to carry out this plan for all subjects of investigation, as the number of observations required usually would be impracticably large. Such provisions may be made to cover the main lines of investigation, and isolated observation points may be used to gain information as to tendencies of other portions of the structure, but of course less reliance must be placed on the results of the latter measurements than where the larger number of observations is made. It would be advantageous, as was done in the Powers Building test and also in the Barr panel test, for two observers to check measurements on the same points. One or both of these checks is very valuable in establishing the correctness of observations. Fig. 3, 4, 5, and 58 illustrate the former method. Fig. 3 gives the load-deformation diagrams for a series of gauge lines in the test of

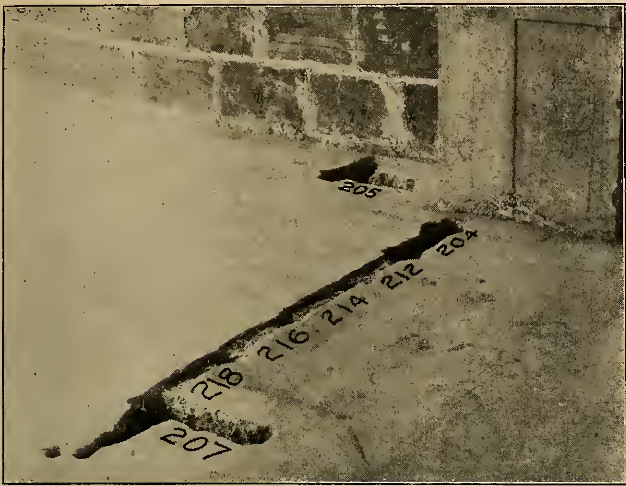

FIG. 4. POWERS BUILDING TEST; LOCATION OF SERIES OF GAUGE LINES.

the Powers Building. Fig. 4 shows the location of these gauge lines with reference to the wall and a column. Fig. 5 shows the same data plotted as deformation against distance from the column instead of against load. It may be seen that the correctness of the load-deformation curve for one of these points, if standing by itself, might be doubted because of the complete change in the character of the curve at a load of 200 lb. per sq. ft. But when these deformations are plotted against distance, the results look so consistent that it is scarcely conceivable that they are seriously incorrect. In the test of the Wenalden Building very high compression deformations were observed in the beams near the sup-

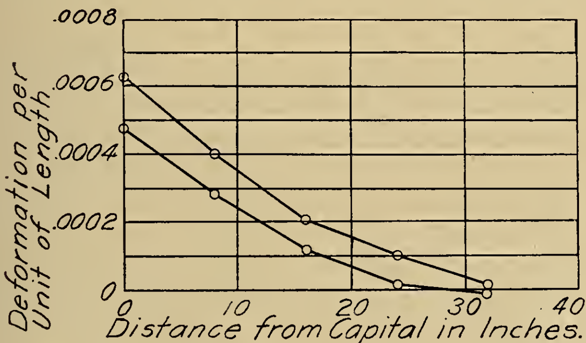

FIG. 5. POWERS BUILDING TEST; DATA OF FIG. 3 PLOTTED AS DISTANCE-DEFORMATION CURVES.

ports. As this was the first test of a beam and girder floor it was considered especially important that in the next test, that on the Turner-Carter Building, evidence be obtained which would give additional information bearing on this high compression in the concrete; accordingly the method of placing observation points at frequent and regular intervals along the ends of the beams was used. The deformations measured are plotted in Fig. 58, p. 80, against the distance from the supporting column, and the results not only tend to show the correctness of these measurements but also to indicate that the high stresses observed in the beams of the Wenalden Building were actually present.

The subjects for investigation will vary largely in different tests. In the tests discussed in this bulletin deformations have been measured with a view to obtaining information on each of the following subjects:

(a) The values of the coefficients of bending moment at the center of the span and at the support of the beam or slab under investigation.

(b) Relative moments at support for various conditions of fixedness.

(c) The extent to which the floor slab acts as a compression flange of the floor beam to produce T-beam action.

(d) Bond stresses.

(e) Diagonal tension.

(f) Stresses in columns.

(g) Time effect under constant load.

(h) The lateral distribution of stress to parts of the structure entirely outside of the loaded area.

(i) The extent to which steel stresses are modified by errors in the assumption that no tension is carried by concrete.

(j) Stresses in slabs of beam-and-girder floors.

(k) Relative stresses in short and long directions of rectangular panels.

Other subjects of investigation have received attention but those mentioned above are the most important ones. Some phenomena have been observed which bring out additional problems. Of these the determination of the amount of arch action present is probably the most important.

It is not to be expected that the moment coefficients can be determined with absolute accuracy. The method used has been to measure deformations on both steel and concrete at the center and supports of the beams and from these measurements to determine the total resisting moment developed at the given section. Equating this resisting moment to the bending moment kWl (where k is the bending moment coefficient), a solution is made for the value of k. Arch action, tension in the concrete and the sharing of bending moment by adjacent beams complicate the

problem. It is suggested that the amount of arch action may be determined in any case by making a special study of the deformations in a cross-section at the mid-span of each beam and on the floor slab across an entire panel. In this study, deformations should be observed on the steel and at various elevations on the concrete so that the position of the neutral axis and of the center of gravity of tensile and compressive stresses respectively may be accurately determined. By this means it should be possible to determine if the sum of the compressive stresses is in excess of that of the tensile stresses. If so, the difference apparently must be the direct thrust due to arch action. A second section may profitably be taken at or near the point of inflection. The same study can be made, though not so satisfactorily, at the ends of the beams. The measurements for thrust will require observations on an extremely large number of gauge lines, and the observations must be extended far enough into the adjacent panels to determine the extent to which they share in carrying the load.

9. *Preparation for the Test.*—In all of these tests it was necessary to cut holes in the concrete in order to expose the steel. Fig. 4 shows holes cut in the concrete of the Powers Building where a series of measurements was taken on a rod which passed through a column. This cutting has been best accomplished by the use of a cold chisel with a very gradually tapering point. This work is a task for common laborers and a long one for inexperienced men. It has been found that a great deal of speed can be developed by practice, hence the importance of completing this part of the work with a single set of workmen.

A saving in mutilation of floors often can be effected by planning the test ahead of time and inserting plugs in the concrete during construction in the proper position for the gauge lines. Removal of the plugs after the concrete has set exposes the steel without the use of a cold chisel. Likewise metal plugs may be set in the concrete at the proper positions for the measurement of concrete stresses and thus save cutting into the concrete to place compression plugs. The point has been raised that by preparation of this kind a chance is given to the contractor to know what panels are to be tested and thus to make the construction of that panel better than others. For this reason there is room for question as to the advisability of using this method. Its employment will depend largely on the purpose of the test and on the conditions under which it is made. In most of the tests under consideration this point has been taken care of by the fact that it was not known until shortly before the test what area was to be loaded.

Drilling of the gauge holes will be discussed in article 14.

An elevated platform which will enable the observer to get close enough to the floor above to take observations of deflection and deformation is necessary. This should be supported at such a height that when the observer stands upon it the points where measurements of deformation are to be taken will be about one inch above his head. For flat slab construction this condition is easily obtained (see Fig. 6),

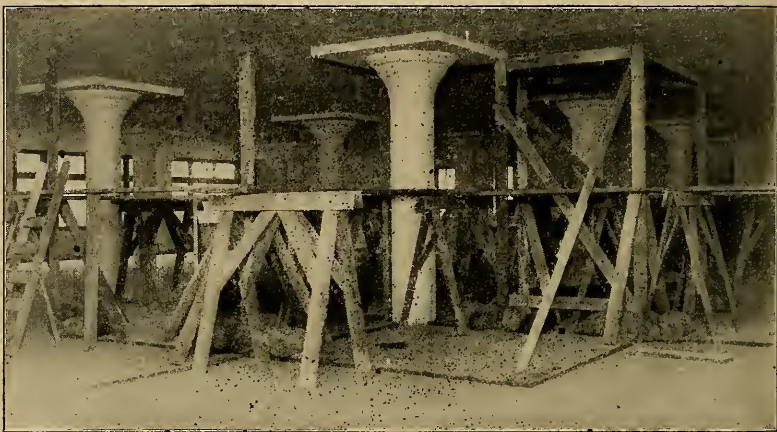

FIG. 6. FRANKS BUILDING TEST; OBSERVATION PLATFORM AND DEFLECTION FRAMEWORK.

but with beam and girder construction where there are measurements on beams, girders and the floor slab, the heights of different gauge lines vary so much that arrangement will need to be made for building certain parts of the platform higher than others (see Fig. 7). It is important that the elevation of the platform should be such that the observer can stand erect while taking the readings, and yet such that the instrument will not be too high for convenient and accurate observation.

Another framework for the purpose of supporting deflection apparatus under the points where measurements of deflection are to be taken is also necessary. In order that the movements of the observers upon the observation platform may not jar the deflection apparatus, the two frameworks must be built independently of each other. In all the tests which have been made, these deflection frames have stood on the floor and have been braced from one to the other in order to make a comparatively rigid framework. Fig. 6 shows scaffolding and deflection frames for the Franks test.

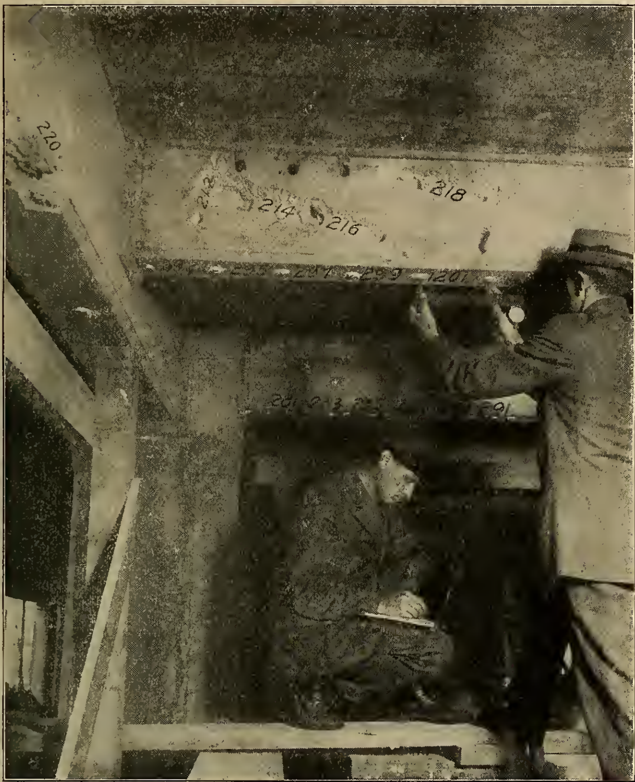

FIG. 7. TURNER-CARTER BUILDING TEST; PHOTOGRAPH SHOWING VARIATION IN HEIGHT OF GAUGE LINES.

The equipment necessarily will consist of the following: cutting and drilling tools, portable lamps for throwing light into observation holes, note books and facilities for doing drafting and for reducing data.

The cutting and drilling tools are sufficiently described in other paragraphs.

Some kind of a portable light is a necessity as gauge lines are often located in dark corners and as observations may be taken at any hour of the day or night. The lamp shown in the photograph of Fig. 16, p. 29, is a hunter's acetylene lamp and is quite satisfactory. The lamp is attached to the forehead and light may be thrown in various directions according to the setting of the clamp attachment. The acetylene tank may be attached to the belt or carried in the pocket.

Loose leaf note books should be provided in which the sheets are as large as conveniently can be handled and filed. The data sheets shown

in Fig. 26, p. 44, are very conveniently ruled in hectograph ink and copied by means of a hectograph. Printed forms might be used, depending on whether the number required would justify the expense of having them printed.

For the most efficient work in computing results and making sketches for records, it is important that an adequate place be provided where quiet may be had, where benches and drafting tables may be used and where instruments and other equipment may be kept. The photograph in Fig. 8 shows the temporary office which was provided in the

Fig. 8. Turner-Carter Building Test; Interior of Office.

Turner-Carter Building test. This is one of the portable office shanties which the company moves to places where work is being done. The photograph shows the interior of the office with the observers and recorders at work reducing the data of the test. This added equipment will add only slightly to the cost of the test but very greatly to the efficiency of the work. Special attention is called to it because the office is an important piece of equipment and it has not always been provided.

10. *Loading.*—In the tests which already have been made, the following loading materials have been used: brick, cement in bags, loose sand in boxes or bins, sand in sacks and pig iron. The material used will almost always be that which is most easily available, because the moving of loading material from any distance adds very greatly to the cost of the test. Leaving consideration of cost out of the question, sand

in sacks seems to be the most satisfactory of the materials above mentioned for loading purposes. Some of the qualities of the materials mentioned are as follows:

(a) Brick: Brick spalls and chips in handling, covering the floor with dust and jagged particles which cause discomfort to the observer in kneeling to take observations. It is important to avoid this because discomfort necessarily decreases the accuracy of his observations. This might be avoided by sweeping, but in sweeping it is difficult to avoid getting dirt into holes where observations are to be taken, and this is just as troublesome as having the dirt on the floor. Fig. 31, p. 50, a photograph of the Wenalden test, shows the use of both brick and cement in the same test. Attention is called to the proximity of the cement sacks to the beams and girders of the floor above. In some cases if cement and brick were used the intensity of the load would be limited by the height of the ceiling.

(b) Cement: Cement sifts through the sacks and the sacks become untied, scattering cement on the floor, filling observation holes and causing much dust in sweeping or cleaning up. The dust is injurious to delicate instruments and annoying to observers and recorders.

(c) Loose Sand in Small Boxes: As sand is usually damp, it does not have the fault of causing dust, and consequently is more easily cleaned up than the other materials mentioned. There are, however, other objections to it. In filling boxes it is difficult to avoid spilling the sand around and between the boxes, and consequently filling the observation holes. On account of the great difficulty in removing loose sand without spilling a great deal of it, it is impracticable to take observations as the load is being removed, therefore it is necessary to remove in one increment the whole load from a given panel. Fig. 46, p. 68, is a photograph of the Turner-Carter test and shows this method of loading.

(d) Sand in Sacks: Sand in sacks constitutes a very satisfactory loading material. An example of its use is shown in Fig. 9, a photograph of the test of the Barr Building test panel. It was piled up to a height of about nine or ten feet, and very little inconvenience was caused by the sacks coming untied or by spilling the sand. The worst difficulty encountered, and this exists with all materials handled in sacks, is that of the sliding of sacks on themselves when the load is piled high. It can be seen in Fig. 9 that bracing was necessary to prevent the sand from sliding together and filling up the aisles. It is a source of danger to those taking observations as, if a slide should occur, it probably would give very little warning and might catch the observer while in such a position that he could not escape. However, this objection would hold

FIG. 9. BARR PANEL TEST; SAND IN SACKS AS A LOADING MATERIAL.

with any material when piled as high as was that in this test. Under any circumstances it is necessary that care be taken and undue risks avoided.

(e) Pig Iron: Pig iron was used as loading material in the test of the Franks Building (see Fig. 10). From the standpoint of the making of the test, the worst objection to it is that, as with the brick, small particles break off and cause annoyance to observers. This is

FIG. 10. FRANKS BUILDING TEST; PIG-IRON AS A LOADING MATERIAL.

less troublesome than with brick and in other ways pig iron is clean. It possesses the great advantage that with its use a very heavy load can be applied without piling the load extremely high.

Tin plate in boxes two feet square, each weighing 200 pounds, was to have been used in a building test. A more nearly ideal loading material would probably be hard to find, but unfortunately this test could not be carried out.

The intensity of the loading will depend mainly on the load for which the building was designed. It will not be possible to make the load absolutely uniform, as aisles usually will be necessary for the purposes of (a) convenience in placing the load, (b) access to gauge lines for the taking of observations, and (c) the prevention of arching in the loading materials. It has been found that it is difficult to cover more than about 75% of the actual area of the floor, and in many cases less than this will be covered. Hence in computing the probable height of the load this fact must be taken into consideration.

Aisles should be so placed that the load, even though partly carried by arching of the material, will cause stresses in the floor which approximately are equal to, and always as severe as, those caused by an actual uniform load. Fig. 11 shows the moment and shear diagrams which would be obtained by loading a simple beam with a total load W distributed over the span in three different ways, as follows:

(a) Solid Line: Total load W, uniformly distributed over full span.

(b) Broken Line: Same load W distributed over one-half of span, giving aisles of equal width at center and support.

(c) Heavy Dotted Line: Same load W distributed over one-half span, half of load being carried by arch action to ends of boxes (shown here as concentrated loads, $\frac{W}{8}$), and the other half being uniformly distributed over the half span.

It will be possible in almost any test to arrange the loading material in such a way as to come within the limits outlined by the three arrangements of load assumed in the preceding illustration, and it is seen that if this is done, the presence of the aisles or of arching to the sides of the boxes or piers, while not affecting the amount of the maximum moment and the maximum shear, would tend to cause them to exist over greater portions of the span than would be the case with an equal uniform load. In this figure aisles equal to one-quarter of the span have been assumed. In no case would they be as large as this, and therefore the moment and shear diagrams actually should conform even more nearly to those for uniform load than is shown in the figure.

Arrangement should be made, if possible, to store the loading material near the test area in order to hasten the work of applying the load after the test begins. The general rule has been to allow loading material to be stored as close as one full panel length from the test area, but the intensity of the storage load has been kept down as much as possible.

The number of laborers which can be used advantageously will depend upon the distance of the point from which the loading material is

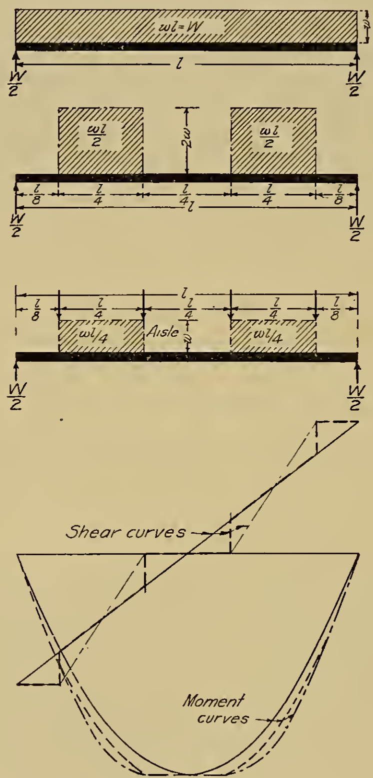

FIG. 11. MOMENT AND SHEAR CURVES FOR THREE ARRANGEMENTS OF LOAD.

to be transferred, upon the size and accessibility of the tested area, upon the amount of work which can be done by them during the intervals between increments of load while observations are being taken, and upon the length of time required to take a series of observations. The direction of the labor should not be left to the one in charge of the test, if it can be avoided, since proper attention to the conduct of the test demands all of his time. In the tests included in this bulletin the number of la-

borers has varied between wide limits, from 5 or 6 in the Powers test to 30 or 35 in the Deere and Webber test.

11. *Extensometers.*—In the past the great obstacle to the measurement of deformations in building tests has been the difficulty of attaching the measuring instruments to either the steel or the concrete on the flat surface of a floor, and recent tests show the necessity of making measurements of steel deformation directly on the steel. A satisfactory method of accomplishing this has been provided by the introduction of the extensometer invented by Professor H. C. Berry of the University of Pennsylvania and adapted to this work by modifications and improvements made at the University of Illinois. This instrument is similar in some respects to the strain gauge designed and used as long ago as 1888 by James E. Howard, Engineer-Physicist of the Bureau of Standards, and until recently Engineer of Tests at Watertown Arsenal.

The great value of this instrument in building tests lies in the following facts: (a) Its use makes it possible to take measurements directly upon the surface of the steel and concrete. (b) With its use there is no apparatus left in place to be damaged or disturbed during loading. (c) Due to the fact that it is portable, measurements may be taken in a large number of places with a single instrument. Measurements have been taken at as many as 268 points in a single test. If fixed instruments were used this would call for an outlay of from $2500 to $5000 for instruments and the impossibility of keeping so many instruments in adjustment under testing conditions would render their use impracticable.

Fig. 12 shows the Illinois extensometer in its present form.* Any

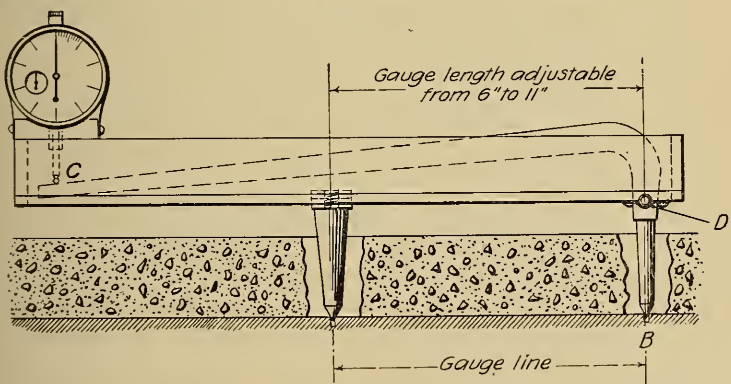

FIG. 12. ILLINOIS TYPE OF BERRY EXTENSOMETER.

*Another form has been devised since the manuscript for this bulletin was prepared and this newer extensometer is now in use in laboratory tests.

movement of the point B due to a change in the length of the gauge line is transmitted to the Ames gauge through vertical movement of point C, by means of the leg BD and the arm DC pivoted at D. The Ames gauge is sensitive to a movement at C of .0001 inch. The ratio of the length CD to the length BD is approximately five and the Ames gauge is thus sensitive to a movement at B of .00002 in. (.0001 inch ÷ 5). However, this must not be taken to mean that the extensometer possesses this degree of accuracy in measuring stresses, since some movement of the point of the leg at B is certain to result from variation in the handling of the instrument.

To obtain the exact ratio between movements at points B and C the instrument is calibrated by means of a Brown and Sharpe screw micrometer. For known movements of the point B readings of the Ames gauge are taken and a calibration curve plotted for the entire range of the instrument.

The first instrument of this type built by the Engineering Experiment Station of the University of Illinois was made for the Deere and Webber test. It was designed by Professor H. F. Moore and Mr. A. R. Lord, and was like the instrument in use at present except that it had a 15 in. gauge length and was made entirely of steel. Later on in making the instrument for general use aluminum was substituted for steel in order to reduce its weight and the gauge length was made variable from 6 in. to 11 in. Since then several minor changes have been made. The legs have been made stiffer in order to reduce the error due to unconsciously applied longitudinal thrust and the points have been made sharper in order to reduce the pressure required in seating the instrument. These improvements have considerably reduced the probable error of observation.

The extensometer loaned by Professor Berry to the University of Illinois in 1910 for use as one of the instruments in the Deere and Webber test is shown in Fig. 13. It differed from the Illinois instrument in that the movement of the multiplying arm was measured by means of a screw micrometer instead of the Ames gauge head, the point of contact of the micrometer plunger and the lever arm being determined by means of a telephone apparatus. The screw micrometer and the frame of the extensometer were insulated from each other and were connected with the poles of a small battery by means of copper wires. A contact between the plunger of the screw micrometer and the multiplying lever completed the circuit and the current set up produced a vibration of the diaphragm of the telephone apparatus carried on the head. This method of observation was very slow and the electrical connection got out of order very easily.

Fig. 13. Original Berry Extensometer in Use.

The use of the Ames gauge head (instead of the screw micrometer and telephone apparatus) adopted by Professor Moore in the instrument used in the Deere and Webber test has greatly facilitated the use of the extensometer. The legs of this instrument also were made longer in order to adapt it to the measurement of deformations of steel embedded in concrete. Both of these modifications later have been used by Professor Berry in instruments which he has put upon the market.

The extensometer more recently designed by Professor Berry is shown in Fig. 14. It is not different in principle from the one just described.

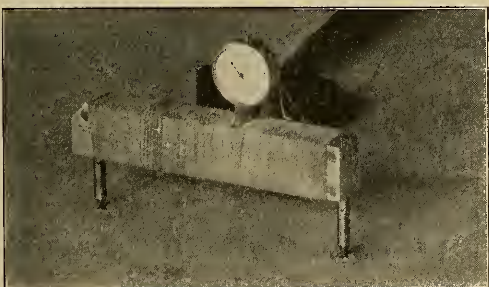

Fig. 14. New Berry Extensometer.

It differs from the Illinois instrument in the following details: (a) Instead of having a uniformly variable gauge length ranging from 6 in. to 11 in., it has two fixed gauge lengths of 2 in. and 8 in. respectively. (b) The instrument shown here has a multiplication ratio of two be-

tween leg and arm, and in order to make this ratio five (as in the Illinois type) it is necessary to use a leg which is only one inch long. With this arrangement the instrument cannot usually be used for measuring deformations in reinforcing bars in place, owing to their depth of embedment. (c) The framework of this instrument is of invar steel or of aluminum. While invar steel makes the weight somewhat greater than that of the aluminum instruments, it has the advantage of reducing the dependence on an invar steel standard bar and the study of the effect of temperature changes in the steel and concrete of the structure is accomplished with greater ease.

Mr. F. J. Trelease of the Corrugated Bar Company has designed an instrument of the Berry type and has used it in at least one test. This instrument, shown in Fig. 15, also has as its main feature a multiplying

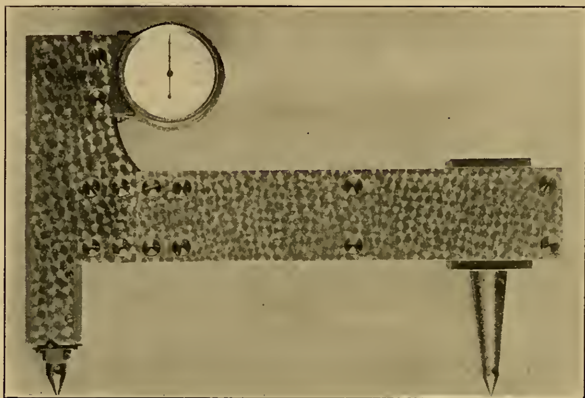

Fig. 15. Extensometer Designed by F. J. Trelease.

lever which actuates the plunger of an Ames gauge head. The principal difference between this instrument and the one shown in Fig. 12 is that the multiplying lever is vertical instead of horizontal. Results have been obtained with it which do not differ much as to accuracy with those of the Illinois type of instrument.

12. *Standard Bar.*—The necessity for a standard bar was first found in the test of the Deere and Webber Building. Variation in temperature was sufficient to cause a change in the length of the instrument as great in many cases as that in the reinforcing steel due to the applied load. Hence it was found necessary to make observations on an unstressed standard bar to show any temperature changes in the length of the instrument. In this test a bar of about ⅝-in. steel was used

as a standard. It was protected from rapid temperature changes by embedment in plaster of paris, and was kept on the floor where the test was being made. In this way it was expected to make the change in the length of the standard bar due to temperature variations about equal to the change in length of the reinforcing steel due to the same cause. To some extent this purpose was accomplished, but as the plaster covering was thin and not very dry, the change in the standard bar must have been much more rapid than that in the reinforcing steel. In the test of the Wenalden Building and of the Barr test panel, precautions were taken to embed a standard bar in concrete. This was done also in the tests of the Powers Building and of the Franks Building, and in addition standard gauge lines were established in parts of the floor not affected by the load. In the Turner-Carter test only the latter method was used. These standard gauge lines have been placed both on the reinforcing steel and in the concrete. Fig. 16 shows the taking of an

FIG. 16. TURNER-CARTER BUILDING TEST; TAKING AN OBSERVATION ON A STANDARD GAUGE LINE.

observation during the Turner-Carter test on a standard gauge line. The standard gauge line is located in a part of the floor entirely away from the loaded area.

The greatest development in the use of the standard gauge line has been in the frequency of reference to it and in the development of an exact system for the calculation of temperature corrections. It was noted previously that a steel extensometer was used in the Deere and Webber test but that in the subsequent tests an aluminum instrument was used. Since the coefficient of expansion for aluminum is almost twice that for steel, it is apparent that dependence on a standard must have been of still greater importance in the later tests. Difficulty was found in interpreting the notes taken on the Wenalden test, but the greater dependence on the standard gauge line and the more systematic use of it since then has very largely overcome this difficulty. In the test of the Larkin Building standard bars of invar steel were used. Invar steel has a very low coefficient of expansion and its use as a standard bar makes it possible to eliminate from the measurements almost all the effects of temperature variation in the extensometer. If it is desired to determine how great are the temperature effects, a standard gauge line can be placed in the floor as before in such a position as not to be affected by the floor load.

It has been the practice in the more recent building tests for each observer to make observations regularly on two standard gauge lines. This is done in order that one may form a check on the other. If only one standard were used, a large accidental change in the readings due for instance to sand in the gauge holes might be mistaken for a temperature effect. If two standards are used, such an accidental change as the above seldom would be the same in both, and the error would be detected. An accident to the instrument would probably cause the same change on both standard gauge lines and the use of the two standards would not help to detect this kind of an error. However, such errors are usually so large as to be apparent in a reading of the standard gauge line and are infrequent as compared with errors due to dirt in the gauge holes.

13. *Deflection Instruments.*—In the building tests referred to in this discussion deflection instruments of two types have been used, one being that used by the Illinois Engineering Experiment Station and the other that used by the Corrugated Bar Company. The former consists of a screw micrometer head of 1-in. travel, connected in tandem with an Ames gauge head micrometer of $\frac{3}{4}$-in. travel. The screw micrometer is designed to cover large variations in deflections and the Ames gauge head small ones. The Ames gauge head shows an increase in reading for a decrease in length of the deflection instrument, and a screw micrometer head, as ordinarily constructed, would show a decrease in read-

ing for a decrease in length. Thus to obtain an observation which involves the readings of both the Ames gauge head and the screw micrometer head it is necessary to take the difference of these readings, but in making calculations a sum is much more easily and accurately handled than a difference. To permit addition, the graduation on the screw micrometer head used in this deflection instrument has been reversed so that it shows an increase for a decrease in length, just as with the Ames gauge head. Fig. 17a shows this deflection instrument and also

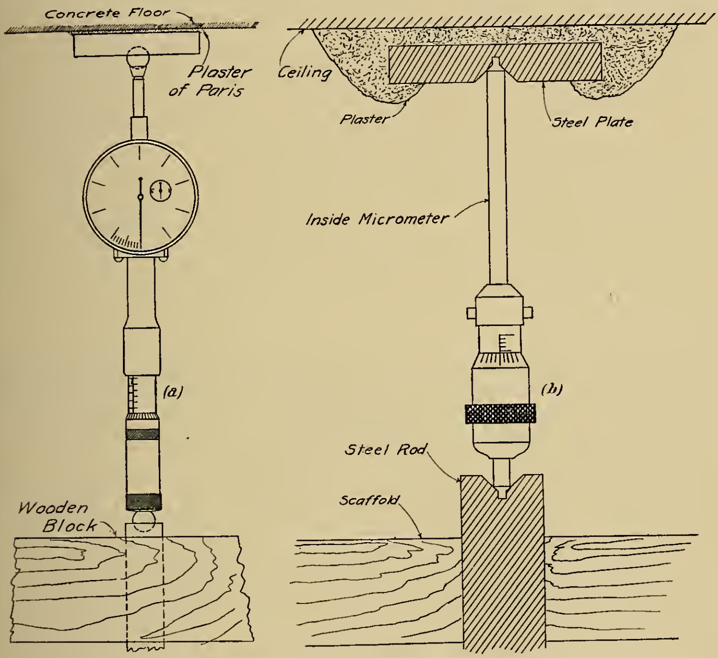

FIG. 17. (a) DEFLECTOMETER; UNIVERSITY OF ILLINOIS TYPE. (b) DEFLECTOMETER USED IN CORRUGATED BAR COMPANY'S TESTS.

the method of using it. A plate having a ½-in. steel ball attached is plastered to the surface, deflections of which are to be measured. A ⅝-in. bolt, which has a steel ball inserted into its upper end, is set into a wooden block (part of the deflection framework) in such a way that its elevation can be adjusted to give any desired zero reading of the deflectometer. Thus at the beginning of a test all the zero deflection readings can be determined, so that for a considerable length of time all the change in deflection will be shown on the Ames gauge without any change of the screw micrometer. As larger changes take place, a second setting of the screw micrometer will probably be necessary. The

great advantage of this instrument is the rapidity with which it can be used. It has been found to work very satisfactorily in most respects.

The deflection instrument used by the Corrugated Bar Company is shown in Fig. 17b and consists of a screw-micrometer depth-gauge by means of which distances for varying loads are measured between the stationary frame and a point on the beam or floor slab. It has the advantage of a much larger range of measurement than is found in the Illinois instrument. In the Barr panel test a gross deflection of more than 3 in. took place. As the Illinois type of deflectometer has a range of only 1¾ in., it could not have been used in this test. This amount of deflection, however, is more than would be likely to occur in the test of a building. The disadvantage of the Corrugated Bar Company instrument is that it requires a longer time to make an observation than does the deflectometer previously described.

14. *Extensometer Observations.*—In obtaining good results with this type of extensometer, a great deal depends upon careful manipulation. Two things which are of great importance in this respect are (a) the preparation of the gauge holes, and (b) care and experience in the use of the instrument.

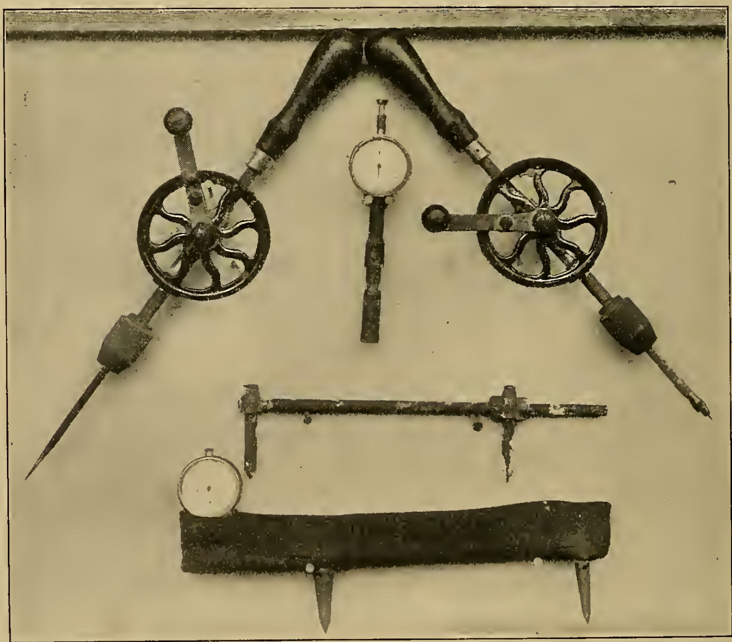

FIG. 18. TURNER-CARTER BUILDING TEST; INSTRUMENTS AND TOOLS.

The placing of the gauge holes so they will come within the range of the extensometer is best accomplished by the use of some kind of gauge marker such, for instance, as is shown in Fig. 18. In the work of the Illinois Engineering Experiment Station the holes are drilled with a No. 54 drill (.055 in. in diameter). At the beginning of the use of the Berry extensometer a number E countersink drill (approximately $^3/_{32}$ in. in diameter) was used; but a smaller one seems to be better, because it is easier to get the properly finished hole, because a slight eccentricity of the gauge holes on the reinforcing rod (see Fig. 19) causes less error in manipulation of the extensometer when a small drill is used, and because, in the case of measurements on small rods, the $^3/_{32}$-in. drill cuts away a large percentage of the steel in the rods.

A breast drill geared so that one revolution of the crank gives about 4¼ revolutions of the drill had been used previously. In the hands of a skilled workman very satisfactory work can be done in this way, but where, as quite frequently will be the case, the drilling has to be done by persons not familiar with this kind of work something better is needed. A drill driven by a flexible cable attached to a small electric motor giving a speed of rotation of 400 r.p.m. or more does better and more rapid work. Where high carbon steel is encountered fewer drills are broken, and when a hole is drilled a better job is usually the result.

After drilling the holes, the edges should be finished to remove the burr and to round off the sharp corners. The tool shown in Fig. 19 is

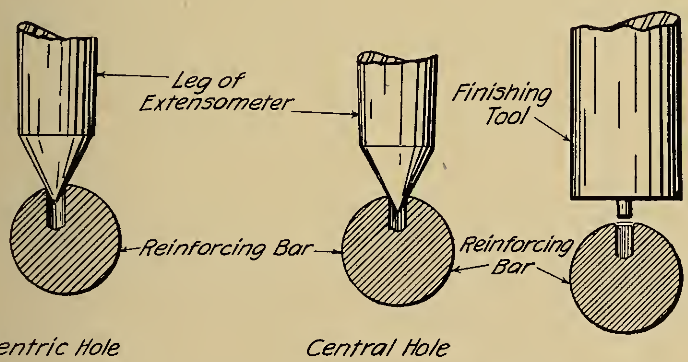

FIG. 19. POSITION AND FINISH OF GAUGE HOLES.

designed to accomplish this purpose. Such a tool should not be a cutting tool but rather a wearing or polishing tool. A pointed magnet to remove steel dust and small fragments of steel torn off in drilling, would be of

use. It is hard to place too much emphasis on the proper preparation of gauge holes.

Observers should be experienced in the use of the Berry extensometer before undertaking work on a field test. The chances of error in the manipulation of the instrument are large. As a rule the deformations measured are small so that the error is likely to be quite a large proportion of the total measurement, hence it is important to reduce errors to the lowest possible limit.

If the observations at zero are as reliable as other observations, a curve may be drawn through all the points of any load-deformation diagram, weighting the zero observations equally with the others; the zero point shown by the intersection of the most probable curve should then be used as the origin. This method involves waiting until the completion of the test to draw these curves. It would be much better to repeat the observations at zero load several times and to give more care and time to their determination than is given at any other load. By this means a check can be had upon the action of the structure as the test progresses, and the construction of the most probable curve will be made more simple. To obtain a satisfactory zero point, then, it is essential that several complete series of observations should be taken with no load on the floor, and it would be well to repeat this through considerable range of temperature to study temperature effect on the steel and on the concrete. This study was attempted in the Deere and Webber test, but the changes both in instruments and in reinforcement were included in the measurements and could not be separated, so no definite conclusions could be drawn. However, with an invar steel standard bar or with an instrument made of invar steel these two sorts of changes can be separated and to some extent at least the effect of temperature determined.

In taking an ordinary observation about five readings should be averaged. In most of the building tests which have been made, individual extensometer readings were recorded, but in certain laboratory tests and in the test of the Larkin Building the practice of averaging the results mentally was followed. This has given very satisfactory results and saves a great deal of time on a test, and with a good recorder the calculations can be kept up with the observations; but the practice should be adopted with caution and only after some experience in this kind of work. In the more recent building tests where individual readings were recorded, the practice followed in making an observation has been to reject all readings until five consecutive readings have been obtained which agree within .0004 in. These five consecutive readings then are averaged to form an observation.

Deflectometer observations have been sufficiently discussed in the description of the deflectometer and will not be considered here again.

15. *Observation of Cracks.*—Up to very recently the observation of cracks has been considered one of the most important features of a test. Although it is not considered so important as formerly when strains were not measured, if carefully done it may form a valuable check on the measured deformations. These observations should be made and recorded for zero load and at each increment of load. This is one of the most tedious parts of the test, and to carry it out faithfully requires a great deal of patience. The examination should be minute and very thorough. One who is not familiar with this kind of work will be likely to miss important indications, and careful supervision should be maintained over this part of the investigation.

Special attention has been called to observation of cracks because of incorrect ideas which apparently prevail with regard to them. It seems to be the opinion of some engineers that the type of construction advocated by themselves is immune from cracks. When it is remembered that plain concrete fails in tension at a unit deformation of about .0001, it is apparent that cracks must form when the stress in the steel is such as to correspond with this deformation, or at about 3000 lb. per sq. in. At this stage the cracks are often too small for detection with the unaided eye, but with care in observation almost always very fine cracks can be seen at stresses ranging between 3000 and 10 000 lb. per sq. in. Thus to report for a floor loaded to twice the design load that no cracks were observed is to admit one of three things; that an excess of steel was used, or that sufficient care in taking observations was lacking, or that not all the facts of the case were reported. It should be borne in mind that the cracks referred to in this discussion are often extremely minute and usually are not visible to a casual observer. Frequently cracks have been traced with a lead pencil to make them distinct for the purpose of sketching, and it seems apparent that some persons visiting the test have mistaken these pencil marks for large cracks. At any rate reports have been circulated as to the existence of large cracks in a test where, to the writers' personal knowledge, there were none but minute cracks.

16. *Accuracy of Deformation Measurements.*—The ratio of multiplication in the Berry extensometer is not exactly equal to the ratio of the length of the arm to the length of the leg, the error being due to the fact that the plunger of the Ames gauge head does not always travel in a line perpendicular to the multiplying lever. However, calculations show that this approximation results in an error in the measurement of

steel stresses equal to only about one-quarter of one per cent for an extreme case. It may be seen later that errors of observation are so large in proportion that this error can be neglected.

In forming a basis for a conclusion as to the accuracy of the figures given out as results of tests, use has been made of the check readings taken by two observers on the same gauge lines and of calculated probable error of the mean of five readings. While it is possible to calculate with some accuracy the probable error of replacing the instrument on the same gauge line time after time at one sitting, it is very difficult to determine the error caused by gradually cramping the quarters of the observer as the loading material piles up. Consequently the probable error calculated from a number of readings taken on the same gauge line at different sittings will in general be larger than that calculated from the same number of readings if taken at a single sitting. However, as experience develops skill in replacing the instrument at a single sitting, experience will also increase the consistency of results obtained at widely different times, and the calculated probable error will be a measure of relative, but not of the actual, accuracy of observation. A determination of errors based on independent checking by a second observer should be expected to eliminate to a large extent errors of all kinds and the greatest dependence should be placed on this kind of results.

In the test of the Powers Building most of the observations taken were checked by a second observer and some of the results are shown in the load-stress curves of Fig. 20. The values shown in solid circles were observed by Mr. F. J. Trelease and those in open circles, by Mr. Slater. The zero reading for the latter is in each case at a load of 50 lb. per sq. ft., and in order to make a direct comparison of results, all these curves must be set over so that their zeros coincide with the stress values at 50 lb. per sq. ft. of Mr. Trelease's curves. Having made this correction the average variation between all the comparable points is about 670 lb. per sq. in. (.0000223 unit deformation), which amounts to a probable error of approximately $\pm$340 lb. per sq. in. ($\pm$.000011 unit deformation).

Fig. 21 shows the results of a series of measurements taken in the same way on the upper and lower surfaces of a 4-in. by 4-in. timber beam loaded with sacks of sand on a 12-ft. span. The points in open circles represent measurements on the top surface and those in crosses on the bottom surface. Determined in the same way, these measurements show an average probable error of approximately $\pm$.000017 unit deformation.

In Fig. 22 is given a curve which shows for each of four building tests the probable error of the average of five readings. Each plotted

point is the average of the probable errors calculated for six different gauge lines at a given load. What this diagram may be expected to

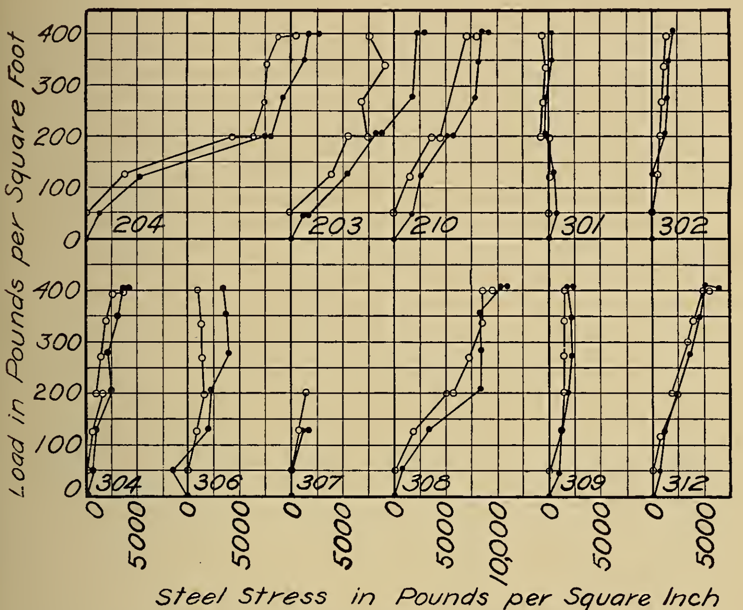

Fig. 20. Powers Building Test; Load-deformation Curves of Two Observers.

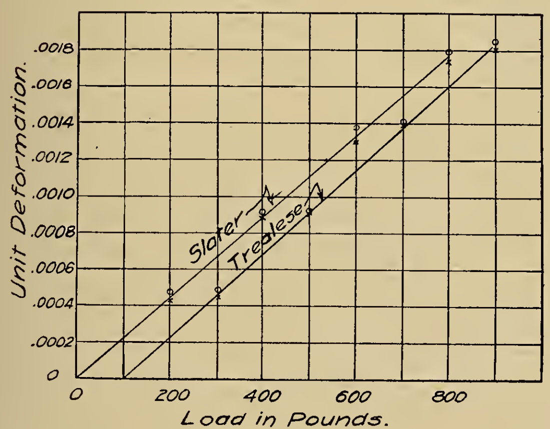

Fig. 21. 4 x 4-inch Timber Beam Test; Load-deformation Curves of Observations Made to Compare Instruments.

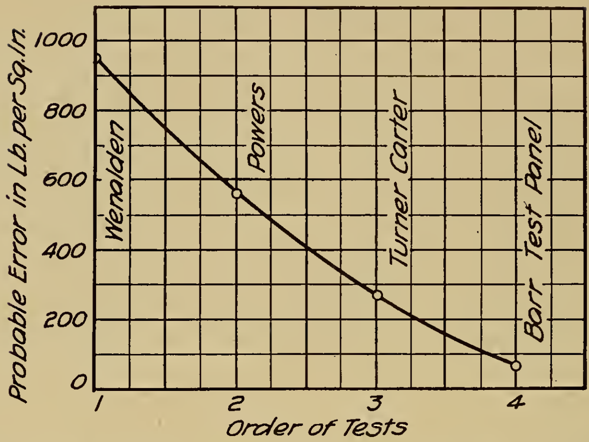

FIG. 22. PROBABLE ERROR; DIAGRAM SHOWING VALUES CALCULATED FROM DATA OF FOUR BUILDING TESTS.

show is the improvement in results with increased experience rather than the actual value of the probable error. The marked improvement in results shown here is due in part to increased skill in the observer and in part to improvement in the instrument itself. Fig. 23 gives a curve showing deformations in steel in a bottom bar of the Barr test panel as shown

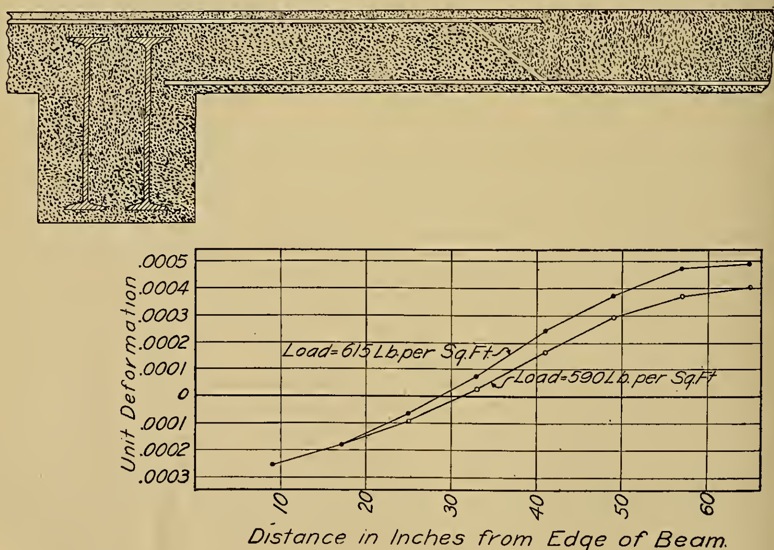

FIG. 23. BARR PANEL TEST; DIAGRAM SHOWING DEFORMATION ALONG BOTTOM REINFORCING BAR.

in the sketch. The points shown as open circles are for a load of 590 lb. per sq. ft. and solid circles are for a load of 615 lb. per sq. ft. This is the best curve the writers have been able to obtain on any building test, and it can not be taken as representative, but rather to illustrate what may be obtained under the best conditions. The regularly varying differences for a small difference of loads indicate that the stresses must have been determined correctly within a very small range.

A study of probable error was made in the Turner-Carter test by the use of a series of 100 observations taken by each of the two observers on two gauge lines selected as likely to give the most and the least accurate results. The results of this study are given in Table 1.

TABLE 1.

PROBABLE ERROR OF THE AVERAGE OF ANY GROUP OF FIVE CONSECUTIVE READINGS.

	Observer	Gauge Line		
		1	2	Average
Unit deformation	H. F. Moore W. A. Slater	.00000687 .0000043	.0000106 .000014	.00000873 .0000091
Stress in steel in lbs. per sq. in.	H. F. Moore W. A. Slater	206 130	318 435	262 282

While these measurements were not all on steel, the probable error has been reduced to terms of stress in steel for convenience of interpretation. It is very interesting to note that the average probable error of ±282 lb. per sq. in. agrees very well with that for the Turner-Carter test as shown in the curve of Fig. 22. The same observer took the data in both cases, but the data for the value shown in Fig. 22 are taken directly from the records of the test and represent the conditions on six typical gauge lines.

From the data in hand it seems safe to conclude that under difficult conditions stresses in steel can be determined with ±500 lb. per sq. in. and that under favorable conditions with careful work it may be determined within ±200 or ±100 lb. per sq. in. The advantage of further increase in accuracy of results lies in the determination of the relations between stresses in different parts of the structure.

17. *Effect of Changes in Temperature on Accuracy of Results.*—Changes of temperature will give measurable changes of length in reinforcing steel, in concrete and in instruments made of ordinary materials. In most of the building tests, corrections have been made for the changes in the instrument due to changes in temperature by means of observations on standard unstressed gauge lines chosen to represent

as nearly as possible the conditions of the steel and the concrete in the part of the structure tested. The method of calculating this correction will be described in a later paragraph. It is there mentioned that in distributing the corrections found by reference to the standard bar, a linear variation from the time of one standard observation to the time of the next standard observation was assumed. Some observations have been made to determine the correctness of this assumption.

To determine the amount of change in length of an aluminum extensometer covered and uncovered, a test was made in which the two instruments were suddenly exposed to a change of temperature of 60° F. A covering which consisted of a double layer of rather heavy felt protected one of the instruments from a sudden change in temperature. The other instrument was entirely unprotected. The results of this test are shown in Fig. 24 with the change of length of the instrument plotted as ordinates against time as abscissas. For these measurements a standard bar of invar steel was used. The coefficient of expansion of this being very small, the change of length measured must have been almost entirely that in the instrument. The curve shows that for an instrument not insulated from temperature changes, only about five minutes is required for the instrument to come to the temperature of the air. For the insulated instrument about 20 minutes was required. This may be interpreted to mean that if an unprotected instrument is used, readings on the standard bar should not be more than five minutes apart.

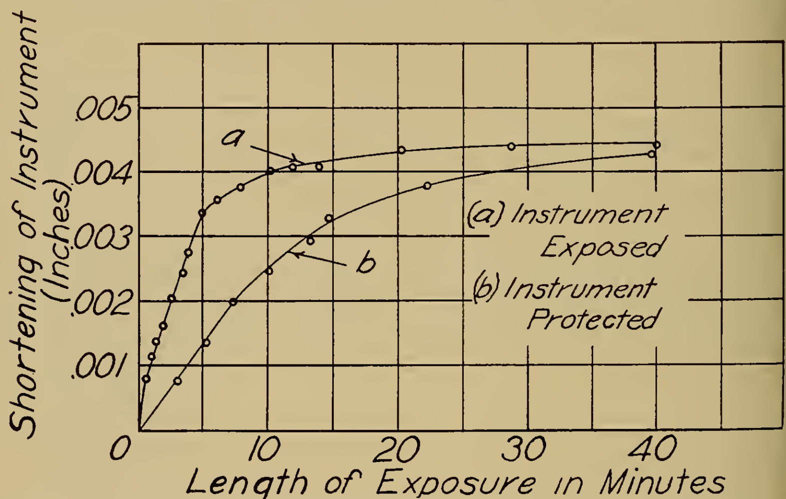

Fig. 24. Diagram Showing Change in Length of Instruments Due to Change in Temperature.

With an instrument protected as was this one, intervals of 20 minutes would not be too much. The amount of change for the case shown here is extreme, as the instrument was suddenly exposed to a change in temperature of about 60° F. This change would seldom be found, and the length of time required to make the change for a smaller difference of temperature may be less, but probably would not vary much for other ranges of temperature. It may be concluded that the method used for distributing the correction is justifiable, since the instrument was protected from sudden change of temperature and the observations on standard bars were usually at intervals not greater than 20 minutes.

The above test shows the effect on the instrument of change in temperature. Another test was made to determine the effect of change in temperature on steel embedded in concrete and on steel exposed to the air. A 3/8-in. square bar of steel entirely unprotected from temperature changes and a 3/8 in. round bar embedded in 1 in. of concrete were exposed to a sudden change of temperature of about 43° F. Measurements were taken on a 6-in. gauge length of each bar at very short intervals of time. The results are shown in Fig. 25. The results of this single

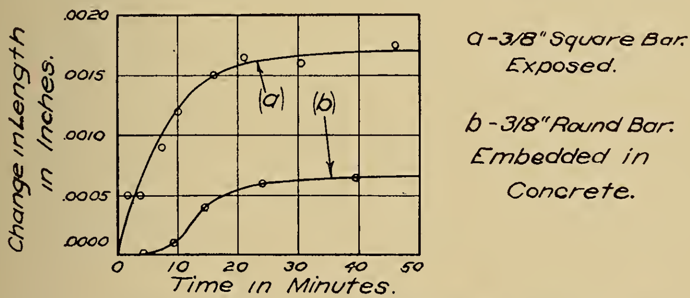

FIG. 25. DIAGRAM SHOWING CHANGE IN LENGTH OF STEEL BAR DUE TO CHANGE IN TEMPERATURE.

test must be used with caution as the total measured change in length was very small and a small error would show up very plainly. However, the curve for the embedded bar agrees in its general characteristics with some of the results obtained by Professor Woolson on "Effect of Heat on Concrete" reported in the 1907 Proceedings of the American Society for Testing Materials. The test indicates that for this range of temperature rather rapid changes may be found in the steel, corresponding to stresses of about 9000 lb. per sq. in. and 3000 lb. per sq. in., respectively, for exposed steel and steel protected as in this case. The range of temperature is extreme and the size of bars smaller than is

TABLE 2.

FORM SHOWING METHOD OF REDUCING DEFORMATION DATA.

Load	Interval	0		1	2	n−2	n−1	n	
	Gauge Line	Standards		Gauge Line Numbers				Standards	
		a	b	101	104	108	116	a	b
Dead Load Only	Uncorrected Average	S_a	S_b	R_1	R_2	R_{n-2}	R_{n-1}	S'_a	S'_b
	Correction	0	0	$\frac{1}{n}\frac{C'_a+C'_b}{2}=C_1$	$\frac{2}{n}\frac{C'_a+C'_b}{2}=C_2$	$\frac{n-2}{n}\frac{C'_a+C'_b}{2}=C_{n-2}$	$\frac{n-1}{n}\frac{C'_a+C'_b}{2}=C_{n-1}$	$S_a-S'_a=C'_a$	$S_b-S'_b=C'_b$
	Corrected Zero Average			$R_1+C_1=A_1$	$R_2+C_2=A_2$	$R_{n-2}+C_{n-2}=A_{n-2}$	$R_{n-1}+C_{n-1}=A_{n-1}$		
Any Live Load P	Uncorrected Average	s_a	s_b	r_1	r_2	r_{n-2}	r_{n-1}	s'_a	s'_b
	Uncorrected Difference			$A_1-r_1=d_1$	$A_2-r_2=d_2$	$A_{n-2}-r_{n-2}=d_{n-2}$	$A_{n-1}-r_{n-1}=d_{n-1}$		
	Correction	$S_a-s_a=c_a$*	$S_b-s_b=c_b$*	$c_{ab}+\frac{1}{n}(c'_{ab}-c_{ab})=c_1$	$c_{ab}+\frac{2}{n}(c'_{ab}-c_{ab})=c_2$	$c_{ab}+\frac{n-2}{n}(c'_{ab}-c_{ab})=c_{n-2}$	$c_{ab}+\frac{n-1}{n}(c'_{ab}-c_{ab})=c_{n-1}$	$S_a-s'_a=c'_a$†	$S_b-s'_b=c'_b$†
	Corrected Difference			$d_1-c_1=e_1$	$d_2-c_2=e_2$	$d_{n-2}-c_{n-2}=e_{n-2}$	$d_{n-1}-c_{n-1}=e_{n-1}$		

* Put $\frac{c_a+c_b}{2}=c_{ab}$

† Put $\frac{c'_a+c'_b}{2}=c'_{ab}$

often found in floor construction; therefore the results found in tests would probably be less extreme. However, this indicates the necessity of attempting to eliminate from the results of the test the effect of temperature changes, especially if the stresses measured are small.

18. *Records and Calculations.*—Since the beginning of the use of the Berry extensometer for testing purposes, as much development has been made in the keeping of notes as in the use of the instrument. Because of a lack of completeness of notes the advantages of the use of the standard bar were not fully realized for some time. Only after the method of keeping notes had been highly systematized was it possible to make properly the corrections which observations on the standard bars indicated should be made. During the time of placing an increment of load the recorder will have considerable time in which to be working up results of the series of observations taken at the previous increment of load, and as the method of making these calculations is quite intricate, a man is required for this work who has ability to do more than merely record.

It is very important on account of the great number of observations taken (about 12000 in the Turner-Carter test) that all records be arranged systematically. The following points are mentioned as being important in this connection: (1) In the field test individual readings should be recorded and their average used as a single observation unless it appears that the proposed abridgment of this procedure (see page 35) may be used safely. (2) Recording readings in the order of their size will assist the recorder in obtaining the correct readings and in rapidly obtaining the average. (3) The exact sequence of observations should be maintained in the record as the calculations of corrections depends largely on this.

Fig. 26 is a sheet for the record of original readings and the results calculated from them.

Calculating corrections and applying them to the results make the reduction of data rather intricate. This work has been reduced to a definite system indicated by the form shown in Table 2. In this system the first observation on the standard bar is used as the reference observation (see definition, p. 10). The corrections are distributed among the gauge lines as though the change in the length were a linear function of the time from one standard bar observation to the next one. These assumptions do not entirely accord with the facts but have been found satisfactory as a working basis. Any other standard bar observation than the first one would do as well for a reference observation except for matters of convenience. It is important that calculations should be

kept up as the work progresses, because it can be done with less labor then than at any other time and because it will be of value to know as the test progresses what results are being secured.

19. *Test Data.*—Table 3 gives general data of the tests referred to in this discussion. The figures giving area of test show the total area

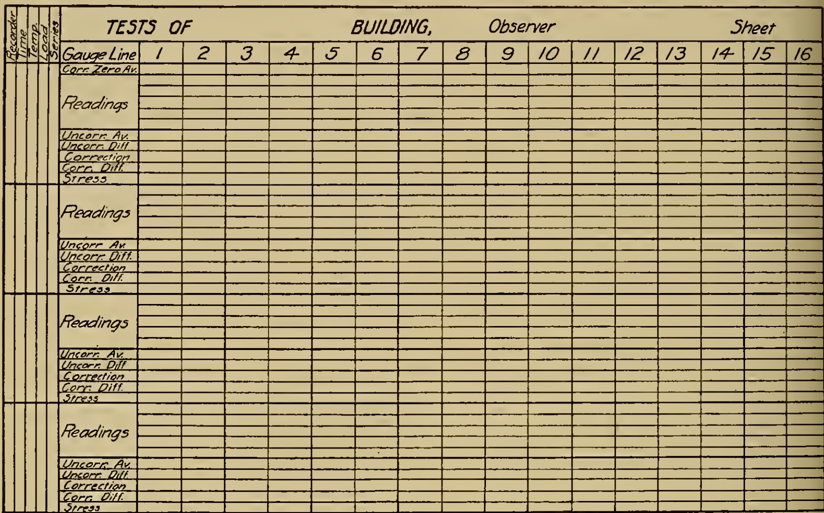

FIG. 26. FORM FOR RECORDS OF ORIGINAL AND CALCULATED NOTES.

of the floor covered, and do not count any area twice, even though loaded twice as was done in the Wenalden Building test. They do include the area of separate single panel tests which were made in the Wenalden and Franks tests.

The maximum test load in lb. per sq. ft. is given in the column under that caption. In some cases this was over only a part of the test area. The part upon which the maximum load was applied bore the following ratios to the total test area: Wenalden 80 per cent, Powers 50 per cent, Franks 40 per cent, Larkin 40 per cent, all others 100 per cent.

The column giving the amount of load handled includes the rehandling due to change of position of loads. The proportionate parts of the loads rehandled in this way were: Wenalden 40 per cent, Powers 50 per cent, Franks 80 per cent, Larkin 73 per cent. In all the other tests no load was rehandled.

The column giving the number of observers includes only those reading deformations. In the Wenalden, Powers and Larkin tests another

TABLE 3.

General Data of Tests

Building	Type	Test Area		Loading Material	Design Load lb. per sq. ft.	Test Load lb. per sq. ft.	Load Handled tons	Number of Gauge Lines	Number of Observers	Days Required for	
		In sq. ft.	Per cent of Total Area							Preparation	Test
Deere and Webber	Flat Slab	2850	14.2	Brick and Cement	225	350	500	26	3	7	5
Wenalden	Beam and Girder	1800	3.5	Brick and Cement	250	400	420	52	3	8	7
Powers	Flat Slab	1070	40.0	Cement	200	400	174	66	2	..	..
Franks	Flat Slab	1959	16.6	Pig Iron	250	624	430	95	3	..	..
Turner-Carter	Beam and Girder	1690	14.1	Sand in Boxes	150	300	260	104	2	7	10
Barr	Reinforced Concrete and Tile	1270	100.0	Sand in Sacks	150	650	348	71	3	..	9
Carleton	Flat Slab	440	1.5	Brick	150	400	90	30	1	1	..
Larkin	Flat Slab	2450	10.0	Sand in Bins	225	618	500	268	3	5	6

observer took deflection readings. In the Powers test and the Barr test, almost all the deformation readings were taken by each of two observers, giving a larger number of gauge lines per observer than appears in the table.

20. *Cost of the Tests.*—An effort was made to learn the cost of making the building tests in which the stresses in the structure were observed, but difficulty was found in separating the items connected with the tests from those incidental to the building construction. The expense of such a test depends upon the size of the test area as well as upon the number of gauge lines used. The loading of a single panel gives little information, and this information may be misleading in regard to the maximum stresses which will be developed in such a panel when the adjacent panels also are loaded. A test of a floor system should involve the loading of as many as five panels; a greater number would be more representative of full loading. The application and removal of 600 000 lb. of load involves considerable expense, especially if this material has to be brought to the building and later taken away. This item may be from $300 to $700 depending upon the distance the loading material is conveyed. The cost of building the platforms and drilling and cutting the holes for the gauge points may be counted to be about $100. A thorough test will take a week's time of the observers and two weeks' time of the one in charge of the test even though the test itself may not run over five days. A well organized party of three observers and three recorders was able to take the observations on 268 gauge lines and record and work up the data as the test progressed. This involved placing the load in four increments and removing it in two increments and the test itself covered a period of six days. To make an adequate report of such a test is itself quite a task, and the expense of this item is considerable. A much smaller amount of work will give special information on a few matters. The data at hand indicate that a thorough test may cost as much as $1500 for all items and in one test mentioned the total cost was more than $2000.

III. The Wenalden Building Test.

21. *The Building.*—The Wenalden Building, Fig. 27, is a ten-story reinforced concrete structure at 18th and Lumber streets, Chicago. It was built by the Ferro-Concrete Construction Company, Cincinnati, Ohio, in accordance with the plans and specifications of Howard Chapman, architect. It is now occupied by Carson, Pirie, Scott and Company, dry goods merchants, as a warehouse. The building is of the beam and girder type. The floor panels are 15 ft. by 20 ft. The gird-

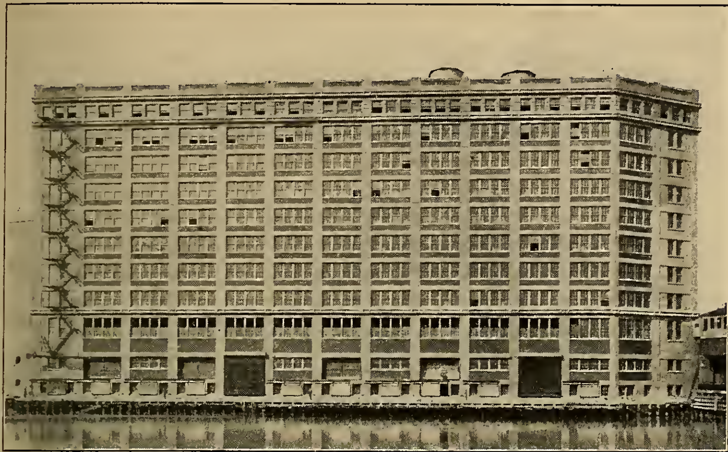

FIG. 27. THE WENALDEN BUILDING.

ers are placed between columns in the short direction. Floor beams extend the long way of the panel, there being two intermediate beams built into and supported by the girders and a column beam built into and supported by the columns. The floor, 3⅞ in. thick (including the top finish), was built continuously with the beams and girders.

The reinforcement is of the form used by the Ferro-Concrete Construction Company. The main reinforcing bars (twisted bars) are carried along the bottom of the beam from the end of a panel to a point beyond the middle of the panel, where they are bent up to the top of the beam and carried horizontally to a corresponding point on the other side of the support, then bent down and continued along the bottom of the beam to the end of the next panel, these reinforcing bars thus having a length of two panels. In the intermediate beams at the bottom and middle there are four rods ¾ in. square and in the side beams one rod ¾ in. square and three rods ⅝ in. square. In the girders there are four rods ⅞ in. square, the disposition of which is similar to that in the beams. By this plan there is twice as much of the main reinforcement in the bottom of the beam or girder at the middle of the span as there is at the top over the supports, except that four ⅜ in. square rods placed in the floor slab are also available for end reinforcement of the intermediate beams. The beams are 6¼ in. and the girders 7½ in. wide. The general position of the reinforcement is shown in Fig. 28. The position of the vertical stirrups is not shown.

The contractors report that the concrete was composed of one part portland cement, 2 parts torpedo sand, and 4 parts crushed limestone.

Although the building was not fully completed when the test was made, the floor tested had been built more than 12 months at the time of the test. The work of cutting the floor and beams for inserting points of measurement proved to be very difficult and showed the concrete to be very hard and of excellent quality.

22. *Method of Testing.*—The test was made on the first floor of the building. This was the only one which could be reached with the loading material. The space chosen was one freest from openings and other

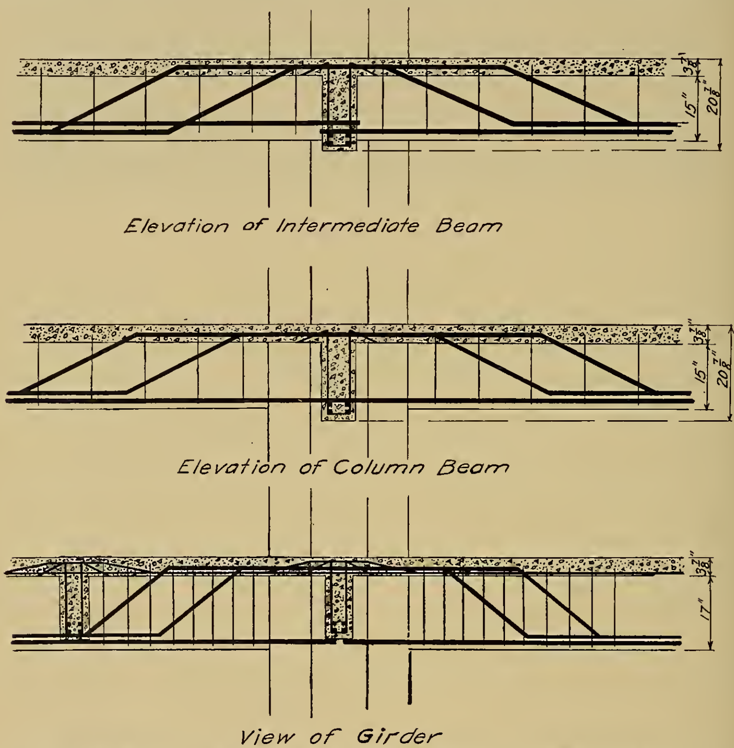

FIG. 28. GENERAL POSITION OF REINFORCEMENT IN WENALDEN BUILDING.

irregularities of construction. At various points at the top and bottom of beams, holes were cut into the concrete until the reinforcement was bared and gauge holes were drilled in the bars 6 in. or 10 in. apart for use in inserting the instruments with which the measurements of elongation were made. Where stresses in the concrete were to be measured, holes were cut in the concrete and short pieces of steel were set in plaster of paris. Gauge holes were drilled in these steel inserts in such a way as to give gauge lines 6 in. or 10 in. long. The position of these points is

ment and in the concrete at various points in the girders, beams and slabs. The most important determinations undertaken in the test were the measurement of the compressive deformations in the concrete at and

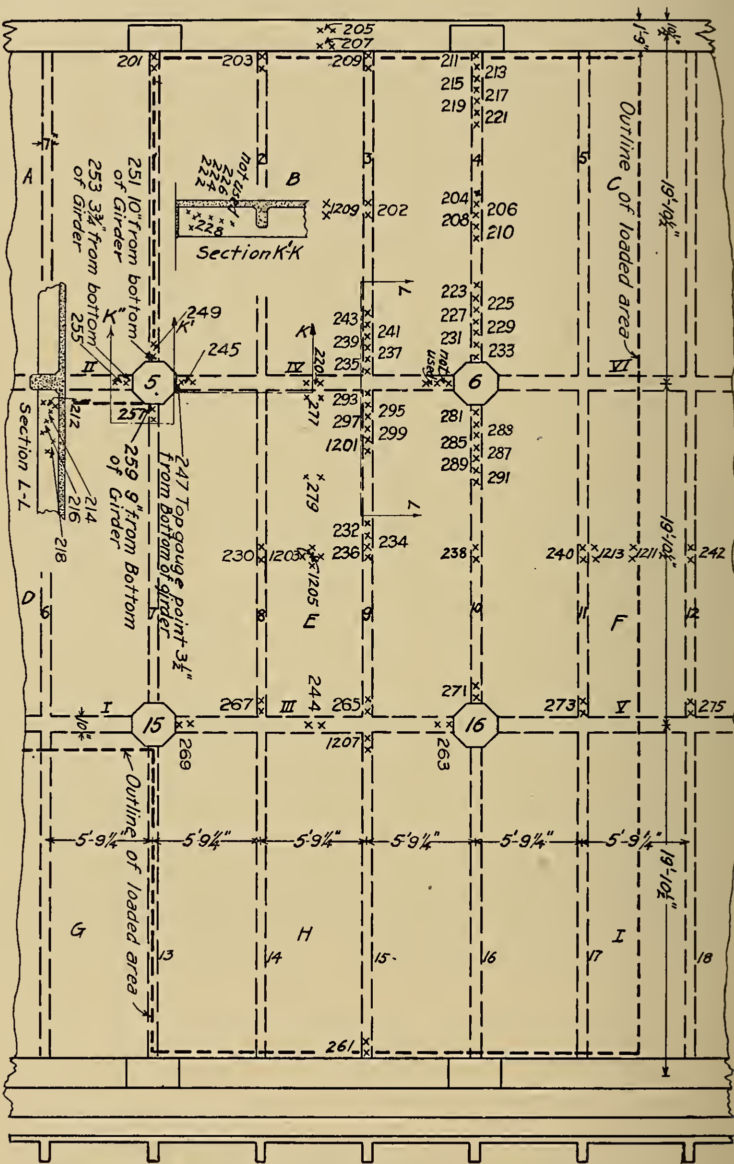

FIG. 42. PLAN SHOWING LOCATION OF GAUGE LINES ON UNDER SIDE OF FLOOR.

near the supports of the beams, the compressive deformations of the concrete at the middle of the beam, and the distribution of these compressive stresses across the top of the slab between beams to determine the extent of T-beam action. The deformation in the reinforcement was

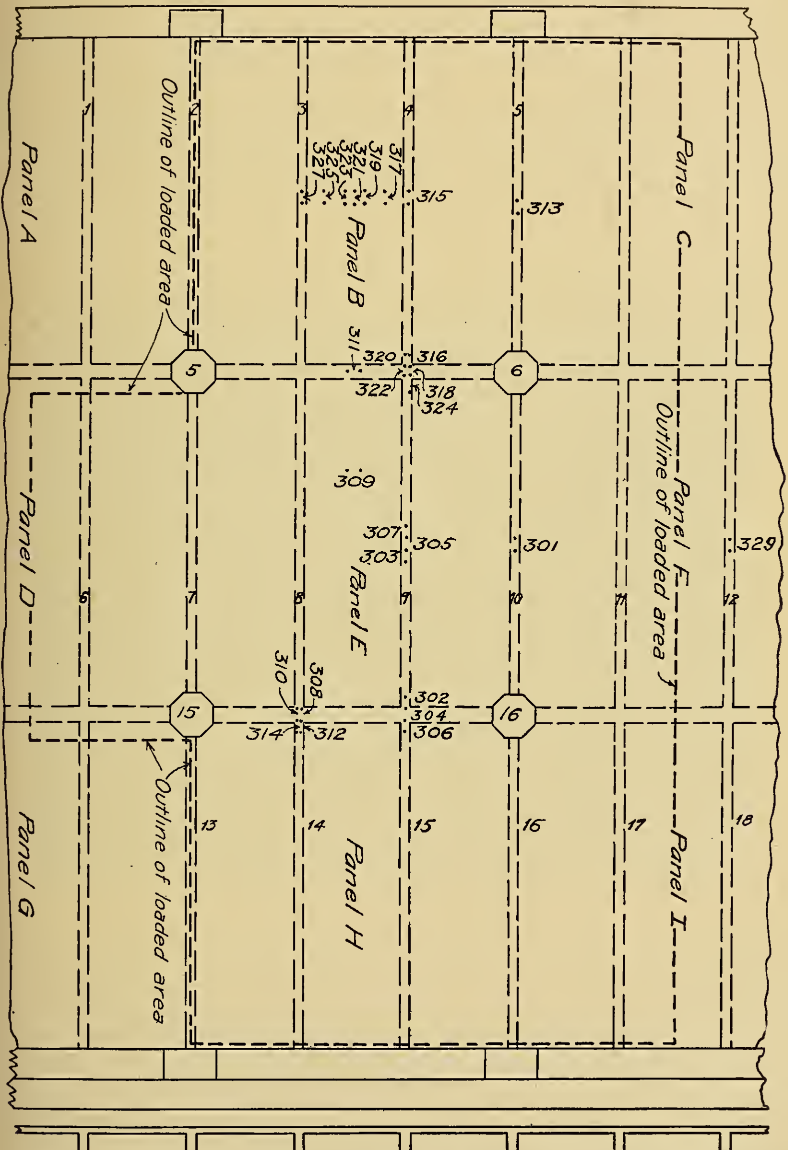

Fig. 43. Plan Showing Location of Gauge Lines on Upper Side of Floor.

shown in Fig. 29 and 30. For the work of measuring deflections, steel balls were affixed to the under side of beams and girders at various places and other balls were placed about 7 in. lower on supports which had been

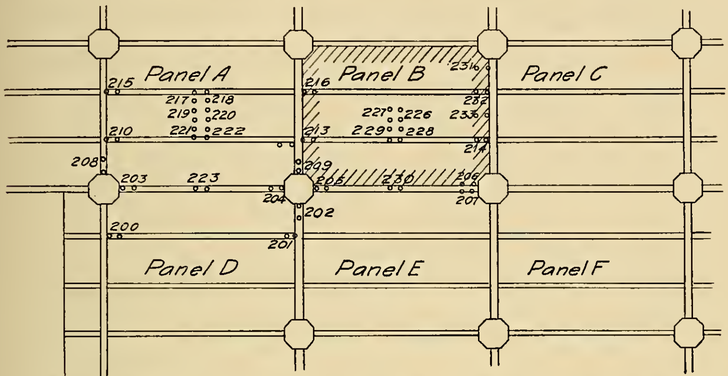

FIG. 29. PLAN SHOWING LOCATION OF GAUGE LINES ON UPPER SIDE OF FLOOR.

built up independently of the observing platforms. A number of these points of deflection were used to determine the inflection points of the beams.

For any observation several instrument readings, usually five, were taken on each gauge line and these were averaged. Measurements were

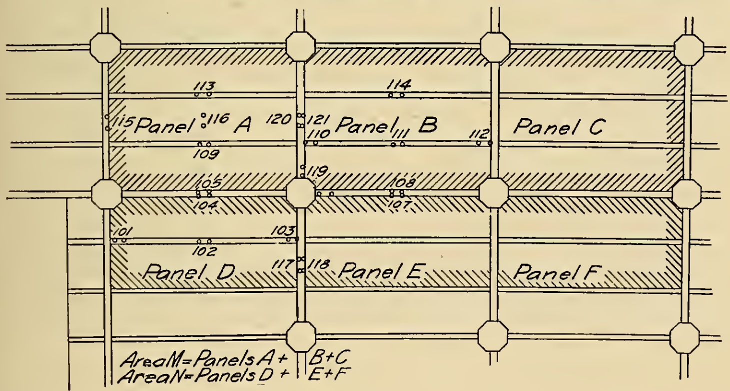

FIG. 30. PLAN SHOWING LOCATION OF GAUGE LINES ON UNDER SIDE OF FLOOR.

made on the standard bar before and after each series of observations to permit corrections for instrumental changes.

23. *Method of Loading.*—The floor was designed for a live load of 200 lb. per sq. ft. and the test load was made 400 lb. per sq. ft. over the panels loaded.

The loading was done by piling brick and bags of cement in piers separated by aisles in such a way as to give access for points of measurements and to prevent arching effects influencing the tests. The load was put on in increments of about 80 lb. per sq. ft. of the total panel area, and a set of observations was taken at each increment of load. Brick was used in the first part of the loading and cement in the later work. The average weight of the brick was determined by weighing a considerable number, and such care was given to determine the number of brick and sacks of cement that it is believed the weights are accurately known.

The following is the general plan of the test. A single panel (*B*, Fig. 29) was first loaded. This load was then removed. The load was then applied on three panels in tandem (*ABC*, Fig. 30). These three panels are termed area *M* in the load-deformation diagrams. Then, leaving this load on, a load was applied along three adjacent panels (*DEF*) covering two-thirds of the width and making in all the equivalent of five loaded panels. This portion of these three panels is termed area

FIG. 31. VIEW OF TEST LOAD IN WENALDEN BUILDING.

N in the load-deformation diagrams. The load was then taken off by increments. The total weight of the load used was 600 000 lb. Fig. 31 is a view of the test load.

A further test was made by loading a single panel at the end of the building, a so-called wall panel.

The loading was begun Monday, July 10, 1911, the loading of five panels was finished at noon on the following Friday, and unloading was

completed on Monday, July 17. The schedule of loading is given in Table 4. The wall panel was loaded on Friday and Saturday, July 14 and 15. The unloading of this panel was finished August 1.

24. *The Deformations and Stresses.*—The results of observations for various gauge lines are plotted in Fig. 32, 33 and 34. From a

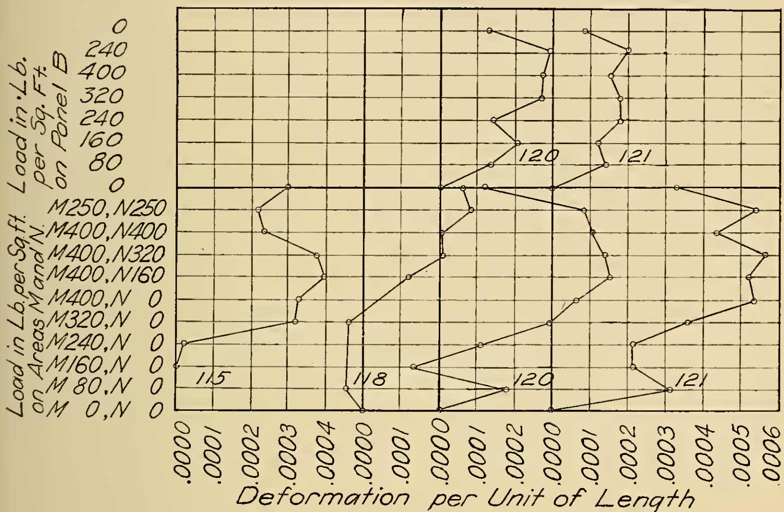

FIG. 32. LOAD-DEFORMATION DIAGRAMS FOR UNDER SIDE OF GIRDER AT MIDDLE.

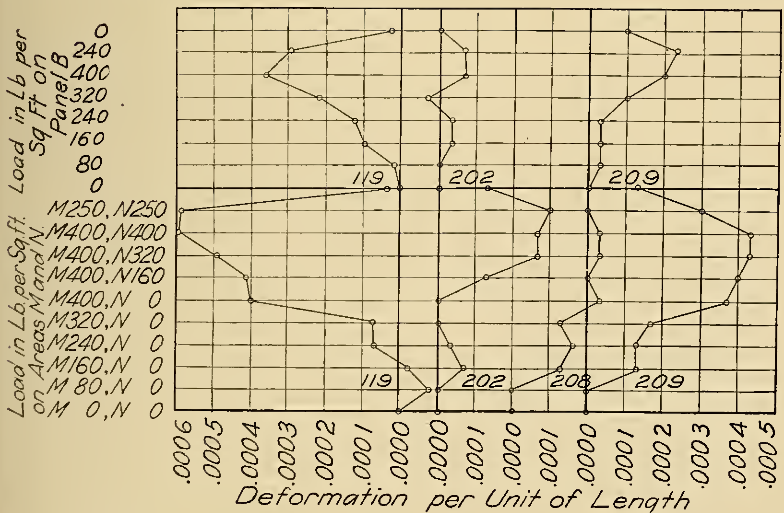

FIG. 33. LOAD-DEFORMATION DIAGRAMS AT END OF GIRDER.

TABLE 4.

Schedule of Loading Operations.

Day	Date	Observations		Loading		Observations	
		Load lb. per sq. ft.	Hours	lb. per sq. ft.	Hours	Load lb. per sq. ft.	Hours
Sunday....	7–9–11	0	5.45 P. M.				
Monday....	7–10–11	0	7.00 to 8.00 A. M.	80 *B*	8.00 to 8.50 A. M.	80 *B*	8.50 to —— A. M.
				160 *B*	—— to 10.25 A. M.	160 *B*	10.25 to —— A. M.
				240 *B*	—— A. M. to 12.10 P. M.	240 *B*	12.10 to —— P. M.
				320 *B*	—— to 3.00 P. M.	320 *B*	3.00 to —— P. M.
				400 *B*	—— to 5.10 P. M.	400 *B*	5.10 to —— P. M.
Tuesday....	7–11–11	400	6.00 to 8.00 A. M.	240 *B*	8.00 A. M. to ——	240 *B*	11.00 A. M. to ——
				0 *B*	12.30 to 3.30 P. M.	0 *B*	3.30 to 4.00 P. M.
Wednesday.	7–12–11	0	6.45 to —— A. M.	80 *A, B* and C.	8.00 to 10.00 A. M.	80 *A, B* and C.	10.00 to 10.45 A. M.
				160 *A, B* and C.	10.45 A. M. to 1.00 P. M.	160 *A, B* and C.	1.00 to —— P. M.
				240 *A, B* and C.	1.30 to —— P. M.	240 *A, B* and C.	—— to 2.45 P. M.
				320 *A, B* and C.	3.20 to 5.00 P. M.	320 *A, B* and C.	5.00 to 6.30 P. M.
Thursday...	7–13–11	320 *A, B* and C.	6.30 to 7.45 A. M.	380 *A, B* and C.	8.00 to 9.45 A. M.	380 *A, B* and C.	9.45 to 10.30 A. M.
				400 *A, B* and C.	10.30 to 11.15 A. M.	400 *A, B* and C.	11.15 A. M. to 12.00 M.
				400 *M* 160 *N*	11.30 A. M. to ——	400 *M* 160 *N*	3.15 to 3.45 P. M.
				400 *M*	3.45 to 4.30 P. M.		
Friday.....	7–14–11			320 *N*	8.00 to 9.30 A. M.		

TABLE 4.

SCHEDULE OF LOADING OPERATIONS—*Continued.*

Day	Date	Observations		Loading		Observations	
		Load lb. per sq. ft.	Hours	lb. per sq. ft.	Hours	Load lb. per sq. ft.	Hours
Friday.....	7–14–11			400 *M* 400 *N*	10.30 to 11.30 A. M.	400 *M* 400 *N*	11.30 A. M. to 12 M.
				80 *G*	2.00 to 3.00 P. M.	80 *G*	3.40 to —— P. M.
				160 *G*	—— to 5.00 P. M.	160 *G*	5.00 to 5.30 P. M.
Saturday...	7–15–11			250 *M* 250 *N*	8.00 to —— A. M.	250 *M* 250 *N*	11.40 A. M. to 1.30 P. M.
				240 *G*	8.15 to 9.00 A. M.	240 *G*	9.00 to —— A. M.
				320 *G*	8.40 to —— A. M.	320 *G*	9.50 to 10.00 A. M.
				380 *G*	—— to 10.45 A. M.	380 *G*	10.45 to 11.15 A. M.
				400 *G*	—— to 11.40 A. M.	400 *G*	—— A. M. to 12.00 M.

study of these it is readily seen that there are irregularities in the measurements and that the general trend of some of the lines must be taken rather than absolute values.

In translating from unit-deformation to unit-stress the modulus of elasticity of steel has been taken at 30 000 000 lb. per sq. in. and that of the concrete has been assumed to be 3 000 000 lb. per sq. in. For simplicity the straight-line stress-deformation relation for concrete has been assumed, though it is evident that this relation does not hold for the higher stresses and that calculated stresses based upon this assumption are in excess of the actual stress. The interpreted stress for a number of gauge lines is recorded in Table 5.

Table 6 gives calculated stresses and calculated bending moment coefficients. The first line of each set gives the calculated stresses in the reinforcement and in the concrete based upon the value of the bending moment quite commonly assumed in design calculations, 1/12 Wl, where W is the total distributed load on the beam and l is the span length. These are printed in italics. In these cases the span length was taken as

TABLE 5.

Stress Indications in Wenalden Building Test.

Stresses are given in pounds per square inch.

	Gauge Line.	Single Panel.	Three Panels.	Five Panels.
Reinforcement at end of girder................	208 209 202	...... 7 000	8 000 12 000	7 000 13 000 9 000
Reinforcement at middle of girder............	115 120 121	...... 8 000 6 000	10 000 14 000 16 000	11 000 17 000 17 000
Concrete at end of girder....................	119	1 100	1 600	2 200
Reinforcement at end of intermediate beam.....	210 211 212 213 214 216	...... 2 000 9 000 9 000 11 000 13 000	9 000 9 000 14 000 16 000 13 000 13 000	10 000 14 000 14 000 16 000 13 000 14 000
Reinforcement at middle of intermediate beam..	109 111 113 114	...... 8 000 ...... 6 000	14 000 7 000 16 000 11 000	16 000 11 000 16 000 11 000
Concrete at end of intermediate beam..........	110 112	...... 1 500	...... 1 700	1 300 2 000
Concrete at middle of intermediate beam.......	217 222 229	Low " "	Low " "	Low " "

TABLE 6.

Maximum Stresses and Moment Coefficients in Wenalden Building Test.

Stresses are given in pounds per square inch.

Member	Reinforcement		Concrete	
	Stress	Coefficient	Stress	Coefficient
Girder, End............................	*44 000*	*1/12*	*1 700*	*1/12*
" End............................	13 000	0.024	2 200	0.106
" Middle.........................	*19 000*	*1/12*	*420*	*1/12*
" Middle.........................	17 000	0.075	.....	..
Intermediate Beam, End................	*36 000*	*1/12*	*1 900*	*1/12*
" End................	16 000	0.037	2 000	0.088
" Middle.............	*22 000*	*1/12*	*440*	*1/12*
" Middle.............	16 000	0.06	.....	..
Column Beam, End......................	*69 000*	*1/12*	.....	..
" End......................	15 000	0.018	.....	..
" Middle...................	*26 000*	*1/12*	.....	..
" Middle...................	11 000	0.035	.....	..

3 in. longer than the clear span. Measurements had been made upon the position of the bars and the depth of the reinforcement, which were not always exactly according to the plans, and the calculations have been

based upon the dimensions found. In the second line of each group the maximum stress obtained by the measurements is given in the column of stresses, and the bending moment coefficient (the coefficient of Wl) corresponding to these stresses is recorded in the adjacent column. In these calculations the common assumptions of design calculations (including the neglect of the tensile strength of the concrete) are followed except that the width of T-beam is taken as equal to the distance from center to center of beams. In calculating the bending moment coefficient from the measured stress, the position of the neutral axis and the value of the moment arm are assumed to be the same as given by the ordinary assumptions. Although the stress in the reinforcement is measured at the surface of a bar of the outer layer, this stress is considered as being the same as that acting at the center of gravity of the group of bars, for the actual variation in the group is unknown and this method will give a bending moment coefficient larger than that found by considering that the stress in the bars of the other layer is smaller.

In the calculations for compressive stresses, the compression reinforcement was considered to take its proportion of the compressive stress though there may be a question whether the embedment in such designs is sufficient to insure this condition.

It will be seen that in the tests with three and five panels loaded the highest stress observed in the reinforcement in the middle of the intermediate beams was 16000 lb. per sq. in. and the highest stress observed at the ends of the beams was 16000 lb. per sq. in. The stresses observed in other bars having similar positions were lower, and probably the highest stress is not representative of the general stresses. However, it may be best to compare on the basis of the highest stresses. The bending moment given in the table as derived from the measured stresses is .06 Wl at the middle of the beam and .037 Wl at the end of the beam. Measurement of the compression of the concrete in this test was less satisfactory than the measurement of the reinforcement deformations, and considerable variation was found at the different points of observation. Not enough gauge lines gave satisfactory measurements to warrant making quantitative conclusions, but the indications of the action of the concrete may be useful. The value of the resisting moment which corresponds to the concrete stresses, on the assumptions made, is .088 Wl for the end of the beam. For the middle of the beam the stresses were small and the indications so irregular that no value of resisting moment can be given.

In the beams at the sides of the panel (column beams) the stresses were in general somewhat lower, but with a full loading a stress of 15000 lb. per sq. in. at the middle of the beam was observed and 11000 lb. per sq. in. at the end.

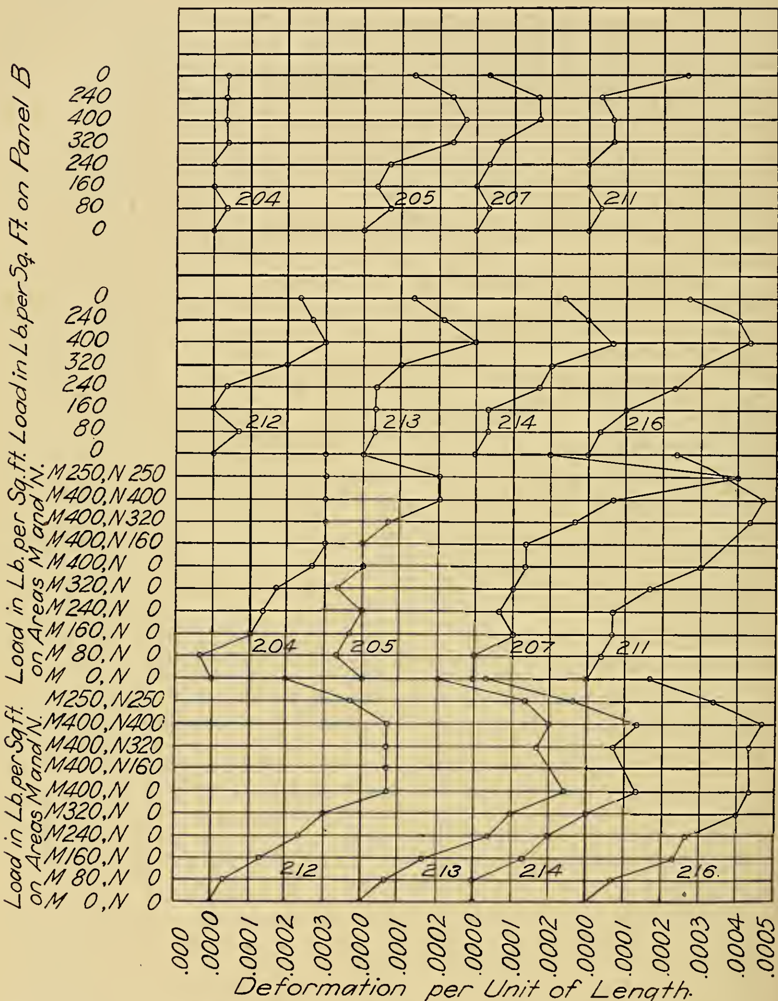

FIG. 34. LOAD-DEFORMATION DIAGRAMS FOR UPPER SIDE OF BEAMS AT END.

Fewer measurements were made on the reinforcement of the girder. A stress of 17 000 lb. per sq. in. was observed at the middle of the girder and 13 000 lb. per sq. in. at the end. On the same assumptions these values correspond to bending moments of .075 *Wl* and .024 *Wl*, respectively. The stress in the concrete at the end of the girder was also very high, but the corresponding bending moment (.106 *Wl*) is not far from the calculated moment for a restrained beam with concentrated

load. It should be noted in this connection that the reinforcement is bent down rapidly into the beam from the face of the column, see Fig. 28.

Calculating with the usual assumptions of beam formulas, the total compressive stress in the concrete at the end of the beam is greater than the total tensile stress in the reinforcement. Two elements probably enter into the results, the tensile strength of concrete, which may be considerable as distributed over the width of the floor, and an arching action of the structure. However, it should be noted that the value of the bending moment coefficient derived from the reinforcement stresses at the middle of the beams and girder is not much less than values commonly used and also that the calculated resisting moment developed at the end of the beam based on the concrete stresses is not far from the amount usually assumed.

Attention should be called to the fact that the compressive stress in the concrete, both that calculated from an assumed bending moment coefficient and that calculated from the measured deformation, is much higher than that to be found by the use of the parabolic stress deformation relation and the actual stress will be less than that given in the table.

Measurements were made on the concrete at the top of the floor slabs in a direction parallel with the beams to find the distribution of compressive stresses between beams. These measurements were not fully satisfactory, but within the limits of accuracy of the measurements, no difference in the amount of shortening over the beam and at points between beams could be determined, and the whole floor evidently acted as a part of the compression member of the T-beam so formed.

25. *Test Cracks.*—The surface of the beams and girders had received a white coat, which permitted very fine cracks to be detected, much finer than may be observed on uncoated concrete. In the test, as the load was applied, fine tension cracks in the concrete through the middle of the length of the beam were observable at stresses in the reinforcement corresponding to the stresses at which load cracks are detected in the tests of beams in laboratory work. To an experienced observer development of the cracks was confirmation of the measurements of the low stresses developed in the reinforcement. Upon removal of load most of these cracks closed until they were not visible to the eye.

As the calculated reaction on the end of a girder was upward of 40000 lb., it will be seen that the vertical shearing stresses were very high. Diagonal tension cracks developed in these girders just outside the junction with the intermediate beams, making an angle of nearly 45° with the horizontal. These cracks did not entirely close on the

removal of the load. No measurements were taken to determine the diagonal deformations. It seems probable that the restraint at the end of the girder and the tensile strength of the concrete acted to prevent the fuller development of these cracks.

No diagonal cracks were observed in the beams.

26. *Deflections.*—Fig. 35 gives the location of the points at which the deflections were measured. Fig. 36 shows the deflections with a load

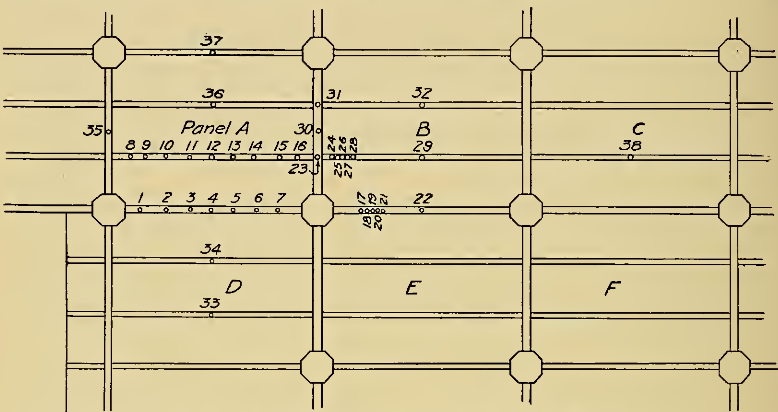

Fig. 35. Location of Deflection Points in Wenalden Building.

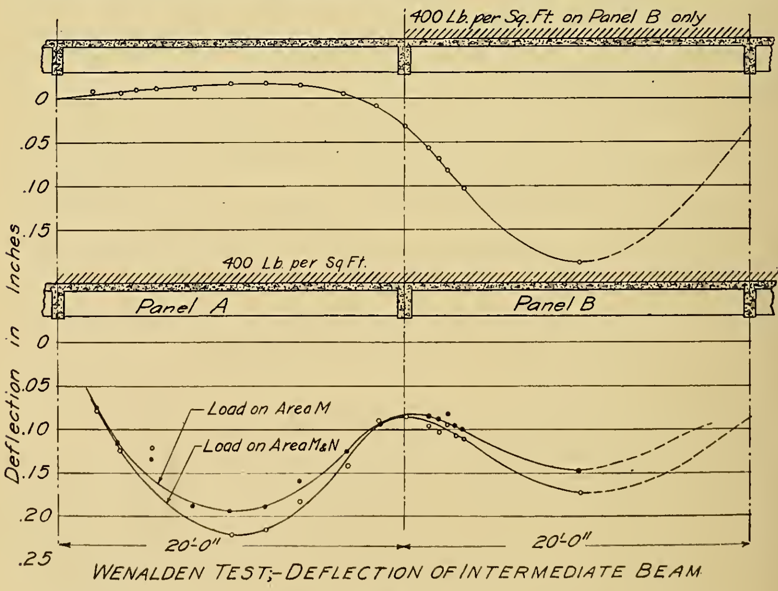

Fig. 36. Diagrams Showing Deflection of Intermediate Beam.

of 400 lb. per sq. ft. for points along an intermediate beam, (1) with one panel loaded (panel *B*, Fig. 29) and (2) with three panels loaded (area *M*, Fig. 30), and (3) with five panels loaded (areas *M* and *N*, Fig. 30). As may be expected, the deflection in the middle panel is greater for one panel loaded than when three panels are loaded.

27. *Wall Panel.*—A single wall panel was loaded and observations were taken on the gauge lines which are shown in Fig. 37. No meas-

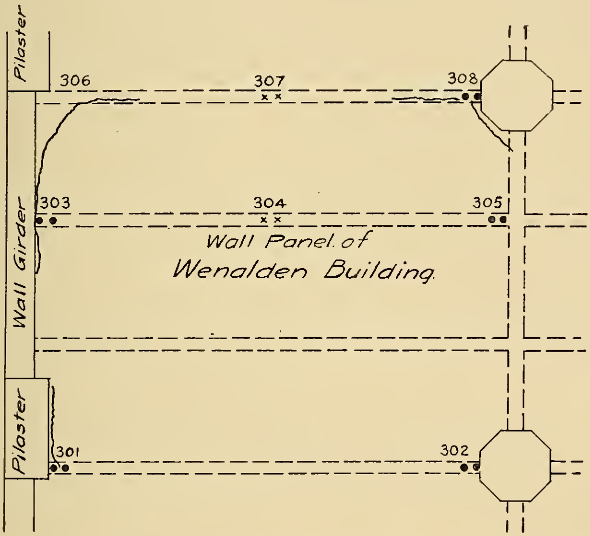

FIG 37. WALL PANEL TEST; PLAN SHOWING LOCATION OF GAUGE LINES.

urements of the compression in the concrete were made. On the reinforcement only a few gauge lines were used. The observed values are plotted in Fig. 38 and Fig. 39. Because of the small number of gauge lines and because of some of the conditions of the test which were not entirely favorable, the results may not be trustworthy quantitatively as compared with the other tests, but the indications are of interest. There is considerable restraint shown at the ends of the beams, that at the pilaster being about the same as that at the column end and that at the wall being greater than that at the girder end. All of these are nearly as large as the values found in the interior panels. The stress in the middle of the beams is considerably greater than that found in the middle of the beams in an interior panel. The deflections are also greater than for interior panels and the deflection curves are of a different character. It is recognized that there are some apparent inconsisten-

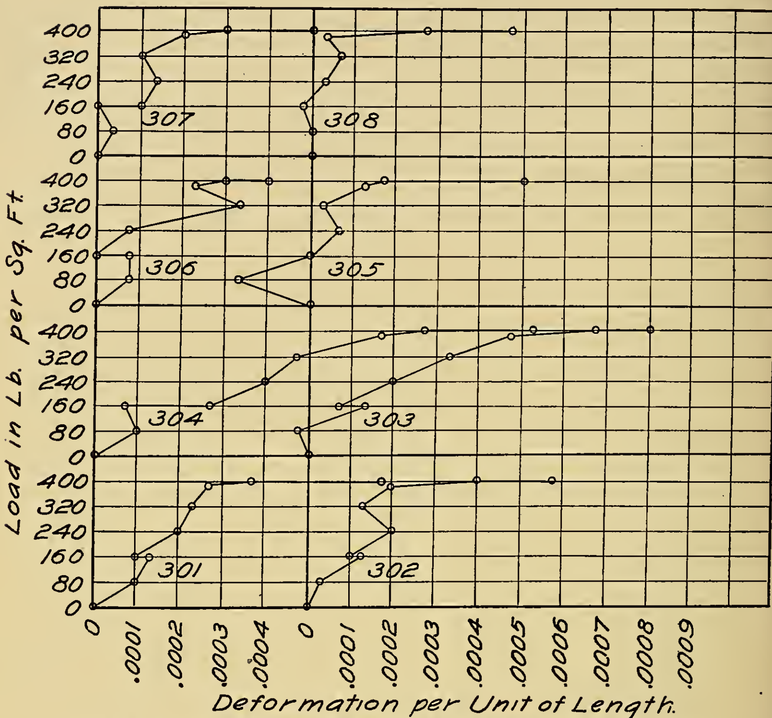

FIG. 38. WALL PANEL TEST; LOAD-DEFORMATION DIAGRAM.

cies in these statements, and the action of wall panels is a matter which should receive full investigation in the future.

28. *Examination of Floor after Test.*—An examination of the floor was made May 6, 1912, to ascertain whether the cutting of the concrete for purposes of observation had caused any permanent disfigurement. On the under surface of slabs and beams where the chances would be greatest for material used for filling the test holes to fall out of place, the concrete was intact. It seems probable that if the surface had been painted over after the repairs were made the patched portions could not have been detected without a minute examination. Although the basement was well lighted in the vicinity of the main test the only indications of the location of cracks were pencil marks where the cracks had been traced to secure ease in sketching their position. These pencil marks had been painted over but showed through the thin coat of white. It is probable that a more minute examination would have detected cracks, but the fact that after removal of the test load not even the diagonal tension cracks were plainly visible bears out the conclusions that the

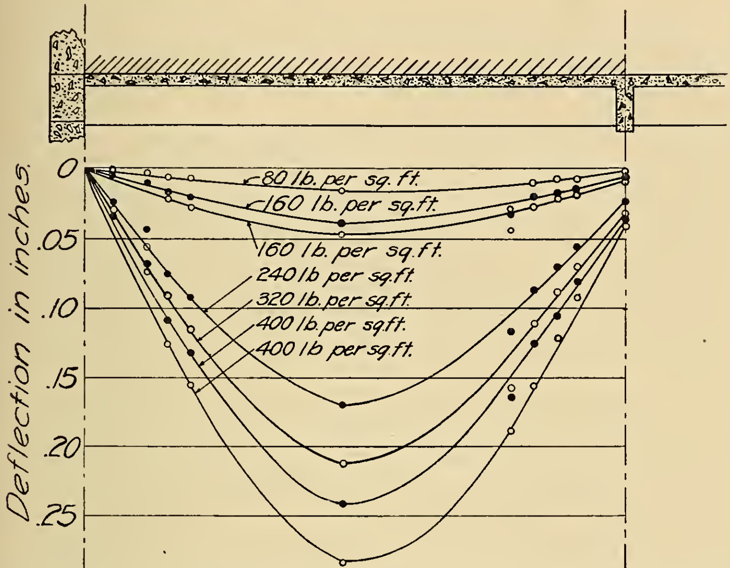

FIG. 39. WALL PANEL TEST; DIAGRAM SHOWING DEFLECTION OF INTERMEDIATE BEAM.

steel stresses caused by the test load were light. The basement under that part of the floor where the wall panel test was made was not so well lighted, hence the examination here was not so significant. On the upper surface of the floor tested, there were cracks which were distinct, but not more so than many which were observed before the test had been made. The area on which the wall panel test was made was inaccessible, being completely covered with merchandise.

IV. THE TURNER-CARTER BUILDING TEST.

29. *The Building.*—The Turner-Carter building (see Fig. 40) is an eight-story reinforced concrete building 60 x 200 ft., located at Willoughby Avenue and Walworth Street, Brooklyn, New York. It was constructed by the Turner Construction Company for the Turner-Carter Company (manufacturers of shoes) in accordance with the plans and specifications of Frank Helmle, architect.

The building is of the beam and girder type. The panels are 17 ft. 4 in. by 19 ft. 6 in. The floor was built continuously with the beams and girders. The girders are 10 in. wide and 24 in. deep including the finished floor. Each panel has two intermediate beams 7 in. wide with a total depth of 18 in. The column beams are the same size as the inter-

FIG. 40. THE TURNER-CARTER BUILDING.

mediate beams. The columns below the test floor are octagonal and are 30 in. from face to face. The position of reinforcement of the beams and girders is shown in Fig. 41. The beams and girders were designed as simple beams, but reinforcement is supplied for continuity, and the construction is such as to give continuity in the beams and girders. The structure was designed for a live load of 150 lb. per sq. ft.

The aggregates were an excellent grade of sand and gravel obtained from the sand banks in Hempstead Harbor on the north shore of Long Island. The gravel ranged in size from $\frac{3}{8}$ to $\frac{7}{8}$ in. For the beam and girder reinforcement bars having an elastic limit of about 50000 lb. per sq. in. were used. The beams have one 1-in. square bar and two $\frac{7}{8}$-in. square bars at the middle and one 1-in. square bar over the support carried about 15 in. beyond the center line of the girder. Ten $\frac{3}{8}$-in. round bars placed in the slab are also available for tension reinforcement in the end of the intermediate beams, as is also one T-bar used for supporting the slab reinforcement during construction. The girders have two 1-in. square and three $\frac{7}{8}$-in. square bars at the middle, placed in two layers, and two 1-in. square bars over the support carried nearly to the farther face of the column.

The floor tested was constructed July 25 so that at the time of the test, September 10 to 20, 1911, the work was about fifty days old.

30. *Method of Testing.*—The feature of the test, as of the Wenalden test, was the measurement of the deformations in the reinforce-

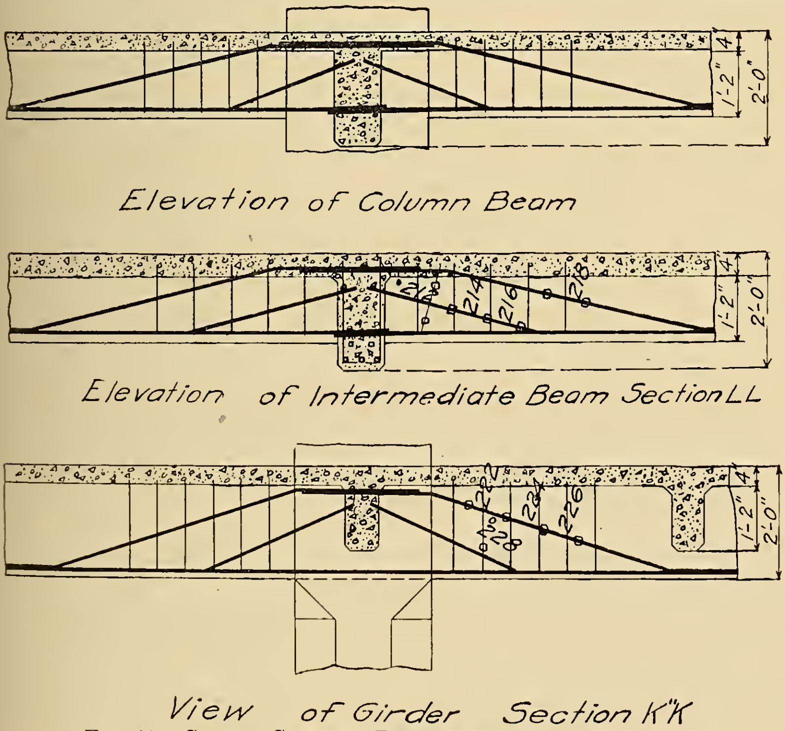

FIG. 41. SKETCH SHOWING REINFORCEMENT OF BEAMS AND GIRDERS AT SUPPORTS.

measured at the centers of the spans and at the ends and also on the inclined portions of the bent-up bars. Various other measurements which it was thought would throw light upon the action of the structure were taken.

31. *Preparation for the Test.*—A week was used in preparing for the test. Platforms supported by scaffolding for the use of observers were built on the second floor. Independent of this was a framework which was supported by the second floor, for use in making measurements of deflection. The boxes for holding the sand were constructed, this being facilitated by a power saw located on the second floor. Considerable time was consumed in drilling holes in the concrete to bare the rein-

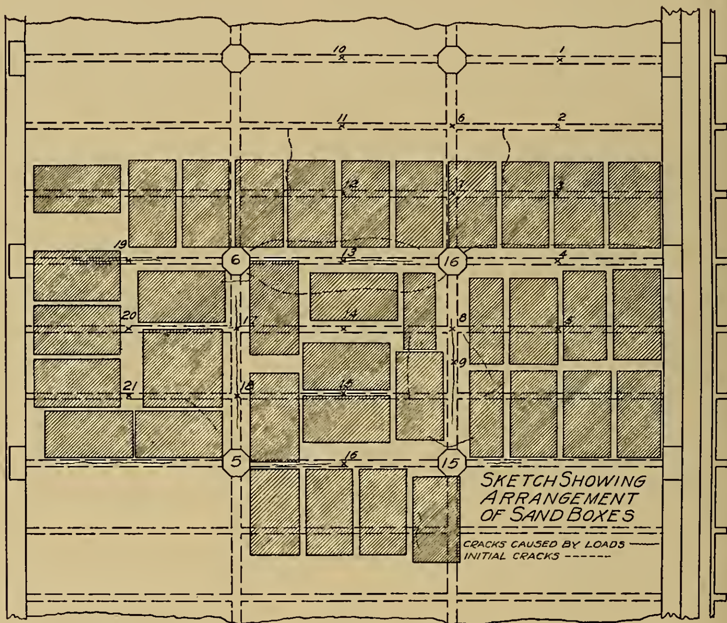

FIG. 44. LOCATION OF SAND BOXES AND FLOOR CRACKS.

forcement. In some cases this was found to be at a considerable depth from the surface. In all nearly two hundred holes were cut in the concrete. Holes were drilled in the reinforcing bars, as heretofore described, for use as gauge points. The gauge length was made 8 in. The position of the gauge lines for the reinforcing bars is shown on Fig. 42 and 43 by the even numbers. For use in the measurement of deformations of the concrete, holes about ½ in. in diameter and 1 in. deep were drilled in the concrete and steel plugs were inserted and set in plaster or

aris. Gauge holes for receiving the points of the extensometers were rilled in these plugs with a No. 54 drill. The position of the gauge ines is shown in Fig. 42 and 43 by the odd numbers. The gauge ength was 8 in.

The deflections were measured between a steel ball set in the under urface of the beam and a ball attached to the framework previously de-cribed. The measurements were made as described in Art. 13.

32. *Method of Loading.*—The test area was on the third floor. The oading material was damp sand which was placed in bottomless boxes. hese boxes were of various sizes and were placed in such a way as to ive a well distributed load. The general size of the box was 4 ft. 6 in. ide, 8 ft. long and 4 ft. 6 in. deep. Fig. 44 shows the position of the oxes and the test area. Fig. 45 is a view with the sand boxes ready for

Fig. 45. View of Sand Boxes.

oading. The boxes were made small enough to permit a good distribu-on of load even though part of the weight of the sand might be carried y arching and friction down the sides. The test area covered three full anels and parts of four others, in all equivalent to five panels. A load-ng space was chosen which it was thought would give the fullest stresses ver the girders and beams on which the principal measurements were ade. In removing the load the outer panels were unloaded first in an

attempt to determine the relation between single panel loading and group loading. The load applied was the equivalent of 300 lb. per sq. ft., double the design live load.

Before beginning the test, a calibration of the heaviness of the sand was made by weighing the sand which had been shoveled into a box of 16 cu. ft. capacity placed on the scales. It was found that there was a difference of about 10 per cent in the weight of sand which had been

FIG. 46. VIEW OF TEST LOAD IN TURNER-CARTER BUILDING.

thrown in loosely and sand which was packed somewhat. During unloading, the entire contents of three of the sand boxes (about 50(cu. ft.) were weighed. This gave an average of 88.6 lb. per cu. ft., agree ing closely with the weights of the unpacked sand previously weighed and this value was used in the calculation of loads.

On a part of the area where the boxes were not carried to a sufficien height and where the space was not covered adequately by them, cemen in sacks was used as loading material.

The supply of sand for the loading had previously been delivered o the same floor, the piles being kept at least one panel away from th location of the test area, and this was distributed over sufficient floo space that the stresses in the beams of the test area could not be affected In applying the load the sand was wheeled in barrows and dumped int

the boxes. As the sand was placed, the sides of the boxes were rapped to break the adhesion of the sand. Some leveling of the sand in the boxes was done, but there was little compacting by tramping or otherwise.

33. *Making the Test.*—A very important element of a test of this kind is the initial observation for fixing the zero point of the test readings. Three sets of observations for a number of gauge lines were made

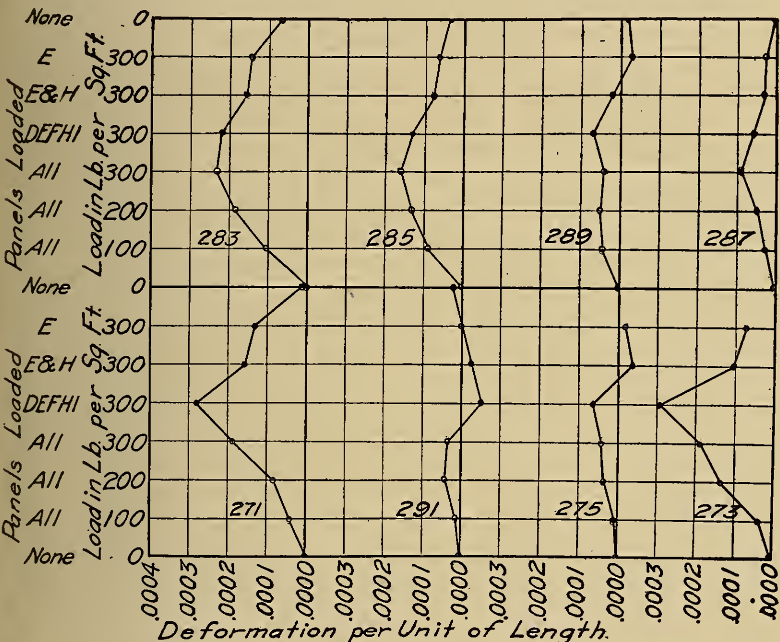

Fig. 47. Load-deformation Diagrams for Under Side of Beams at End.

before the beginning of the test, on the afternoon of September 10 and the forenoon of September 11. Where discrepancies were found new observations were made. Even with this number of observations there are uncertainties in some initial readings. Experience confirms the view that before any load is placed the initial readings which have been taken should be worked up and observations repeated until all discrepancies and uncertainties have been removed.

Readings were taken immediately after the completion of each increment of load and again immediately before the beginning of placing another increment of load. This usually corresponded with evening readings and morning readings. A series of readings was also taken with the full test load on. These extended over a period of 48 hours. A similar method was used in the process of removing the load.

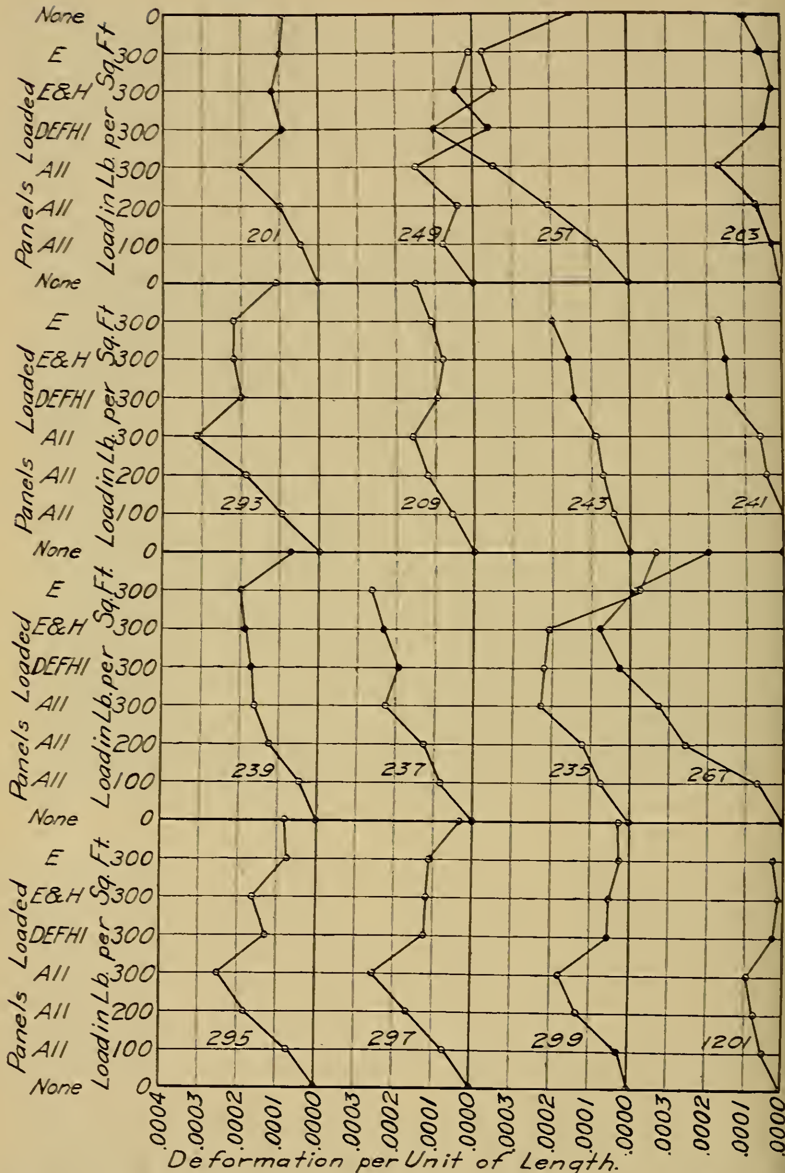

FIG. 48. LOAD-DEFORMATION DIAGRAMS FOR UNDER SIDE OF BEAMS AT EN

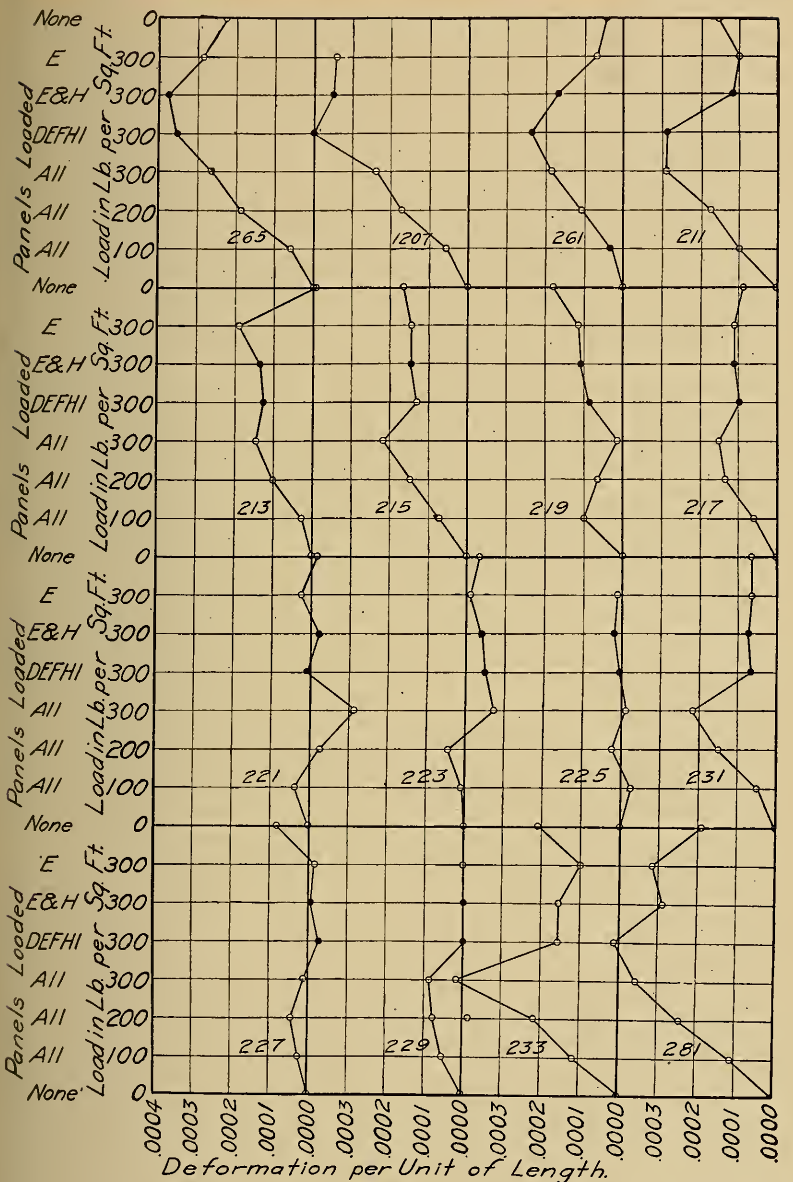

FIG. 49. LOAD-DEFORMATION DIAGRAMS FOR UNDER SIDE OF BEAMS AT END.

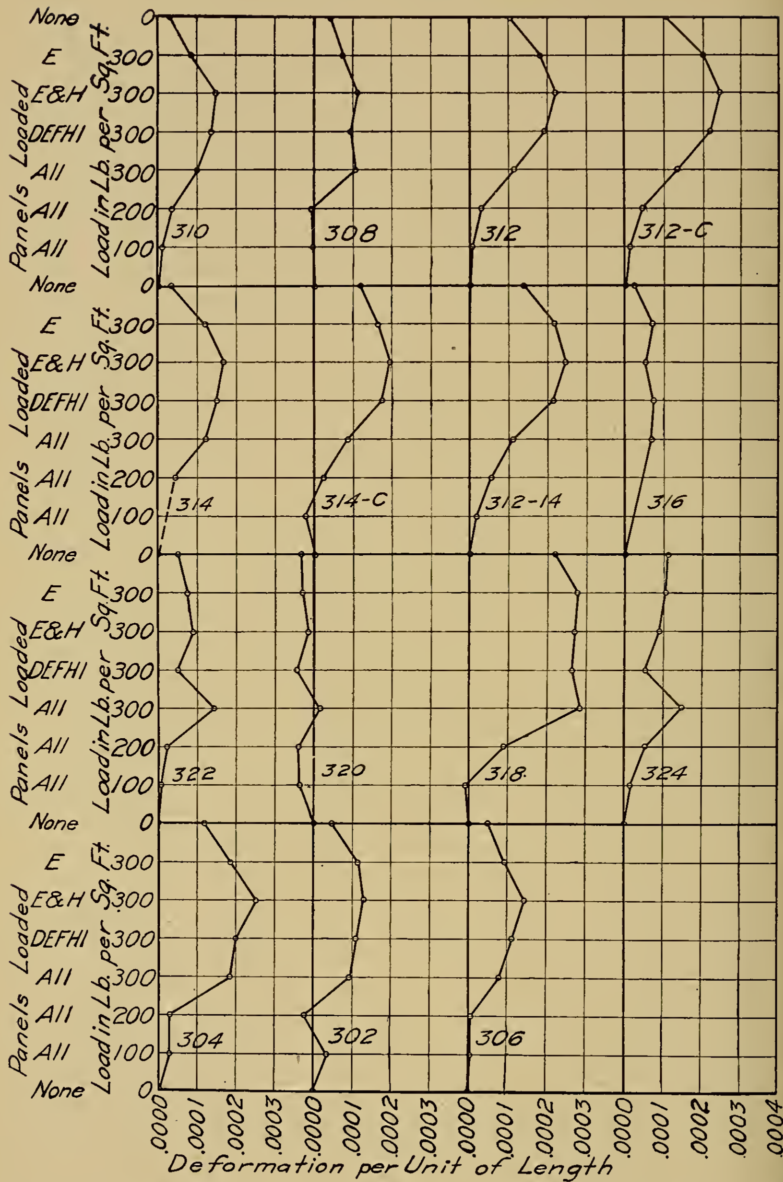

FIG. 50. LOAD-DEFORMATION DIAGRAMS FOR UPPER SIDE OF BEAMS AT END.

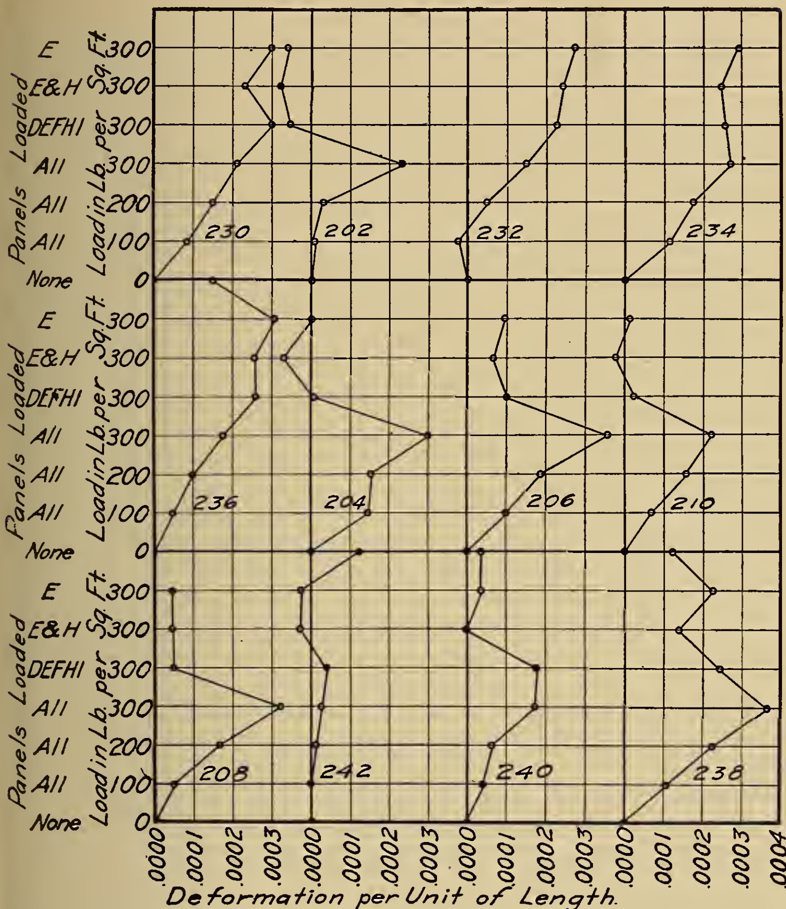

FIG. 51. LOAD-DEFORMATION DIAGRAMS FOR UNDER SIDE OF BEAMS AT MIDDLE.

Table 7 shows the loading schedule. The load was applied in increments of 100 lb. per sq. ft. based upon the whole test area. The application of the load consumed three days. The full load was left on 48 hours. The unloading schedule is shown also in Table 7. In the unloading, the load on panels *B* and *C* was first removed, then the load on panel *D, F* and *I,* followed by the removal of the load on panel *H.* Fig. 46 is a view at a load of 300 lb. per sq. ft. over the test area. The total load was over 500 000 lb.

34. *Deformations and Stresses.*—The results of observations on various gauge lines for the beams and girders are plotted in Fig. 47 to 54. Fig. 55 gives the deformations in the concrete on the under side of the

TABLE 7.

Schedule of Loading Operations in Turner-Carter Building Test.

Day	Date	Observations		Loading		Observations	
		Load lb. per sq. ft.	Hours	lb. per sq. ft.	Hours	Load lb. per sq. ft.	Hours
Loading Schedule.							
Sunday....	9–10–11	0	12 M. to 2 P. M.				
Monday....	9–11–11	0	7.20 A. M. to 12 M.	100	1.30 to 6.00 P. M.	100	6.10 to 8.00 P. M.
Tuesday....	9–12–11	100	6.30 A. M. to 8.15 A. M.	200	10.30 A. M. to 3.00 P. M.	200	3.10 to 5.30 P. M.
Wednesday.	9–13–11	200	6.20 to 8.20 A. M.	300	9.00 A. M. to 3.30 P. M.	300	3.50 to 5.50 P. M
						300	10.30 to 11.30 P. M.
Thursday...	9–14–11	300	8.00 to 8.30 A. M.			300	3.00 to 3.30 P. M.
Unloading Schedule.							
Friday.....	9–15–11	300	7.30 to 9.30 A. M.	300 on *D*, *E*, *F*, *H* and *I*.	3.30 to 7.30 P. M.	300 on *D*, *E*, *F*, *H* and *I*.	8.00 to 8.30 P. M.
Saturday...	9–16–11	300 on *D*, *E*, *F*, *H* and *I*.	7.20 to 9.15 A. M.	300 on *E* and *H*.	9.30 to 11.45 A. M.	300 on *E* and *H*.	6.30 to 8.00 P. M.
Monday....	9–18–11	300 on *E* and *H*.	6.15 to 9.20 A. M.	300 on *E* only.	9.30 A. M. to 12.00 M.	300 on *E* only.	12.15 to 1.50 P. M.
							4.15 to 8.00 P. M.
Tuesday....	9–19–11					300 on *E* only.	4.50 to 6.50 P M.
Wednesday.	9–20–11	300 on *E* only.	8.30 A. M. to 12.30 P. M.	Zero.	1.00 to 3.40 P. M.	Zero on all panels.	4.00 to 5.40 P. M.

floor slab and Fig. 56 those on the upper side. Fig. 57 records measurements made on the bent-up bars and stirrups.

As already stated, the location of the gauge lines is shown on Fig. 42 and 43, the odd numbers referring to measurement on the concrete, the

even numbers to measurement on the reinforcement. The numbers in the two hundreds are gauge lines on the under side or second story side, and the numbers in the three hundreds are on the upper side or third story side.

Stresses and bending moment coefficients are tabulated in Tables 8

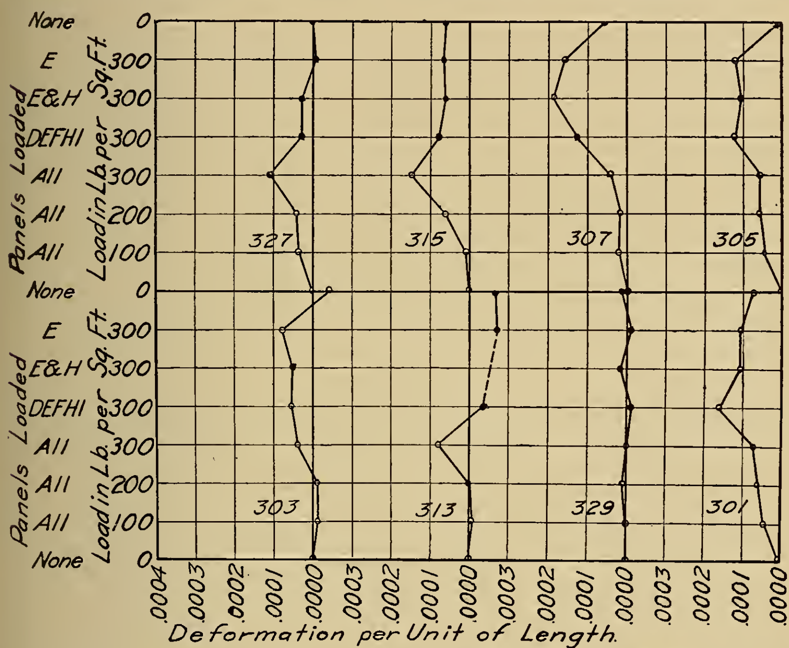

FIG. 52. LOAD-DEFORMATION DIAGRAMS FOR UPPER SIDE OF BEAMS AT MIDDLE.

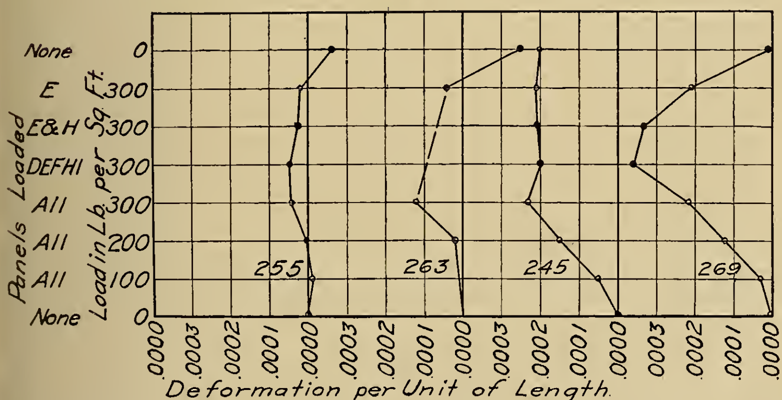

FIG. 53. LOAD-DEFORMATION DIAGRAMS FOR UNDER SIDE OF GIRDERS AT END.

TABLE 8.

STRESS INDICATIONS IN TURNER-CARTER BUILDING TEST.

Stresses are given in pounds per square inch.

Member	Gauge Line	Reinforcement	Gauge Line	Concrete
End of girder	...	......	269	900
Middle of girder	220	8 000	311	Little
"	244	9 000	...	......
End of beam	304	8 000	265	1 100
"	318	8 000	267	1 100
"	310	4 000	281	1 000
"	...	......	293	800
Middle of beam	202	7 000	301	350
"	206	11 000	305	350
"	230	9 000	313	200
"	234	8 000	315	300
"	236	8 000	...	......
"	238	11 000	...	......
"	240	5 000	...	......
Bent up bar in girder	222	5 000	...	......
"	224	5 000	...	......
Bent up bar in beam	214	—3 000	...	......

TABLE 9.

MAXIMUM STRESSES AND MOMENT COEFFICIENTS IN TURNER-CARTER BUILDING TEST.

Stresses are given in pounds per square inch.

Member	Reinforcement		Concrete	
	Stress	Coefficient	Stress	Coefficient
Girder, End	*31 000*	*1/12*	*1 200*	*1/12*
" End	......	..	900	0.06
" Middle	*12 500*	*1/12*	*300*	*1/12*
" Middle	8 000	0.05	Little	.....
Intermediate Beam, End	*21 500*	*1/12*	*1 300*	*1/12*
" End	8 000	0.03	1 100	0.07
" Middle	*18 500*	*1/12*	*380*	*1/12*
" Middle	11 000	0.05	350	0.077
Column Beam, End	*19 600*	*1/12*	*1 200*	*1/12*
" End	......	..	950	0.064
" Middle	*17 000*	*1/12*	*350*	*1/12*
" Middle	10 000	0.05	225	0.054

and 9. The stresses calculated on the basis of a bending moment coefficient of 1/12, the more usual one in designing are printed in italics.

The suggestions given for caution and care in interpreting measurements should be applied to this test.

35. *Beams.*—For the tensile stresses in the reinforcement at the middle of the intermediate beams at the full load of 300 lb. per sq. ft., the highest stress observed was 11000 lb. per sq. in. and the average

stress recorded may be said to be 8500 lb. per sq. in. At the ends of the intermediate beams, the highest stress observed in the reinforcement was

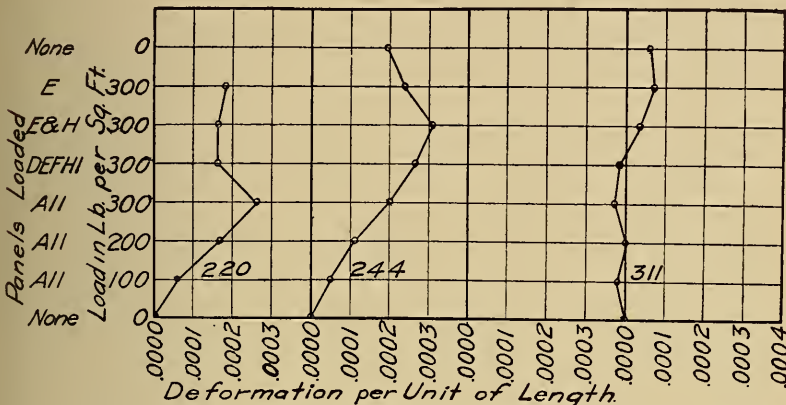

FIG. 54. LOAD-DEFORMATION DIAGRAMS FOR UPPER SIDE AND UNDER SIDE OF GIRDERS AT MIDDLE.

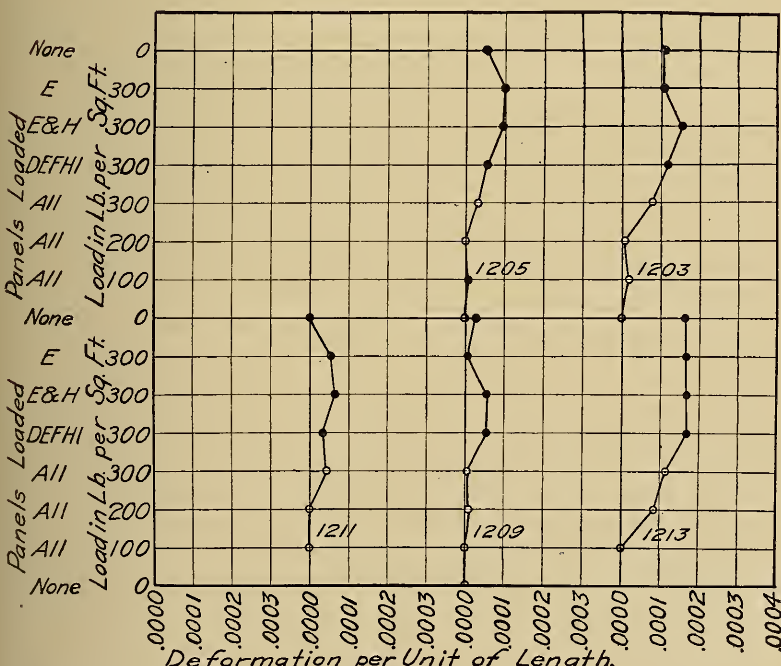

FIG. 55. LOAD-DEFORMATION DIAGRAMS FOR CONCRETE ON UNDER SIDE OF SLAB.

8000 lb. per sq. in., and the general value may be said to be 7500 lb. per sq. in. Using the assumptions for resisting moment ordinarily taken in design calculations, these stresses may be considered to correspond to a bending moment coefficient of .05 *Wl* for the maximum stress at the middle of the beam and .03 *Wl* for the maximum stress at the end of the beam, if the tensile strength of the concrete be not considered.

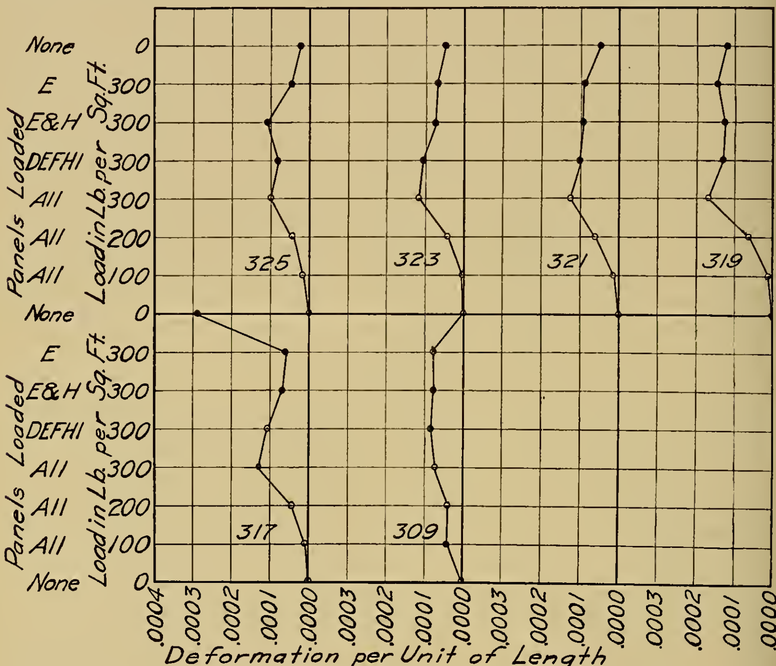

FIG. 56. LOAD-DEFORMATION DIAGRAMS FOR CONCRETE ON UPPER SIDE OF SLAB.

Assuming a modulus of elasticity for the concrete of 2 500 000 lb. per sq. in., the concrete on the compression side of the beams at the middle showed a compressive stress of 350 lb. per sq. in. and at the end of the beam 1100 lb. per sq. in. It is apparent that the total compressive stress in the concrete is greater than the total tensile stress in the reinforcement of the beams. A possible explanation is that end thrust exists, involving so-called arch action in the beams and floor structure, and that the tensile stress is relieved by the presence of this thrust. The tensile strength of the concrete must have a large effect on the resisting moment. The coefficient for *Wl* in the expression for bending moment, necessary to give a compressive stress equal to the maximum measured in the con-

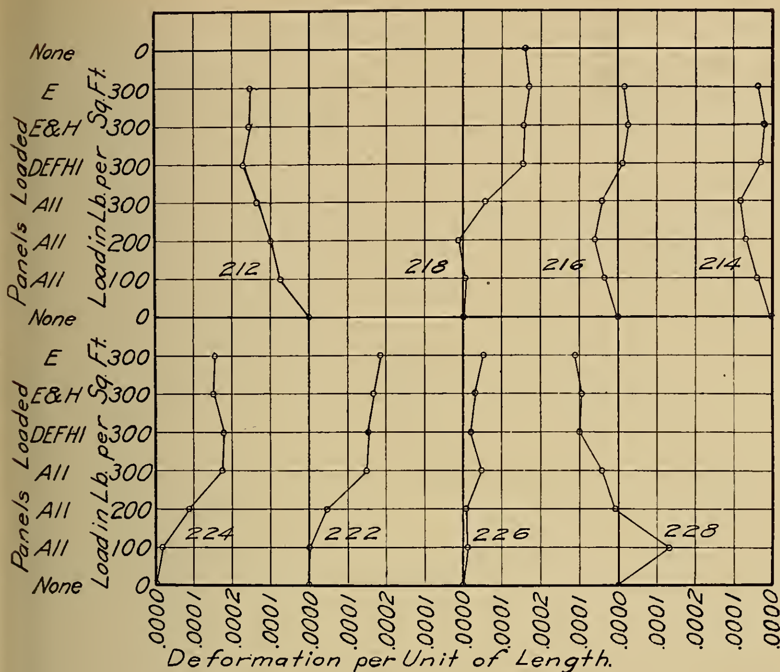

FIG. 57. LOAD-DEFORMATION DIAGRAMS FOR BENT-UP BARS AND STIRRUPS.

crete, on the assumptions made, is .077 for the middle of the beam and .07 for the end of the beam. These coefficients are lower than the value of 1/12 usually assumed in design of such beams.

36. *Girders.*—For the tensile stresses at the middle of the girders the observations showed about 8000 lb. per sq. in. in the reinforcement at the middle. This corresponds to a bending moment coefficient of .05, again neglecting the tensile strength of the concrete. The reinforcement at the end of the girder was inaccessible.

Assuming a modulus of elasticity of 2 500 000 lb. per sq. in., the concrete on the compressive side of the beam at the support showed a compressive stress of 900 lb. per sq. in. The reading at the middle of the beam showed very little compression. Assuming that the loads on the girder are concentrated at the points where the intermediate beams are connected, and making the same assumption of distribution of stress as before, the coefficient of bending moment was .06. It seems probable that the compression at the middle of the span must be distributed over a considerable width of floor, or larger readings of compression would have been obtained.

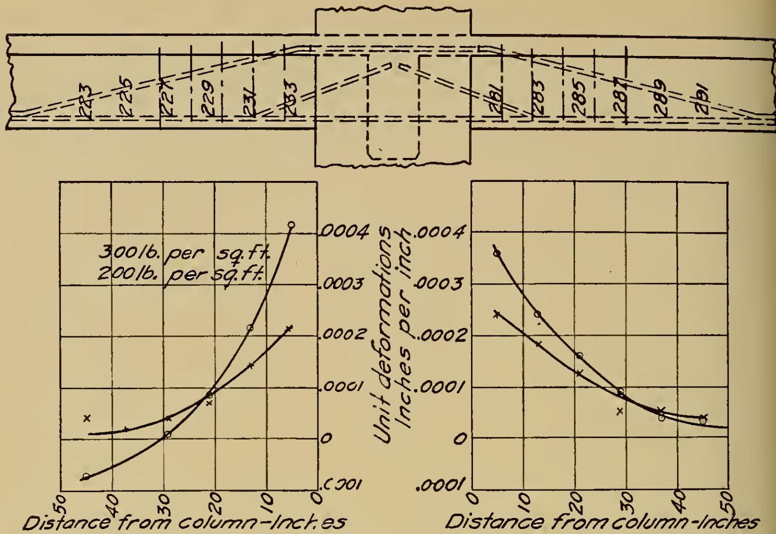

FIG. 58. DIAGRAM SHOWING DISTRIBUTION OF COMPRESSIVE DEFORMATION IN BOTTOM OF COLUMN BEAM.

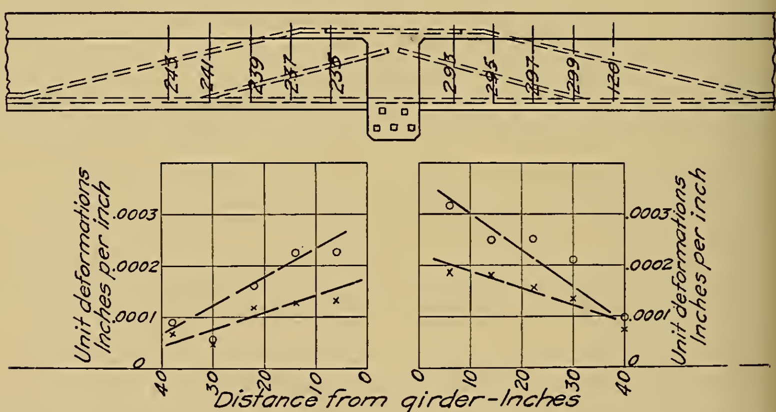

FIG. 59. DIAGRAM SHOWING DISTRIBUTION OF COMPRESSIVE DEFORMATION IN INTERMEDIATE BEAM.

37. *Decrease in Compression with Distance from Support.*—In four beams measurements of compressive deformations were taken at a series of gauge lines from the support to a location near the point of inflection. The position of these points is shown in Fig. 42. The gauge lines No. 223, 225, 227, 229, 231 and 233 are on one side of column No. 6, and 281, 283, 285, 287, 289 and 291 are on the other side of column

No. 6. It may be expected that there will be full restraint for the end of the beams. Gauge lines 243, 241, 239, 237 and 235 are on one side of a girder and 293, 295, 297, 299 and 1201 are on the other side. The unit-deformations for these gauge lines at loads of 200 lb. per sq. ft. and 300 lb. per sq. ft. are plotted in Fig. 58 and 59.

The measurements recorded for the column beams show considerably more compressive stress than do those for the intermediate beams, per-

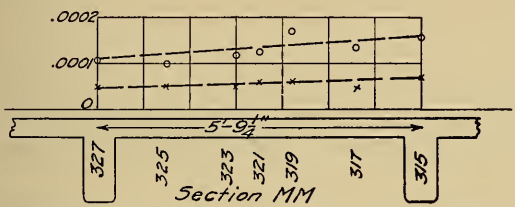

Fig. 60. Diagram Showing Distribution of Compressive Deformation Across Flange of T-beams.

haps one-third more. This difference in stress may be due partly to the deflection of the girder, and to the deflection of the intermediate beam between its support and a point opposite the end of the column beam, which would permit a larger part of the load to be carried by the column beam. It may be due somewhat to the fact that reinforcing bars are bent down from a point at the end of the column beam, while in the intermediate beams the bars run horizontally for a foot from the face of the girder.

The direction of the lines in Fig. 58 and Fig. 59 indicates a zero stress at about 45 in. from the face of column in the column beams and at about 50 in. from the face of the girder in the intermediate beams. In both cases the results locate the point of inflection at about 0.22 of the clear span.

38. *T-beam Action.*—The distribution of compressive stresses in the T-beam formed by a beam and the floor slab (which involves the distances away from the beam for which compressive stresses are developed) has been a fruitful source of discussion. Measurements parallel to the axis of the beam were taken on the upper surface of the floor slab immediately above beams and at intervals between them. These gauge lines are No. 315, 317, 319, 321, 323, 325 and 327 (see Fig. 42). The deformations are shown in Fig. 52 and 56. The amount of these deformations at points across the slab for loads of 200 lb. and 300 lb. per sq. ft. is shown in Fig. 60. It is apparent that a somewhat higher stress existed in one beam than in the other. Taking this into consideration,

the compressive stress varies quite uniformly from one beam to the other, and the full width of the floor slab may be said to be effective in taking compression. The overhang (counting to the mid-point between beams) is 6½ times the thickness of slab. It will be noticed that the conclusions are the same as given for the Wenalden building test.

Readings were also taken on the under side of the floor slabs parallel to the beams at three places (No. 1205, 1211 and 1213), but the conditions attending the location of these points do not permit conclusions to be drawn.

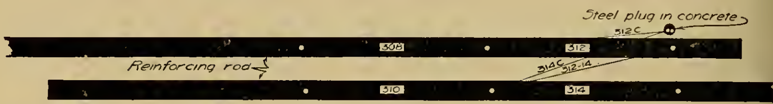

FIG. 61. ARRANGEMENT OF GAUGE LINES TO TEST FOR MOVEMENT OF BAR RELATIVE TO CONCRETE.

39. *Floor Slab.*—Measurements were taken on the floor slab in the direction of its span at three places on the under side and at one place on the upper side immediately above one of the lower measurements. These gauge lines were No. 277 on the under side of the slab close to a girder (Fig. 42), No. 279 on the under side of the slab 5 ft. from the edge of the girder, No. 309 (Fig. 43) on the upper surface immediately above No. 279, and No. 1203 (Fig. 42) on the under side half way between girders. The measurements are plotted in Fig. 55 and 56. As might be expected from being close to the girder and near the level of its neutral axis, No. 277 showed little deformation. The pair of gauge lines (No. 279 and 309) shows less deformation than would be calculated by the ordinary beam formula, but perhaps not less than would be the case if the tensile strength of the concrete is considered to be quite effective. The reading of No. 1203 was even smaller than No. 279. All the stresses found in the floor slab were low. The deformations parallel to the beams were discussed under T-beams.

40. *Bond Stresses.*—At the ends of the beams the reinforcing bars lapped over the center line of the girder a distance of 15 in. An effort was made to determine whether there was a movement of one of these bars with reference to the adjoining concrete and with reference to the adjoining bar; also whether the deformation in the stub end of the reinforcing bar was the same as in the adjoining bar. Fig. 61 shows the location of the reinforcing bars with reference to each other, and the position of the gauge lines. No. 312-14 in comparison with No. 312 and 314 will indicate any relative movement of one bar with respect to

the other, and No. 312c and 314c in comparison with No. 312 and 314, respectively, will indicate any movement of the bars with respect to the concrete.

It appears possible that the initial reading of No. 314 is slightly in error, and the remarks already made about quantitative interpretation of results and the chances for variations in stresses in adjacent bars or in adjoining concrete should be borne in mind in studying the results. It seems evident that No. 314 (on the lapped bar) records considerably less stress than (Fig. 50, p. 74) No. 312. The measurements indicate a possibility that the right-hand point of gauge line No. 314 has moved to the right relatively to the right-hand point of No. 312, though this amount may not be more than the amount of initial slip necessary to develop the requisite bond stress. The measurements taken have no bearing on whether the left-hand point of No. 314 has moved. The measurements also indicate that there was no motion of the left-hand point on the reinforcing bar (No. 312 gauge line) relatively to the concrete at its side, though it must be borne in mind that the point taken was so close to the bar that only slip and not distortion of concrete could be measured.

41. *Web Deformations.*—No diagonal tension cracks were visible on any of the beams or girders.

In girder 4 measurements were taken on the diagonal portion of a reinforcing bar, one of the bars which is provided to take negative bending moment. This is shown in Fig. 42, Section K-K. The gauge lines are No. 222, 224 and 226. The position of the gauge lines is also shown in Fig. 41. The measurements are plotted in Fig. 57. It was impracticable to measure the deformation at a point closer to the support. The measurements show about the same stress at No. 222 and 224, perhaps 000 lb. per sq. in. The stress at No. 226 is materially less. It is not improbable that there was tension in this rod throughout its length. As there was considerable compression measured in the gauge lines on the bottom of the girder below No. 222, it seems probable that a crack was formed in the top of the floor slab somewhere above No. 222, but as this space was filled in with bags of cement no observation was made during the test, and inspection of this space after the load was removed seems to have been overlooked. At the other end of the girder, near column 6, a fine test crack was found on the upper surface of the floor 2 in. from the face of the column extending across the width of the girder and beyond. This extended through the floor. A similar crack was observed on girder near column 15.

Gauge line No. 228 is on a stirrup (see Fig. 41). This stirrup is in an inclined position. It is not known what bar it is intended to be

connected with, nor whether there is connection with a tension bar. The gauge line is in a region of the beam where horizontal compressive stresses may be expected. The measurement in the stirrup at the first increment of load shows tension (see Fig. 57) and subsequent increments give compression. It should be noted that readings could not be taken on the upper end of the stirrup. If the upper ends are merely bent out into the floor slab it is hard to see that the stirrup may be expected to be useful in transmitting web stresses.

In beam 9 (see Fig. 42, section L-L, gauge line No. 218) measurement was taken on the diagonal portion of a reinforcing bar which is carried through the girder at its top and a few inches beyond. See also Fig. 41. This shows a tension of 3000 to 5000 lb. per sq. in. (See Fig. 57.) This bar was inaccessible from the top of the floor, but the gauge lines on the companion bar (No. 324 and 318) show about 5000 and 9000 lb. per sq. in. Measurements in the diagonal portion of a single-bend bar (gauge lines No. 216 and 214, Fig. 42) which extends only to the center of the supporting girder indicate a small compression in the bar (see Fig. 57). A stirrup, which like the one in the girder was close to the end of the beam and was inclined so that its lower end was nearer the support than its upper, showed shortening of the stirrup (see gauge line No. 212, Fig. 41, 42 and 57). In both cases, the arrangement was such that the stirrup could hardly be effective.

The amount of the vertical shear in the beams and girders was such that diagonal tension cracks might be expected except for the small tensile stresses in the top of the girder and the end constraint which seems to have been developed in both beams and girders.

42. *Deflections.*—The deflections of the beams (including that due to deflection of girder) and the deflections of girders are given in Fig. 62. The location of the deflection points is shown in Fig. 44. The effect of time upon the deflection is shown by the increase in deflection under constant load. The change when portions of the load had been removed may be due in part to the time element and in part to the effect of location of the load on the panels. The deflections seem relatively small, especially when compared with deflections obtained in laboratory tests of beams carrying the same loads. The conditions were such that the supports were subject to possible displacement by workmen.

43. *Effect of Number of Panels Loaded.*—In taking off the load, the outer panels were unloaded first, and observations were taken on the remaining panels in an attempt to determine the relation between single panel loading and group loading. Panels B and C were first unloaded (see Fig. 42), then panels D and F, then panels H and I, and finally

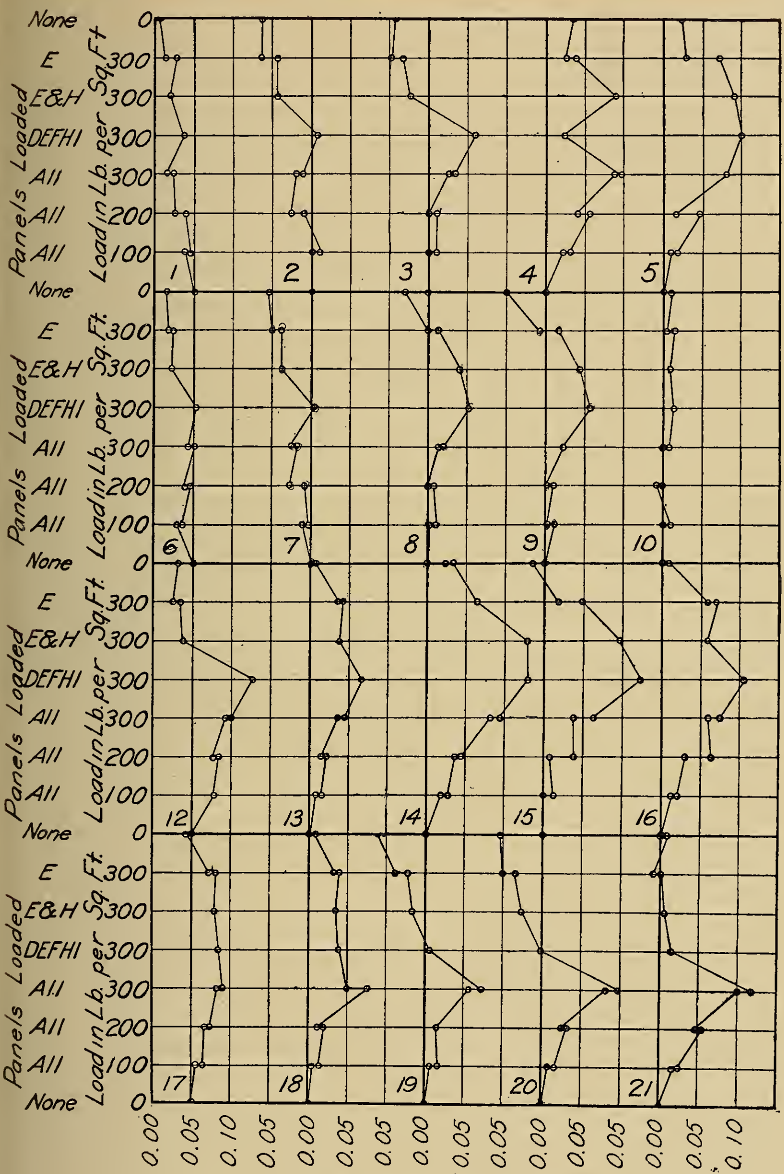

FIG. 62. LOAD-DEFLECTION DIAGRAMS.

panel E. The deformations at each of these stages are shown in the load-deformation diagrams. If at each stage of the loading the average of the deformations at all the points having a similar location (say the points on the under side of the beams at the south end of the test area) be taken the effect of area loaded may be judged by the ratios of these values to the corresponding ones at full load. If the beams be considered as freely supported (without restraint) and their weight be neglected, and if it be assumed that no time is required for adjustment of members to the load coming upon them, it should be possible in many cases to forecast the effect of a change in the area loaded. Comparing the ratios referred to above (no diagrams reproduced here) with what might be expected on the basis of the above assumptions, it is found that in most cases the direction of the changes in stress agrees with predictions. The amount of change to be expected can not be predicted because of complications in the division of load between elements of the structure. A point worthy of note is that the stresses at the center of the panel E are 30 per cent less when only panel E is loaded than when the whole test area is loaded. This must be due to the fact that with the removal of the load which rested on the side panels F and D the column beams at the edge of panel E recover a considerable part of their deflection, and because of their smaller deflection they will receive the effect of a greater proportion of the panel load than taken before, thus relieving the interior beams somewhat. The stresses were decreased at this stage more than they were increased later by the removal of the load in the end panel H. This indicates that the stiffness of the floor system permits considerable lateral distribution of the load-carrying stress. The removal of the load in panel H increased the stress in the beams at the center of panel E much as though the beams were continuous and freely supported. This would indicate that the most severe condition of loading affecting the center of the beams is brought about by loading several panels which lie side by side and are not separated by girders.

44. *Effect of Time on Stresses Developed.*—To determine the effect of time-under-load on the amount of deformation developed, observations were taken at each stage of the loading after the load had been in position for from 8 to 12 hours and also at intervals of 8 to 16 hours when all panels were fully loaded. The latter investigation continued over 48 hours. The results found during the loading seem to indicate in a general way a tendency for the deformations at the ends of beams both above and below to increase and also those at the centers of the beams above, but on the lower surface of the beams at the center the tendency was to decrease. No reason is apparent why the changes on the

compression and tension surfaces at the center of the beam should be in opposite directions and it is probable that this result is erratic. With full load the time effect at only a few gauge lines was observed; only two of them were at the center of the beams, both being below and none above. These measurements also indicate an increase in deformation at the ends of beams both above and below. At the center of beam below, one gauge line shows an increase and the other a decrease in deformation thus giving no results.

45. *Columns.*—Readings were taken on the four faces of column No. 5 just below the girders, but the results are not consistent enough to warrant attempting to draw conclusions.

46. *Test Cracks.*—Fine tension cracks were observed in the lower part of the beams and girders. The location of the observed cracks is shown on Fig. 63. The appearance of these fine cracks is similar to those

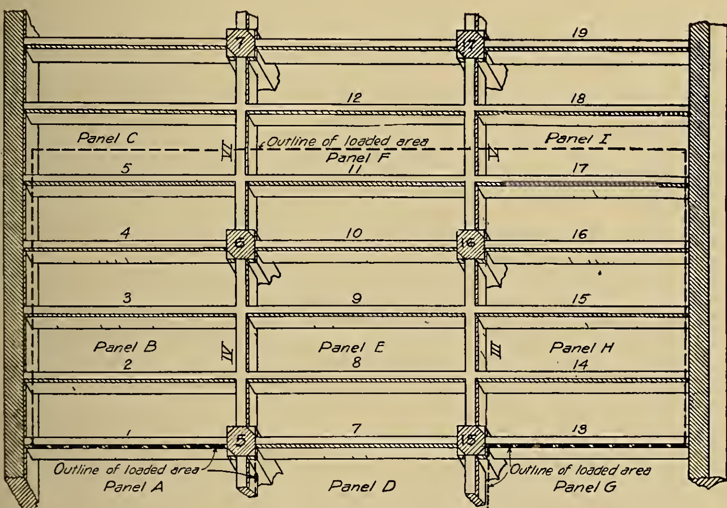

FIG. 63. CABINET PROJECTION SHOWING BEAMS AND GIRDERS AND POSITION OF TEST CRACKS.

observed in laboratory tests. They would not be noticed without specially careful examination.

The floor cracks already mentioned indicate the development of the tensile stresses in the beams and girders at the support.

It was not possible to give full attention to every feature upon which information was sought, and in some cases isolated points were used with a view of determining tendencies, and in these naturally there is less certainty in the indications.

V. The Deere and Webber Building Test.

47. *The Building.*—The Deere and Webber Building is an eleven-story and basement warehouse at Minneapolis, Minnesota, owned by the Deere and Webber Company. It was built by the Leonard Construction Company of Chicago. Fig. 64 is a view of the building at the time of test. Fig. 65 shows the floor plan of the building and the location of the panels loaded. The dimensions of the panels are 18 ft. 8 in. by 19 ft. 1 in. A 1-2-4 mixture was used, the slab thickness measuring 9 3/16 in. The floor was designed by the Concrete Steel Products Company for a live load of 225 lb. per sq. ft., and the details of the reinforcement are shown in Fig. 66. The floor tested was the fourth from the ground and the conditions were not such as to make a high showing of strength. Owing to a failure in the supply of aggregates during the construction of this floor, an abnormal number of bulkhead separations occur in the slab, as is shown in Fig. 65. Such separations

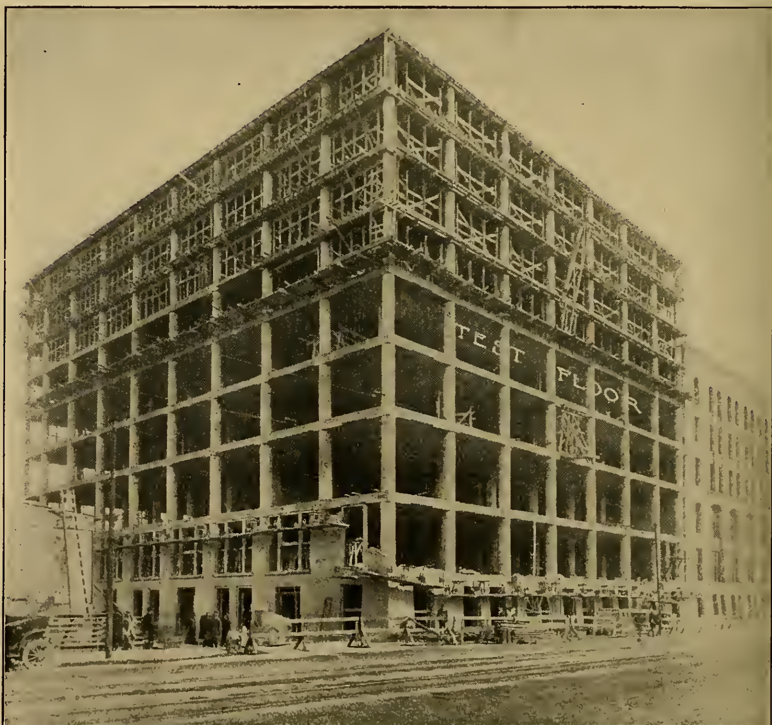

Fig. 64. Deere and Webber Building at the Time of Test.

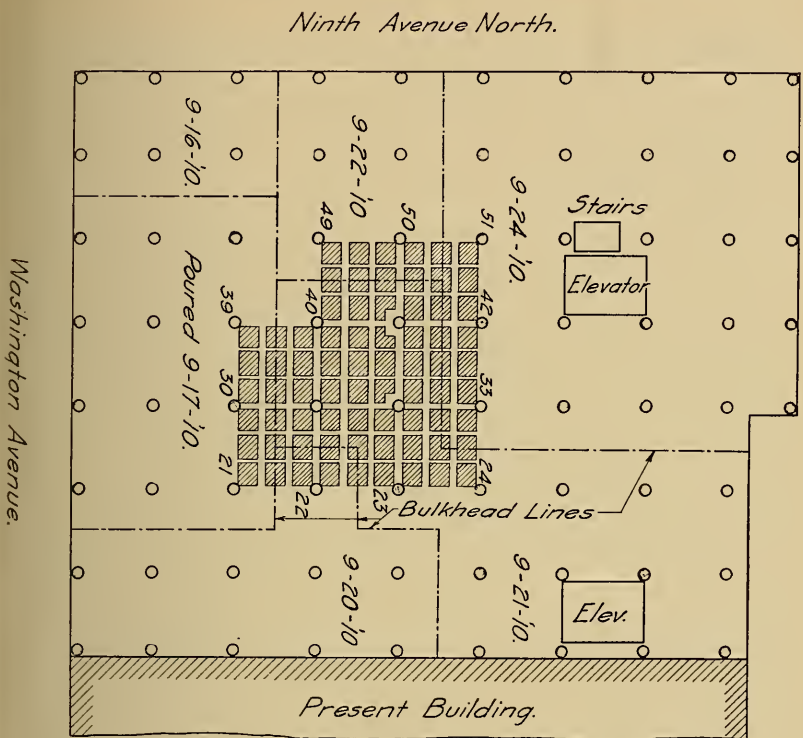

FIG. 65. PLAN OF FLOOR SHOWING LOCATION OF PANELS TESTED.

occur in every panel under load except one. The concrete was only 40 days old at the beginning of the test. In general the conditions were such as to give slightly higher stresses than would be expected had the slabs been well seasoned and normally poured.

48. *Method of Testing.*—Fig. 66 and Table 10 show the position of points at which measurements of deformation were made. The numbers given are those used in recording and plotting the data in the tables and diagrams. The total number of readings was in excess of 3300. The falsework for instruments and observers is shown in Fig. 67. For measuring deflections the instrument shown in Fig. 68 was used. A polished steel ball was attached to the ceiling, another was carried on an upright, and the instrument was inserted between them. Measurements were made in this manner to the nearest .001 inch with accuracy. For measuring the deformation in the reinforcement at the center of the span a clamp was rigidly attached to the slab rod (the concrete being removed at one point for this purpose), and a Wissler dial was carried

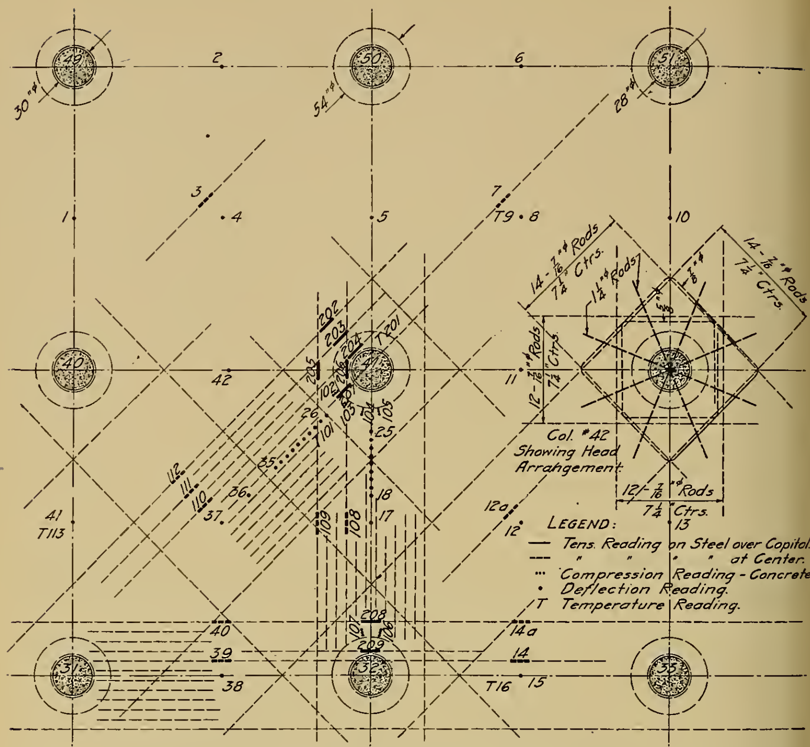

FIG. 66. ARRANGEMENT OF REINFORCEMENT AND LOCATION OF OBSERVATION POINTS.

on the clamp (Fig. 69). A fine silk-covered copper wire was attached to the rod at a distance of 15 in. from the clamp and passed immediately below the rod, over an idler on the clamp, and then over the drum of the dial. As this wire was 1/16 in. below the under surface of the slab rod, the deformations observed were only slightly in excess of the deformation in the rod. The wire was placed in this position because experience in the laboratory has demonstrated that measurements taken below the slab depending upon the position of the neutral axis for correction, are subject to considerable error. By this arrangement the deformation was measured to an indicated .0002 in. on a gauge length of 15 in. The measurement was less responsive to slight changes than were the other measurements made.

For measurements of deformation in the reinforcement over the column capital the University of Illinois type of Berry extensometer built for this test (Article 11) was used. A gauge length of 15 in. was

TABLE 10.

Data on Position of Rods on which Deformations were Measured.

Gauge Line	Band	Position in Band	Embedment inches*		Layer of Steel over Column
3	Diagonal	3d rod from center	....	$\frac{11}{16}$	
7	"	" " " "	$\frac{15}{16}$	....	
12a	"	2d " " "	....	$\frac{3}{8}$	
14	Cross	" " " "	$\frac{5}{16}$	....	
14a	"	Outer rod of band	....	$\frac{1}{2}$	
39	"	2d rod from center	$\frac{3}{8}$	....	
40	"	Outer rod of band	....	$\frac{11}{16}$	
108	"	3d rod from center	$\frac{15}{16}$	....	
109	"	Outer rod of band	....	$\frac{13}{16}$	
110	Diagonal	3d rod from center	$\frac{3}{4}$	....	
111	"	5th rod from center	....	$\frac{13}{16}$	
112	"	Outer rod of band	$\frac{9}{16}$	....	
202	"	" " " "	....	$2\frac{3}{4}$	2d layer from top
203	"	5th rod from center	$2\frac{1}{16}$	....	" " " "
204	"	3d " " "	....	$1\frac{7}{8}$	" " " "
207	"	1st " " "	$2\frac{3}{8}$	....	" " " "
205	Cross	Outer rod of band	....	$3\frac{1}{4}$	3d " " "
206	"	3d rod from center	3	....	" " " "
208	"	Outer rod of band	....	$2\frac{3}{4}$	" " " "
209	"	3d rod from center	$2\frac{1}{8}$	....	" " " "

*Measurement from surface to center of rod.

used. For measuring deformations in the concrete the original Berry 6-in. extensometer (Fig. 13) was used.

49. *Loading and Testing.*—In applying the load care was taken that no serious arch action in the load be possible. In the earlier stages brick were piled in piers, as shown in Fig. 65 and in Fig. 70, with open aisles from 8 in. to 16 in. wide between the piers. For the later loading, cement in bags was used as loading material, the piers being kept separate as before. The load given in the tables is in all cases the total load on the panel divided by the area of the panel, the intensity of the load under the pier being greater. The aisles gave an opportunity for making observations upon the concrete and reinforcement.

To correct for temperature variations one entire day was spent in observing effects due to temperature alone, and the large Berry extensometer was read on a standard bar before and after each series of slab readings.

The test continued for six days from October 30 to November 4, inclusive, 1910. Eight panels were loaded. First readings were taken on all instruments with the floor unloaded and then a load equal to 75 lb. per sq. ft. was applied over the entire eight panels. Another series of observations was taken and the load increased to 150 lb. per sq. ft. In this manner alternate observations and loadings were continued for three

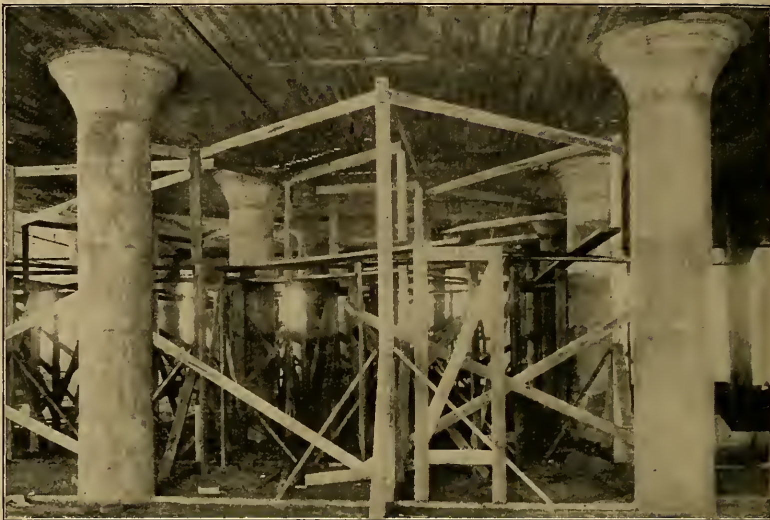

FIG. 67. FALSE WORK FOR INSTRUMENTS AND OBSERVERS.

FIG. 68. DEFLECTOMETER IN PLACE.

FIG. 69. WISSLER DIAL FOR MEASURING DEFORMATION IN REINFORCEMENT.

FIG. 70. VIEW OF MAXIMUM TEST LOAD.

TABLE 11.

Deflection of Slab in Inches at Points Midway between Columns.

Load in lb. per sq. ft.		Observation Point Number															
Center Panel	Outer Panels	37	4	8	12	17	42	5	11	15	38	41	1	2	6	10	13
0	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
75	75	29	29	32	16	16	13	18	18	22	20	24	7	11	11	15	10
150	150	61	73	79	66	39	30	57	45	44	38	49	22	22	17	27	17
187.5	187.5	74	101	111	95	52	35	81	63	58	57	64	33	26	19	34	20
187.5	187.5	0.085	0.121	0.123	0.105	0.063	0.049	0.089	0.070	0.072	0.076	0.077	0.046	0.041	0.041	0.050	0.038
225	225	98	139	143	119	74	61	107	82	83	83	86	52	45	43	56	45
262.5	262.5	124	173	180	149	89	75	138	103	103	93	111	64	48	45	69	50
300	300	168	225	242	186	105	101	183	128	126	93	140	78	48	57	88	57
300	300	177	234	252	193	110	109	189	132	135	99	148	80	52	62	96	62
300	300	0.180	0.242	0.258	0.198	0.113	0.112	0.193	0.134	0.139	0.106	0.152	0.087	0.059	0.068	0.101	0.064
350	350	224	272	293	231	133	132	218	157	156	121	178	97	70	77	117	76
350	350	217	281	301	235	135	131	224	161	161	123	182	102	71	79	121	81
350	350	220	285	306	240	137	138	227	163	162	125	186	101	73	79	122	80
350	350	223	288	309	242	138	143	229	163	162	125	186	102	73	80	121	80
350	350	224	....	....	244	139	137	231	165	163	126	188	102	73	81	123	81
350	350	227	....	....	246	141	140	235	167	165	128	191	103	74	82	125	83
350	350	0.234	0.299	0.321	0.250	0.146	0.146	0.238	0.169	0.170	0.134	0.196	0.107	0.078	0.088	0.130	0.090
350	187.5	246	223	241	188	138	134	172	124	125	127	181	80	63	70	104	73
350	0	266	115	114	97	124	112	72	50	54	115	156	34	58	41	58	43
350	0	274	116	111	96	126	117	68	47	54	122	160	36	42	46	61	48

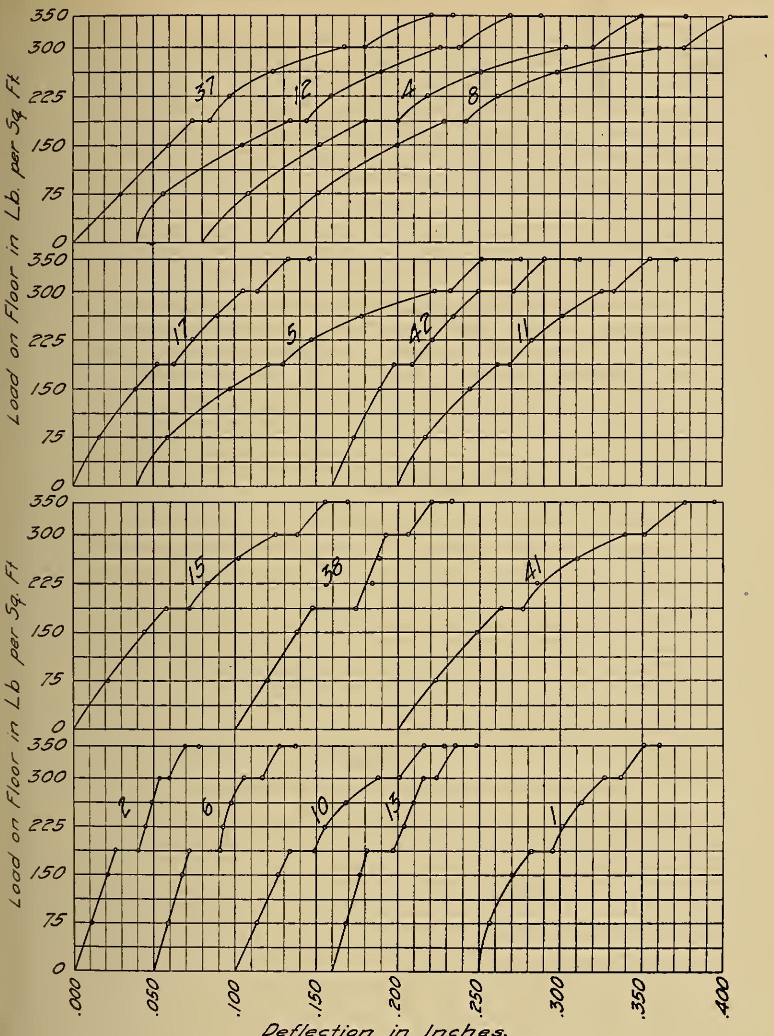

Fig. 71. Diagram of Deflections.

days. Over-night readings were taken on one or two occasions in the evening, about midnight and in the morning. The maximum load of 350 lb. per sq. ft. was allowed to remain on the floor about 22 hours, readings being taken at frequent intervals during that time. In the process of unloading the outer panels were first cleared, and finally the load was removed from the center panel. Readings were taken at intervals during the progress of the unloading. The data obtained are presented in Tables 10-15 and plotted in Fig. 71-74.

50. *Deflections.*—Fig. 71 shows graphically the deflections at sixteen points, the same data being recorded in Table 11. On the second diagram of Fig. 71 note the comparison between readings 5 and 11, where bulkheads existed, and readings 17 and 42, where no bulkheads were present. Other instances of the marked effect of bulkheads on the stiffness of the slab may be seen in the plotted data. It may also be said in general that the deflections were greater in the outer panels than in the center panels, in part due to the bulkheads in these outer panels, and in part to the tendency to higher stresses and deflections

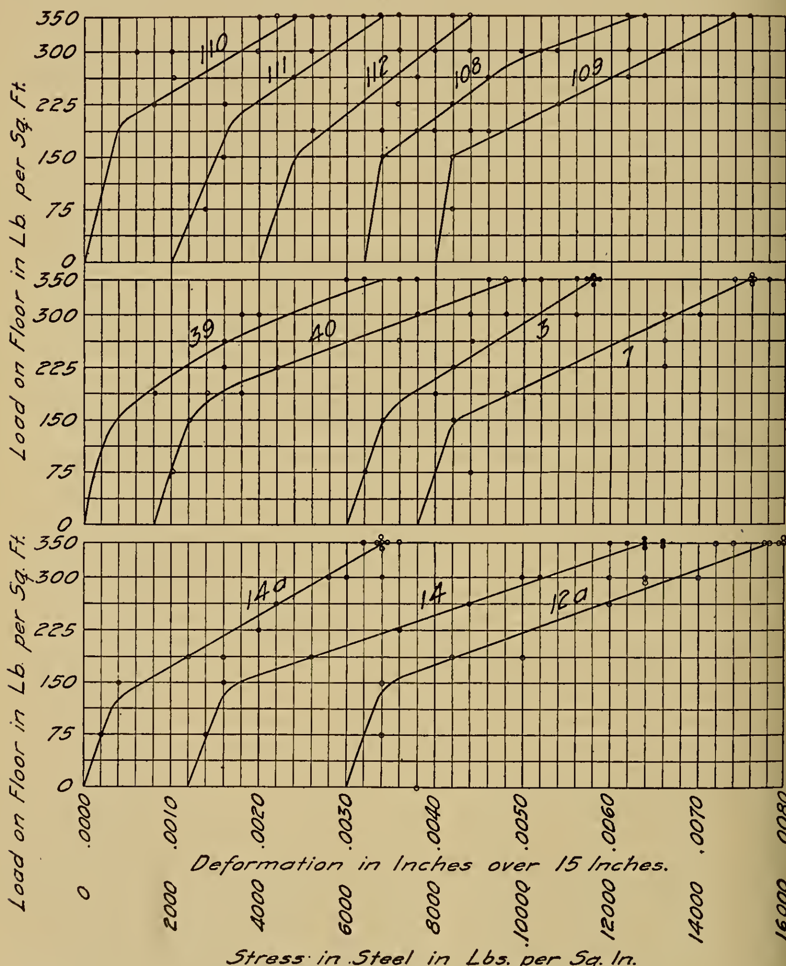

FIG. 72. DIAGRAM SHOWING STRESS IN REINFORCEMENT AT CENTER OF SPAN.

in end panels. The deflections probably would have been smaller with well cured concrete and in considering deflections it must be remembered that this slab was only 43 days old when the maximum load was placed upon it. The maximum deflection found was .32 in., which is 1/1000 of the span. This was at a bulkhead in an outer panel. In the center panel the deflection for all eight panels loaded, was .227 in., or 1/1400 of the span, which increased to .274 in. or 1/1200 of the span, when the load was removed from the outer panels.

51. *Stress in Reinforcement at Center.*—Fig. 72 and Table 12 give the data on the measured deformations in the reinforcement at the center of the spans. The table is reduced to unit deformations while the diagrams show total deformation over the lengths gauged. The stresses observed at the center were very low. On the upper diagram in Fig. 72 are shown deformations in the center panel and it is to be noted that these are, in general, smaller than those in the outer panels. This would seem to indicate that the reinforcement at the center of the span should be designed for one panel loaded, as this apparently gives a worse condition at the center than full loading. The observed stresses indicate that the diagonal and cross band rods took practically the same stress.

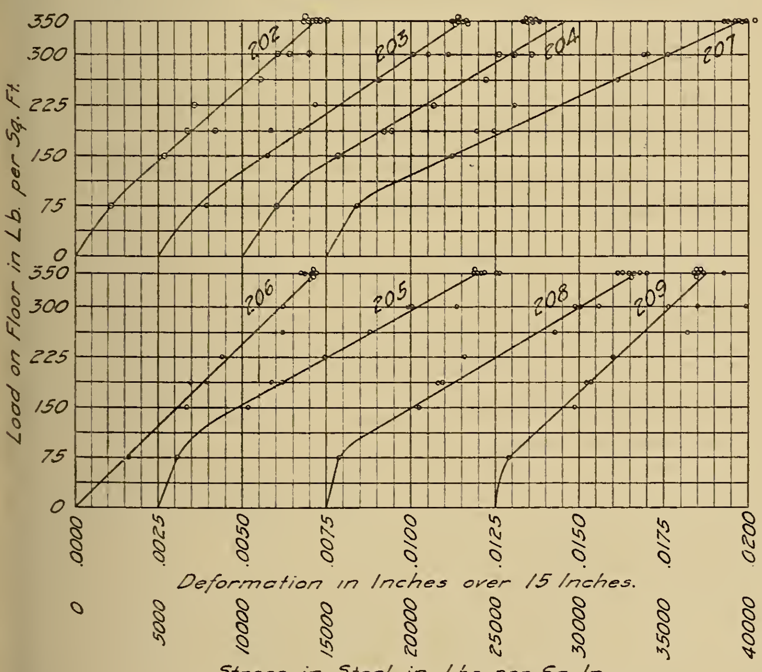

FIG. 73. DIAGRAM SHOWING STRESS IN REINFORCEMENT OVER CAPITAL.

TABLE 12.

Unit-deformation in Reinforcement at Center of Span between Columns.

Load in lb. per sq. ft.		Gauge Line											
Center Panel	Outer Panels	39	40	108	109	14	14a	110	111	112	7	3	12a
0	0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
75	75	– 1	1	– 3	1	0	1	– 5	3	– 3	– 1	1	– 3
150	150	– 1	3	1	1	1	3	– 5	4	– 4	– 3	3	– 3
187.5	187.5	0	4	4	3	8	8	– 3	– 1	1	1	7	3
187.5	187.5	0.00005	0.00007	0.00005	0.00004	0.00013	0.00011	–.00001	–.00001	0.00007	0.00007	0.00007	0.00008
225	225	11	9	7	9	15	13	5	4	8	12	8	1
262.5	262.5	11	19	9	15	20	15	7	9	9	12	9	15
300	300	21	20	12	15	24	19	7	3	11	12	12	17
300	300	13	20	12	13	25	20	4	3	8	15	12	17
300	300	0.00021	0.00024	0.00015	0.00017	0.00031	0.00023	0.00013	0.00011	0.00013	0.00021	0.00017	0.00021
350	350	21	27	20	24	31	24	15	17	13	21	19	23
350	350	20	29	20	24	32	23	15	15	12	20	19	24
350	350	21	25	21	24	33	23	13	16	13	20	19	28
350	350	24	28	20	23	35	23	16	16	12	20	19	27
350	350	25	27	21	23	33	23	19	16	11	19	17	28
350	350	21	27	20	24	33	23	17	12	9	19	19	28
350	350	0.00020	0.00025	0.00021	0.00024	0.00035	0.00023	0.00016	0.00011	0.00009	0.00020	0.00019	0.00027
350	187.5	19	25	23	26	31	17	19	9	12	17	17	19
350	0	17	28	21	26	11	– 1	21	15	15	9	4	3

52. *Stress in Reinforcement at Column Capital.*—Table 13 and Fig. 73 give the data on the stress in the reinforcement over the column capital. The upper diagram covers diagonal rods, the lower diagram cross band rods. Among the diagonal rods it may be noted that the stress in rod No. 207 was measured over the edge of the capital while that in No. 203 and 204 was measured opposite the center of the column. The higher stress in No. 207 would seem to indicate that the stress in these rods decreases, passing from the critical section at the

TABLE 13.

Unit-deformation in Reinforcement over Capital.

Load in lb. per sq. ft.		Gauge Line							
Center Panel	Outer Panels	202	203	204	207	205	206	208	209
0	0	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
75	75	14	8	7	6	4	11	2	2
150	150	24	22	19	25	18	22	19	14
187.5	187.5	29	28	30	33	24	26	23	18
187.5	187.5	.00035	.00022	.00028	.00030	.00022	.00023	.00022	.00018
225	225	30	31	38	37	26	29	28	23
262.5	262.5	44	44	48	58	42	42	45	52
300	300	47	50	50	63	50	42	50	50
300	300	49	54	54	63	50	42	49	34
300	300	.00053	.00058	.00057	.00067	.00059	.00047	.00054	.00040
350	350	52	60	58	72	62	48	57	39
350	350	54	59	56	73	62	47	61	45
350	350	55	60	58	75	64	47	60	40
350	350	53	58	57	75	62	45	58	40
350	350	52	60	57	72	68	47	59	40
350	350	54	61	56	76	67	48	61	41
350	350	.00056	.00061	.00056	.00078	.00064	.00049	.00063	.00041
350	187.5	53	55	54	80	64	48	60	41
350	0	50	48	51	77	62	48	55	39

edge of the capital to any section nearer the center of the column. This is as would be expected. The stresses found from these readings indicate clearly that the slab should be designed for a maximum moment over the support and not at the center. In the design of this building some 75% more reinforcement was provided over the support than was used at the center.

53. *Stress in Concrete at Edge of Capital.*—Fig. 74 and Table 14 give deformations observed in the concrete at the edge of the column capital. Owing to the fact that when the slab was poured there was no intention of testing, no specimens of the concrete were available from which to determine the modulus of elasticity. Hence it is necessary to assume a value for concrete about 40 to 45 days old cured in fall weather at Minneapolis. From experiments made at the University of

TABLE 14.

UNIT-DEFORMATION IN CONCRETE AT EDGE OF CAPITAL.

Load in lb. per sq. ft.		Gauge Line					
Center Panel	Outer Panels	102	103	104	105	106	107
0	0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
75	75	11	17	9	7	8	7
150	150	21	28	21	22	17	20
187.5	187.5	24	30	24	16	18	20
187.5	187.5	0.00015	0.00021	0.00018	0.00013	0.00015	0.00012
225	225	15	25	20	17	14	20
262.5	262.5	28	37	31	28	25	31
300	300	36	43	38	36	32	34
300	300	39	44	38	33	30	33
300	300	0.00032	0.00038	0.00033	0.00029	0.00024	0.00031
350	350	41	47	40	38	32	38
350	350	43	47	42	39	33	38
350	350	43	49	43	42	35	39
350	350	44	45	44	42	34	37
350	350	47	48	46	42	34	40
350	350	49	50	48	41	36	40
350	350	0.00047	0.00045	0.00046	0.00040	0.00034	0.00040
350	187.5	47	49	44	36	31	37
350	0	44	44	38	23	22	30

Illinois concrete of the same age cured under laboratory conditions showed a modulus of 1875000 lb. per sq. in., and in Table 15 this modulus has been used as giving the value for the concrete stress. In Fig. 74 a stress of 100 lb. per sq. in. corresponds to a deformation of .00032 in. if a modulus of 1875000 be assumed.

An interesting feature shown in the curves is the falling off in the concrete deformation when the load was allowed to remain over night. The decrease is less marked at higher loads than at low loads,

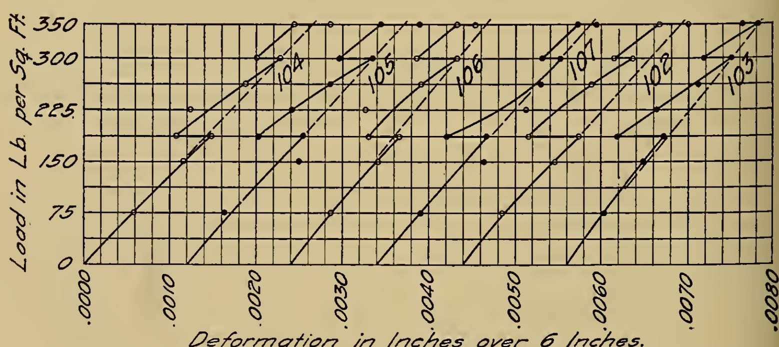

FIG. 74. DIAGRAM SHOWING STRESS IN CONCRETE AT EDGE OF CAPITAL.

while readings taken at very frequent intervals while the maximum load was on the floor showed that at first the stress steadily increased and the decrease did not begin until some time after the load was applied. The phenomenon is of interest as showing the readjustment in stresses which takes place under load even in the least plastic constructions. In general the concrete stresses checked those found in the reinforcement over the support.

54. *Summary of Stresses.*—Table 15 gives a summary of the stresses found at various points under the design load of 225 lb. per sq. ft. and also under the maximum load applied of 350 lb. per sq. ft.

TABLE 15.

Summary of Stresses.

Stresses are given in pounds per square inch.

		Design Load—225 lb. per sq. ft.			Maximum Load—350 lb. per sq. ft.		
		L. L.	D. L.	Total	L. L.	D. L.	Total
Reinforcement Over Head:							
Diagonal Band.	Maximum..	13 800	6 900	20 700	24 200	6 900	31 100
	Average....	11 000	5 500	16 500	18 800	5 500	24 300
Cross Band.	Maximum..	10 000	5 000	15 000	18 800	5 000	23 800
	Average....	9 000	4 500	13 500	17 200	4 500	21 700
Reinforcement at Center:							
Diagonal Band.	Maximum..	2 400	1 200	3 600	4 800	1 200	6 000
	Average....	2 000	1 000	3 000	4 800	1 000	5 800
Cross Band.	Maximum..	2 800	1 400	4 200	8 000	1 400	9 400
	Average....	2 500	1 300	3 800	6 600	1 300	7 900
Outer Panels.	Maximum..	4 600	2 300	6 900	10 400	2 300	12 700
	Average....	3 800	1 900	5 700	8 000	1 900	9 900
Concrete at Capital:							
Diagonal Direction.	Maximum..	530	265	795	800	265	1 065
	Average....	500	250	750	750	250	1 000
Cross Direction.	Maximum..	500	250	750	800	250	1 050
	Average....	468	234	700	750	234	984

Concrete Stresses based on E_c = 1,875,000 lb. per sq. in.

In making up this table the dead load stresses have been taken as one-half the indicated live load stress at the design load (the dead weight of the slab being half this load). This is a maximum assumption and probably is somewhat in excess of the true value, as the concrete was not broken in tension until after a live load of 75 lb. per sq. ft. was applied.

55. *Cracks.*—Very careful observations were made to discover and record all cracks. A reading glass was used to aid the eye, and dust was removed by means of bellows. It is easy *not* to discover cracks in such a test, and with casual observations very likely but few of these cracks would be noted. In fact all of them were very fine. At a load of 262.5 lb. per sq. ft. a crack was observed at the bulkhead where two days had elapsed between the pouring of the adjacent floor sections.

At 300 lb. per sq. ft. cracks appeared at the other bulkheads. Very fine cracks were also found in the center panel where no bulkhead existed and over the edge of the capital at column No. 41, these being very faint and hard to trace for any distance. At 350 lb. per sq. ft. there could be traced out the cracks shown in Fig. 75 in which the dotted lines represent cracks in the ceiling below. The cracks about the column head are of interest as indicating the position of the critical

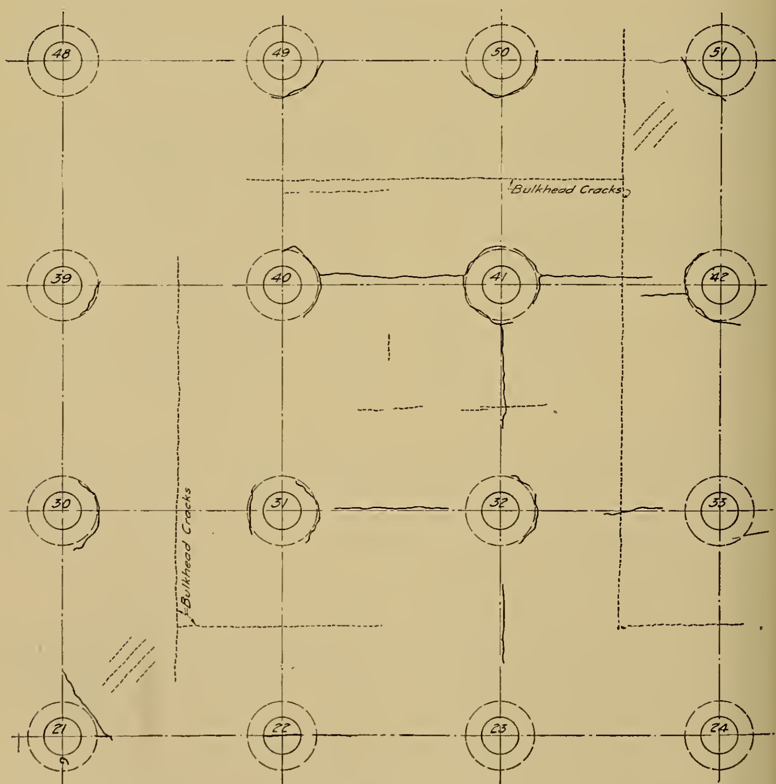

FIG 75. LOCATION OF CRACKS TRACEABLE AT LOAD OF 350 LB. PER SQ. FT.

section for which moments should be figured in analyses. These averaged about 2 or 3 in. outside the edge of the capital.

At column No. 51 the position of the crack would seem to indicate that for a single panel loaded the critical section moves nearer the support, resulting in higher stresses at the center. This crack and similar ones, as at columns No. 49, 39 and 21, were very faint, indicating a lower stress in the reinforcement over the support at such points. The cracks shown running diagonally near columns No. 21 and 51 were in all cases directly beneath slab rods.

Another set of cracks which developed only under the maximum load of 350 lb. per sq. ft. is significant. These cracks ran along the center line of the cross bands, being easily traced in the portion about half way between columns, growing fainter toward the columns, and disappearing entirely in most cases before reaching the crack over the edge of the capital. Evidently there is negative bending moment at these sections. These cracks, we believe, had not been observed before, probably because other building tests have not been so extensive, and because cracks have not ordinarily been very carefully observed.

56. *Comments.*—The most important result of the Deere and Webber Building test lay in the demonstration that a field test of a reinforced concrete building may be made with the reasonable expectation of securing reliable and useful data on the stresses developed in the steel and in the concrete. The test gives certain well-defined indications. It shows that the bending moment at the support is much greater than that at the center of the span. It indicates, by the position of the cracks, a critical section for which moments should be calculated. It indicates that the stresses at the center of the span are lower than analyses would lead one to expect. It indicates that bulkheads act to increase deflections and stresses. It indicates that the reinforcement at the center receives its maximum stress for the condition of load on one panel only.

VI. General Comments.

57. *General Comments.*—The tests described in the bulletin are of such a nature and cover so much ground that it is impracticable to summarize results or to formulate specific conclusions in any brief way. In the body of the text, the results of tests have been stated and described in detail, the action of structures discussed and conclusions drawn. The data are given in full in the tables and diagrams. In general the conclusions may be considered to be applicable to structures of similar construction. Possibly some of the conclusions, easily recognized in the text, will require further tests to determine whether they are generally appli-

cable. The information obtained in these tests will be found of value in the settlement of a number of questions which are in dispute, and the results when taken in connection with other tests may be expected to be of considerable assistance in developing analyses and determining constants for use in the design of reinforced concrete structures. Many of the results of the tests have a bearing upon the unsettled problems and even on matters which many have considered to be not open to question.

The tests here recorded have shown the practicability of measuring the deformations or strains in critical parts or members of a reinforced concrete structure when subjected to load. Methods have been developed for making measurements and tests in a way that will give trustworthy data. Difficulties have been overcome, and many of the precautions found necessary have been formulated. Skill and experience are essential in making such tests, and the difficulties encountered are of a wider range than those met in the best laboratory practice. As in other tests, caution must be exercised in drawing conclusions, and judgment must be used in interpreting results. The presence of low stresses should not be taken as being conclusively indicative of low bending moments; and the action of tension in the concrete, of horizontal thrust distributed over large distances, and of other agencies may need consideration. Action under partial load, as when a single panel is loaded, must be recognized to differ from that under full load.

Such problems as the distribution of bending moments along the length of the beam, the distribution of stresses over areas outside of those usually assumed as forming the beam, the presence of secondary stresses and of web stresses in structures as they are fabricated, will be solved only when adequate field tests have been made. The analysis of structures and the determination of the resistance of individual members or parts require the making of assumptions and the choice of constants, and the proper determination of these may be made only with full knowledge of the properties of the materials found by laboratory tests and of the action of the fabricated structure as shown in adequate field tests.

PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

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BULLETIN NO. 65

THE STEAM CONSUMPTION OF LOCOMOTIVE ENGINES FROM THE INDICATOR DIAGRAMS

BY

J. PAUL CLAYTON

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS

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UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

ULLETIN No. 65 JANUARY, 1913

HE STEAM CONSUMPTION OF LOCOMOTIVE ENGINES FROM THE INDICATOR DIAGRAMS

By J. Paul Clayton, Assistant, Mechanical Engineering Department, Engineering Experiment Station

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THE STEAM CONSUMPTION OF LOCOMOTIVE ENGINES FROM THE INDICATOR DIAGRAMS

I. Introduction

1. *Preliminary.*—Steam locomotives in service on the road are tested from time to time to obtain data as to their capacity and efficiency, both being matters of vital importance in the successful and economical operation of trains. The tests are made for various purposes, among which the most important are those for the determination of the indicated or dynamometer horse power, the measurement of the steam and coal consumption per horse power developed either in the cylinders or at the tender drawbar, and the adjustment of the valve gear.

Two general methods of testing locomotives for economy and capacity are in use: (1) On the test plant, where all quantities measured may be ascertained with great accuracy for any given set of conditions and where conditions can be controlled. (2) On the road in regular service by measuring the total coal and water used for all purposes, and by determining the power developed by the engine either in the cylinders by means of the indicator, or at the tender drawbar through the aid of a dynamometer car.

The first method is the only one which permits the accurate determination of the effects produced by the varying conditions of service, including the steam consumption per indicated horse power hour developed at a given rate of power.

The second method is of use only to measure the capacity and average economy over a given run or over a long period of time. The steam consumed at the several rates of power cannot be segregated in this method due to the continual change of profile, necessitating corresponding changes in the rates of power, and to operating restrictions imposed by different service conditions.

The use of steam for auxiliaries such as the air pump, the train-heating system, blower, electric train-lighting sets, and also the loss of steam or water through the safety valves, blow-off valves, and boiler leaks render impossible the determination of the actual steam consumed per unit of power developed by the engines, since from 2% to possibly 15% or 20% of the steam generated is used by the auxiliaries, or is lost. It is thus practically impossible under ordinary conditions to measure the actual steam consumption of the engines.

While indicator diagrams are generally taken during a locomotive test, the use of these diagrams has been limited to the determination

of the indicated horse power, the correctness of the adjustment of the valve gear, the effect of the exhaust nozzle on the back pressure, and the diagram factor as a guide for future engine design. It has been considered impossible to obtain a reliable estimate of the steam consumed by the engines from the diagrams, because of the variable and unknown "initial condensation" of steam up to the point of cut-off. Further, no data have been secured from the diagrams regarding piston and valve leakage, and it is practically impossible correctly to locate on the diagram the events of the stroke, cut-off, release, true compression, and admission.

2. *Development of a New Analysis of Cylinder Performance.*—As the result of an investigation of the forms of the expansion and compression curves which occur in indicator diagrams, new methods were developed for obtaining a measure of the actual steam consumption from the diagram alone, for detecting leakage into or out from the cylinder while the engine is in normal operation, for determining the amount of the clearance displacement, and for determining the location of the events of the cycle.

The development of these methods has been described in detail in Bulletin No. 58 of the Engineering Experiment Station and in a paper[1] before the American Society of Mechanical Engineers. A short résumé of the results will be given here.

The investigation disclosed the fact that the indicator diagram contains in itself the evidence necessary for an almost complete analysis of cylinder performance, the results of which have not heretofore been considered possible.

In obtaining these results the indicator diagram was transferred to logarithmic cross-section paper and a figure constructed which is called a logarithmic diagram. By the aid of this diagram it was found that the expansion and compression curves of all elastic media used in practice obey substantially the polytropic law $PV^n = C$, or in other words that the curves become straight lines on logarithmic paper. From this fact there were developed rational methods of approximating the clearance of a cylinder, of closely locating the cyclic events, and of detecting moderate leakage with the engine in regular operation.

It was discovered that the value of n in the law $PV^n = C$ is controlled directly in steam cylinders by the quality of the steam mixture which is present at cut-off, called x_c, and that the relation of x_c and n is practically independent of cylinder size and of engine speed for the same class of engine as regards jacketing and back pressure. This fact

[1] A. S. M. E. Journal, p. 539, April, 1912.

enables us to determine the actual amount of steam and water present in a cylinder at cut-off from the experimentally determined relations of x_c and n, and thus to determine from the diagram the actual steam consumed. This method of determining steam consumption will be called the $n - x_c$ method in subsequent discussion.

For details of the processes described and for the exhibit of the facts leading to the results mentioned, the reader is referred to Bulletin No. 58.

3. *Application of the Analysis to Locomotives.*—In view of the difficulties of determining the actual steam consumption of locomotive engines on the road, and of segregating the consumption at different rates of power, the advantages to be derived from applying the new analysis to locomotives are apparent.

For this application it is extremely fortunate that there are in existence several hundred complete tests of almost all common types of simple and compound locomotives. These tests were run on locomotive test plants under laboratory conditions and were carried on with great care.

4. *Sources of Data.*—The tests which have been analyzed in this investigation comprise the Purdue tests made by Dr. W. F. M. Goss, the St. Louis tests made by the Pennsylvania Railroad System, and later tests made at Altoona, Pennsylvania, by the Pennsylvania Railroad Company.

The Purdue tests were run at the locomotive testing plant of Purdue University, Lafayette, Indiana, on a 16 in. x 24 in. simple locomotive. The first series of tests analyzed were made in 1904-1905 with saturated steam under different pressures varying from 120 lb. to 240 lb. gage, and are noted in "High Steam Pressures in Locomotive Service," by Dr. W. F. M. Goss. In the second series of tests which are described in "Superheated Steam in Locomotive Service," by the same author, a Cole superheater giving about 160° F. of superheat was used under various pressures.

The St. Louis tests which are described in "Locomotive Tests and Exhibits," published by the Pennsylvania Railroad Company, were run at the Louisiana Purchase Exposition at St. Louis, Missouri, in 1904. The test plant was built by the Pennsylvania Railroad System, and erected in the Transportation Building of the Exposition. Two simple and six compound locomotives were tested at this plant, and the results from all of these tests have been analyzed.

The Altoona tests were run on the same plant as the St. Louis tests after that plant had been removed to its permanent location at Altoona, Pennsylvania. Of the tests made there since 1904, two simple locomotives

using saturated steam and one simple locomotive using steam superheated about 250° F. were among those analyzed.

The locomotives from which the tests were obtained comprise practically all important types in use in America and Europe except the Mallet compound locomotive. Six simple and six compound locomotives are represented.

5. *Results of the Analysis of Locomotive Diagrams.*—The development of the $n - x_c$ method of determining from the indicator diagrams the actual steam consumed, and of the other methods relating to leakage, clearance, and cyclic events results in enlarging the scope of locomotive tests and in making more accurate and valuable the information to be gained from them.

It also makes more valuable the application of the results of locomotive tests on test plants to the conditions obtaining upon the road. The application of these methods to road tests accomplishes the following results:

(1) The steam consumption of the engines may be obtained from the indicator diagram alone to a degree of accuracy greater than that obtained by most full road tests.

(2) The $n - x_c$ method of accounting for the actual steam consumption of the engines in connection with the regular road test will give reliable information as to the amount of steam used by auxiliaries and that lost and wasted, such information being exceedingly difficult to obtain at the present time.

(3) The existence, and in some cases the amount of leakage through valves and into and out from the cylinders may be ascertained; the spring of valve gears may be determined from the logarithmic diagrams as shown by the change in location of the cyclic events under various conditions.

(4) The clearance may be determined from the diagram to a satisfactory degree of accuracy.

6. *Acknowledgment.*—Acknowledgment is made to Dr. W. F. M. Goss for the use of material from the tests which he conducted at Purdue University; to Purdue University through Dean C. H. Benjamin for the use of the original indicator diagrams from the Purdue tests; to the Pennsylvania Railroad Company through Mr. C. D. Young, Engineer of Tests, for the use of the original indicator diagrams and of material from the St. Louis tests, and also from later tests made at Altoona.

Valuable suggestions have been received from Dr. W. F. M. Goss, Professor C. R. Richards, and Mr. F. W. Marquis.

Thanks are also due to Mr. A. F. Westlund for his efficient services in preparing the data, and in checking the manuscript.

II. Application to Simple Locomotives of the $n - x_c$ Method for Determining Steam Consumption

7. *Operating Conditions and Methods of Application to Tests.*—All of the tests of simple and compound locomotives analyzed were run with the locomotive operating under approximately constant conditions of speed, cut-off, and boiler pressure as shown by the laboratory designation. The duration of tests was in most cases from two to three hours, although some few were run for only 30 minutes. All readings except the weight of coal, this weight not being used in this investigation, and the indicator diagrams were taken every ten minutes.

All water lost from injector overflow was returned to the feed tank, or was caught, weighed, and allowance was made; all steam lost from safety valves, calorimeters, etc., was allowed for and subtracted from the quantity used per hour by engines; correction was not made in the log for the moisture carried by the steam, as the actual amount of steam and water present was desired; allowance was made for differences in height of water in the gage glass between the start and end of a test; therefore as far as can be ascertained, the weight of total steam and water used by engines per hour was correct, barring boiler leakage and the usual experimental errors incident to short boiler tests.

To represent the tests, one complete set of indicator diagrams taken at the same reading was selected, with an average mean effective pressure nearest to the average mean effective pressure of the whole test. These readings were taken when the boiler pressure was not more than 4 lb. from the average pressure prevailing during the test. Almost all of the sets of diagrams were within ½ of 1% of the average mean effective pressure for the test, hence the set selected could be taken fairly to represent the whole test.

From the set so selected logarithmic diagrams were constructed in accordance with the method described in Appendix I. The average values for n, x_c, and the absolute pressure at cut-off, called p, were determined from the logarithmic diagrams in the manner described in Appendix I, Article 3.

For the purpose of obtaining reliable results the following conditions were imposed:

(1) The steam pressure was maintained fairly constant. Tests having bad "steam failures" were rejected.

(2) The feeding of water was regular for the very short tests, short tests having irregular feeding were not used.

(3) Representative diagrams were chosen.

(4) Tests rejected by the Pennsylvania Railroad for evident inconsistencies were not used.

(5) Boiler pressures which varied between wide limits were not used. Only those diagrams were used which were taken when the boiler pressure did not vary more than 4 lb. from the average for the test.

All tests not rejected for the conditions mentioned were analyzed and used. The tests rejected amounted to 5% of the total number available.

8. *Relations of x_c and n for Simple Locomotive Engines.*—The relations of x_c and n from the cylinders of six typical simple locomotives were examined. The tests gave data as to the effect on the relation of change of cut-off pressure, speed, cylinder size, the use of saturated and superheated steam, and of various types of valves as they affected valve leakage and steam distribution.

A total of 189 tests was analyzed. The general classification of the locomotives tested and the number of tests analyzed for each one is given in Table 1. The principal dimensions of the six locomotives are given in Appendix II.

The general logs of the 189 tests are given in Tables 5-10 in Appendix II. Only those quantities are given which relate to the water fed and the values of x_c and n.

9. *Relations of x_c and n for the Purdue Locomotive.*—The values of x_c and n from the 104 tests of the Purdue locomotive are plotted in Fig. 1. These tests were run at the boiler pressures of 120, 160, 180, 200, 220, and 240 lb., and for each pressure at speeds of 97.5, 146, 195, and 243.5 r. p. m. The points comprise 35 tests with superheated steam, and 69 tests with saturated steam. The cut-off varied from 12% to 46% of the length of the stroke. In spite of the widely varying conditions of pressure, speed, cut-off, and quality of steam used, it is seen that the general relation of x_c and n is well defined.

The values of x_c and n for the 19 tests run at 200 lb. pressure are plotted in Fig. 2, showing the effect of various speeds for tests run at the same boiler pressure. This figure contains lines which will be explained in connection with the determination of the influence of speed upon the relations of x_c and n.

10. *Relations of x_c and n for all Simple Locomotives Tested.*—All of the values of x_c and n from the 189 tests of the six locomotives were plotted in Fig. 3. This figure proves the relation of x_c and n for one class of engine, and it also proves the comparative independence of this

TABLE 1

General Data for Simple Locomotives Tested.

Number or Name of Locomotive	Owner	Classification of Railroad Company	Type	Where Tested	Size of Cylinders, inches	Boiler Pressure	Quality of Steam Used	Type of Valves	Speeds Tested. r. p.m.	Number of Tests Analyzed
Purdue	Purdue University	Schenectady No. 2 and 3	American Eight Wheel	Purdue University Lafayette, Ind.	16 x24	120–160–180 200–220–240	Saturated and Superheated	Balanced Slide	97.5–146–195–243.5	104
7510	Penn. R.R.	K2	Pacific	Altoona, Pa.	24 x 26	205	Saturated	Piston	80–100–120–160–180	28
3395	Penn. R.R.	K29	Pacific	Altoona, Pa.	27 x 28	200	Superheated	Piston	100-120-160-180-240-280	18
5266	Penn. R.R.	E2a	Atlantic	Altoona, Pa.	20½ x 26	205	Saturated	Double-Ported Balanced Slide	80–120–160–200	13
1499	Penn. R.R.	H6a	Consolidation	St. Louis, Mo.	22 x 28	205	Saturated	Balanced Slide	40–80–120–160	11
734	L.S.& M.S. R.R.	B1	Consolidation	St. Louis, Mo.	21 x 30	200	Saturated	Double-Ported Balanced Slide	40–80–160	15

TABLE 2

General Data for Compound Locomotives Tested.

Number	Owner	Classification of Railroad	Type	Where Tested	Size of Cylinders, Inches	Boiler Pressure	Quality of Steam Used	Type of Valves	Speeds Tested	Number of Tests Analyzed H.P.	L.P
585	M.C.R.R.	W	Consolidation	St. Louis, Mo.	2 Cylinders 23 & 35 x 32	210	Saturated	H. P. Piston; L. P. Double-Ported, Balanced Slide.	40–80–160	9	9
929	A.T. & S.F. R. R.	900	Santa Fe	St. Louis, Mo.	4 Cylinders 19 & 32 x 32	225	Saturated	Piston	40–60–80	9	9
2512	Penn. R.R.	De Glehn Compound	Atlantic	St. Louis, Mo.	4 Cylinders $14\frac{3}{16}$ & $23\frac{11}{16}$x25¼	225	Saturated	H.P. Balanced D Slide. L.P. Not Balanced	80–160–240–280	8	5
535	A.T. & S.F. R.R.	507	Atlantic	St. Louis, Mo.	4 Cylinders 15 & 25 x 26	220	Saturated	Piston	80–160–240–280	8	10
628	Royal Prussian Railway	S8	Atlantic	St. Louis, Mo.	4 Cylinders $14\frac{5}{32}$ & 22 x 23⅝	200	Superheated	H.P. Piston; L.P. Double-Ported, Balanced Slide	80–160–240–280	9	9
3000	N.Y.C.&H. R.R.R. Co.	I1	Atlantic	St. Louis, Mo.	4 Cylinders 15½ & 26 x 26	220	Saturated	Piston	80–160–240–280–320	9	11

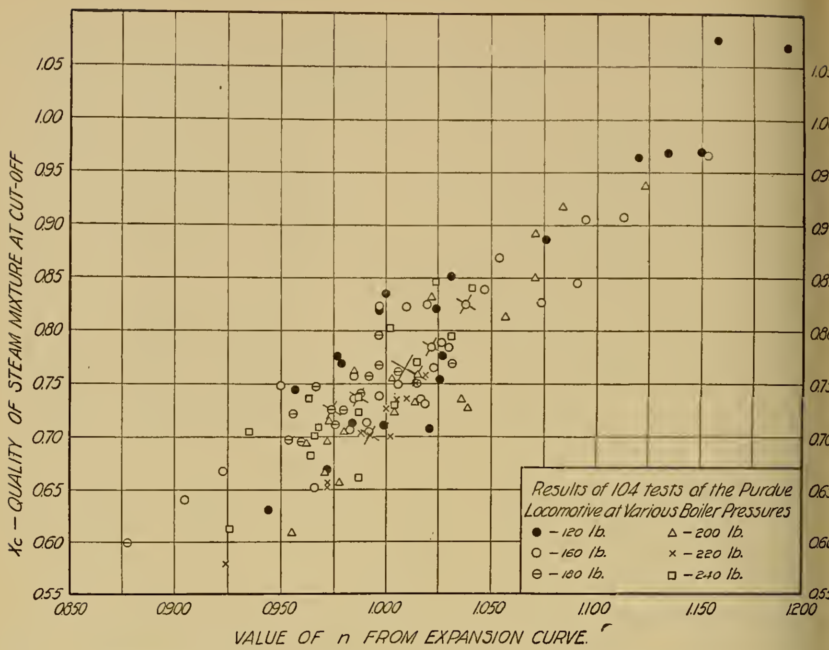

FIG. 1. GENERAL RELATIONS BETWEEN x_c AND n FOR THE PURDUE LOCOMOTIVE.

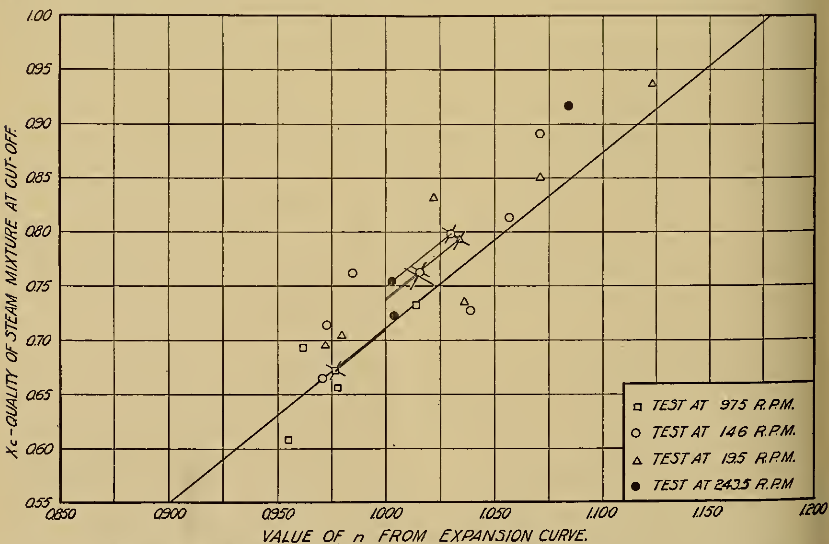

FIG. 2. RELATIONS OF x_c AND n FOR VARIOUS SPEEDS AT 200 LB. GAGE PRESSURE. FROM THE PURDUE LOCOMOTIVE.

relation from the effect of unduly varying pressures, speeds, sizes, types of valves, and quality of steam used.

In the tests shown, the speeds varied from 40 to 280 r. p. m.; the boiler pressures from 120 to 240 lb.; the displacement of the cylinders from 2.8 to 9.1 cu. ft. (the cylinder sizes varying from 16 in. x 24 in. up to 27 in. x 28 in.); the quality of the steam from saturation to 250° F. of superheat and the cut-off from 12% to 50% of the stroke.

The tests were made with locomotives having common slide valves, balanced slide valves, and piston valves. Almost every condition which is ever present in a simple locomotive cylinder is present except speeds below 40 and above 280 r. p. m., and lengths of cut-off over 50%.

The conditions mentioned were not obtained for the following reasons: Speeds below 40 r. p. m., which were made in general at the limit of adhesion, were not attempted because of the very uneven rotational speed which caused slipping. This was due to lack of sufficient flywheel inertia effect, this effect being present on the test plant and not on the road because on the road the whole mass of the locomotive and train caused the wheels to revolve at a uniform rate. The torque exerted by the brakes attached to the supporting wheels was fairly uniform, and the speeding up and slowing down of the drivers during parts of a revolution caused slipping, thereby necessitating the regrinding of the surface of the supporting wheels.

Lengths of cut-off exceeding 50% were not attempted because of the danger of slipping, as the coefficient of adhesion of the driving wheels upon the supporting wheels of test plants is far less than the coefficient for clean dry rails.

Speeds exceeding 280 r. p. m. were obtained, but the effect of the inertia of the indicator pencil motion at speeds of 320 r. p. m. and over, distorted the expansion curves so greatly that good values of n could not be obtained from them.

In determining the curve to represent the general relations of x_c and n shown in Fig. 3, several factors must be examined. It will be seen that as 55% of the tests were from one locomotive and if each test were given equal weight, then this locomotive, although the smallest one tested, would govern over one-half of the evidence on which the relations for all locomotives would be founded. The tests run on each locomotive were numerous enough that the average value of x_c and n for each one could be closely determined for its entire range of action. As the locomotives tested represented practically all of the typical designs of simple engines in modern use as to size, pressure, and valves,

it was decided to locate the "center of gravity" of the tests for each locomotive and to give equal weight to the tests from each.

The center of gravity, or average value of the values of x_c and n for each locomotive was obtained by averaging the co-ordinates of x_c and n, and plotting the result as a cross through the same kind of point which was used to represent the tests of each locomotive.

After the center of gravity of the tests of each locomotive was obtained, the location of the center of gravity for all the locomotives was determined in a similar manner by using the centers found for each one, the center for all tests being plotted as a large cross in Fig. 3.

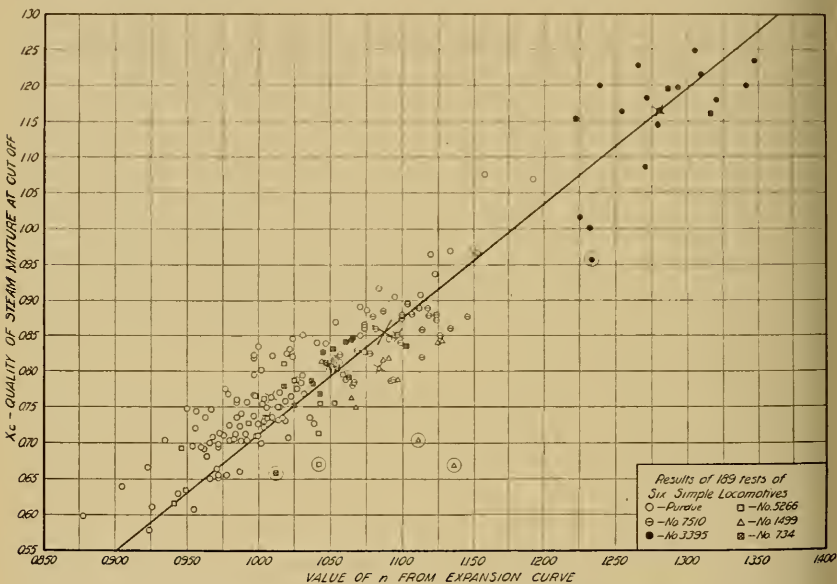

FIG. 3. RELATIONS OF x_c AND n FOR ALL SIMPLE LOCOMOTIVES TESTED.

A straight line was then drawn through the center of gravity of the tests of all locomotives, and was made to pass through the center of the tests of locomotive No. 3395.

A straight line was used from the following considerations: Previous experience[1] with the relations of x_c and n in Corliss engines had shown that the relation is a natural straight line for any one engine, and that the relations of other engines of the same class might differ a small amount as to the exact values obtained but that the slope of the line obtained was substantially the same. The relation of x_c and n

[1] Bulletin No. 58, p. 12. Engineering Experiment Station.

for adiabatic expansion as developed in Bulletin No. 58, is also a straight line having a different slope.

The line as drawn also represents the centers of the tests of all the locomotives with an average deviation of 2.4%, measured from the zero value of x_c. The average deviation of the points above the curve is about the same as that of the points below the curve.

The center of gravity of all tests is shown by a concentric circle which is 2.4% higher than the curve, due to the fact that the Purdue locomotive has 55% of all the tests, and is the highest point in distance measured above the line.

The points shown which are drawn with a circle around them were not considered in drawing the line, as they were undoubtedly subject to certain inconsistencies in testing, the exact causes of which have not been determined.

The equation of the line selected is $x_c = 1.620\ n - 0.909$. This equation expresses the results of all tests (except the five encircled points) to an average deviation of 5%, regardless of sign, and to a normal maximum deviation of about 9%.

It must be remembered, however, that all the errors of very short boiler tests are present and their effect is included in the values of x_c which have been found. For this reason it is believed that the errors given are considerably larger than the true ones, and that the line given represents the true value of x_c for a given value of n much more closely than appears from the errors given.

The points plotted in Fig. 3, of which the average condition is represented by the line drawn, were examined for the effect on the value of x_c, for constant values of n, of varying the speed, cut-off pressure, back pressure, size of cylinder, the type of valves used as regards leakage, the use of saturated and superheated steam, and the length of cut-off. The seven variables given comprise all of the variables present in the tests of the six engines examined and also in each engine under different conditions of construction and of operation.

11. *Effect on the Relations of x_c and n of Varying the Speed.*—In locomotive service the speed of engines under sustained conditions of operation varies from about 40 up to perhaps 320 r. p. m., the latter speed corresponding to a piston speed of 1500 ft. per min. for a 28 in. stroke of the piston.

In stationary practice the speed of engines having the same length of stroke or longer is between 60 and 150 r. p. m., thus having a range of speed of only 90 r. p. m. between limits. Locomotive engines, however, have a range of speed of about 280 r. p. m. between limits,

showing that the range is three times that of stationary engines of equivalent size or power. It may be expected, therefore, that varying the speed would affect in some degree the relations of x_c and n for locomotive engines.

The method employed to examine the effect of change of speed upon the relations of x_c and n is illustrated in Fig. 4, which is derived from

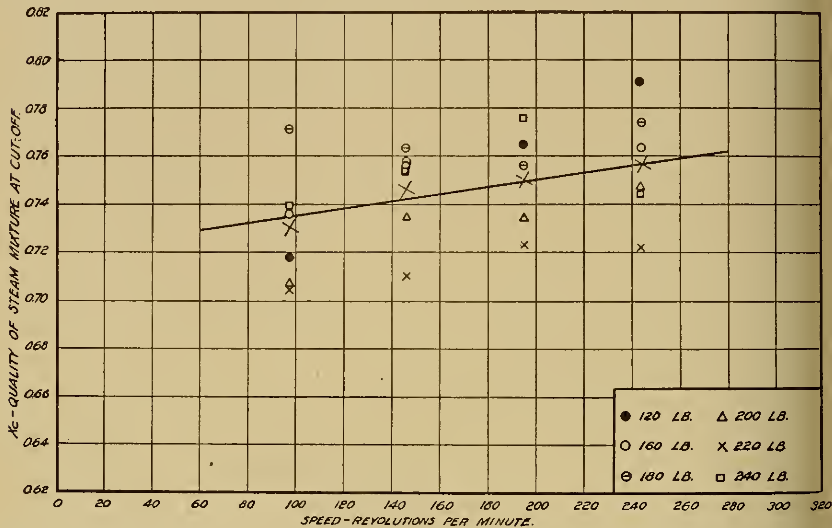

Fig. 4. Relations of x_c and Speed from the Purdue Locomotive for the Constant Value of $n = 1.000$

Fig. 2. Fig. 2 gives the points derived from all tests of the Purdue locomotive run at 200 lb. boiler pressure. The center of gravity of each group of tests run at one speed was obtained and was plotted as a cross through a point representing the speed examined. The full line representing the average relation for all locomotives is drawn as shown. Through the center of gravity for each speed a line was drawn, parallel to the full line, to the intersection of the line $n = 1.000$. The point of intersection therefore gives the value of x_c, for the constant value of $n = 1.000$, at the speed examined. The points of intersection thus obtained were plotted in Fig. 4, and are represented by the triangles. Points obtained from the tests at other pressures were also plotted in Fig. 4. This figure represents the relation of x_c and speed at the value of $n = 1.000$ for the series of pressures used.

It is evident from the points plotted in Fig. 4 that there is a definite tendency for a high value of x_c to be obtained as the speed increases, the value of n remaining the same.

The process just described was repeated for the tests of all the locomotives and, together with the centers of gravity of the Purdue tests, was plotted in Fig. 5. A group of points was obtained which like Fig. 4 showed that the average effect of increasing the speed was to increase the value of x_c, for constant values of n.

To represent the average effect of change of speed upon x_c, the points were divided into two groups and the center of gravity of each one was found as indicated by the crosses; a line was then drawn through the two centers as shown. The average deviation of the points irrespective of sign is 3.2% (measured from the zero of x_c), each point being weighted in proportion to the number of tests it represented, except the Purdue tests, which were rated at 4 tests each.

From Fig. 3 it is seen that the line crosses the value $n = 1.000$ at $x_c = 0.711$. On drawing the line $x_c = 0.711$ in Fig. 5 it is found to

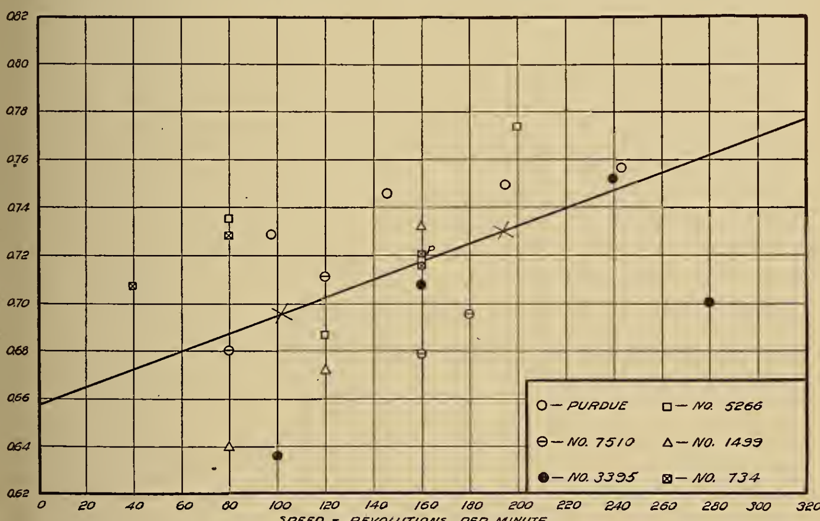

FIG. 5. RELATIONS OF x_c AND SPEED FROM ALL SIMPLE LOCOMOTIVES FOR THE CONSTANT VALUE OF $n = 1.000$

intersect the line representing the relations of x_c and speed at 143 r. p. m. This intersection indicates that the average relation of x_c and n represented by the line $x_c = 1.620\, n - 0.909$ is true for a speed of between 140 and 150 r. p. m. It also indicates: (1) That the average points obtained at speeds below 140 r. p. m. fall below the line; (2) That the average points obtained at speeds above 150 r. p. m. fall above the line; (3) That by making corrections for the speed at which the indicator diagrams were obtained, the average deviation resulting from determin-

ing the value of x_c by the equations given is lowered from 5.0% to 3.2%

The equation $x_c = 1.620\ n - 0.909$ was modified therefore by adding a corrective term for speed which expresses the relation of x_c and speed shown in Fig. 5. The modified equation is

$$x_c = 1.620\ n - 0.909 - 0.00037\ (143 - S)$$

where $S =$ speed in r. p. m.

The corrective term drops out at 143 r. p. m., it lowers the value of x_c for speeds below 143 r. p. m., and raises the value of x_c for higher speeds.

The causes which operate to produce the effect upon the value of x_c due to change of speed are discussed in Section IV.

12. *Effect of Varying the Cut-off Pressure.*—The effect on the value of x_c due to variation of the cut-off pressure of the diagram, speed and n being constant, was first observed from the Purdue tests, which include a wide range of cut-off pressures resulting from boiler pressures varying from 120 to 240 lb.

The points used were plotted in Fig. 6 and were obtained from Fig. 4

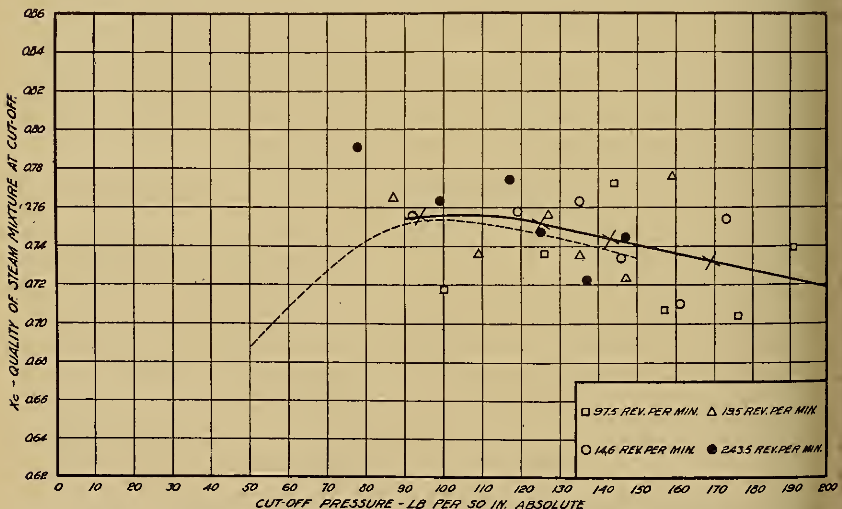

FIG. 6. RELATIONS OF x_c AND CUT-OFF PRESSURE FROM THE PURDUE LOCOMOTIVE FOR CONSTANT SPEEDS AND THE VALUE OF $n = 1.000$.

as follows, the example being taken from the tests at 20 miles per hour; the cut-off pressures for the tests run at 220 lb. pressure and at the speed of 97.5 r. p. m. (20 miles per hour) were averaged and found to be 176 lb. absolute; hence for n=1.000 and speed S=97.5 r. p. m., the value of x_c obtained is 0.704. In a similar manner the tests at

200 lb. pressure gave an average cut-off pressure of 157 lb. absolute and a value of 0.707 for x_c. This procedure gave the relation of x_c and cut-off pressure at the constant speed of 97.5 r. p. m. and $n = 1.000$.

In like manner the points for the other speeds were plotted in Fig. 6, thus giving the relation desired for a series of constant speeds. The points obtained were divided into four groups of six points each and the center of gravity of each group found as indicated. The centers were joined by a smooth curve. The curve as drawn indicated that the value of x_c at constant values of S and n reached an average maximum value for a cut-off pressure of about 105 lb. absolute, and that above 125 lb. a straight line was obtained.

It appears therefore that as the cut-off pressure increases the value of x_c decreases for constant values of S and n.

The effect on the relations of x_c and n, due to change of cut-off pressure, was compared with the same effect found in the previous tests[1] on a Corliss engine and the relations for this engine for the value of $n = 1.052$ and the speed of 120 r. p. m. was drawn dotted as shown.

The result of this comparison is a striking corroboration of the curve as drawn. The curves are almost exactly parallel in the range common to both. Both have a maximum point only 5 lb. apart, and both have about the same slope for pressures above 120 lb. The form of the curve representing the effect of cut-off pressure is thus fairly well established.

The points in Fig. 5 were examined for the effect of cut-off pressure in a way similar to that described for the Purdue tests, and the results are given in Fig. 7. The centers of gravity obtained in Fig. 6 were transferred to Fig. 7 to represent the Purdue tests.

The points of Fig. 7 give the average relation for all locomotives of x_c and cut-off pressure for a series of constant speeds and for $n = 1.000$. The group of points obtained shows the same trend as the Purdue tests although the vertical displacement due to speed is present and makes the relation somewhat indefinite.

It was assumed that the Purdue tests shown in Fig. 6, because of their large range of cut-off pressures and similarity to previous relations, represented the form of the curve, but was placed too high on the plot to represent the average relation for all locomotives. Therefore the center of gravity of all the points of Fig. 7 was found and a curve drawn through this center similar in form to the Purdue relation.

It is seen from Fig. 7 that the range of cut-off pressures in simple locomotives is between 120 and 180 lb. absolute, while the majority lie

[1] Bulletin No. 58, p.15, Engineering Experiment Station. Journal A. S. M. E., April, 1912, p. 550.

between 130 to 160 lb. The average cut-off is about 145 lb., this point being close to the average value of $x_c = 0.711$ for $n = 1.000$ given in Fig. 3.

It is also evident that the difference in the value of x_c for the average cut-off, 145 lb., to an average high value, 165 lb., is only 1.6%, while to an average low value, 125 lb., it is only 1.3%.

The effect of varying the cut-off pressure on the relation of x_c and n is so small of itself, and so much smaller than the effect due to change of speed, that it need not be considered.

13. *Effect of Varying the Size of Cylinders.*—The size of the cylinders varied from 16 in. x 24 in. up to 27 in. x 28 in., the latter having a displacement of 3.25 times the smaller one. No effect on the relation of x_c and n was found that could be traced to any change of size of the cylinders.

In applying the $n - x_c$ method to stationary engines, as noted in Bulletin No. 58, the sizes varied from 10.5 in. x 12 in. up to 34.2 in. x 60 in., the latter having 53.5 times the displacement of the former. No effect in the relation of x_c and n was found from this cause.

14. *Effect of Various Types of Valves.*—The locomotive engines tested were equipped with plain balanced D slide valves, balanced double-ported slide valves, and piston valves. It was thought that there might be some difference in the apparent relations of x_c and n due to the different types of valves allowing steam to leak at different rates from the steam chest direct to the exhaust passage.

The results indicate that the locomotive having the least apparent valve leakage had balanced D slide valves as did also the one having the greatest valve leakage. The results of the two piston valves tested indicate that they both leaked apparently at a rate intermediate between the two extreme cases, their rate of leakage being close to the average for all valves. None of the valves tested leaked materially into or out from the cylinder.

The probable leakage occurring through the valves is discussed in Section IV.

15. *Effect of the Use of Superheated and of Saturated Steam.*—The use of superheated steam in cylinders in place of saturated steam results in a higher value of x_c for the same conditions of speed and cut-off. The higher value of x_c which is obtained results, in turn, in a higher value of n. When the same value of x_c is obtained at the same speed with either saturated or superheated steam, the same value of n has been found to result.

16. *Effect of Varying the Length of Cut-off.*—The length of cut-off used in the tests varied from 12% to 50%. It has been found that, other conditions being the same, if a given value of x_c occurs in one test at 12% cut-off, and in another test on the same locomotive at 46% cut-off, the value of n resulting is the same, showing that the length of cut-off has no measurable effect on the relation of x_c and n.

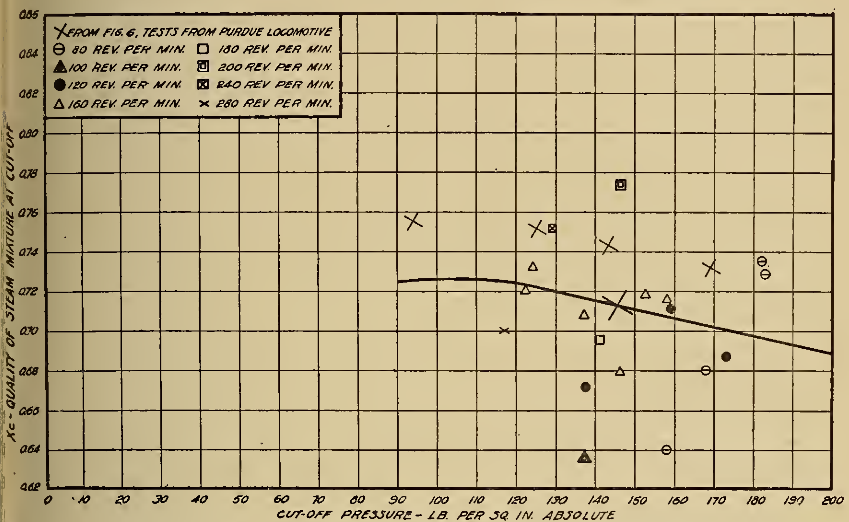

FIG. 7. RELATIONS OF x_c AND CUT-OFF PRESSURE FROM ALL SIMPLE LOCOMOTIVES FOR CONSTANT SPEEDS AND THE VALUE OF $n = 1.000$

17. *Effect of Varying the Back Pressure.*—The back pressure of the steam in the cylinders varied from 15 to 36 lb. absolute, a range of 21 lb. Varying the back pressure between these limits appears to have no effect on the relation of x_c and n.

When the back pressure is as high as 100 lb. absolute, however, as in the high pressure cylinders of compound engines, the relation of x_c and n is changed considerably, other conditions being the same.

III. APPLICATION OF THE $n - x_c$ METHOD TO COMPOUND LOCOMOTIVES

18. *Operating Conditions and Methods of Application to Tests.*—The same general test conditions prevailed for the compound locomotives as for the simple locomotives.

The method of selecting a set of indicator diagrams to represent the average conditions of a test was as follows: The average power of the test was represented by a combined sum made up of the sum of the values of the mean effective pressures for the low pressure cylinders, after each was multiplied by the value of the cylinder ratio, plus the

sum of the values of the mean effective pressures for the high pressure cylinders. The set chosen, taken at the same reading, was one in which this combined sum was nearest to the sum of the test averages obtained in the same manner, and in which the boiler pressure was not more than 4 lb. from the average pressure prevailing during the test. The method of selection used eliminates any unequal division or work in the cylinders. The desired condition was, in most cases, fulfilled to within an average of ½ of 1% of the mean condition.

For the purpose of obtaining reliable and accurate results the following conditions, in addition to those mentioned on page 9, were imposed:

(1) Expansion lines showing extreme effects of inertia or sticky indicator pistons were not used, as fair values of n could not be obtained.

(2) The range of pressure from cut-off to release during expansion exceeded 12 lb. for low pressure indicator diagrams and 60 lb. for high pressure diagrams in order to provide a sufficient length of curve to be able accurately to determine the value of n.

The tests rejected for the reasons outlined above and those given on page 9 amounted to 10% of the total number available.

For the purposes of this investigation the high pressure and low pressure indicator diagrams were treated as separate tests, each receiving, however, the same weight of steam per revolution from the boiler.

19. *Relations of x_c and n for Compound Locomotives.*—Indicator diagrams from six typical compound locomotives were examined to determine the relations of x_c and n. Data were obtained as to the effect on the $n — x_c$ relation of change of speed, cut-off and back pressure, the use of saturated and superheated steam, the type of valves employed, and the effect of various rates of leakage of steam from the steam chest directly under the valves to the exhaust passage without this steam having entered the cylinder. This class of leakage in subsequent discussion will be called simply valve leakage.

The conditions existing during the tests varied between wide limits. The speeds varied from 40 to 320 r. p. m. The boiler pressures varied from 200 to 225 lb. The pressure in the receivers between the high and low pressure cylinders varied from 25 to 95 lb. gage. The back pressure in the low pressure cylinders varied from 1 to 20 lb. above the atmosphere. The cut-off varied from 25% to 65% of the length of the stroke. The valves used included plain, balanced, and balanced double-ported slide valves, plain piston valves, and special piston valves controlling the steam distribution to both high and low pressure cylinders. The receiver capacity varied from 0.4 to 2.2 times the displacement of the low pressure cylinders. The size of the high pressure cylinders varied

from 14.16 in. x 23.65 in. to 23 x 32 in.; the low pressure cylinders varied from 22 in. x 23.65 in. to 35 in. x 32 in.

A total of 52 high pressure and 53 low pressure tests are presented. For very serious inconsistencies, due apparently to excessive valve leakage amounting to as high as 18.5% of the total steam used, all tests of locomotive No. 929 and all low pressure tests of No. 535 were not used to obtain the general relation of x_c and n. An analysis of the tests of these locomotives in comparison with the other tests is given in Section IV. After deducting the tests not used, there remained a total of 43 tests from the high pressure cylinders of five locomotives and 34 tests from the low pressure cylinders of four locomotives.

The general classification of the locomotives tested and the number of tests analyzed for each one are given in Table 2. The principal dimensions of the six locomotives are given in Appendix II.

The general logs of the 105 tests are given in Tables 11-16 in Appendix II. Only those quantities are given which relate to the water fed and to the values of x_c and n.

20. *Relations of x_c and n for High Pressure Cylinders.*—The values of x_c and n for all high pressure tests are plotted in Fig. 8.

The values obtained exhibit a much larger variation in position than those obtained from the simple locomotives in Fig. 3. The large variation obtained was found to be due to the addition of an important variable not present in the simple locomotives, namely, widely varying back pressures in the cylinders due to the varying receiver pressures employed in the different locomotives tested.

The center of gravity of the tests of each locomotive, the center of all tests, and the center obtained from the centers for each locomotive were all obtained in a manner similar to that fully described on page 13. The center of tests for No. 929 was plotted, but was not used. It is the point shown encircled in Fig. 8.

Unfortunately the range in the values of x_c obtained was so small relatively that a definite slope for a line to represent the average relations for all tests could not be obtained. It was decided, however, in the light of previous experience that the relations for the high pressure cylinders of locomotives possessed a slope similar to that obtained from simple locomotives, as the same general design and conditions prevail for both types.

A line was drawn, therefore, as shown in Fig. 8, to represent the average relation obtained from the high pressure cylinders of typical compound locomotives. The equation of this line is

$$x_c = 1.620\ n - 0.827.$$

The average deviation of all points from this line is 8.0%, while the normal maximum deviation is 12.0%.

The crosses representing the center of the tests for each locomotive are located at distances varying considerably from the average line due to varying back pressures for the same cut-off pressures and also to varying rates of valve leakage. The effect of both of these variables is treated later.

21. *Effect of Varying the Speed.*—The effect of varying the speed on the relations of x_c and n was investigated by the same general method as that employed for simple locomotives, except that, in order to eliminate the vertical displacement of the center representing all the tests for each locomotive, the values of x_c for this center were each corrected to the average line in Fig. 8.

The points obtained are plotted in Fig. 9. The method of correcting for vertical displacement of the center for the tests of each locomotive is as follows: A line was passed through this center parallel to the average line for all tests and extended to the intersection with the value of $n = 1.000$; the value of x_c at the intersection was read off; the amount in the value of x_c that the center was above or below the line

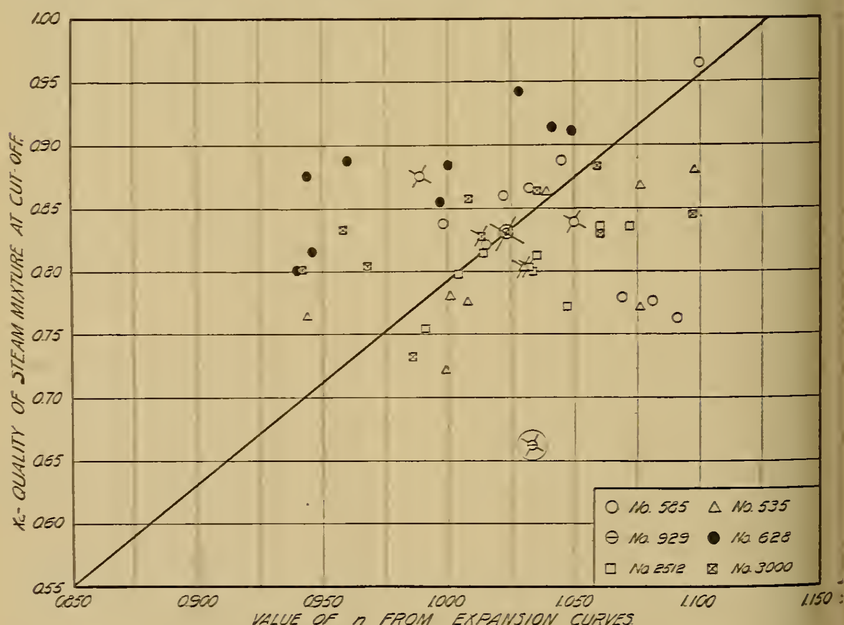

Fig. 8. Relations of x_c and n for High Pressure Cylinders of Compound Locomotives.

was subtracted or added respectively to all readings for the intersections of the lines passed through the center of tests at each speed. Thus if a center for the tests of one locomotive was located 0.06 in the value of x_c above the line, the value of 0.06 was subtracted from the value at the intersection of each speed center at the value of $n = 1.000$.

The average effect of change of speed was determined as the straight line in Fig. 9. The average deviation of the points from the line is 3.4%. The result of accounting for the effect of speed on the equation expressing the relations of x_c and n is as follows:

$$x_c = 1.620\ n - 0.827 - 0.00034\ (150 - S)$$

where S=speed of engine in r. p. m.

The speed effect obtained is very similar to the same effect for simple locomotives, but with a slightly different slope.

22. *Effect of Varying the Range of Pressure.*—It was found that other conditions remaining the same, raising the back pressure in the high pressure cylinder increased the value of x_c for the same value of n. It was also found that lowering the cut-off pressure, the back pressure remaining the same, had the same effect, though to a smaller degree. This last effect is corroborated by the same effect observed in the relations of x_c and n for simple cylinders given in Fig. 7.

Since the effects of raising the back pressure and of lowering the cut-off pressure each change the value of x_c in the same direction, they are cumulative and can be combined as the range of pressure from cut-off to the average back pressure. It was also found after careful examination that this range of pressure was the best measure of the observed effects for both the high and low pressure cylinders.

The back pressure in the high pressure cylinders is taken from the indicator diagrams as the average pressure at mid-stroke when the receiver capacity is large and the back pressure is therefore fairly uniform. When the receiver capacity is relatively small and the back pressure therefore is extremly variable, it is obtained from the diagrams at a point near the beginning of compression.

For the purpose of ascertaining the effect of varying the range of pressure on the relation of x_c and n, the effect due to change of speed was removed from the values of x_c obtained from the intersection of the line extended from the center of tests at one speed.

To show the effect desired the points obtained are plotted in Fig. 10. The corrections made for the effect of speed were obtained from Fig. 9. The method used was as follows: The average speed of all the tests of one locomotive was found and the value of x_c read off from the line in Fig. 9 for this speed; the correction for speeds below the average

was made by adding to the value of x_c obtained, the difference between the value of x_c taken from the line at this speed, and the value of x_c found for the average speed; the correction for speeds above the average was made by subtracting in a similar manner the difference obtained.

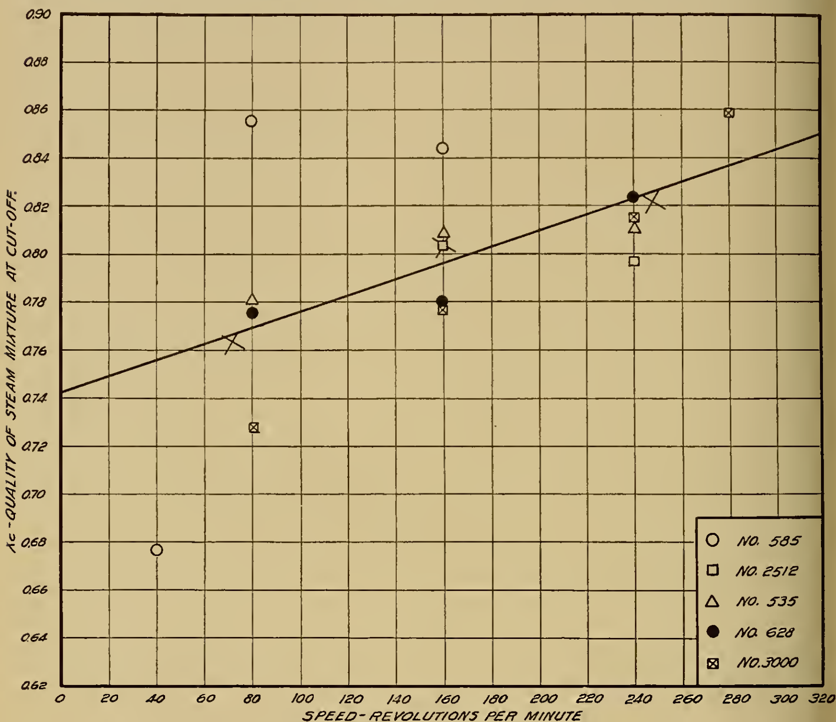

FIG. 9. RELATIONS OF x_c AND SPEED FOR HIGH PRESSURE CYLINDERS OF COMPOUND LOCOMOTIVES FOR CONSTANT VALUE OF $n = 1.000$, AND FOR CENTER OF TESTS CORRECTED TO AVERAGE LINE.

For example, if the average speed for the tests of one of the locomotives is 160 r. p. m., the average value of x_c at $n = 1.000$ from Fig. 9 is 0.796. If the center of tests at 80 r. p. m. be considered, the average value of x_c at 80 r. p. m. from Fig 9 is 0.769, being lower than the value of x_c at 160 r. p. m. by 0.027. The value of x_c from the center of tests at 80 r. p. m., after being increased by 0.027 to eliminate the average effect of speed, would be plotted in Fig. 10.

In examining the effect under consideration it is apparent that the relations of x_c and n for simple cylinders and for the high pressure cylinders of compound locomotives differ only in the effect produced by different ranges of pressures; that is, if the back pressure in the high pressure cylinder were lowered to about atmospheric pressure,

this cylinder would operate, under the same conditions, as a simple cylinder and would therefore have the same relations of x_c and n. The effect of range of pressure for high pressure cylinders and for simple cylinders is, therefore, a continuous one.

The centers of all tests of each locomotive, the centers of tests at each speed for each locomotive, and two concentric circles and a cross representing the average relation for all simple locomotives are plotted

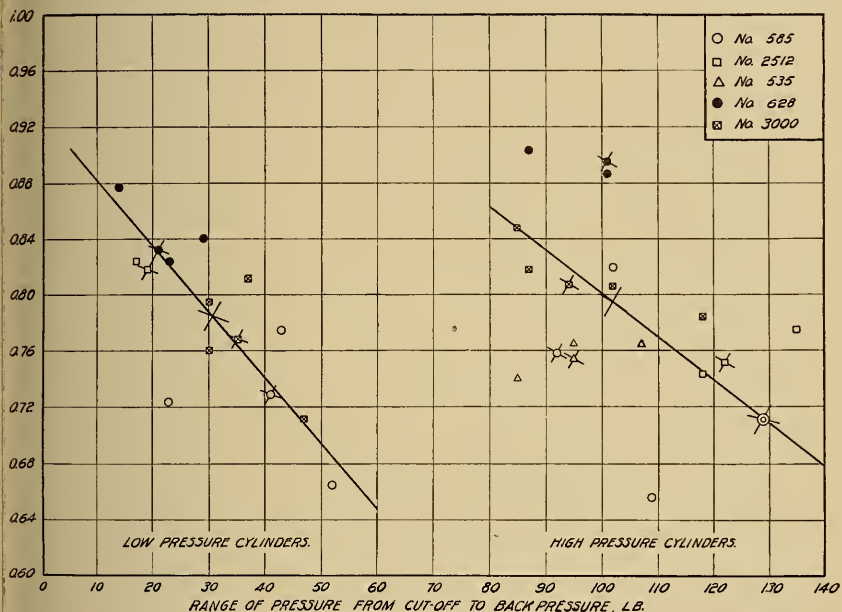

FIG. 10. RELATIONS OF x_c AND RANGE OF PRESSURE (CUT-OFF TO AVERAGE BACK PRESSURE) FROM COMPOUND LOCOMOTIVES FOR CONSTANT SPEEDS AND THE VALUE OF $n=1.000$

in Fig. 10. The center of all high pressure points plotted was obtained and is plotted as a cross.

A line passing through the center of high pressure tests and the center of simple tests was drawn to represent the average effect of range of pressure on the relation of x_c and n.

The equation after correcting for the average effect of range of pressure takes the form

$$x_c = 1.620\,n - 0.827 - 0.00034\,(150 - S) + 0.0031\,(102.5 - R)$$

where R = range of pressure from cut-off to the back pressure in lb. per sq. in.

The average value of x_c is obtained at 102.5 lb. range of pressure. The value of x_c is higher for ranges of pressure below 102.5 lb., and

lower for ranges above 102.5 lb. The average deviation of the points from the line is 6.2%. This large deviation apparently shows the effects of varying rates of valve leakage as explained in Section IV.

23. *Relations of x_c and n for Low Pressure Cylinders.*—The methods employed to obtain the general relations of x_c and n for low pressure cylinders, and the effect on this relation of change of speed and range of pressure, were similar in all respects to the methods described for the high pressure cylinders.

The values of x_c and n obtained are all plotted in Fig. 11. The values obtained from locomotives No. 929 and No. 535 were not used on account of excessive valve leakage.

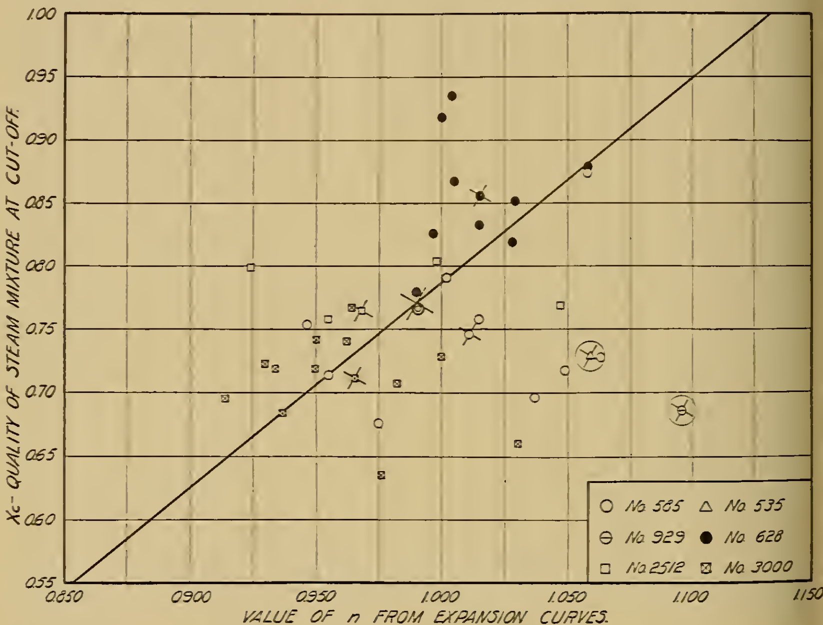

FIG. 11. RELATIONS OF x_c AND n FOR LOW PRESSURE CYLINDERS OF COMPOUND LOCOMOTIVES.

The line shown in Fig. 11 was drawn as outlined for the high pressure cylinders, using the same slope as the line obtained for the simple cylinders. The equation of the line is

$$x_c = 1.620\ n - 0.833.$$

The average deviation of all the points from this line is 7.8%.

24. *Effect of Varying the Speed.*—The points obtained which show the effect of varying the speed on the relation of x_c and n are plotted

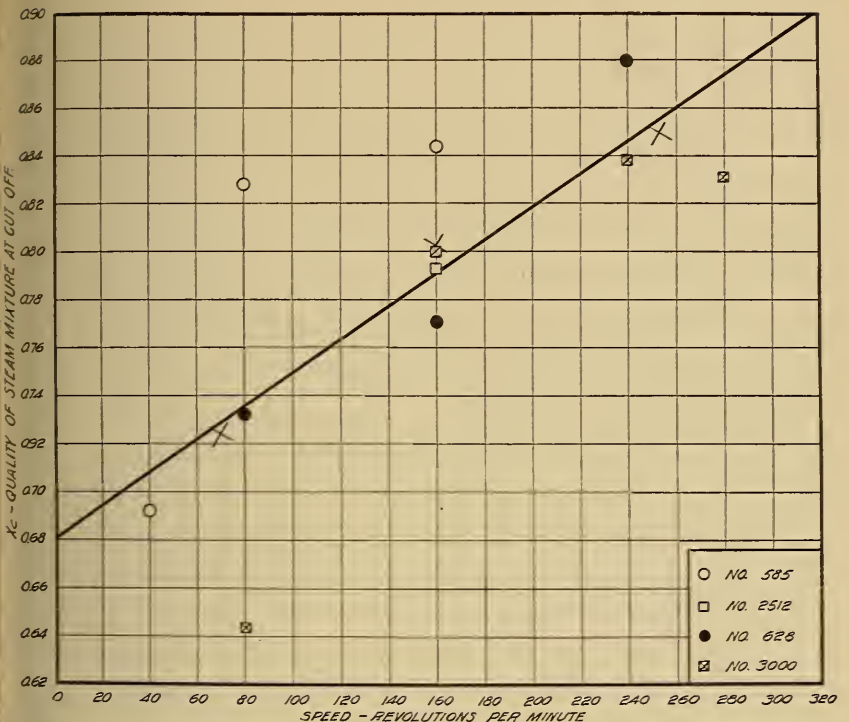

FIG. 12. RELATIONS OF x_c AND SPEED FOR LOW PRESSURE CYLIN DERS OF COMPOUND LOCOMOTIVES FOR CONSTANT VALUE OF $n=1.000$, AND FOR CENTER OF TESTS CORRECTED TO AVERAGE LINE.

in Fig. 12. The equation after correcting for the effect of change of speed is

$$x_c=1.620\ n-0.833-0.00069\ (154-S)$$

The average deviation of the points from this line is 4.5%.

25. *Effect of Varying the Range of Pressure.*—The points obtained which show the effect on the relation of x_c and n of changing the range of pressure are plotted in Fig. 10. The equation as further modified by this effect is

$$x_c=1.620\ n-0.833-0.00069\ (154-S)+0.0047\ (30-R)$$

The average deviation of the points from this line is 3.3%.

This extremely close agreement shows that the effects of varying the speed and range of pressure comprise all the important variables.

IV. ANALYSIS OF THE $n - x_c$ RELATIONS

26. *Causes Which Affect the $n - x_c$ Relations.*—The data which have been presented in the preceding pages will here be examined critically as to the causes which affect the relation of x_c and n due to

varying the range of pressure from cut-off to the back pressure and to varying the speed of the engine. These two variables are the only ones which cause an appreciable effect on the $n - x_c$ relation for locomotive engines.

It is well here to consider and compare the operating conditions of pressure and speed obtaining in stationary and in simple locomotive engines.

The boiler pressure of simple locomotives is always closely constant. The great majority have pressures of about 200 lb. gage, although the newer designs using highly superheated steam employ pressures of from 165 to 190 lb. gage. Pressures of from 200 to 230 lb. are employed in compound locomotives. The pressure used in stationary engines varies from 80 to 200 lb. gage.

The speed of locomotive engines in sustained operation varies from about 40 to 320 r. p. m. The speed of simple stationary engines of equal power varies from 60 to 150 r. p. m.

Relatively speaking, therefore, simple locomotive engines may be classed as constant pressure and variable speed; while compound locomotive engines may be classed as variable pressure and variable speed. Simple and compound stationary engines, however, may be classed as constant speed and variable pressure types.

From this classification it is apparent that the effects on the $n - x_c$ relation to be considered are as follows:

(1) In simple and compound stationary engines the effect of range of pressure is important while speed may be neglected.

(2) In simple locomotive engines the effect of speed is important while range of pressure may be neglected. (See Fig. 6 and 7.)

(3) In compound locomotive engines the effects of both range of pressure and speed are important. (See Fig. 9, 10, 12.)

The valves of a locomotive engine, when compared with stationary engines of the same size, are extremely large in proportion to the size of the cylinder. This is due to the high speeds which it is necessary to employ in order to obtain large specific capacities from the cylinders. As much as 2500 indicated horse power has been obtained continuously from two 27 in. x 28 in. cylinders. (See tests of No. 3395.) The piston valves of this locomotive are 16 in. in diameter.

If it be true as stated by various experimenters that single valves as a class leak steam directly under the valve from the steam chest to the exhaust passage, without this steam having entered the cylinder, then locomotive engines with their large valves might be expected seriously to show the effects of valve leakage at low speeds.

In Corliss and four valve engines any steam that leaks from the steam chest through the valves must leak into the cylinders.

27. *Effect on the Relations of x_c and n of Varying the Range of Pressure.*—The effect of lowering the range of pressure from cut-off to the back pressure is always to give smaller values of n for the same value of x_c, other conditions remaining the same. This result is due to the following reasons: For a given value of x_c, the value of n is the result of the volumes obtained by adiabatic expansion of the steam and water present plus an increase of volume of steam due to the return of heat from the cylinder walls and the consequent re-evaporation of part of the water condensed during admission. The rate of re-evaporation is controlled by the range of temperature due to the range of pressure, and it is greater or less according as the range of pressure decreases or increases respectively.

The effect of the range of pressure upon the $n - x_c$ relations for simple locomotives is small, as will be seen in Fig. 6 and 7; it is larger for the high pressure cylinders of compound locomotives, as is shown in Fig. 10; it is still larger for low pressure cylinders, as is also shown in Fig. 10. The slopes of the lines in these figures are 0.12, 0.76, and 1.17 respectively, showing the increasing effect of this variable as the range of pressure decreases.

28. *Effect on the $n - x_c$ Relations of Varying the Speed.*—The effect on the relations of x_c and n of increasing the speed is to give higher values of x_c for the same value of n, other conditions remaining the same.

The relations of x_c and n for Corliss engines did not show any definite increase in the value of x_c for speeds of from 90 to 150 r. p. m.

The effect on the value of x_c due to increase of speed is well shown for the relations of the Purdue locomotive in Fig. 4. In this figure it is seen that the average value of x_c increased from 0.735 at 100 r. p. m. to 0.756 at 240 r. p. m., a change in value of 0.021.

The average effect due to changes of speed for all simple locomotives, shown in Fig. 5, is from 0.695 at 100 r. p. m. to 0.747 at 240 r. p. m., an increase of 0.52, which is 25 times the increase found for the Purdue locomotive.

It is apparent that the center of all tests of the Purdue locomotive, which showed the least increase of x_c for increase of speed, is relatively the highest center plotted in Fig. 3. It will be seen also from Fig. 5 that the points of No. 1499 show the largest increase of x_c for increase of speed, and that its center of tests plotted in Fig. 3 is relatively the lowest.

The only phenomenon which accounts consistently for the observed effects due to speed is valve leakage. By valve leakage is meant the leakage of steam directly through the valves without its having entered the cylinder.

As has been stated, locomotives are peculiarly open to valve leakage at low and moderate speeds because of their relatively large valves and the use of very high pressures. The large valves expose a large surface through which steam may leak while the high pressures employed tend to increase the leakage to much larger amounts than those found in low pressure stationary engines with small valves. The experiments[1] which have been made on valve leakage show that the leakage is proportional, in a given engine, to the outside perimeter of the valve seat between the steam chest and the exhaust passage and also to the difference of pressure between these two regions. Varying the speed of operating the valves was found to have no effect on the amount of leakage. It was found also that although a valve might be perfectly tight while at rest yet it would leak a large amount when in motion, due to the breaking up of the oil film and to condensation and re-evaporation from the bridges and surfaces of the valve seat after they had been exposed first to steam at the initial pressure and then to the exhaust pressure.

Since valve leakage is proportional only to the perimeter of the valve seat and to the difference of pressure, it would, therefore, be a constant weight per unit of time for any one engine. Hence it follows, as valve leakage is constant per unit of time, that increasing the speed of the engine and thus using more steam in the cylinders would lessen the effect of valve leakage, measured as the per cent. of the total steam used. For example, if an engine running 50 r. p. m. leaks 500 lb. of steam per hour through its valve while receiving 4500 lb. per hour in the cylinder, the leakage is 10% of the total steam supplied; on the other hand, if it runs 150 r. p. m. and uses therefore three times the steam in the cylinders, 13,500 lb., then the valve leakage is only 3.6% of the total steam supplied.

All the evidence at hand shows that the existence of valve leakage and its relatively diminishing amount with increase of speed are the only conditions which account satisfactorily for the observed facts as shown in Fig. 4, 5, 9, and 12. That is, it is probable that there is very little if any effect on the $n—x_c$ relations of varying the speed, but the existence of leakage and its increasing relative amount when speeds are decreased lead to an apparent value of x_c which is lower than the real value for the steam actually admitted to the cylinders. It is also

[1] Leakage through a Piston Valve, by George Mitchell, Power, Oct. 11, 1910, p. 1805. Callender and Nicholson's Experiments on Valve Leakage.

probable that the value of x_c for engines having the larger values of valve leakage would increase faster with increase of speed than engines having very small leakage, because the increase of speed would increase the apparent value of x_c at a much faster rate. This condition is exactly the one which occurs, as is shown in Fig. 3, 4, 5, 9, and 12.

The slope of the line representing the effect of varying the speed for any one locomotive is apparently the best index as to the amount of valve leakage which exists.

It is probable that the relative location of the centers of tests in Fig. 3 is also a reliable index to the amount of valve leakage. The line drawn in Fig. 3 represents the average relation of the apparent value of x_c and the value of n. If a center of the tests for one locomotive lies above this line, then it apparently has a higher average value of x_c for the same value of n; the reverse is true if the center lies below the line. Since the effect of any leakage that may take place is to decrease the value of x_c for a given value of n, then the greater the eakage the lower would be the value of x_c.

The most striking case of the effects which have been described is ound in the tests of No. 929, having tandem compound cylinders. The cylinders of No. 929 were almost exactly the same size as those f No. 585, yet, at the same speeds, cut-offs, and pressures the steam onsumption of No. 929 averaged 19.6% more than that of No. 585. is the conditions of the tests of these two locomotives were almost xactly similar, they should have had practically the same steam conumption. It is, therefore, almost absolutely certain that a large proortion of the steam delivered by the boiler of No. 929 never entered s high and low pressure cylinders, where it could be used in producing ower, but that it leaked directly through the valves. The effect of ny such leakage was charged to the value of x_c which was obtained, nd this value in consequence appears to be very much smaller than ne values obtained from any of the other compound locomotives, alough all of them except the No. 585 had much smaller cylinders. or instance, in the high pressure cylinders the average value of x_c btained for the No. 929 is 0.662, while the average value of x_c for all her compounds is over 0.80. The same condition holds for the low ressure cylinders, although to a lesser degree.

In the light of the facts which have been presented and also from evious experience it is practically certain that the values of n obtained r the No. 929 are accompanied by values of x_c for the steam actually esent in the cylinders as shown by the line in Fig. 11. It is also actically certain that had no leakage occurred, the steam consump-

tion of the No. 929 would have been practically the same as for the No. 585 under the same conditions.

It was assumed, therefore, that the value of n obtained represented the values of x_c actually existing in the cylinders, and that the steam consumption of the No. 929 for the steam actually used in the cylinders was the same as for the No. 585.

The average steam consumption of No. 929 was 23.8 lb. per i. h. p. hr., while that of No. 585 for the tests run at the same speeds was 19.9 lb., showing that No. 929 consumed 19.6% more steam than did the No. 585 for the same conditions.

The average quality of x_c in the high pressure cylinders of the No. 929 was found to be 0.836, as computed from the values of n, S, and R, using the equation given on page 29. In a similar manner the value of x_c for the low pressure cylinders was found to be 0.817, making the average value of x_c for both the high and low pressure cylinders, 0.827.

The average apparent value of x_c from the tests for both high and low pressure cylinders was 0.674. Since this value of x_c is lower than 0.827, more steam to the amount $\frac{0.827-0.674}{0.827} = 18.5\%$ was used than was accounted for in the cylinders.

The results of these computations is a striking proof that not only was large valve leakage taking place, but that its amount has been established by two independent methods, which corroborate each other to a marked degree of accuracy. No. 929 consumed 19.6% more steam than the No. 585, while from the relations of x_c and n it is seen that 18.5% more steam was used than that which was accounted for in the cylinders. This analysis shows conclusively why the No. 929 had such poor economy in steam consumption when compared with all the other compound locomotives tested.

An examination of the design of the valves in No. 929 was made to ascertain if there were any peculiar features present in these valves which would account for the large valve leakage. The design is shown in cross-section in the report of the St. Louis tests, page 390. It will be seen that there are two annular passages containing live steam from which the high pressure cylinder receives its supply. The high pressure valve is in effect two inside admission piston valves made in one and it is seen that the live steam has four possible avenues of leakage into the receiver as against only two in the ordinary piston valve. These four faces of the valve undoubtedly permit at least twice the leakage which occurs through an ordinary piston valve with only two faces

and they are doubtless responsible for the enormous valve leakage which has been shown to exist. The low pressure valve is an outside admission piston valve, having only two faces which would permit of leakage. The results show, however, that it leaked to about the same degree as the high pressure valve, so that its surface on the valve bushing must not have been in good condition or the piston rings were not yet worn to a good bearing.

The only other locomotive which had valves similar in design to those on the high pressure cylinders of No. 929 were the low pressure cylinders of No. 535, shown on page 540 of the report of the St. Louis tests. This locomotive has one valve which controls the steam distribution to both the high and low pressure cylinders. The portion of the valve serving the high pressure cylinder is in effect a simple inside admission piston valve, and as seen in Fig. 8 and 9 no unusual amount of leakage was present in the high pressure portion of the valve. The portion of the valve serving the low pressure cylinder is in effect two outside admission piston valves with four faces subject to leakage from steam in the receiver direct to the two annular exhaust passages. The values of x_c and n from the low pressure cylinders show the effects of very bad valve leakage at 80 r. p. m., becoming less in proportion at 160 r. p. m. and negligible at 240 r. p. m. The effects of the large valve leakage at 80 r. p. m. are also apparent in the increased steam consumption[1] of No. 535 over that obtained in the other compound locomotives. At 160 r. p. m. the effects of leakage begin to grow proportionately much less and at this and higher speeds the steam consumption decreased to the amount generally consumed by compound locomotives having their valves in good condition.

The valve leakage found for No. 929 is corroborated by the actual performance on the road of the class of locomotives to which No. 929 belongs. It was learned from an independent source that when one of these locomotives is starting a full tonnage train, the "blow," or leakage through the valves, is so great that a distinctive and continuous roar of leaking steam is heard.

The analysis which has been given for the low points in Fig. 8 and 11 shows that the variations in the vertical distance of the center of tests for each locomotive from the average line is almost certainly due to the fact that the valves of each locomotive have their own rate of leakage, the leakage for some valves being less than the average, while that of others is greater.

The amount of valve leakage existing for a particular locomotive

[1] Locomotive Tests and Exhibits, pp. 707, 708, 710.

probably depends on the design and relative size of the valves, and also on the condition of the surfaces of the valve seats, balance rings, and balance plates for slide valves, and of the surfaces of the valve bushings and piston rings for piston valves.

Referring to Fig. 3 the relatively high point of the Purdue locomotive and its flat speed effect curve in Fig. 4 probably means that its valves were tighter than any of the others, due to its being kept in laboratory condition. On the other hand, the low point of No. 1499 in Fig. 3 and its steep speed effect curve in Fig. 5 is probably due to the fact that this locomotive was new and had not been broken in before it was tested, consequently the valves had not worn to a good surface, and the average leakage was larger than that for any of the other simple locomotives tested.

29. *Values of x_c.*—The assumption has been made throughout this investigation that the steam saved in the cylinders in compression was always dry saturated at the beginning of compression and the values of x_c given were computed on this basis.

It is probable, however, in the simple locomotives, that the compression steam is slightly wet for engines using saturated steam and is superheated for engines using highly superheated steam.

If these two conditions exist the values of x_c as given are slightly wrong. Any error from this cause does not affect the results, however, if the convention of dry steam at compression is uniformly followed. The values of x_c obtained by following this convention are comparable though not absolutely correct.

30. *Values of x_c and n Found for Various Types of Locomotives—Their Use in Design.*—Fig. 3, 8, and 11 show the average values of x_c and n which are obtained from typical simple and compound locomotives. Where conventional indicator diagrams are laid out before the locomotive is built, the data show the values of n which should be used for the expansion curves. The value of n=1.0 still may be used for the compression curves without serious errors.

The value of n for the expansion curves of locomotives using steam superheated 250° F. should be assumed as an average value of 1.3.

V. Directions for Determining the Steam Consumption of Locomotive Engines from the Indicator Diagrams

31. *Preparation of Indicator Diagrams for the Application of the n—x_c Method for Determining Steam Consumption.*—Possibly 95% of the locomotives in use in the United States and Canada are of the type using simple cylinders. The compound locomotive, with the singl

exception of the Mallet type, has dwindled in numbers and importance since the introduction of the high superheat simple locomotive.

The directions for applying the $n - x_c$ method will therefore be outlined in full for the simple type, and the change in the method for its application to the compound type will be briefly mentioned.

Since the $n - x_c$ method depends entirely on the indicator diagram, great care must be observed in taking these diagrams. The indicator itself must be an accurate instrument in the best possible condition. The indicator pipe connections must be large, short, direct, and well covered with heat insulating material. An extensive investigation by Dr. W. F. M. Goss[1] shows that long and indirect pipe connections materially alter the form and character of the expansion curves. A theoretically correct reducing motion, free from lost motion, must be used so as to reproduce the actual expansion. Very short cords for the indicator drum must be used to avoid cord stretch. The arrangement of having one indicator at each end of the cylinder is always to be preferred if the road clearances allow it.

It is to be remembered that the method accounts for the actual weight of steam and water present in one revolution only, as represented by the set of diagrams analyzed.

Since the method accounts for the consumption on the basis of one revolution, indicator diagrams for road tests must be taken at regular intervals of distance, perhaps ½ or 1 mile apart. The mile post can be used to indicate the time for taking diagrams. The rate of speed must be observed as each set of diagrams is taken, as the speed has an influence on the relations of x_c and n.

After the test is over all diagrams are integrated and the average horsepower obtained in the usual manner. To apply the $n - x_c$ method the diagrams are divided into groups of similar lengths of cut-off, four or more groups generally being sufficient for this purpose. Since the effect of speed on the $n - x_c$ relations is a linear one, the average speeds may be used without error. From each group as outlined above select two sets of diagrams, each taken at the same reading, which are nearest in area to the average area of the group and which also are nearest to the average speed which prevailed for the diagrams in the group.

Construct logarithmic diagrams for each set as described in detail in Appendix I. From these diagrams the average value of n for one set of diagrams is determined.

[1] Trans. Am. Society of M. E., Vol. 17, p. 398.

The relations of x_c, n, and S for simple locomotives are plotted in the chart of Fig. 13. The values of n and S are located on the chart and the intersection of the lines representing them is located. The intersection is then seen to lie between two lines of constant quality, or value of x_c, the exact value of x_c being determined by interpolation. The value of x_c found gives the quality of the steam mixture at cut-off. From the steam accounted for per revolution at cut-off by the indicator, and the value of x_c found for this steam, the actual weight of steam and water in the cylinder is determined. The weight of dry steam saved per revolution in compression is determined as described in Appendix I. The total weight of steam and water per revolution found to be present, less the weight of compression steam saved per revolution, gives the weight of steam supplied per revolution by the boiler for average conditons of valve leakage. Calculations similar to those described but in the reverse order are given in detail in Appendix I. Similar calculations are also given in Bulletin 58.

After all the sets chosen are analyzed the average consumption at different values of cut-off is determined on the basis of consumption in one revolution.

To determine the water consumed by the engine over a given run or during a given time, the average steam consumption per revolution for each group is multiplied, first by the number of sets of diagrams in any one group of the different cut-off values, and second by the number of revolutions that the drivers have made between the taking of cards. In this manner, if diagrams are taken every mile, and the number of revolutions made by an 80 inch driver would be 252, the calculation described gives the total water consumed by one group. The consumption for the other groups, after being computed in the same manner are all added and the total water used for the period is obtained.

Instantaneous rates of consumption in total weight of steam per hour or per indicated horse power hour can be obtained from each set of diagrams taken.

32. *Other Applications of the $n - x_c$ Method.*—If it is desired to test an engine for valve leakage, the locomotive should be tested on a test plant or under road conditions where the steam used for other purposes is known. The steam used by the engines is accounted for by the $n - x_c$ method, the total water fed to the boiler is measured, and this includes the amount used in the cylinders; the amount that leaked through the valves over the average leakage of the valves when in good condition is accounted for by the difference between the two measurements if the boiler is tight.

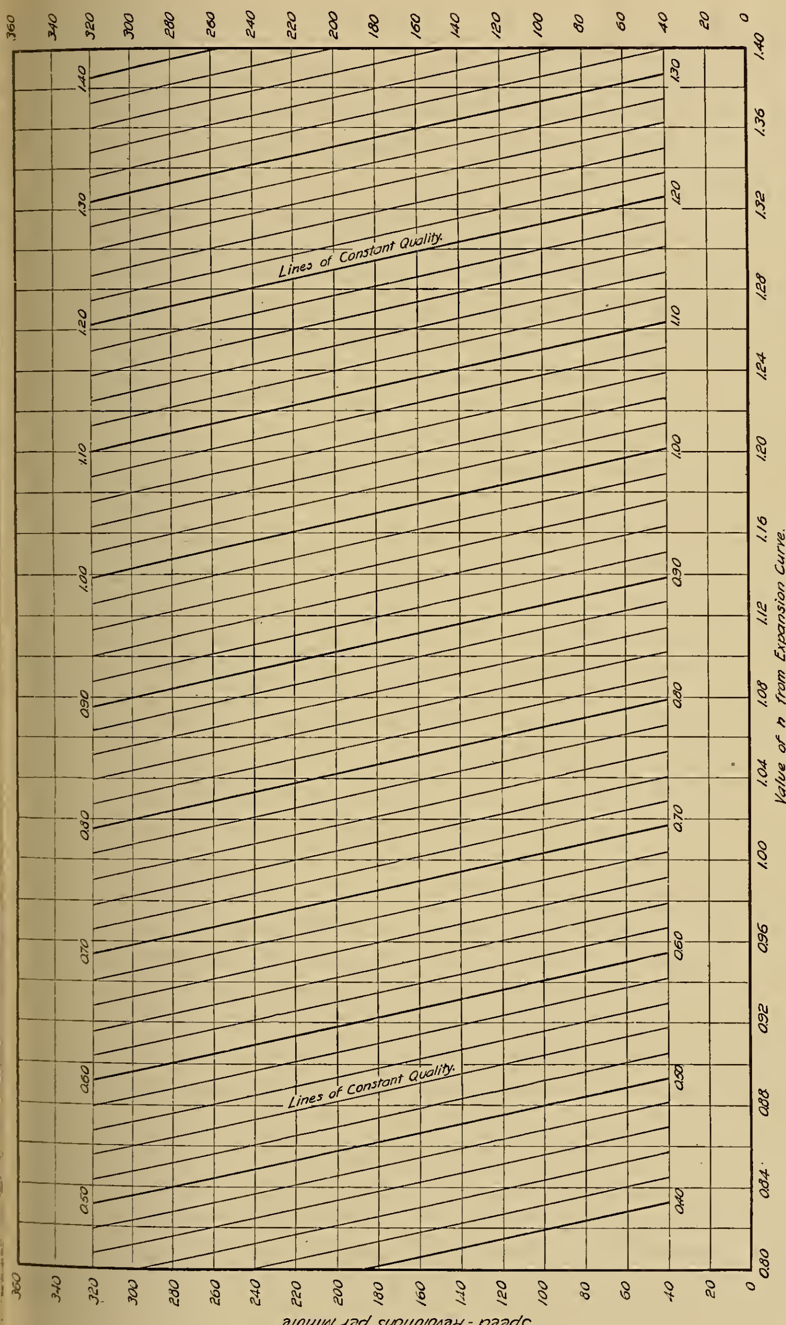

FIG. 13. THE RELATION BETWEEN SPEED AND THE VALUE OF n FOR SIMPLE LOCOMOTIVES WITH CONSTANT QUALITY OF STEAM IN THE CYLINDER AT CUT-OFF.

Locomotives in winter passenger service use such a large proportion of the steam generated by the boiler in heating the train, running the air pump, etc., that in combination with the increased train resistance in cold weather, schedules are maintained with great difficulty. It is practically impossible to measure the steam used in this way by any methods heretofore employed.

The $n - x_c$ method may be employed for this purpose by accounting for the difference between the total water fed to the boiler and that used by the engines. The difference is that which is used to heat the train, to operate the air-pump, head-light sets, train-lighting sets, blower and also that steam and water which is lost through the safety valves, blow-off valves, and boiler leaks. The $n - x_c$ method provides the only means believed to exist by which these uses of steam for auxiliaries or that which is lost may be measured.

33. *Application of the $n - x_c$ Method to Compound Locomotive Engines.*—The application of the $n - x_c$ method to compound locomotive engines differs from the method of application to simple locomotive engines only in detail, because of the difference in construction.

The total steam used by the engines may be obtained from either the high pressure or low pressure diagrams, as the same steam is used by both cylinders. If only the total steam used by the engines is desired, and not the quantity per indicated horse power hour, then the low pressure diagrams are all that are required.

It is possible to take both the high and low pressure diagrams without difficulty when all the cylinders are located outside of the frames, but it is generally difficult to take diagrams from inside cylinders.

The low pressure diagrams are better for the application of the $n - x_c$ method than the high pressure diagrams, because of the relative smoothness of the expansion curves from the low pressure cylinders. Of course the method may be applied to both cylinders, the determinations then serving as a check on each other.

One additional important variable must be considered in compound diagrams, namely the range of pressure from cut-off to the back pressure.

The method can be applied to Mallet locomotives by applying the regular high and low pressure relations of x_c and n. Either or both the high pressure and low pressure diagrams may be used.

The relations of x_c, n, S, and R for high pressure cylinders are given for compound engines on page 27, and for low pressure cylinders on page 29.

34. *The Degree of Accuracy Obtained by the Use of the $n - x_c$ Method.*—The steam consumption of all of the 185 tests of the six simple locomotives examined may be recomputed from the relations of x_c and n to an average difference of 3.7% from the test results.

In the tests of the compound locomotives, the average difference for the high pressure tests is about 8%, and for the low pressure tests 3.8%.

It is extremely probable that at least one fourth of the per cent. difference given is due to irregularities in measuring the difference in the height of the water in the boiler gage glass at the start and at the finish of the test. It must be remembered that all of the boiler tests were only ½ to 3 hours in length, so that an error in the gage glass correction means a noticeable difference in the water actually evaporated.

If one fourth of the difference is credited to errors of the boiler measurements then the average differences are 2.8%, 6% and 2.9% for the simple cylinders, the high pressure, and low pressure cylinders respectively. These differences show that the $n - x_c$ method is close enough for all the purposes of locomotive tests. The high difference of the high pressure cylinders is due to the difficulty of obtaining good values of n because of inertia in the expansion curves, and also to variable valve leakage.

VI. Application of the Logarithmic Method to the Determination of the Leakage, Clearance, and Location of Cyclic Events in Locomotive Tests

The methods which have been developed from the logarithmic diagram for detecting leakage into or out from the cylinder, for determining the clearance, and for closely locating the events of the cycle, such as cut-off, lead, etc., have been fully described in Bulletin 58 of the Engineering Experiment Station.[1] The reader is referred to the previous work for the methods employed.

35. *Uses of the Method for Detecting Leakage.*—The method of detecting leakage from the logarithmic diagram may be employed in locomotive tests to detect piston "blows," and valve leaks either into or out from the cylinder. In some cases it is possible to compute the amount of steam which leaked.

36. *Uses of the Method for Determining Clearance.*—If the clearance of a locomotive cylinder be desired, it may be determined from the logarithmic diagram to within 5% to 10% of the actual clearance volume.

[1] Also in Journal A. S. M. E. April, 1912.

37. *Uses of the Method for Closely Locating the Cyclic Events.*—It has been generally known that the valve gears of locomotives give a later cut-off at high speed than at low speed with the reverse lever in the same notch. This is due to the spring or lag caused by the lost motion of worn parts and to inertia. The Stephenson valve gear is the worst offender in valve gear spring due to the indirect motion which is necessary.

The most accurate method of measuring the amount of this spring in use at present is to take valve ellipse diagrams from the valve rod. Experiments using this method have been carried on by the University of Illinois. The Pennsylvania Railroad System has attempted to measure the amount of this spring from the difference of cut-off as shown by the indicator diagrams for high and low speeds with the reverse lever in the same notch.

To show the amount of the spring of valve gears the following is quoted from Bulletin No. 5, Pennsylvania Railroad System, Tests of an E2a Locomotive, No. 5266. The tests of this same locomotive are included in this investigation.

"It is apparent that tests at different speeds, while run with the reverse lever in the same notch, do not have the same actual cut-off in the cylinders, but the cut-off point becomes later as the speed increases, due, probably, to a springing of the valve motion. This effect is so marked that the locomotive will run forward at the higher speeds with the reverse lever in one of the notches of the backward motion. The cut-off increases from 15.7% at 80 r. p. m. to 21.4% at 320 r. p. m., while the nominal cut-off or reverse lever notch remains the same."

The extremely close location of the cyclic events, cut-off, release, the beginning of compression, and lead obtainable from the logarithmic diagram enables the study of the spring of valve gears to be carried on with only the indicator diagrams taken at various speeds with the same reverse lever position.

VII. Conclusions

The data support the following conclusions:

(1) The actual steam consumption of simple locomotive engines in regular service on the road, when the valves are in good condition, may be determined from the indicator diagrams alone to within an average difference of from 3 to 4 per cent. of the consumption as measured on test plants.

(2) The actual steam consumption of compound locomotive engines in regular service on the road, when the valves are in good con-

dition, may be determined from the low pressure indicator diagrams alone to within an average difference of from 3 to 4 per cent. of the consumption as measured on test plants. It also may be determined from the high pressure diagrams to within an average difference of 6% from the test results.

(3) The weight of steam lost by valve leakage in excess of that usual for valves in good condition may be determined by accounting for the difference between the steam used by the cylinders as shown by the $n - x_c$ method and the water fed to the boiler if the boiler is tight and if steam is used for no other purpose than operating the engines.

(4) The weight of steam used for other purposes than operating the engines may be determined in road tests, when the valves are in good condition, by accounting for the difference between the steam used by the cylinders as shown by the $n - x_c$ method and the water fed to the boiler.

APPENDIX I

The Logarithmic Diagram

1. *Description of Logarithmic Cross-section Paper.*—Logarithmic cross-section paper differs from rectangular cross-section paper in that the distances from the origin are proportional to the logarithms of the numbers to be plotted instead of the numbers themselves. This system of co-ordinates gives an uneven scale similar to that on a slide rule.

The numbers of the divisions on logarithmic paper are placed opposite the lines corresponding to their logarithms, as on a slide rule, instead of the values of the logarithms of the numbers. This fact aids in plotting, as the logarithms are employed without having to ascertain their values.

The logarithmic cross-section paper used in this investigation consists of four squares arranged two each way. These squares are five inches each way, making the four squares together ten inches each way. The use of four squares enables values to be plotted ranging from 0.1 to 10.0, 1.0 to 100.0, etc., thus giving a range of ten times the values obtainable if only one square were used.

2. *Construction of the Logarithmic Diagrams.*—The co-ordinates of the indicator diagram are proportional to pressure and stroke, the latter being proportional to the volume displaced by the piston. The co-ordinates of several points on the indicator diagrams are found in terms of absolute pressures, preferably in pounds per square inch, and absolute volumes, preferably in cubic feet. The scale of units employed is not material as long as it starts at the line of zero pressure, or of zero volume. However, the units are more easily manipulated afterwards if they are the same as are used in the steam tables.

The method of transferring the indicator diagram to the logarithmic form is described in detail for the diagrams of Test 110 of the Purdue locomotive, given in Fig. 14*a*, Crank End. The diagram is shown in outline by *ABYX*. Perpendiculars *QR* and *EX* are drawn to the atmospheric line *EQ*, and pass through the extreme stroke positions of the diagram. The distance *EQ* is then the length of the diagram. *OM* is laid off perpendicular to the atmospheric line *EQ* (extended) which was drawn by the indicator pencil. *OM* is the line of zero volume, and is drawn at a distance *FE* from the admission end *EX* of the diagram, the distance *FE* being the same length in per cent. of the line *EQ*, or length of the diagram, as the proportion that the per cent. of clearance,

or waste space, of the cylinder bears to the piston displacement. In this case the length of the diagram is 3.70 inches, and the clearance is 7.98%. The length *FE* is therefore 0.0798 x 3.70, or 2.95 inches. *ON* is the line of zero pressure and is drawn a distance *FO* below the atmospheric line, to the scale of the spring used in obtaining the *PV*-diagram. This distance is proportional to the barometer reading, corrected for temperature, prevailing at the day and place of the engine test. In this case the corrected barometer reading is 14.7 lb. per sq. in. absolute, so the distance *FO* is $\frac{14.7}{150.0}=0.098$ in.

From *ON* points are laid off on *QR* and *EX* corresponding to the absolute pressures at the intervals where it is desired to read off the corresponding volumes. Fine lines are drawn connecting similar pressure points, as 40–40, 60–60, etc. The volumes *G–A′, F–B, H–D, G–C,* etc., are read off in hundredths of an inch to the nearest half hundredth. The tabular form used in this investigation is given in Table 3 for the diagrams of Fig. 14*a* taken in Test 110 of the Purdue locomotive. Thus the length *G–A* is read off as 1.20 inches, and is given under the column for 40 lb. pressure headed Comp., meaning the compression curve for the crank end diagram. The volumes in inches are then multiplied by the constant ratio which one inch of length of the diagram bears to the displacement of the piston. From Table 3 it is seen that the piston displacement of the crank end is 2.72 cu. ft., and the length of diagram 3.70 inches; hence, the ratio is $\frac{2.72}{3.70}$ or 0.735 cu. ft. of piston displacement per inch of diagram length. The length *G–A* in cu. ft. of displacement now becomes 1.20 in. x 0.735 cu. ft. per inch, the volume of steam present at this point. This process is repeated at intervals until the co-ordinates of from ten to thirty points are determined. In the diagram shown in Fig. 14*a* the co-ordinates of 27 points were found.

The co-ordinates of *P* and *V* are then plotted on logarithmic cross-section paper, as shown in Fig. 14*b*, which are the logarithmic diagrams derived from the indicator diagrams of Fig. 14*a*. The points plotted in Fig. 14*b* are taken from the columns headed cu. ft. at the pressures shown. A smooth curve is drawn through the points thus plotted, and the diagram is in shape to be studied.

It will be seen that the expansion curve becomes a straight line in the logarithmic diagram, as does also the compression curve. The value of *n*, which is the slope or measure of inclination of this line to the horizontal, is found as follows: in Fig. 14*b*, from a point *X* on the expansion line draw *OX* parallel to the axis of log. *P*, and draw *OY*

parallel to the axis of log. V. The value of n is then the value of the ratio $\frac{OY}{OX}$.

3. *Method of Obtaining the Values of x_c and n from the Diagrams.*—The manner of computing the value of x_c and n is given for Test 110 of the Purdue locomotive, the representative indicator diagrams of which are given in Fig. 14*a*, and the logarithmic diagrams in Fig. 14*b*. The results of the computations for all tests are given in Tables 5 and 16.

The absolute pressure of cut-off was determined from both the indicator card and logarithmic diagrams by inspection for each end of both cylinders. The cut-off pressures from the four cards were averaged and this average was used to obtain the value of x_c for the test.

The logarithmic diagrams of Test 110 of the Purdue locomotive, shown in Fig. 14*b*, are the basis of the calculations for the value of x_c. The calculations which follow are given in detail in the same form as they were made for all tests.

Computations for the Values of x_c and n for Test 110 of the Purdue Locomotive

Volumes obtained from Fig. 14*b*.

Steam present at cut-off pressure of 138.0 lb. absolute.

Volume of steam present, right head end	0.666 cu. ft.
Volume of steam present, right crank end	0.603 cu. ft.
Volume of steam present, left head end	0.624 cu. ft.
Volume of steam present, left crank end	0.617 cu. ft.
Total, right and left sides	2.510 cu. ft.

Specific volume* of steam at 138.0 lb. absolute = 3.263 cu. ft. per lb.

$$\text{Weight of steam present at cut-off} = \frac{2.510}{3.263} = 0.769 \text{ lb.}$$

Steam retained in compression at 15.0 lb. absolute.

Volume of steam present, right head end	2.470 cu. ft.
Volume of steam present, right crank end	1.893 cu. ft.
Volume of steam present, left head end	2.018 cu. ft.
Volume of steam present, left crank end	1.817 cu. ft.
Total, right and left sides	8.198 cu. ft.

Specific volume of steam at 16.0 lb. absolute = 24.79 cu. ft. per lb.

$$\text{Weight of steam retained in compression } \frac{8.198}{24.79} = 0.331 \text{ lb.}$$

From Table 5 the weight of steam and water supplied = 6938 lb.

Revolutions per hour = 11670

$$\text{Weight of steam and water supplied per revolution} = \frac{6938}{11670} = 0.5945 \text{ lb.}$$

* Marks and Davis Steam Tables.

Total weight of mixture present per revolution.

0.5945 lb. supplied
0.3310 lb. retained in compression
0.9255 lb. total present

$$x_c = \frac{0.769}{0.9255} = 0.831$$, or 83.1% of the total weight of mixture present as steam at cut-off.

Value of n from Fig. 14*b*.—

$$\text{Right head end } \frac{OX}{OY} = \frac{2.975}{3.055} = 0.974$$

$$\text{Right crank end } \frac{OX}{OY} = \frac{2.91}{2.775} = 1.048$$

$$\text{Left head end } \frac{OX}{OY} = \frac{2.875}{2.86} = 1.005$$

$$\text{Left crank end } \frac{OX}{OY} = \frac{2.975}{2.800} = \frac{1.062}{4.089}$$

$$\text{Average value of } n = \frac{4.089}{4.000} = 1.022$$

When the actual cut-off pressure was less than 138.0 lb. the line of constant weight of steam mixture on the logarithmic diagram was extended to this pressure, and the calculations made.

TABLE 3

Construction of the Logarithmic Diagrams of Test No. 110, of the Purdue Locomotive.†

No.	Absolute Pressure, Lb. sq. in.	Head End: Inches Comp.	Head End: Inches Exp.	Head End: Cu. ft. Comp.	Head End: Cu. ft. Exp.	Crank End: Inches Comp. X to P*	Crank End: Inches Exp. X to P*	Crank End: Cu. ft. Comp.	Crank End: Cu. ft. Exp.
1	230	0.34	0.31	0.263	0.240	.....	.....	.....	.....
2	220	0.35	0.27	0.271	0.209	.....	.....	.....	.....
3	210	0.365	0.32	0.283	0.248	0.325	0.30	0.239	0.221
4	200	0.38	0.41	0.294	0.317	0.335	0.38	0.246	0.278
5	190	0.40	0.54	0.310	0.418	0.345	0.48	0.254	0.353
6	170	0.425	0.68	0.329	0.526	0.38	0.65	0.279	0.478
7	150	0.455	0.795	0.352	0.615	0.405	0.76	0.298	0.559
8	140	0.465	0.845	0.360	0.654	0.42	0.81	0.309	0.596
9	130	0.49	0.91	0.379	0.704	0.45	0.87	0.331	0.639
10	120	0.53	0.99	0.416	0.766	0.48	0.93	0.353	0.684
11	100	0.655	1.20	0.507	0.929	0.56	1.125	0.412	0.827
12	80	0.795	1.495	0.615	1.157	0.68	1.36	0.500	1.000
13	60	1.00	2.06	0.774	1.594	0.855	1.815	0.628	1.334
14	40	1.44	3.07	1.115	2.376	1.20	2.69	0.882	1.977
15	30	1.83	3.33	1.417	2.577	1.55	3.18	1.139	2.337

	H. E.	Cr. E.
Length of indicator diagram........................	3.62 in.	3.70 in.
Ratio of clarance to Piston displacement (same end)...	0.0744	0.0798
Length of diagram propotrional to clearance ratio.....	0.269 in.	0.295 in.
Piston displacement (cylinder 16″x24″)..............	2.802 cu. ft.	2.720 cu. ft.
Clearance volume..................................	0.208 cu. ft.	0.217 cu. ft.
Ratio, cu. ft. per inch of length on diagram...........	0.774	0.735
Scale of indicator spring per in. of ordinate...........	150 lb.	150 lb.

*Letters refer to Fig. 14*a*, crank end. †Final results given in Table 5.

The point of compression was selected in the following manner from Fig. 14*b* for the reasons explained fully in Bulletin 58. The straight line of the compression curve in the logarithmic diagram, or the line of constant weight of steam mixture, was prolonged dotted to the back pressure as shown. The intersection of the compression line, prolonged, with the back pressure line, extended (in this test 16 lb.), was taken as the volume of dry steam retained in compression. This method generally gives less steam retained than the ordinary method.

Steam was found to be superheated at cut-off for practically all of the tests of No. 3395. The manner of measuring the value of x_c for superheated steam at cut-off was the same as for saturated steam; that is, the ratio of the volume of steam present at cut-off per lb. of the total weight of mixture present divided by the specific volume of dry saturated steam at cut-off pressure. To illustrate, it will be assumed that a quality of 1.25 is found for a test having a cut-off pressure of 160 lb. absolute. The specific volume of dry saturated steam at this pressure is 2.834 cu. ft. per lb. The value of x_c of 1.25 means that the specific volume per lb. of the total weight present was found to be 3.542 cu. ft. per lb. On examining the superheat portion of the steam tables, it is seen that the specific volume of 3.542 cu. ft. per lb. at 160 lb. absolute corresponds to steam at 152° superheat.

The value of x_c is based on the specific volume of dry saturated steam because the qualities obtained when the steam is superheated at cut-off are of the same order of magnitude as the qualities for wet steam, hence the curve does not "break" at a quality of 1.00.

4. *Typical Indicator and Logarithmic Diagrams from the Locomotives Tested.*—Typical indicator and logarithmic diagrams from the right side of each of the locomotives tested are given in Fig. 14-25. The figures of the indicator and logarithmic diagrams from the same locomotive are numbered the same but the letters *a* and *b* are used in addition to the figure number for the indicator and logarithmic diagrams respectively.

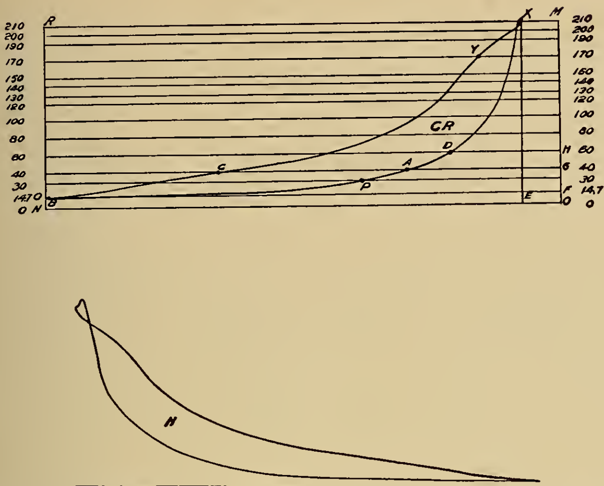

FIG. 14a. REPRESENTATIVE INDICATOR DIAGRAMS FROM THE PURDUE LOCOMOTIVE, TEST No. 110.

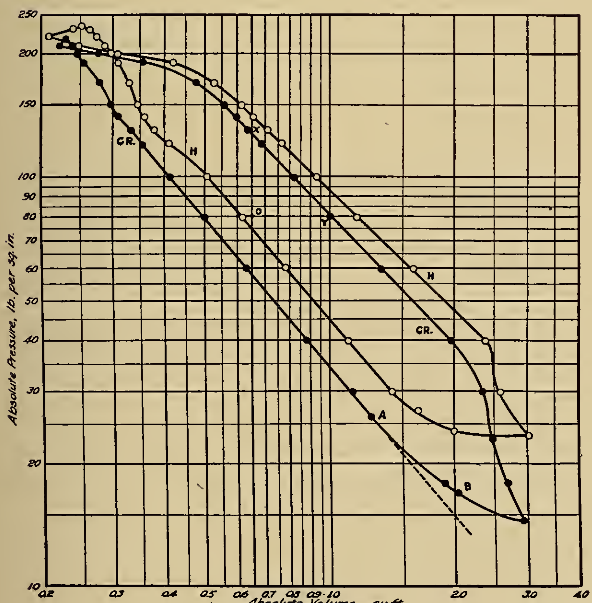

FIG. 14b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM PURDUE LOCOMOTIVE, TEST No. 110.

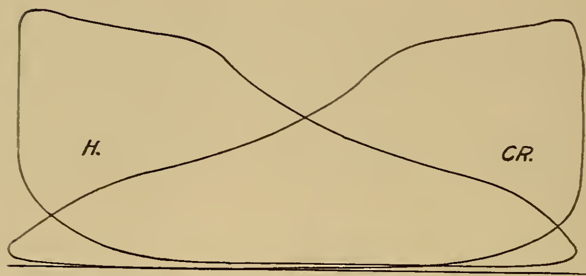

FIG. 15a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOCOMOTIVE 1499, TEST No. 118.

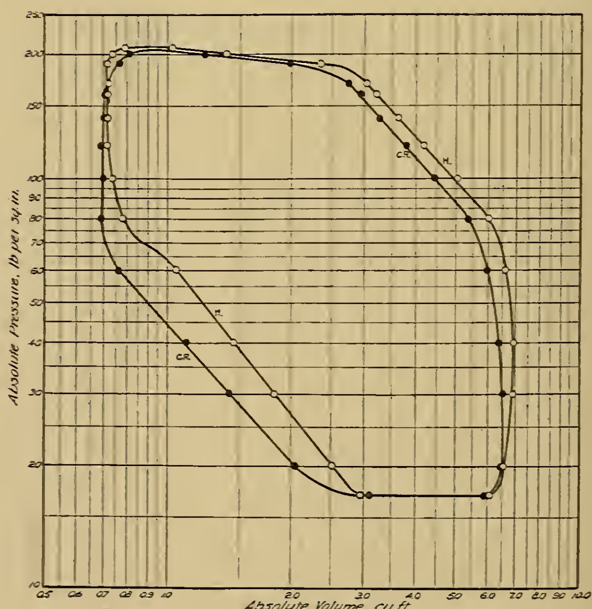

FIG. 15b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 14[illegible] TEST No. 118.

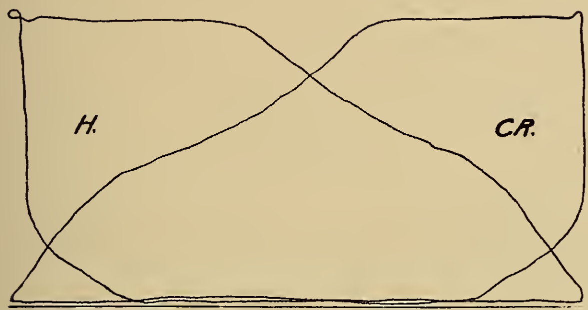

FIG. 16a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOCOMOTIVE 734, TEST NO. 203.

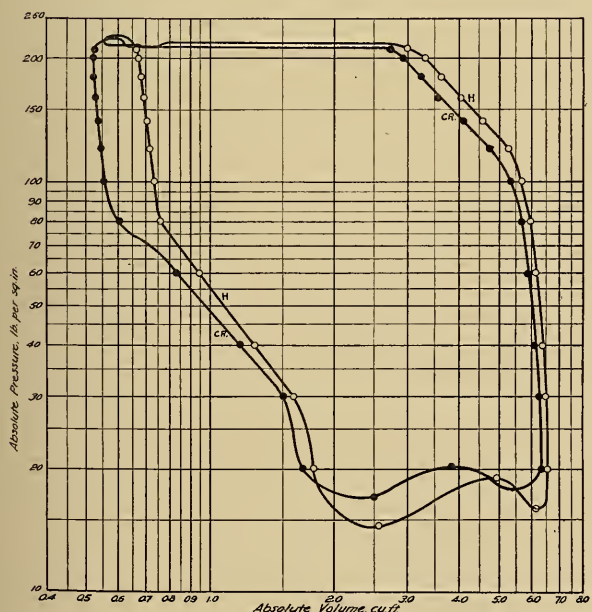

FIG. 16b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 734, TEST NO. 203.

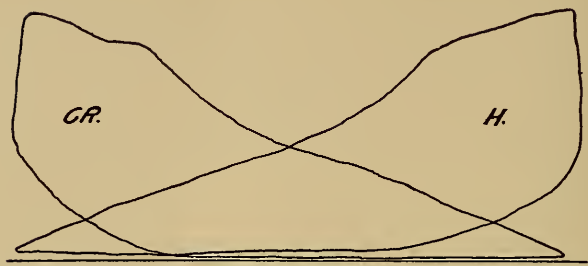

Fig. 17a. Representative Indicator Diagrams from Locomotive 5266, Test No. 917.

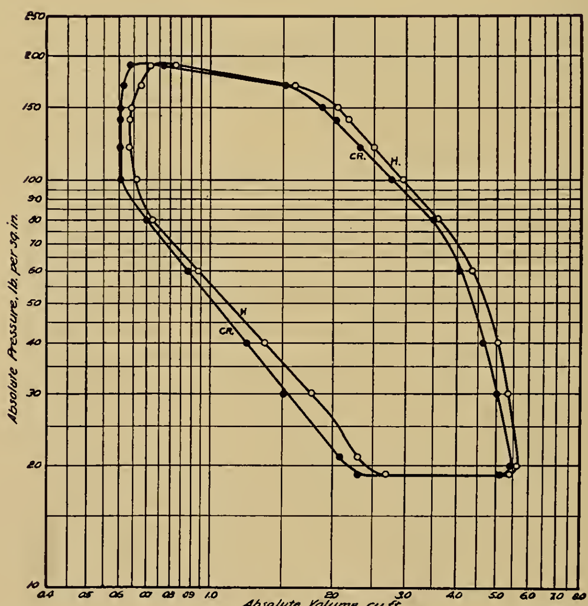

Fig. 17b. Representative Logarithmic Diagrams from Locomotive 526[illegible] Test No. 917.

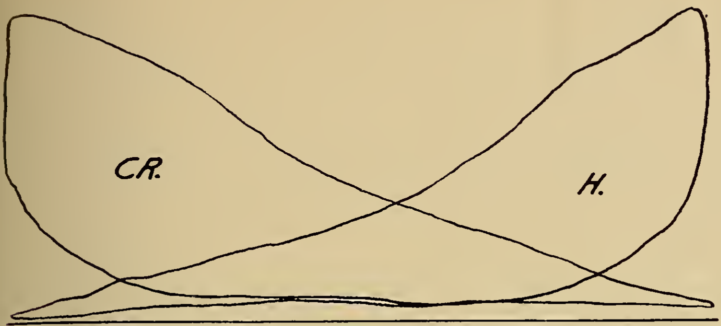

Fig. 18a. Representative Indicator Diagrams from Locomotive 7510, Test No. 1613.

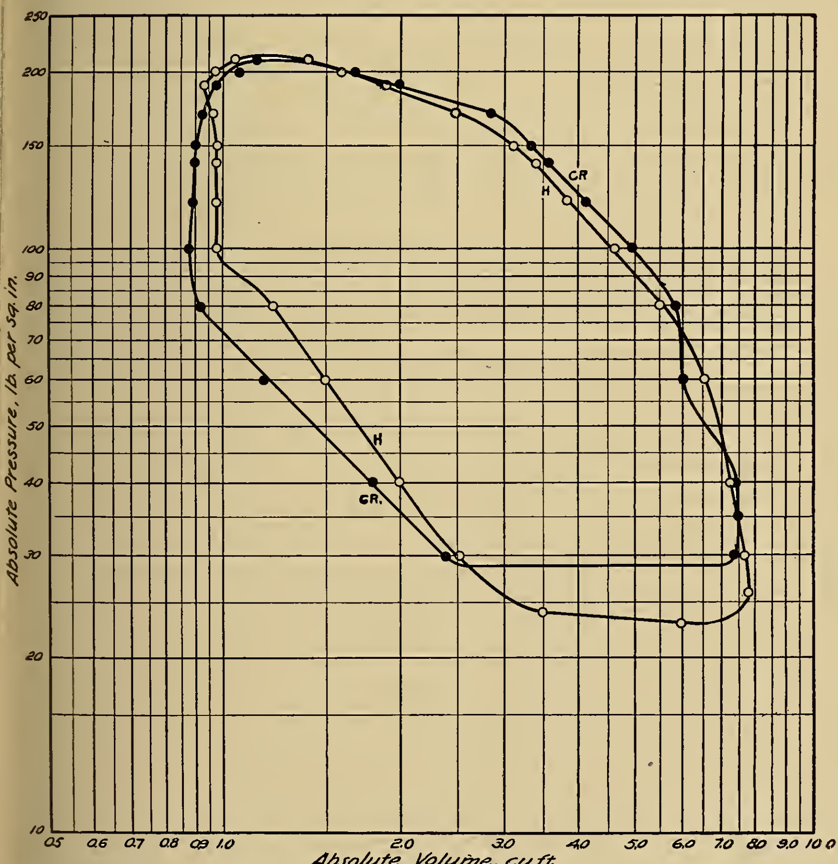

Fig. 18b. Representative Logarithmic Diagrams from Locomotive 7510, Test No. 1613.

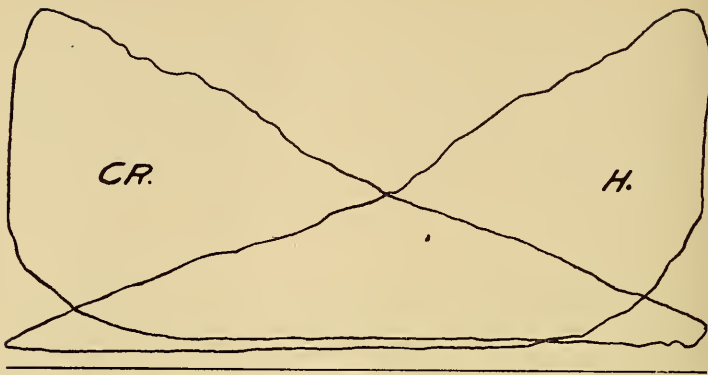

FIG. 19a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOCOMOTIVE 3395, TEST NO. 2413.

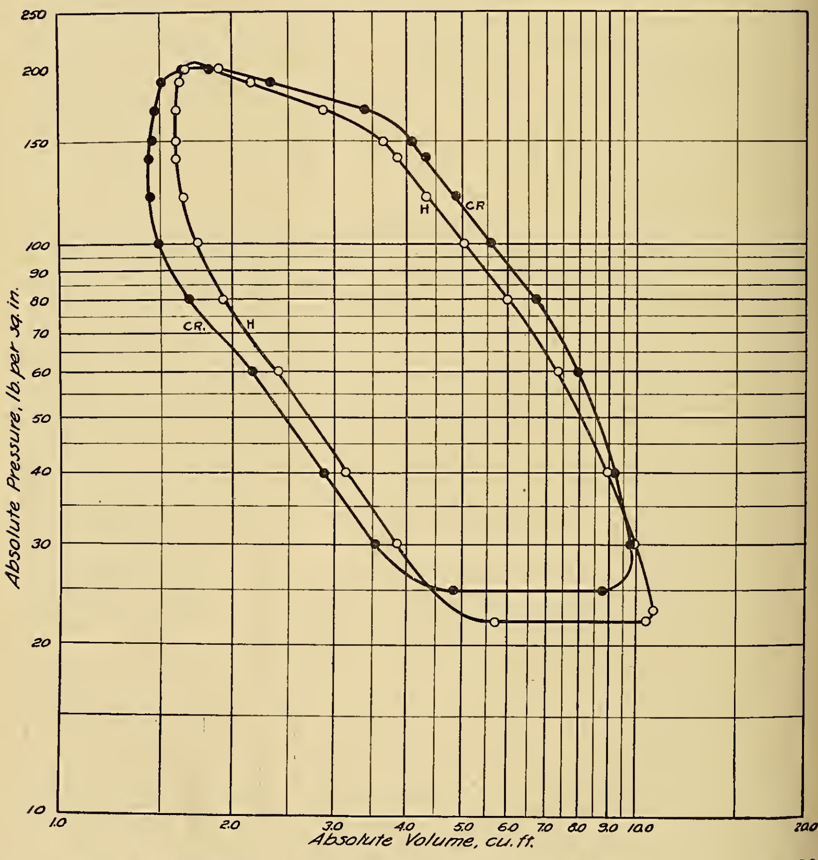

FIG. 19b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 3395 TEST NO. 2413.

FIG. 20a. REPRESENTATIVE INDICATOR DIAGRAMS FROM HIGH PRESSURE CYLINDER OF LOCOMOTIVE 585, TEST NO. 308.

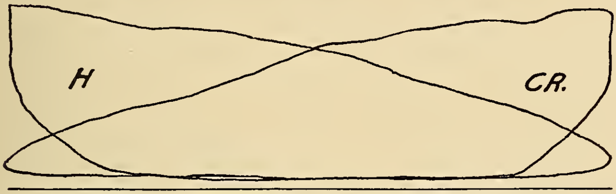

FIG. 20a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOW PRESSURE CYLINDER OF LOCOMOTIVE 585, TEST NO. 308.

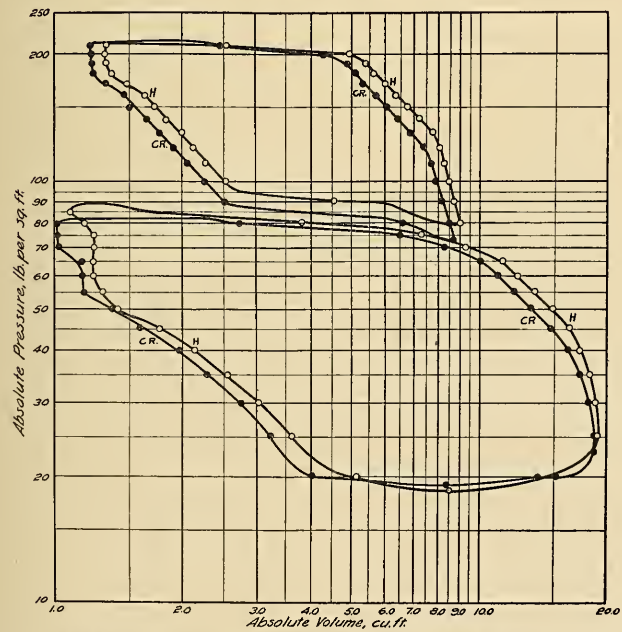

FIG. 20b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 585, TEST NO. 308.

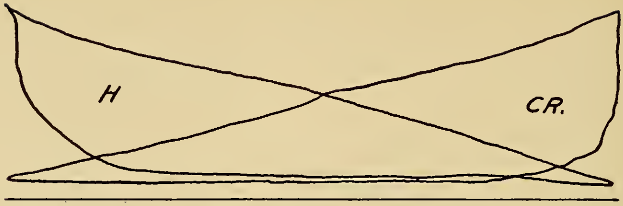

FIG. 21a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOW PRESSURE CYLINDER OF LOCOMOTIVE 929, TEST NO. 408.

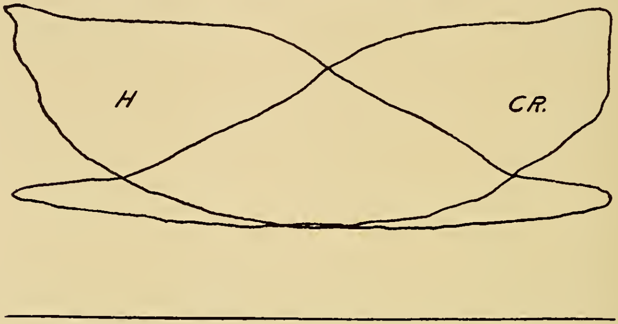

FIG. 21a. REPRESENTATIVE INDICATOR DIAGRAMS FROM HIGH PRESSURE CYLINDER OF LOCOMOTIVE 929, TEST NO. 408.

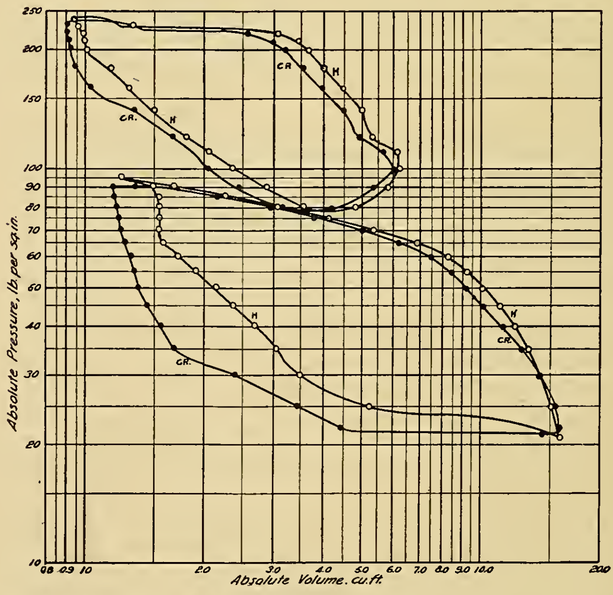

FIG. 21b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 929 TEST NO. 408.

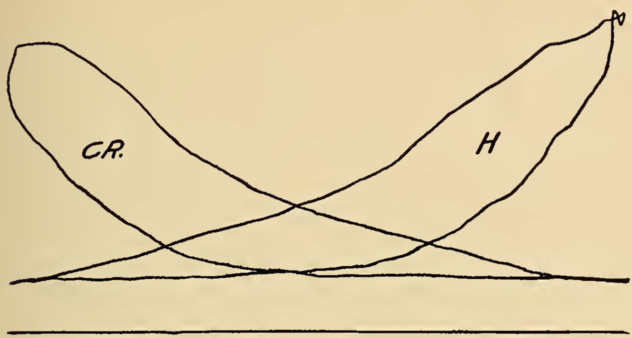

FIG. 22a. REPRESENTATIVE INDICATOR DIAGRAMS FROM HIGH PRESSURE CYLINDER OF LOCOMOTIVE 2512, TEST NO. 510.

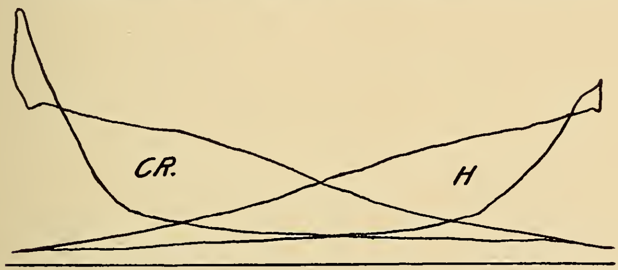

FIG. 22a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOW PRESSURE CYLINDER OF LOCOMOTIVE 2521, TEST NO. 510.

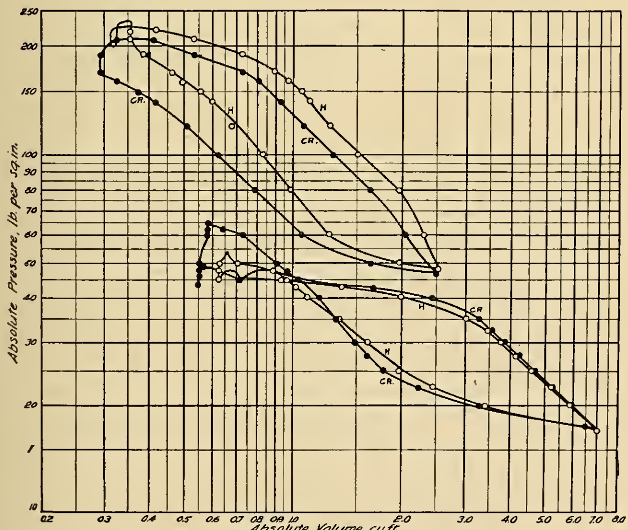

FIG. 22b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 2512, TEST NO. 510.

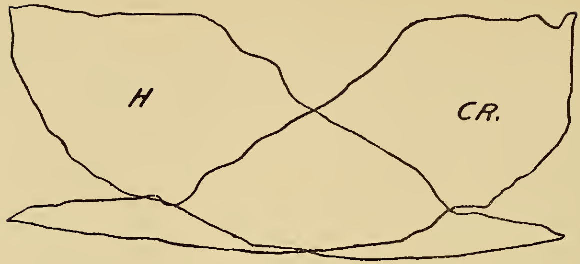

FIG. 23a. REPRESENTATIVE INDICATOR DIAGRAMS FROM HIGH PRESSURE CYLINDER OF LOCOMOTIVE 535, TEST NO. 603.

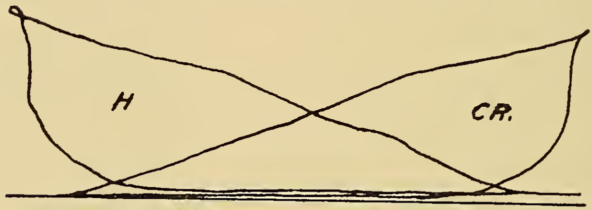

FIG. 23a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOW PRESSURE CYLINDER OF LOCOMOTIVE 535, TEST NO. 603.

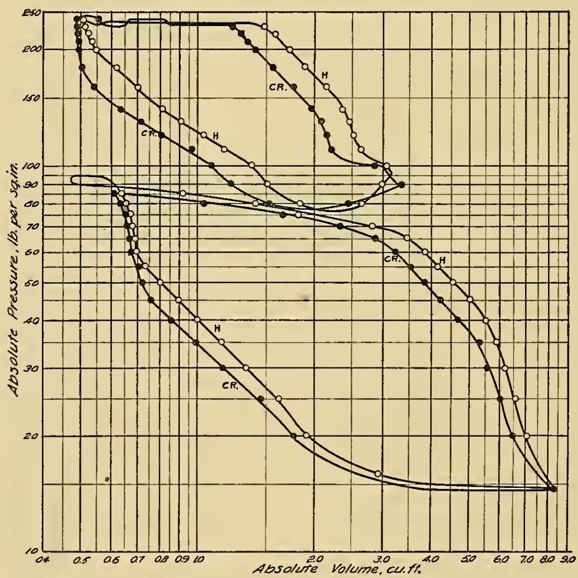

FIG. 23b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 535, TEST NO. 603.

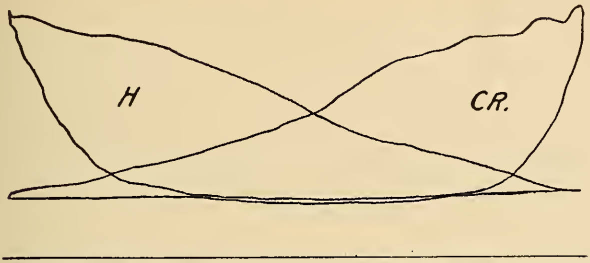

FIG. 24a. REPRESENTATIVE INDICATOR DIAGRAMS FROM HIGH PRESSURE CYLINDER OF LOCOMOTIVE 628, TEST NO. 711.

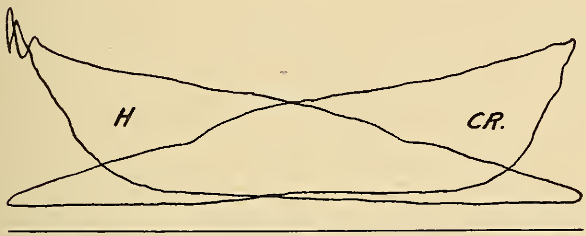

FIG. 24a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOW PRESSURE CYLINDER OF LOCOMOTIVE 628, TEST NO. 711.

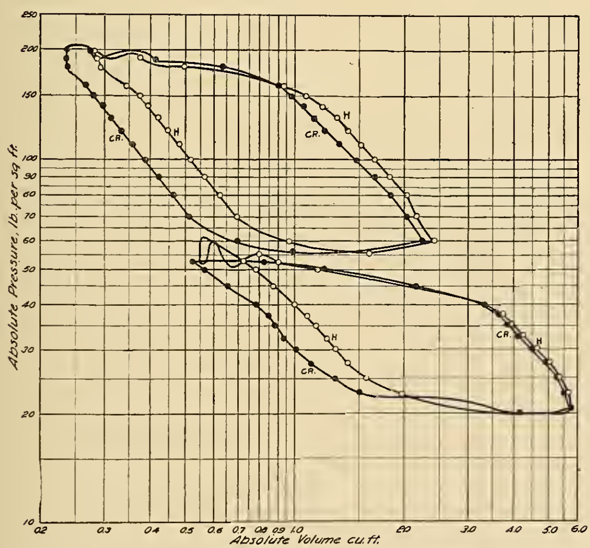

FIG. 24b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 628, TEST NO. 711.

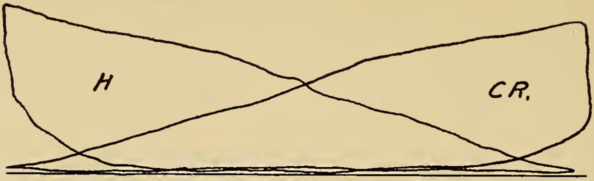

FIG. 25a. REPRESENTATIVE INDICATOR DIAGRAMS FROM LOW PRESSURE CYLINDER OF LOCOMOTIVE 3000, TEST NO. 802.

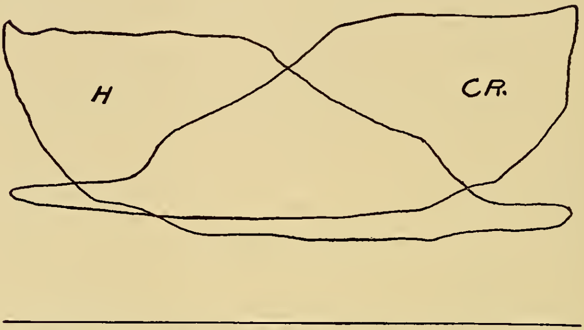

FIG. 25a. REPRESENTATIVE INDICATOR DIAGRAMS FROM HIGH PRESSURE CYLINDER OF LOCOMOTIVE 3000, TEST NO. 802.

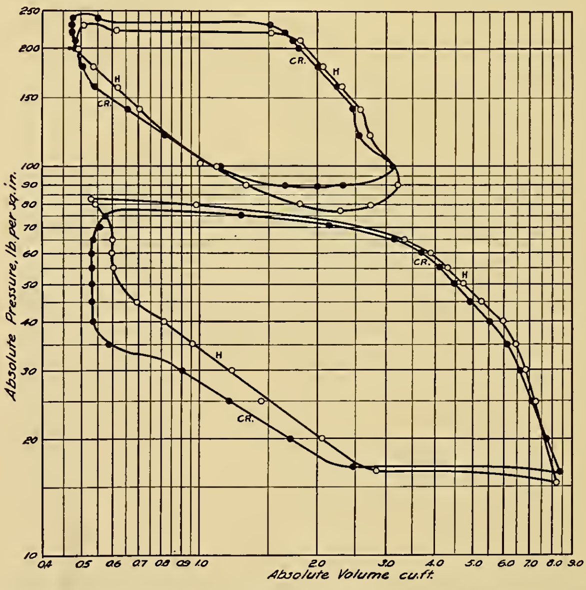

FIG. 25b. REPRESENTATIVE LOGARITHMIC DIAGRAMS FROM LOCOMOTIVE 3000, TEST NO. 802.

APPENDIX II

The Tests

1. *Principal Dimensions of the Locomotives Tested.*—The principal dimensions of the locomotives tested are given in Table 4.

2. *Logs of the Tests.*—The logs of the tests containing the quantities necessary for this investigation are given in Tables 5-16.

TABLE 4—PRINCIPAL DIMENS

No. of Items*	Locomotive Number	Purdue #2	Purdue #3	1499	734	52
	Type	American 8 wheels	American 8 wheels	Consolidation	Consolidation	Atla
	Class	Schenectady No. 2	Schenectady No. 3	H 6 a	B 1	E
	Owned by	Purdue University	Purdue University	P.R.R. Co.	L.S. & M.S R.R. Co.	P.R.R
	Tested at	La Fayette Indiana.	La Fayette Indiana.	St. Louis, Mo.	St. Louis, Mo.	Alto P
	Driving Wheels					
1	Number of pairs	2	2	4	4	
2	Approximate diameter, inches	70	70	56	63	
	Engine Truck Wheels					
14	Number	4	4	2	2	
15	Diameter, inches	69	69	29.78	36.68	
	Trailing Wheels					
16	Diameter, inches	....	....	....	....	
	Wheel Bases					
17	Driving wheel bases	8.5	8.5	16.550	17.342	7.
18	Total wheel base	23.0	23.0	24.770	25.572	30.
	Weight of Engine with Water at 2nd. Gage Cock and Normal Fire, in lb.					
20	On truck	48,000	48,000	21,000	18,700	37,1
21	On 1st. drivers	....	....	44,000	39,325	53,3
22	On 2nd. drivers	....	....	43,200	39,775	56,6
23	On 3rd. drivers	....	....	43,500	40,275	..
24	On 4th. drivers	....	....	42,500	43,225	..
25	On 5th. drivers	....	....	....	....	..
26	On trailers	....	....	....	....	37,0
27	Total	109,000	109,000	194,200	181,300	184,1
28	Total on drivers	61,000	61,000	173,200	162,600	110,0
	Cylinders					
29	High pressure, number	2	2	2	2	
30	High pressure, number	....	....	....	....	...
31	Arrangement	Outside	Outside	Outside	Outside	Outs
	Diameter, inches					
32	High pressure, right	16.022	16.022	21.997	21.004	20.51
33	High pressure, left	16.160	16.160	21.993	21.023	20.81
34	Low pressure, right	....	....	....	....	...
35	Low pressure, left	....	....	....	....	...
	Stroke at Piston, Feet					
36	High pressure, right	2	2	2.332	2.499	2.16
37	High pressure, left	2	2	2.332	2.500	2.16
38	Low pressure, right	....	....	....	....	...
39	Low pressure, left	....	....	....	....	...
	Clearance Per cent of Piston Displacement					
40	H. P., right, head end	7.44	7.44	11.83	9.21	12.
41	H. P., right, crank end	7.98	7.98	10.80	9.21	12.
42	H. P., left, head end	7.34	7.34	11.31	9.26	12.
43	H. P., left, crank end	7.63	7.63	11.25	9.36	11.
44	L. P., right, head end	....	....	....	....	...
45	L. P., right, crank end	....	....	....	....	...
46	L. P., left, head end	....	....	....	....	...
47	L. P., left, crank end	....	....	....	....	...
	Receiver, Cubic Feet					
48	Volume, right side	....	....	....	....	...
49	Volume, left side	....	....	....	....	...
	Piston Rods, Diameter, inches					
74	High pressure, right	2.75	2.75	3.997	3.737	3.47
75	High pressure, left	2.75	2.75	3.998	3.733	3.50
76	Low pressure, right	....	....	....	....	...
77	Low pressure, left	....	....	....	....	...
	Tail Rods, Diameter, inches					
78	High pressure, right	....	....	....	....	...
79	High pressure, left	....	....	....	....	...
80	Low pressure, right	....	....	....	....	...
81	Low pressure, left	....	....	....	....	...

*Item numbers are the same as those given in the Reports of Tests at the St. Louis Exposition.

ALL LOCOMOTIVES TESTED.

510	3395	585	929	2512	535	628	3000	No. of Items*
cific	Pacific	Consolidation	Santa Fe	Atlantic	Atlantic	Atlantic	Atlantic	
-2	K-29	W	900	De Glehn Compound	507	S8	I1	
R. Co.	P.R.R. Co.	M.C.R.R.	A.T. & S.F. R.R.	P.R.R. Co.	A. T. & S.F. R.R.	Royal Prussian R.R.	N.Y.C. & H. R.R.R.	
oona, Pa.	Altoona, Pa.	St. Louis, Mo.	St. Louis, Mo.	St. Louis, Mo.	St. Louis, Mo.	St. Louis, Mo.	St. Louis, Mo.	
3	3	4	5	2	2	2	2	1
80	80	63	56.5	80	79	78	79	2
4	4	2	2	4	4	4	4	14
36	36	33	28.5	37.824	34.25	39.48	36.48	15
56	54	....	39.9	60.780	50.2	39.48	50.0	16
.84	13.85	16.99	19.768	7.058	6.834	6.890	7.00	17
.21	36.50	25.79	35.885	28.569	30.288	29.540	27.75	18
4334	50,500	24,500	25,860	41,250	51,880	39,200	50,000	20
5467	66,000	44,200	53,110	44,550	52,000	33,625	55,100	21
6000	65,800	44,800	58,100	43,300	47,200	31,725	54,900	22
6167	66,000	39,800	45,760	....	....	....	....	23
...	....	35,700	36,110	....	....	....	....	24
...	....	....	40,680	....	....	....	....	25
4133	68,700	....	26,120	34,900	50,420	28,800	40,000	26
7,101	317,000	189,000	285,740	164,000	201,500	133,350	200,000	27
7634	197,800	164,500	233,760	82,850	99,200	65,350	110,000	28
2	2	1	2	2	2	2	2	29
...	....	1	2	2	2	2	2	30
tside	Outside	Outside Cross Compound	Outside Tandem Compound	L. P. Inside H. P. Outside De Glehn Compound	L. P. Outside H.P. Inside Vau Clain Compound	L. P. Outside H. P. Inside Hanover Compound	L. P. Outside H.P. Inside Cole Compound	31
995	27.037	....	19.129	14.1847	15.034	14.166	15.510	32
994	27	23.096	19.165	14.1834	15.037	14.164	15.512	33
...	....	35.106	32.000	23.6550	25.023	22.048	26.006	34
...	....	....	32.000	23.6536	25.020	22.078	26.013	35
170	2.332	....	2.671	2.0975	2.166	1.968	2.163	36
163	2.338	2.667	2.668	2.0975	2.167	1.966	2.163	37
...	....	2.671	2.671	2.1025	2.167	1.969	2.166	38
...	....	....	2.668	2.1050	2.169	1.969	2.166	39
3.2	15.1	....	17.217	13.59	17.80	12.8	17.1	40
3.5	15.3	....	17.111	13.02	19.10	11.5	16.9	41
13.3	13.4	17.09	17.172	13.83	18.20	11.9	16.7	42
13.6	13.9	16.28	17.064	12.65	19.20	10.7	16.8	43
...	....	5.76	8.275	9.92	6.40	10.5	6.7	44
...	....	5.66	8.458	9.12	6.58	10.2	6.5	45
...	....	....	8.284	11.27	6.58	10.8	6.4	46
...	....	....	8.467	8.68	6.77	10.3	6.5	47
...	....	19.0	....	14.52	3.07	4.13	6.11	48
...	....	19.0	....	14.52	3.05	4.13	6.10	49
497	4.502	....	3.250	2.6763	3.255	3.147	3.000	74
499	4.501	3.998	3.250	2.6769	3.245	3.152	3.000	75
...	....	4.022	4.461	2.6755	3.245	2.755	3.002	76
...	....	....	4.461	2.6759	3.235	2.753	3.002	77
...	3.0	....	....	....	....	....	....	78
...	3.0	....	....	....	....	....	....	79
...	....	....	3.25	2.3609	....	1.968	....	80
...	....	....	3.25	2.3610	....	1.992	....	81

TABLE 4—PRINCIPAL DIMENSI[ON]S

No. of Items*	Locomotive Number	Purdue No. 2	Purdue No. 3	1499	734	52[illegible]
	Valves					
82	Type	"D" Slide	"D" Slide	"D" Slide	"D" Slide	Wilso[illegible] anced [illegible] ble P[illegible] Slid[illegible]
83	Design	Richardson Balanced	Richardson Balanced	Richardson Balanced	Allen Richardson	Ame[illegible] Bala[illegible] Valve[illegible]
84	Per cent of balanced to total area	....	....	Left 58.97 Right 58.99	Left 53.13 Right 52.99	75.
85	Type of Link motion	Stephenson	Stephenson	Stephenson	Stephenson	Steph[illegible]on
	Miscellaneous					
106	Cylinder lagging material	....	....	Magnesia	Magnesia	Mag[illegible]a
107	Cylinder jacket material	....	....	Sheet steel	Sheet iron	Sheet [illegible]n
	Boiler					
113	Type	Extended Wagon Type	Extended Wagon Type	Belpaire Wide Fire-box	Extended Wagon Top Radial Stays	Belp[illegible] Wide [illegible]e-bo[illegible]
114	Outside diameter, 1st ring, inches	52	52	71	68.375	67
	Tubes					
115	Number of Tubes	200	111	373	338	315
115	Number of Flues	....	16	....	....	..
116	Outside diameter of tubes, inches	2	2	2	2	2
116	Outside diameter of flues, inches	....	5	....	....	..
117	Thickness, inches	No. 12 Gage	No. 12 Gage	0.135	0.120	0[illegible]5
118	Length between tube sheets, inches	138	138	164.50	178.937	179
119	Total fire area, square	....	....	6.089	5.71	5
124	Boiler Pressure	250	250	205	200	205
	Superheater					
125	Number of Tubes	....	32	....	....	..
126	Outside diameter, inches	....	1.25	....	....	..
127	Thickness, inches	....	....	....	....	..
128	Length of tubes, inches	....	207.24	....	....	..
129	Type of Superheater	....	Cole	....	....	..
	Grates					
144	Style	....	....	Rocking Finger	Rocking	Roc[illegible] Fin[illegible]
145	Total area, square feet	17	17	49.21	33.76	55
146	Total area, dead grates, square feet	....	....	0.0	0.0	6
147	Width of air spaces, inches	1.25	1.25	0.69	0.69	0
	Heating Surface, Square feet					
154	Of the tubes, water side	1196.00	897	2677.27	2638.97	2471
155	Of the tubes, fire side	1086.00	817	2315.86	2322.30	2162
156	Of the fire-box, fire side	126	126	166.40	218.92	156
157	Of the Superheater, fire side	....	193	....	....	..
158	Total, based on inside of fire-box and inside of tubes	1212.00	1136	2482.26	2541.22	2319
159	Total, based on inside of fire-box and outside of tubes	1322.00	1216	2843.67	2857.89	2627
	Constants for Piston Displacement, Cubic feet					
188	High Pressure Cylinder, right, head end	2.802	2.802	6.155	6.013	4
189	High Pressure Cylinder, right, crank end	2.720	2.720	5.952	5.823	4
190	High Pressure Cylinder, left, head end	2.849	2.849	6.153	6.025	5
191	High Pressure Cylinder, left, crank end	2.766	2.766	5.950	5.834	4
192	Low Pressure Cylinder, right, head end	....	....	....	....	..
193	Low Pressure Cylinder right, crank end	....	....	....	....	..
194	Low Pressure Cylinder, left, head end	....	....	....	....	..
195	Low Pressure Cylinder, left, crank end	....	....	....	....	..

*Item numbers are the same as those given in the Reports of Tests at St. Louis Exposition.

OF ALL LOCOMOTIVES TESTED.—(*Continued.*)

10	3395	585	929	2512	535	628	3000	No. of Items*
ton	Piston	H.P.Piston L.P. "D" Slide	Piston	"D" Slide	Piston	H. P. Piston L.P. Allen Balanced	Piston	82
..	American Locomotive Company	H. P. Outside Admission; L.P. Allen Richardson	Baldwin Locomotive Works	L. P. Not Balanced; H.P. Balanced Ring	Baldwin Locomotive Works	Hanover	American Locomotive Company.	83
..	10.0	H. P. 90.63 L. P. 55.88	H. P. 72.5 L. P. 85.1	H. P. 40.92	H. P. 70.59 L. P. 75.31	H. P. 75.6 L. P. 44.7	H. P. 91.22 L. P. 90.79	84
W heart	Walscheart	Stephenson	Stephenson	Walscheart	Stephenson	Heusinger	Stephenson	85
M nesia	Magnesia	Magnesia	Magnesia	Magnesia	Magnesia	Hair	Magnesia	106
S t iron	Sheet iron	Sheet steel	Sheet iron	Sheet iron	Sheet iron	Sheet iron	Sheet iron	107
B aire ide F -box	Radial Stay Wide Fire-box	Straight Top Wide Fire-box	Wagon Top Radial Stay Wide Fire-box	Belpaire Straight Top	Wagon Top Radial Stay Wide Fire-box	Straight Top Wide Fire-box	Straight Top Wide Fire-box	113
.0	....	70.125	81.05	59.664	69	62.24	72.25	114
3	241	363	393	139 Serve Tubes 8 Ribs No Flues	273	241	390	115
..	40	....	....		....	....	....	115
.25	2.25	2	2.25	2.75	2.25	2	2	116
..	5.50	....	....	....	....	....	....	116
.125	0.125	0.12	0.125	0.075	0.125	0.098	0.125	117
.40	262.88	190.375	238.500	176.136	225.137	143.78	191.295	118
.83	9.19	6.13	8.570	4.691	5.955	2.50	6.514	119
5	200	210	225	225	220	200	220	124
..	160	....	....	....	....	241	....	125
..	1.438	....	....	....	....	2	....	126
..	0.148	....	....	....	....	0.098	....	127
..	251.75	....	....	....	....	29.92	....	128
..	Schmidt	....	....	....	....	Pielock	....	129
king nger	Rocking Finger	Rocking	Rocking Finger	Stationary Wrought Iron Bars	Rocking Finger	Stationary	Rocking Finger	144
.41	50.08 Incl. Retorts 70.73	49.43	58.41	33.39	48.36	29.06	49.90	145
.0	1.82	0.0	0.0	0.0	0.0	0.0	0.0	146
.78	0.375	0.75	0.94	0.50	1.06	0.5	0.75	147
4 .06	4373.31	3015.34	4601.00	1468.87	3016.71	1511.94	3255.27	154
3 .23	3968.27	2653.51	4089.77	2479.20	2681.75	1363.77	2848.36	155
.47	232.84	165.69	216.36	177.28	220.30	105.59	151.69	156
..	1301.87	....	....	....	....	283.79	....	157
4 .70	4201.11	2819.20	4306.13	2656.48	2902.05	1753.15	3000.05	158
4 .53	4606.15	3181.03	4817.36	1646.15	3237.01	1932 16	3406.96	159
.86	9.19	....	5.330	2.3020	2.6702	2.1539	2.8880	188
.57	9.05	....	5.177	2.2199	2.5450	2.0475	2.7318	189
.79	9.18	7.761	5.344	2.3014	2.6724	2.1511	2.8387	190
.55	9.04	7.528	5.191	2.2190	2.5480	2.0446	2.7325	191
..	....	17.954	14.762	6.3520	7.4006	5.1786	7.9898	192
..	....	17.718	14.627	6.3340	7.2761	5.1389	7.8833	193
..	....	....	14.746	6.3595	7.4056	5.1918	7.9941	194
..	....	....	14.611	6.3413	7.2818	5.1534	7.8876	195

TABLE 5—TESTS OF PURDUE LOCOMOTI

Number of Test	Laboratory Designation	Total Moist or Superheated Steam Passing thru Cylinders per hour	Total I. H. P.	Moist or Superheated Steam per I. H. P. hr., pounds	Pressure-lb. per sq. in. In Boiler by Gage	Pressure-lb. per sq. in. In Branch-pipe by Gage	Pressure-lb. per sq. in. Barometer	Degrees of Superheat in Branch pipe
1	2	3	4	5	6	7	8	9
TESTS AT 200 LB. GAGE PRESSURE								
29	20-2-200	6329	223.47	28.32	200.2	196.9	14.4	...
30	20-4-200	7549	287.62	26.24	199.6	196.9	14.4	...
31	20-6-200	9778	375.82	26.01	199.7	.....	14.5	...
32	20-8-200	12443	472.90	26.31	200.3	195.6	14.4	...
107	30-2-200	6431	280.44	22.93	200.1	196.8	14.6	131
108	30-4-200	8278	383.44	21.59	200.5	198.3	14.4	152
109	30-6-200	10306	516.97	19.93	200.2	199.1	14.5	169
33	30-2-200	7266	268.95	27.01	199.4	200.2	14.2	...
34	30-4-200	9434	367.09	25.70	200.0	199.3	14.4	...
35	30-6-200	12186	489.21	24.91	199.4	196.7	14.4	...
110	40-2-200	6938	309.58	22.41	200.4	198.6	14.7	138
111	40-4-200	9844	466.51	21.09	200.1	193.7	14.3	152
112	40-6-200	11817	588.63	20.07	200.0	198.4	14.4	177
37	40-2-200	8243	306.58	26.88	200.4	195.9	14.5	...
38	40-4-200	11073	448.88	24.66	201.6	.....	14.5	...
39	40-6-200	14786	605.25	24.43	199.4	194.4	14.2	...
113	50-4-200	9766	487.46	20.03	201.1	198.0	14.3	159
41	50-2-200	8471	329.05	25.74	200.5	197.8	14.4	...
42	50-4-200	11972	464.20	25.78	200.6	196.2	14.3	...
TESTS AT 220 LB. GAGE PRESSURE								
13	20-2-220	7066	255.33	27.65	221.6	218.4	14.4	...
14	20-4-220	8846	342.85	25.80	219.8	216.6	14.5	...
15	20-6-220	11008	431.44	25.51	220.1	215.2	14.2	...
16	20-8-220	13788	533.02	25.86	220.6	218.4	14.4	...
17	30-2-220	8522	320.32	26.60	220.5	219.7	14.5	...
18	30-4-220	10720	446.49	24.23	220.0	219.1	14.4	...
19	30-6-220	13192	559.49	23.59	218.8	215.1	14.4	...
21	40-2-220	9505	371.46	25.58	220.7	217.7	14.3	...
22	40-4-220	12058	509.07	23.68	218.6	212.8	14.4	...
24	50-2-220	9960	378.79	26.29	220.8	218.0	14.5	...
25	50-4-220	13540	562.30	24.08	220.0	.....	14.4	...
TESTS AT 240 LB. GAGE PRESSURE								
1	20-2-240	7268	276.45	26.29	241.5	240.9	14.5	...
2	20-4-240	9932	392.52	25.33	242.2	.....	14.4	...
3	20-6-240	11247	466.82	24.09	238.3	237.2	14.4	...
101	30-2-240	8662	369.76	23.43	241.0	237.3	14.5	137
102	30-4-240	10539	476.69	22.11	238.9	236.1	14.3	152
103	30-5-240	11580	534.07	21.68	234.3	225.4	14.4	149
5	30-2-240	9462	371.29	25.48	240.0	.....	14.6	...
6	30-4-240	11500	470.64	24.43	240.2	235.1	14.5	...
104	40-2-240	9078	415.09	21.89	236.4	231.4	14.5	127
105	40-4-240	11758	551.36	21.32	239.7	235.7	14.6	153
8	40-2-240	10192	421.78	24.16	242.0	240.5	14.3	...
9	40-4-240	13511	566.30	23.86	241.0	239.2	14.4	...
106	50-2-240	10038	463.87	21.64	235.8	231.4	14.4	142
11	50-2-240	11627	465.50	24.97	242.0	.....	14.5	...

HENECTADY Nos. 2 AND 3.

Speed		Weight of Steam per Revolution, lb.				Quality of Steam Mixture at Cut-off, x_c, parts of unity	Average Value of n, From Expansion Curves	Av. Absolute Pressure, lb.-sq. in.	
Revolutions per min.	Revolutions per hr.	Passing thru Cylinders	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present			At Cut-off	Back Pressure of Exhaust
10	11	14	15	16	17	18	19	20	21
			TESTS AT 200 LB. GAGE PRESSURE						
7.09	5825	1.0865	0.829	0.2780	1.3645	0.608	0.955	159	16
6.97	5815	1.2979	1.017	0.2520	1.5499	0.656	0.978	153	16
7.52	5880	1.6730	1.305	0.2105	1.8835	0.693	0.962	154	17
7.25	5835	2.1325	1.702	0.1915	2.3240	0.732	1.014	161	17
6.90	8814	0.7296	0.757	0.2650	0.9946	0.761	0.985	143	16
6.20	8775	0.9439	0.973	0.2520	1.1959	0.813	1.057	144	16
8.00	8880	1.1605	1.254	0.2475	1.4080	0.891	1.071	150	17
6.11	8766	0.8288	0.753	0.3025	1.1313	0.665	0.971	150	16
6.19	8772	1.0756	0.969	0.2815	1.3571	0.714	0.973	146	16
6.14	8768	1.3896	1.201	0.2620	1.6516	0.727	1.039	144	17
4.50	11670	0.5945	0.769	0.3310	0.9255	0.831	1.022	138	16
6.88	11810	0.8333	0.935	0.2670	1.1003	0.850	1.071	131	16
8.80	11925	0.9906	1.188	0.2760	1.2666	0.937	1.123	131	17
4.78	11680	0.7053	0.711	0.3170	1.0223	0.695	0.972	136	16
4.85	11690	0.9471	0.894	0.3220	1.2691	0.704	0.980	138	16
4.67	11670	1.2658	1.159	0.3105	1.5763	0.735	1.036	133	17
4.90	14090	0.6929	0.934	0.3250	1.0179	0.917	1.084	124	16
3.38	14610	0.5800	0.684	0.3270	0.9070	0.754	1.003	128	16
3.85	14625	0.8183	0.831	0.3320	1.1503	0.722	1.004	123	16
			TESTS AT 220 LB. GAGE PRESSURE						
6.83	5810	1.3028	0.910	0.2700	1.5728	0.579	0.924	169	16
7.57	5852	1.5110	1.156	0.2500	1.7610	0.656	0.972	173	16
7.39	5840	1.8839	1.472	0.2180	2.1019	0.700	1.002	178	17
7.80	5878	2.3496	1.890	0.2205	2.5701	0.735	1.005	185	17
6.05	8760	0.9726	0.834	0.3070	1.2796	0.652	0.972	170	16
6.15	8761	1.2225	1.074	0.2910	1.5135	0.710	1.000	155	16
6.26	8775	1.5033	1.360	0.2900	1.7933	0.758	1.019	158	17
5.29	11710	0.8110	0.795	0.3200	1.1310	0.703	0.988	148	16
4.71	11680	1.0320	0.981	0.3190	1.3510	0.726	1.000	146	16
3.41	14604	0.6819	0.737	0.3280	1.0099	0.736	1.010	138	16
3.43	14606	0.9270	0.944	0.3326	1.2590	0.750	1.014	135	16
			TESTS AT 240 LB. GAGE PRESSURE						
4.49	5670	1.2819	0.968	0.2980	1.5799	0.612	0.926	189	16
7.72	5861	1.6940	1.292	0.2610	1.9550	0.661	0.987	194	16
7.09	5825	1.9300	1.540	0.2525	2.1825	0.705	0.935	190	17
7.10	8825	0.9814	0.960	0.3215	1.3029	0.736	0.963	180	16
1.65	8500	1.2400	1.165	0.2740	1.5140	0.770	1.015	170	16
3.29	8595	1.3468	1.283	0.2675	1.6143	0.795	1.031	168	17
6.15	8765	1.0790	0.957	0.3240	1.4030	0.682	0.964	176	16
6.00	8760	1.3120	1.131	0.3025	1.6145	0.701	0.966	171	16
3.55	11620	0.7817	0.873	0.3070	1.0887	0.802	1.002	160	16
3.00	11580	1.0150	1.125	0.3150	1.3300	0.846	1.024	155	16
4.46	11660	0.8735	0.850	0.3260	1.1995	0.709	0.968	166	16
4.63	11680	1.1565	1.076	0.3315	1.4880	0.723	0.987	155	16
6.41	14790	0.6789	0.826	0.3045	0.9834	0.840	1.041	146	16
8.38	14910	0.7801	0.827	0.3515	1.1316	0.730	1.004	148	16

TABLE 5—Tests of Purdue Locomotiv

Number of Test	Laboratory Designation	Total Moist or Superheated Steam Passing thru Cylinders per hour	Total I. H. P.	Moist or Superheated Steam per I. H. P. hr., pounds	Pressure, lb. per sq. in. In Boiler by Gage	In Branch-pipe by Gage	Barometer	Degrees of Superheat in Branch pipe Degrees
1	2	3	4	5	6	7	8	9
				TESTS AT 120 LB. GAGE PRESSURE				
* 85	20– 4–120†	4351	134.00	32.47	120.4	117.9	14.3	....
86	20– 8–120	7180	252.79	28.40	121.3	118.0	14.2	....
87	20–12–120	10312	356.91	28.88	120.0	115.6	14.3	....
129	30– 4–120	4454	163.30	27.27	120.2	118.7	14.5	117.
130	30– 8–120	7726	335.27	23.05	120.2	.....	....	156.
131	30–10–120	9043	409.72	22.07	119.9	120.9	14.4	174.
132	30–14–120	11672	517.49	22.56	120.2	119.5	14.2	191.
88	30– 4–120	5238	171.02	30.63	120.5	119.1	14.4	....
89	30– 8–120	8939	325.49	27.46	120.4	117.4	14.3	....
90	30–14–120	15584	514.01	30.31	120.1	114.3	14.4	....
133	40– 4–120	4714	178.58	26.39	120.1	119.0	14.6	120.
134	40– 8–120	8530	382.89	22.27	120.3	120.0	14.4	170.
135	40–12–120	12156	547.57	22.20	120.6	118.3	14.7	190.
91	40– 4–120	5556	184.04	30.18	120.3	117.4	14.4	....
92	40– 8–120	10702	389.05	27.51	120.5	117.1	14.5	....
93	40–12–120	15830	554.77	28.52	119.9	119.1	14.4	....
136	50– 8–120	9368	411.80	22.74	120.6	117.5	14.7	169.
94	50– 4–120	5957	176.03	33.84	119.9	118.6	14.4	....
95	50– 8–120	12024	427.59	28.12	120.3	117.8	14.5	....
96	50–11–120	16149	553.53	29.17	120.1	110.0	14.4	....
				TESTS AT 160 LB. GAGE PRESSURE				
114	20– 2–160	4521	166.77	27.11	160.2	.....	14.7	125.
115	20– 4–160	5504	232.15	23.71	160.5	.....	14.5	133.
116	20– 6–160	6913	300.16	23.03	159.9	.....	14.4	153.
117	20– 8–160	8367	366.71	22.82	159.5	.....	14.3	145.
67	20– 4–160	6153	219.52	28.03	160.2	159.1	14.4	....
68	20– 6–160	7802	298.41	26.14	160.2	158.5	14.4	....
69	20– 8–160	9494	345.05	27.52	159.3	157.3	14.3	....
118	30– 2–160	5014	191.53	26.18	160.8	160.6	14.4	125.
119	30– 4–160	6543	285.39	22.93	160.2	159.7	14.4	141.
120	30– 6–160	8565	387.36	22.11	160.4	.....	14.5	162.
121	30– 8–160	10214	481.82	21.20	160.1	158.6	14.3	172.
71	30– 4–160	7311	272.11	26.86	160.7	160.3	14.4	....
72	30– 6–160	9705	383.86	25.28	159.7	158.6	14.5	....
73	30– 8–160	11947	465.05	25.69	162.2	159.9	14.4	....
122	40– 2–160	5378	211.46	25.46	161.1	160.0	14.5	123.
123	40– 4–160	7389	343.28	21.52	159.9	.....	14.7	154.
124	40– 6–160	9213	429.80	21.44	160.0	.....	14.4	164.
125	40– 8–160	12145	598.70	20.29	160.4	.....	14.4	180.
76	40– 4–160	8408	317.72	26.48	161.0	156.9	14.3	....
77	40– 6–160	11145	431.66	25.82	160.6	154.7	14.4	....
78	40– 8–160	14380	543.87	26.44	161.1	158.0	14.5	....
126	50– 2–160	5238	180.24	29.06	161.2	159.6	14.5	118.
127	50– 4–160	7621	343.05	22.21	160.4	159.4	14.8	152.
80	50– 4–160	9137	338.27	27.01	159.8	158.4	14.4	....
81	50– 6–160	12432	475.87	26.12	160.0	155.6	14.4	....
				TESTS AT 180 LB. GAGE PRESSURE				
46	20– 2–180	5525	191.97	28.78	183.0	181.2	14.6	...
47	20– 4–180	7054	263.59	26.76	181.6	179.0	14.5	...
48	20– 6–180	8499	334.07	25.44	180.0	176.5	14.5	...
49	20– 8–180	10668	411.70	25.91	180.3	177.7	14.5	...
51	30– 2–180	6257	235.15	26.54	181.6	178.6	14.2	...
52	30– 4–180	8090	317.15	25.36	178.6	178.9	14.5	...
53	30– 6–180	9685	393.27	24.62	170.0	170.0	14.6	...
54	30– 8–180	13447	546.26	24.61	179.4	175.5	14.4	...
56	40– 2–180	6711	259.13	25.89	182.3	178.9	14.5	...
57	40– 4–180	9301	386.18	24.08	181.1	178.8	14.5	...
58	40– 6–180	12401	524.08	23.68	179.8	176.2	14.5	...
59	40– 8–180	15771	609.91	25.85	177.7	177.4	14.2	...
61	50– 2–180	7133	268.02	26.61	182.3	181.6	14.3	...
62	50– 4–180	10032	410.62	24.43	181.3	176.7	14.5	...
63	50– 6–180	13766	553.30	24.87	180.7	175.5	14.3	...

* Tests 1–97 run with saturated steam. 101–136 with superheated steam. † Laboratory Design... gauge pressure.

Schenectady Nos. 2 and 3.—*(Continued.)*

Speed		Weight of Steam per Revolution, lb.				Quality of Steam Mixture at Cut-off, x_c, parts of unity	Average Value of n, From Expansion Curves	Av. Absolute Pressure, lb.-sq. in.	
Revolutions per min.	Revolutions per hr.	Passing thru Cylinders	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present			At Cut-off	Back Pressure of Exhaust
10	11	14	15	16	17	18	19	20	21
				TESTS AT 120 LB. GAGE PRESSURE					
7.33	5840	0.7450	0.622	0.2420	0.9870	0.630	0.944	98	16
7.08	5825	1.2320	1.005	0.1835	1.4155	0.710	0.999	98	17
7.04	5822	1.7710	1.502	0.1645	1.9356	0.776	1.027	104	19
6.20	8772	0.5075	0.567	0.2550	0.7625	0.744	0.957	89	16
7.37	8842	0.8737	0.951	0.1990	1.0727	0.886	1.076	90	17
5.13	8708	1.0384	1.167	0.1665	1.2049	0.969	1.134	90	18
6.03	8762	1.3322	1.600	0.1560	1.4882	1.075	1.158	99	20
6.15	8769	0.5973	0.568	0.2510	0.8483	0.669	0.972	90	16
6.45	8787	1.0720	0.910	0.2145	1.2865	0.707	1.021	88	17
5.81	8749	1.7813	1.611	0.1820	1.9633	0.821	1.024	100	20
4.52	11671	0.4039	0.562	0.2570	0.6609	0.851	1.031	85	16
4.55	11673	0.7305	0.904	0.2075	0.9380	0.964	1.120	85	17
4.61	11677	1.0219	1.315	0.2080	1.2299	1.069	1.192	91	19
5.03	11702	0.4747	0.531	0.2695	0.7442	0.713	0.984	85	16
5.01	11701	0.9145	0.875	0.2450	1.1595	0.754	1.026	83	17
5.38	11723	1.3503	1.296	0.2320	1.5823	0.819	0.997	91	19
3.87	14632	0.6406	0.852	0.2380	0.8786	0.970	1.150	76	17
3.71	14623	0.4074	0.525	0.2755	0.6829	0.769	0.979	76	16
3.45	14607	0.8232	0.852	0.2750	1.0982	0.776	0.977	79	17
6.10	14766	1.0936	1.110	0.2365	1.3301	0.835	1.000	80	19
				TESTS AT 160 LB. GAGE PRESSURE					
7.15	5829	0.7755	0.683	0.2480	1.0235	0.667	0.923	128	16
7.06	5824	0.9451	0.851	0.2110	1.1561	0.736	0.985	127	16
7.44	5846	1.1823	1.075	0.1805	1.3628	0.789	1.027	126	17
7.61	5860	1.4285	1.348	0.1660	1.5945	0.845	1.091	127	17
7.34	5840	1.0536	0.824	0.2120	1.2656	0.651	0.966	123	16
7.49	5849	1.3245	1.084	0.2100	1.5345	0.706	0.983	129	17
7.39	5843	1.6229	1.295	0.1925	1.8154	0.713	0.991	125	17
6.44	8786	0.5707	0.625	0.2650	0.8357	0.748	0.950	124	16
5.45	8727	0.7497	0.800	0.2220	0.9717	0.823	0.997	115	16
6.11	8767	0.9769	0.993	0.2240	1.2009	0.827	1.074	115	17
6.27	8776	1.1635	1.250	0.2150	1.3785	0.907	1.113	119	17
5.96	8758	0.8323	0.763	0.2520	1.0843	0.704	0.992	119	16
6.11	8767	1.0071	0.985	0.2480	1.2551	0.784	1.030	119	17
6.42	8785	1.3594	1.218	0.2475	1.6069	0.757	0.985	119	17
9.26	11956	0.4498	0.615	0.2955	0.7453	0.825	1.020	113	16
6.02	11761	0.6282	0.783	0.3055	0.9337	0.839	1.047	106	16
5.25	11715	0.7864	0.936	0.2485	1.0349	0.905	1.095	109	17
5.27	11716	1.0364	1.207	0.2130	1.2494	0.966	1.153	111	17
5.27	11716	0.7176	0.740	0.2940	1.0116	0.731	1.019	107	16
4.68	11681	0.9541	0.919	0.2960	1.2501	0.735	1.017	110	17
5.85	11751	1.2237	1.122	0.2745	1.4982	0.749	1.006	107	17
3.17	14590	0.3589	0.547	0.3070	0.6659	0.822	1.010	105	16
2.91	14575	0.5228	0.703	0.2860	0.8088	0.869	1.054	100	16
3.61	14617	0.6251	0.671	0.2840	0.9091	0.738	0.997	96	16
3.86	14632	0.8497	0.874	0.2920	1.1417	0.765	1.023	96	17
				TESTS AT 180 LB. GAGE PRESSURE					
7.74	5862	0.9472	0.739	0.2870	1.2342	0.599	0.878	146	16
7.74	5870	1.2016	0.943	0.2705	1.4721	0.640	0.905	139	16
7.69	5860	1.4510	1.193	0.2265	1.6775	0.711	0.976	145	17
7.66	5859	1.8234	1.475	0.2085	2.0319	0.725	0.980	146	17
6.76	8805	0.7105	0.700	0.2940	1.0045	0.697	0.954	144	16
5.37	8720	0.9275	0.847	0.2910	1.2185	0.695	0.960	134	16
6.00	8760	1.1050	1.015	0.2640	1.3690	0.741	0.988	125	17
6.76	8805	1.5480	1.360	0.2475	1.7955	0.757	0.992	126	17
4.92	11690	0.5737	0.660	0.3095	0.8832	0.747	0.967	129	16
5.12	11710	0.7940	0.833	0.3165	1.1105	0.750	1.015	128	16
3.64	11618	1.0673	1.026	0.3560	1.4233	0.721	0.956	125	17
6.72	11615	1.3354	1.251	0.2910	1.6264	0.769	1.032	125	17
3.55	14610	0.4880	0.635	0.3100	0.7980	0.796	0.997	125	16
2.77	14560	0.6885	0.771	0.3245	1.0130	0.761	1.006	116	16
7.06	14825	0.9280	0.958	0.3205	1.2485	0.767	0.997	109	17

otes in order, miles per hour, number of notches reverse lever is forward of dead-center position, and boiler

TABLE 6—Tests of Penn. R. R. Class I

Number of Test	Laboratory Designation	Total Moist Steam passing thru Cylinders per hr.	Total I. H. P.	Moist Steam in lb. per I. H. P. hr.	Pressure, lb. per sq. in.		Speed	
					In Boiler by Gage	Barometer	Revolutions per min.	Revolutions
1	2	3	4	5	6	8	10	11
110	40-20-F†	10445	365.7	28.57	200.4	14.5	40.33	24
111	40-30-F	12493	454.5	27.49	203.4	14.5	40.42	24
103	80-20-F	16748	650.0	25.77	197.6	14.4	92.74	55
109	80-20-F	14995	587.6	25.52	191.6	14.4	81.59	48
112	80-30-F	18796	779.3	24.11	200.7	14.5	79.73	47
118	80-37-F	23176	930.5	24.90	201.4	14.5	80.69	48
116	120-30-F	23046	975.1	23.63	198.7	14.5	120.12	72
115	120-35-F	24799	1036.1	23.92	193.1	14.4	120.63	72
102	160-20-F	20068	803.3	24.99	189.3	14.5	160.33	96
105	160-27-F	22750	951.4	23.91	181.6	14.6	157.64	94
117	160-35-P	25496	1023.7	24.90	201.8	14.5	160.63	96

TABLE 7—Tests of L. S. & M. S. Class 1

201	40-20-F	8942	299.4	29.88	192.8	14.6	40.34	24
203	40-40-F	15174	550.4	27.58	199.6	14.5	40.11	24
204	80-45-F	23732	901.1	26.33	178.0	14.5	79.99	47
205	80-20-F	13810	527.0	26.20	200.3	14.6	80.46	48
206	80-30-F	19412	782.6	24.81	201.8	14.5	80.07	48
208	80-40-F	24283	962.5	25.22	204.0	14.5	80.00	48
209	160-20-F	21948	865.6	25.36	201.5	14.4	159.31	95
210	160-23-F	23763	953.7	24.92	198.5	14.4	159.94	95
211	160-27-F	26149	994.8	26.28	178.4	14.4	160.01	96
212	160-26-F	25481	1053.9	24.18	199.4	14.5	160.32	96
213	160-40-P	24770	886.8	27.92	202.6	14.5	160.39	96
217	160-40-P	24300	799.5	30.40	199.3	14.5	159.23	95
218	160-40-P	24495	865.2	28.31	200.8	14.5	160.10	96
219	160-30-P	23822	923.4	25.81	200.7	14.6	158.82	95
220	160-26-P	23287	942.4	24.90	201.6	14.5	160.05	96

TABLE 8—Tests of Penn R. R. Class] A

901	80-15-F	14227	419.8	33.89	201.3	14.1	80.00	48
902	80-20-F	15545	477.2	32.56	200.1	14.2	80.00	48
904	80-25-F	18124	585.6	30.98	198.5	14.2	80.00	48
906	80-30-F	22030	727.9	30.30	202.6	14.2	80.00	48
908	120-20-F	19747	687.6	28.72	201.0	14.1	120.00	72
910	120-25-F	22946	851.1	26.98	200.5	14.1	120.00	72
912	120-30-F	26941	1015.4	26.52	202.7	14.1	120.00	72
913	160-15-F	20222	748.8	27.01	198.0	14.2	160.00	96
914	160-20-F	21166	826.8	25.60	202.9	14.3	160.00	96
916	160-25-F	25791	1011.6	25.49	200.0	14.4	160.00	96
917	160-27-F	28242	1055.0	26.78	188.4	14.2	160.00	96
920	200-20-F	25557	1018.6	25.09	202.0	14.1	200.00	120
922	200-25-F	29468	1223.7	24.08	202.1	14.3	200.00	120

†Laboratory Designation denotes in order, revolutions per minute, cut-off in per-cent of stroke, and f

Locomotive No. 1499, Consolidation Type.

Speed		Weight of Steam per Revolution, lb.				Quality of Steam Mixture at Cut-off, x_c, parts of unity	Average Value of n, From Expansion Curves	Av. Absolute Pressure, lb.-sq. in.	
Speed, ft. per min.	Miles per hr.	Passing thru Cylinders	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present			At Cut-off	Back Pressure of Exhaust
12	13	14	15	16	17	18	19	20	21
8.1	6.70	4.315	3.251	0.548	4.863	0.669	1.136	175	15
8.5	6.72	5.150	3.967	0.486	5.637	0.704	1.111	180	15
2.6	15.40	3.007	2.770	0.631	3.638	0.762	1.065	158	16
0.5	13.55	3.062	2.780	0.649	3.711	0.749	1.068	155	15
1.8	13.24	3.928	3.520	0.540	4.468	0.788	1.097	160	16
6.3	13.40	4.784	4.405	0.459	5.243	0.840	1.125	161	17
0.2	19.95	3.200	3.128	0.622	3.822	0.818	1.091	145	18
2.6	20.04	3.427	3.286	0.604	4.031	0.815	1.087	132	18
7.7	26.63	2.085	2.275	0.744	2.829	0.805	1.049	124	18
5.3	26.20	2.405	2.520	0.690	3.095	0.814	1.044	124	19
9.2	26.68	2.646	2.793	0.673	3.319	0.842	1.128	108	19

Locomotive No. 734, Consolidation Type.

1.7	7.56	3.698	2.842	0.616	4.314	0.659	1.012	195	17
0.5	7.52	6.303	5.695	0.452	6.755	0.844	1.065	200	17
9.9	14.99	4.949	4.676	0.574	5.523	0.847	1.066	169	22
2.2	15.09	2.863	2.721	0.754	3.617	0.753	1.025	191	19
0.3	15.01	4.043	3.698	0.679	4.722	0.783	1.038	186	20
9.9	14.99	5.058	4.780	0.686	5.744	0.832	1.052	185	23
6.4	29.87	2.298	2.588	1.063	3.361	0.769	1.043	170	23
9.6	29.98	2.476	2.670	0.892	3.368	0.792	1.063	157	22
9.9	30.00	2.723	2.837	0.882	3.605	0.787	1.037	149	24
1.0	30.04	2.648	2.910	0.872	3.520	0.827	1.045	156	24
1.8	30.07	2.572	2.774	0.722	3.294	0.842	1.061	106	23
5.0	29.85	2.543	2.560	0.744	3.287	0.780	1.018	100	22
0.4	30.01	2.549	2.599	0.645	3.194	0.813	1.047	105	22
3.9	29.77	2.501	2.792	0.839	3.340	0.836	1.103	134	23
0.1	30.00	2.444	2.701	0.912	3.356	0.805	1.055	165	21

Locomotive No. 5266, Atlantic Type.

5.2	19.10	2.964	2.131	0.492	3.456	0.617	0.941	180	16
5.2	19.10	3.238	2.359	0.473	3.711	0.635	0.949	175	16
5.2	19.10	3.776	2.838	0.450	4.226	0.671	1.042	180	16
6.2	19.10	4.592	3.461	0.395	4.987	0.694	0.946	190	17
9.2	28.65	2.741	2.310	0.495	3.236	0.714	1.042	173	18
9.2	28.65	3.188	2.780	0.490	3.678	0.756	1.053	176	18
9.2	28.65	3.742	3.285	0.429	4.171	0.788	1.025	171	17
2.4	38.20	2.108	1.911	0.517	2.625	0.728	0.993	150	18
2.4	38.20	2.203	2.082	0.532	2.735	0.762	1.004	149	17
2.4	38.20	2.686	2.508	0.466	3.152	0.796	1.059	161	17
2.4	38.20	2.942	2 589	0.488	3.430	0.755	1.043	150	19
5.6	47.75	2.130	2.047	0.540	2.670	0.766	0.998	147	19
5.6	47.75	2.453	2.421	0.532	2.985	0.811	1.018	146	21

pa al throttle opening.

TABLE 9—TESTS OF PENN. R. R. CLASS

Number of Test	Laboratory Designation	Total Moist Steam passing thru Cylinders per hr.	Total I. H. P.	Moist Steam in lb. per I. H. P. hr.	Pressure, lb. per sq. in. In Boiler by Gage	Pressure, lb. per sq. in. Barometer	Speed Revolutions per min.	Speed Revolutions
1	2	3	4	5	6	8	10	1
1609	80–20–F	18910	747.5	25.29	203.1	14.1	80	48
1636	80–25–F	20008	793.3	25.23	204.8	14.1	80	48
1638	80–15–F	16676	643.0	25.93	205.2	14.1	80	48
1601	80–15–F	15745	605.0	26.02	196.8	14.2	80	48
1602	80–25–F	21384	848.0	25.20	202.4	14.1	80	48
1607	80–25–F	21413	852.6	25.13	202.8	14.2	80	48
1637	80–15–F	16009	612.3	26.05	203.8	14.1	80	48
1643	80–25–F	20049	833.9	24.02	203.3	14.1	80	48
1646	80–15–F	14918	573.2	26.03	205.4	14.2	80	48
1610	100–30–F	29201	1172.0	24.92	203.3	14.2	100	60
1603	120–25–F	29057	1202.9	24.17	202.7	14.3	120	72
1605	120–35–F	36363	1455.3	24.98	203.5	14.1	120	72
1644	120–30–F	28840	1188.9	24.26	205.2	14.1	120	72
1611	120–30–F	33951	1371.0	24.77	203.6	14.0	120	72
1635	120–30–F	29023	1227.9	23.62	203.2	14.0	120	72
1647	120–30–F	29660	1199.2	24.72	204.1	14.2	120	72
1604	160–25–F	34569	1475.8	23.41	204.8	14.1	160	96
1622	160–25–F	32491	1201.8	27.03	202.2	14.2	160	96
1634	160–35–F	40745	1657.1	24.58	202.5	14.0	160	96
1649	160–30–F	31858	1309.0	24.32	193.8	14.2	160	96
1606	160–35–F	42240	1699.4	24.86	202.4	14.2	160	96
1608	160–25–F	33072	1384.2	23.90	204.4	14.2	160	96
1612	160–25–F	33133	1400.9	23.66	204.1	14.0	160	96
1624	180–30–F	35903	1497.3	23.98	197.5	14.0	180	108
1631	180–35–F	42896	1723.2	24.91	203.1	14.0	180	108
1613	180–35–F	44130	1773.8	24.89	201.2	14.2	180	108
1633	180–35–F	43407	1773.5	24.51	203.0	14.0	180	108
1645	180–35–F	44276	1754.5	25.24	202.6	14.2	180	108

TABLE 10—TESTS OF PENN. R. R. CLASS E

Number of Test	Laboratory Designation	Total Superheated Steam passing thru Cylinders per hr.	Total I. H. P.	Superheated Steam in lb. per I. H. P. hr.	Pressure, lb. per sq. in. In Boiler by Gage	Pressure, lb. per sq. in. In Branch Pipe by Gage	Pressure, lb. per sq. in. Barometer	Degrees of Superheat in Branch Pipe	Speed Revolutions per min.
1	2	3	4	5	6	7	8	9	10
2434	100–20–F	19525	941.0	20.68	200.1	197.0	13.8	176.2	100
2402	100–23–F	23756	1100.5	21.58	192.4	189.0	14.3	177.6	100
2403	100–23–F	22257	1090.7	20.40	187.5	185.5	14.2	183.2	100
2435	120–25–F	26740	1386.5	19.28	200.1	195.4	13.8	200.3	120
2416	160–31–F	33747	1840.7	18.33	200.4	193.7	14.1	234.4	160
2436	160–31–F	31866	1762.5	18.07	199.8	192.2	14.1	226.0	160
2414	160–31–F	30941	1833.5	16.88	202.0	194.3	14.3	252.0	160
2413	180–31–F	32449	1978.3	16.40	201.2	194.3	14.4	231.4	180
2422	180–38–F	40405	2246.5	17.99	200.5	191.2	14.0	253.7	180
2417	180–38–F	39317	2141.0	18.35	190.9	181.7	14.1	252.8	180
2438	180–40–F	43387	2242.6	19.32	200.0	184.3	14.3	244.4	180
2437	180–42–F	44540	2269.1	19.64	197.4	181.7	14.1	250.3	180
2441	240–25–F	35142	1955.1	17.97	199.9	189.4	14.2	217.7	240
2430	240–31–F	38564	2207.7	17.47	199.9	190.1	14.0	233.1	240
2439	240–35–F	40382	2283.6	17.66	200.6	185.6	14.3	247.0	240
2433	240–35–F	42610	2251.3	18.11	200.5	186.8	14.1	240.2	240
2440	280–35–F	44338	2405.8	18.43	201.5	185.3	14.1	241.2	280
2442	280–35–F	45333	2359.4	19.22	199.8	182.0	14.1	236.6	280

Locomotive No. 7510, Pacific Type.

Speed		Weight of Steam per Revolution, lb.				Quality of Steam Mixture at Cut-off, x_c, parts of unity	Average Value of n, From Expansion Curves	Av. Absolute Pressure, lb.-sq. in.	
Speed, ft. per min.	Miles per hr.	Passing thru Cylinders	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present			At Cut-off	Back Pressure of Exhaust
2	13	14	15	16	17	18	19	20	21
6.6	19.03	3.940	3.688	0.555	4.495	0.820	1.114	169	17
6.6	19.03	4.168	3.920	0.562	4.730	0.829	1.069	180	18
6.6	19.03	3.474	3.218	0.622	4.096	0.785	1.067	171	19
6.6	19.03	3.280	3.030	0.566	3.846	0.788	1.061	168	18
6.6	19.03	4.456	4.169	0.505	4.961	0.840	1.099	169	17
6.6	19.03	4.462	4.130	0.543	5.005	0.825	1.078	168	18
6.6	19.03	3.336	3.132	0.637	3.973	0.788	1.094	170	17
6.6	19.03	4.178	3.900	0.560	4.738	0.823	1.057	166	18
6.6	18.95	3.108	2.865	0.568	3.676	0.780	1.066	148	19
3.3	23.78	4.867	4.710	0.522	5.389	0.874	1.100	172	20
0.0	28.54	4.037	3.978	0.560	4.597	0.865	1.074	160	20
0.0	28.54	5.050	4.990	0.564	5.614	0.889	1.118	170	22
0.0	28.43	4.006	3.990	0.533	4.539	0.880	1.107	155	22
0.0	28.54	4.718	4.691	0.523	5.241	0.895	1.104	165	22
0.0	28.54	4.032	4.020	0.539	4.571	0.879	1.124	156	20
0.0	28.43	4.119	3.995	0.533	4.652	0.859	1.082	148	21
3.3	38.05	3.601	3.667	0.568	4.169	0.879	1.100	144	22
3.3	38.05	3.384	3.204	0.696	4.080	0.786	1.092	139	23
3.3	38.05	4.245	4.240	0.592	4.837	0.877	1.146	150	26
3.3	37.91	3.318	3.437	0.599	3.917	0.877	1.080	139	23
3.3	38.05	4.402	4.486	0.642	5.044	0.889	1.118	149	26
3.3	38.05	3.447	3.579	0.654	4.101	0.872	1.124	148	23
3.3	38.05	3.452	3.520	0.644	4.096	0.860	1.134	152	21
9.9	42.81	3.325	3.412	0.692	4.017	0.850	1.127	136	24
9.9	42.81	3.971	3.993	0.687	4.658	0.858	1.114	147	28
9.9	42.81	4.087	4.243	0.707	4.794	0.885	1.087	143	28
9.9	42.81	4.020	4.074	0.622	4.642	0.878	1.119	140	26
9.9	42.65	4.099	4.080	0.643	4.742	0.860	1.074	140	28

Locomotive No. 3395, Pacific Type

Speed		Weight of Steam per Revolution, lb.				Quality of Steam Mixture at Cut-off, x_c, parts of unity	Average Value of n, From Expansion Curves	Av. Absolute Pressure, lb. sq. in.	
ft. per min.	Miles per hr.	Passing thru Cylinders	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present			At Cut-off	Back Pressure of Exhaust
12	13	14	15	16	17	18	19	20	21
67.0	23.66	3.254	4.055	0.742	3.996	1.015	1.225	150	16
67.0	23.76	3.959	4.442	0.691	4.650	0.956	1.233	149	16
67.0	23.76	3.710	4.378	0.668	4.378	1.000	1.232	141	15
60.4	28.39	3.715	4.908	0.805	4.520	1.086	1.270	146	22
47.2	38.02	3.515	5.020	0.869	4.384	1.145	1.279	140	22
47.2	37.86	3.319	5.260	0.892	4.211	1.249	1.305	136	25
47.2	38.02	3.224	5.120	0.922	4.146	1.235	1.347	134	23
40.6	42.77	3.003	4.890	0.979	3.982	1.228	1.265	140	22
40.6	42.77	3.741	5.770	1.002	4.734	1.215	1.309	136	28
40.6	42.77	3.641	5.500	0.959	4.600	1.195	1.286	128	27
40.6	42.59	4.016	6.100	1.078	5.094	1.197	1.293	142	31
40.6	42.59	4.124	6.180	1.023	5.147	1.200	1.341	134	32
20.8	56.78	2.440	4.201	1.202	3.642	1.154	1.222	135	28
20.8	57.03	2.678	4.518	1.150	3.828	1.180	1.320	125	28
20.8	56.78	2.802	4.853	1.240	4.042	1.200	1.239	132	33
20.8	57.03	2.960	4.910	1.196	4.156	1.182	1.271	123	33
07.6	66.25	2.640	4.751	1.451	4.091	1.161	1.316	114	36
07.6	66.25	2.699	4.730	1.364	4.063	1.164	1.254	120	36

TABLE 11—Tests of M. C. R.

Number of Test	Laboratory Designation	Total Moist Steam passing thru Cylinders per hr.	Total I. H. P.	Moist Steam per I. H. P. hr., lb.	Pressure, lb. per sq. in. In Boiler by Gage	Pressure, lb. per sq. in. Barometer	Speed Revolutions per min.	Speed Revolutions per hr.	Speed Piston Speed, ft. per min.	Speed Miles per hr.	High Pressure Cylinder Wt. of Steam per Revolution Passing thru Cylinders	High Pressure Cylinder Wt. of Steam per Revolution Shown by Indicator
1	2	3	4	5	6	8	10	11	12	13	14	15
301	40-43-F	9052	442.5	20.45	209.2	14.4	40.00	2400	213.5	7.49	3.771	3.83
302	40-45-F	9710	477.4	20.32	209.1	14.4	39.99	2399	213.5	7.49	4.048	4.06
303	40-48-F	10498	512.0	20.50	210.1	14.5	40.01	2401	213.6	7.49	4.371	4.32
305	80-45-F	16631	840.6	19.78	209.6	14.5	79.86	4792	426.3	14.96	3.471	3.88
306	80-42-F	14997	734.9	20.40	205.8	14.5	80.18	4811	428.0	15.01	3.118	3.55
308	80-53-F	18583	932.2	19.93	206.4	14.5	80.00	4800	427.1	14.98	3.872	4.25
309	80-57-F	21087	1040.7	20.25	210.9	14.5	79.98	4799	426.9	14.98	4.394	4.78
312	160-47-F	20883	890.1	23.45	209.0	14.6	160.02	9601	854.3	29.97	2.173	3.29
316	160-62-F	24673	1001.3	24.61	175.1	14.5	159.97	9598	854.1	29.96	2.572	3.58
317	160-50-P	19241	910.4	21.12	208.7	14.6	159.95	9597	853.9	29.96	2.006	3.15
318	160-61-P	22349	937.7	23.84	210.7	14.6	160.03	9602	854.3	29.97	2.327	3.16
319	160-65-P	22805	934.2	24.42	208.3	14.6	160.01	9601	854.3	29.97	2.377	3.05

TABLE 12—Tests of A. T. & S. F.

401	40-27-F	10485	391.6	26.80	215.4	14.5	40.01	2400.6	213.58	6.719	4.368	4.06
402	40-35-F	12824	510.8	25.10	213.5	14.6	40.00	2400.0	213.55	6.718	5.342	4.71
403	40-40-F	14982	633.6	23.68	213.5	14.6	39.97	2398.2	213.41	6.714	6.260	5.45
410	60-30-F	13348	511.3	26.10	214.1	14.6	60.00	3600.0	320.34	10.077	3.708	3.92
411	60-35-F	16567	705.2	23.50	216.3	14.5	60.00	3600.0	320.34	10.077	4.600	4.60
412	60-40-F	19635	888.8	22.09	216.7	14.5	60.64	3638.4	323.75	10.184	5.400	5.31
405	80-30-F	15110	631.4	23.92	214.2	14.4	80.00	4800.0	427.13	13.435	3.150	3.85
407	80-40-F	23107	1088.8	21.22	213.0	14.6	80.00	4800.0	427.13	13.435	4.815	5.20
408	80-55-F	31485	1257.9	25.01	213.5	14.5	81.28	4876.8	434.00	13.652	6.456	6.06

TABLE 13—Tests of Penn R. R.

502	80-$\frac{35}{60}$-F	9323	495.7	18.81	215.2	14.5	80.05	4803.0	336.3	19.14	1.940	.....
505	160-$\frac{25}{45}$-F	10580	524.5	20.18	219.7	14.4	159.96	9597.6	672.1	38.25	1.103	1.305
506	160-$\frac{25}{45}$-F	11219	524.5	21.38	219.6	14.4	160.00	9600.0	672.7	38.26	1.168	1.280
507	160-$\frac{40}{60}$-F	16070	809.3	19.85	214.6	14.7	160.00	9600.0	672.7	38.26	1.674	1.720
508	160-$\frac{50}{70}$-F	19724	944.6	20.89	206.4	14.5	160.00	9600.0	672.7	38.26	2.055	2.075
510	240-$\frac{25}{45}$-F	13246	596.8	22.20	212.5	14.6	240.00	14400.0	1008.3	57.39	0.920	1.266
511	240-$\frac{30}{55}$-F	15014	653.2	23.00	218.9	14.4	240.00	14400.0	1008.3	57.39	1.042	1.305
512	240-$\frac{30}{60}$-F	17574	802.3	21.90	217.5	14.4	239.99	14399.4	1008.2	57.39	1.221	1.470
513	280-$\frac{30}{55}$-F	18710	682.5	27.44	215.0	14.4	279.99	16799.4	1176.1	66.96	1.114	1.311

lass W Locomotive No. 585.

High Pressure Cylinder						Low Pressure Cylinder						
t. of Steam r Revolution		Quality of Steam Mixture at Cut-off, x_c, parts of unity	Average Value of n, From Expansion Curves	Av. Abs. Pressure, lb.-sq. in.		Weight of Steam per Revolution,			Quality of Steam Mixture at Cut-off, x_c parts of unity	Average Value of n, From Expansion Curves	Av. Abs. Pressure, lb.-sq. in.	
ompression from Indicator	Total Mixture Actually Present			At Cut-off	Back Pressure of Exhaust near Compression	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture actually present			At Cut-off	Back Pressure of Exhaust near Compression
16	17	18	19	20	21	22	23	24	25	26	27	28
.268	5.039	0.762	1.091	200	93	3.071	0.450	4.221	0.728	1.063	64	16
.188	5.236	0.776	1.081	200	90	3.126	0.438	4.486	0.697	1.037	65	15
.181	5.552	0.779	1.069	200	90	3.410	0.376	4.747	0.718	1.099	71	15
.163	4.634	0.838	0.998	190	91	2.845	0.514	3.985	0.714	0.955	62	18
.208	4.326	0.821	1.015	190	87	2.542	0.646	3.764	0.676	0.975	60	18
.072	4.944	0.860	1.022	185	87	3.252	0.440	4.312	0.754	0.947	60	18
.988	5.382	0.888	1.045	198	92	3.695	0.481	4.875	0.758	1.015	64	19
.544	3.717	0.866	1.032	150	104	2.382	0.842	3.015	0.791	1.002	48	24
.138	3.710	0.965	1.100	140	96	2.900	0.749	3.321	0.874	1.058	50	27
.386	3.386	0.933	1.090	150	97	2.344	0.771	2.777	0.845	1.011	45	22
.165	3.492	0.906	1.074	140	100	2.504	0.796	3.123	0.802	0.920	45	25
.919	3.296	0.928	1.033	115	87	2.582	0.700	3.077	0.827	1.050	45	28

lass 900 Locomotive No. 929.

.489	6.857	0.592	1.002	214	112	3.550	0.943	5.311	0.669	1.168	64	18
.220	7.562	0.623	1.081	209	111	3.970	0.852	6.194	0.641	1.208	58	18
.041	8.301	0.656	1.092	212	114	4.588	0.745	7.005	0.655	1.079	64	18
.526	6.234	0.630	0.993	210	111	3.059	1.086	4.794	0.638	1.036	58	18
.350	6.950	0.662	0.997	205	114	3.848	0.875	5.475	0.703	1.088	60	17
.112	7.512	0.708	1.051	209	115	4.454	0.934	6.334	0.704	1.109	62	17
.760	5.910	0.652	0.948	195	112	3.138	1.111	4.261	0.737	1.041	60	18
.322	7.137	0.729	1.051	205	112	4.410	0.939	5.754	0.767	1.085	62	19
.139	8.595	0.705	1.078	205	101	4.864	0.948	7.404	0.657	1.054	58	22

e Glehn Locomotive No. 2512.

....	.....	.....	.....	...	...	1.695	0.266	2.206	0.769	1.047	42	15
.534	1.637	0.798	1.004	184	42	.....	.....	.....	.....	.....	..	..
.528	1.696	0.755	0.991	173	43	1.080	0.385	1.553	0.696	0.914	30	17
.478	2.152	0.800	1.033	188	49	1.502	0.310	1.984	0.758	0.955	37	18
.426	2.481	0.836	1.060	181	53	1.900	0.311	2.366	0.804	0.998	42	21
.635	1.555	0.815	1.014	160	53	1.118	0.480	1.400	0.799	0.924	32	19
.564	1.606	0.813	1.035	169	46	.....	.....	.....	.....	.....	..	..
.539	1.760	0.836	1.072	173	48	.....	.....	.....	.....	.....	..	..
.587	1.701	0.772	1.047	168	49	.....	.....	.....	.....	.....	..	..

TABLE 14—Tests of A. T. & S.

Number of Test	Laboratory Designation	Total Moist Steam passing thru Cylinders per hr.	Total I. H. P.	Moist Steam per I. H. P. hr., lb.	Pressure in lb. per sq. in. — In Boiler by Gage	Pressure in lb. per sq. in. — Barometer	Speed — Revolutions per min.	Speed — Revolutions per hr.	Speed — Piston Speed, ft. per min.	Speed — Miles per hr.	High Press Cylinder — Wt. of Stea per Revolut — Passing thru Cylinders	High Press Cylinder — Wt. of Stea per Revolut — Shown by
1	2	3	4	5	6	8	10	11	12	13	14	1
601	80-30-F	8537	356.2	23.94	217.6	14.6	80.00	4800.0	346.8	18.79	1.778	2.C
602	80-35-F	10520	479.0	21.96	215.7	14.6	80.00	4800.0	346.8	18.79	2.191	...
603	80-45-F	12668	470.4	22.20	221.0	14.4	80.00	4800.0	346.8	18.79	2.640	2.6
604	80-55-F	16850	808.4	20.85	220.4	14.5	80.01	4800.6	346.8	18.79	3.511	3.4
605	160-35-F	17290	877.1	19.71	220.6	14.6	160.00	9600.0	693.6	37.59	1.803	2.3
606	160-45-F	20435	999.9	20.43	219.2	14.5	160.00	9600.0	693.6	37.59	2.131	2.5
607	160-55-F	25492	1296.1	19.65	221.9	14.6	160.00	9600.0	693.6	37.59	2.658	...
609	240-50-F	28645	1414.6	20.25	221.2	14.5	239.89	14393.4	1039.7	56.35	1.989	2.6
611	240-53-F	33691	1621.5	20.76	221.0	14.5	240.02	14401.2	1040.4	56.38	2.338	2.9
613	280-50-F	30680	1459.7	21.01	219.5	14.6	280.00	16800.0	1213.6	65.77	1.825	2.5

TABLE 15—Tests of N. Y. C. & H.

801	80-35-F	11938	567.4	21.05	209.4	14.5	79.82	4789.2	345.5	18.72	2.495	2.5
802	80-45-F	14797	714.4	20.70	210.8	14.6	80.00	4800.0	346.3	18.76	3.082	3.1
805	160-35-F	19182	967.0	19.85	222.2	14.6	160.00	9600.0	622.6	37.52	1.998	2.4
806	160-45-F	25314	1253.0	20.21	220.4	14.5	160.00	9600.0	622.6	37.52	2.638	3.0
807	160-55-F	33321	1490.5	22.35	221.4	14.4	160.00	9600.0	622.6	37.52	3.473	...
809	240-35-F	24361	1142.8	21.31	218.7	14.5	240.00	14400.0	1039.1	56.29	1.691	2.3
811	240-50-F	36614	1629.8	22.48	219.6	14.5	239.99	14399.9	1039.0	56.28	2.542	3.0
812	240-55-F	40708	1641.4	24.80	212.3	14.5	240.00	14400.0	1039.1	56.29	2.827	...
813	280-35-F	26886	1102.3	22.52	220.0	14.5	280.11	16806.6	1212.5	65.69	1.598	2.2
814	280-40-F	31132	1368.9	22.75	220.2	14.5	279.97	16798.2	1212.0	65.66	1.854	2.6
815	320-40-F	32475	1335.7	24.39	222.3	14.5	320.00	19200.0	1385.3	75.05	1.690	2.3

TABLE 16—Tests of Hanover Compoun

Number of Test	Laboratory Designation	Total Superheated Steam passing thru Cylinders per hr. lb.	Total I. H. P.	Superheated Steam I. H. P. hr. in lb.	Pressure—lb. per sq. in. — In Boiler by Gage	Pressure—lb. per sq. in. — In Branch-pipe by Gage	Pressure—lb. per sq. in. — Barometer	Degrees of Superheat in Branch pipe	Speed — Revolutions per min.	Speed — Revolutions per hr.	Speed — Piston Speed ft. per Minute	Speed — Miles per Hour
1	2	3	4	5	6	7	8	9	10	11	12	13
701	80-35-F	6795	375.6	18.09	198.3	194.6	14.4	84.45	80.00	4800	314.9	18.5
702	80-45-F	8564	480.5	17.84	202.0	198.3	14.4	86.97	80.00	4800	314.9	18.5
705	160-35-F	10470	622.9	16.81	198.8	194.4	14.5	82.18	160.00	9600	629.7	37.1
706	160-40-F	12097	728.9	16.59	202.6	197.4	14.5	103.98	160.06	9603.6	630.0	37.1
707	160-45-F	14533	813.7	17.86	198.8	193.6	14.4	116.20	160.00	9600	629.7	37.1
708	160-45-F	14550	801.3	18.16	197.0	190.9	14.5	97.19	160.00	9600	629.7	37.1
709	240-30-F	10521	631.2	16.67	204.1	198.5	14.5	88.36	240.00	14400	944.7	55.7
711	240-40-F	15343	816.4	18.78	187.0	179.9	14.7	101.46	240.00	14400	944.7	55.7
712	280-30-F	14655	688.4	21.29	204.2	196.6	14.5	71.28	280.27	16816.2	1103.1	65.0

(LASS 507. LOCOMOTIVE NO. 535.

High Pressure Cylinder						Low Pressure Cylinder						
[W]t. of Steam [pe]r Revolution				Av. Abs. Pressure, lb.-sq. in.		Weight of Steam per revolution					Av. Abs. Pressure, lb.-sq. in.	
[Retained in Compres]sion from Indicator	Total Mixture Actually Present	Quality of Steam Mixture at Cut-off, parts of unity	Average Value of n, From Expansion Curves	At Cut-off	Back Pressure of Exhaust near Compression	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present	Quality of Steam Mixture at Cut-off, parts of unity	Average Value of n, From Expansion curves	At Cut-off	Back Pressure of Exhaust near Compression
16	17	18	19	20	21	22	23	24	25	26	27	28
.898	2.676	0.764	0.944	218	106	1.389	0.487	2.265	0.614	1.042	59	15
...	.....	.....	.....	...	...	1.767	0.436	2.627	0.673	1.066	66	15
.987	3.627	0.721	0.999	218	109	1.976	0.391	3.031	0.652	1.062	62	15
.943	4.454	0.771	1.076	219	118	2.725	0.331	2.842	0.709	1.140	64	16
.267	3.070	0.780	1.001	210	100	1.704	0.481	2.284	0.746	1.065	59	16
.155	3.286	0.776	1.008	194	114	1.945	0.480	2.611	0.745	1.041	65	18
....	.....	.....	.....	...	...	2.385	0.461	3.119	0.765	1.073	63	18
.116	3.105	0.868	1.076	195	115	2.072	0.594	2.583	0.803	1.055	59	20
.068	3.406	0.863	1.039	195	104	2.352	0.584	2.922	0.805	1.024	60	26
.099	2.924	0.880	1.098	179	104	1.920	0.642	2.467	0.779	1.025	55	25

. R. CLASS I1 LOCOMOTIVE NO. 3000.

.011	3.506	0.732	0.986	208	95	1.870	0.451	2.946	0.635	0.976	62	16
.771	3.793	0.830	1.060	212	89	2.318	0.427	3.509	0.661	1.030	63	16
.066	3.064	0.802	0.942	201	105	1.762	0.580	2.578	0.684	0.937	58	19
.924	3.562	0.845	1.097	204	96	2.250	0.546	3.184	0.707	0.982	58	22
....	.....	.....	.....	...	...	2.910	0.576	4.049	0.719	0.934	63	27
.188	2.879	0.804	0.968	199	114	1.758	0.741	2.432	0.723	0.930	59	25
.936	3.478	0.864	1.035	191	102	2.420	0.723	3.267	0.741	0.962	60	31
....	.....	.....	.....	...	...	2.626	0.713	3.540	0.742	0.950	62	34
.145	2.743	0.834	0.958	202	113	1.718	0.791	2.389	0.719	0.950	59	26
.195	3.049	0.858	1.008	199	119	2.064	0.836	2.690	0.767	0.964	59	31
.209	2.899	0.884	1.059	188	121	1.972	1.018	2.708	0.729	1.000	55	30

LASS S8 LOCOMOTIVE NO. 628.

	High Pressure Cylinder							Low Pressure Cylinder						
	Weight of Steam per Revolution					Av. Abs. Pressure, lb.-sq. in.		Weight of Steam per Revolution					Av. Abs. Pressure, lb.-sq. in.	
[Passing] thru Cylinders	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present	Quality of Steam Mixture at Cut-off Parts of Unity	Average Value of n from Expansion Curves	At Cut-off	Back Pressure of Exhaust near compression	Shown by Indicator at Cut-off	Retained in Compression from Indicator	Total Mixture Actually Present	Quality of Steam Mixture at Cut-off Parts of Unity	Average Value of n from Expansion Curves	At Cut-off	Back Pressure of Exhaust near Compression
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1.416	1.465	0.417	1.833	0.801	0.940	188	54	1.287	0.237	1.653	0.779	0.990	42	16
1.784	1.843	0.374	2.158	0.855	0.997	188	55	1.605	0.177	1.961	0.819	1.038	47	15
1.091	1.333	0.430	1.521	0.876	0.944	156	59	1.184	0.299	1.390	0.852	1.029	40	17
1.259	1.514	0.397	1.656	0.914	1.041	160	58	1.331	0.278	1.537	0.867	1.005	40	18
1.514	1.700	0.353	1.867	0.911	1.049	160	55	1.478	0.261	1.775	0.833	1.015	42	19
1.516	1.683	0.386	1.902	0.885	1.000	160	59	1.578	0.279	1.795	0.879	1.058	42	21
0.730	1.091	0.499	1.229	0.888	0.960	145	56	0.990	0.348	1.078	0.918	1.000	36	18
1.065	1.366	0.385	1.450	0.942	1.028	140	55	1.302	0.328	1.393	0.935	1.004	37	22
0.872	1.188	0.498	1.370	0.816	0.946	146	63	1.055	0.406	1.278	0.826	0.997	35	21

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THE PROPERTIES OF SATURATED AND SUPERHEATED AMMONIA VAPOR

BY
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AND
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UNIVERSITY OF ILLINOIS
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 66 JANUARY, 1913

THE PROPERTIES OF SATURATED AND SUPERHEATED AMMONIA VAPOR

BY G. A. GOODENOUGH, PROFESSOR OF THERMODYNAMICS, AND WM. EARL MOSHER, RESEARCH FELLOW IN MECHANICAL ENGINEERING.*

CONTENTS

*In the preparation of this bulletin, Professor Goodenough has exercised general supervision over the work and has furnished, for the most part, the analytical methods used in attacking the various problems. Mr. Mosher has applied these methods and correlated the experimental results. To him is due the selection of constants and the various discussions of the validity and accuracy of the formulas employed.—THE EDITOR.

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THE PROPERTIES OF SATURATED AND SUPERHEATED AMMONIA VAPOR

I. Introduction

1. *Historical Review.*—The vapor of anhydrous ammonia first became of interest in the field of Mechanical Engineering with the advent of Carré's absorption and Linde's compression refrigerating machines. With the development and increased application of the refrigeration industry this vapor has become more important and an accurate knowledge of its properties is highly desirable.

As in the case of steam and many other technically important vapors, the first reliable experimental knowledge of the properties of ammonia was derived from the work of Regnault.[1*] He determined experimentally the relation between the pressure and the temperature of the saturated vapor and expressed it by means of empirical formulas. He also determined the relative volumes of the superheated vapor at different pressures along an isotherm for the temperature 8.1° C.,[2] the specific heat,[3] the theoretical density[3] and the experimental density[4] of the gas. The determinations made by Regnault of the specific heat of the liquid and the latent heat of vaporization were lost in the reign of the Commune in 1870; twelve of the determinations of the latter magnitude, however, have been found.[5]

The first attempt to establish the fundamental equations for the vapor of ammonia and to compute tables of its properties was made by Ledoux.[6] These equations and resulting tables, resting as they do upon unsound hypotheses and meager experimental data, are, in the light of later investigations, quite inaccurate. The next theoretical investigation of the subject was undertaken by Zeuner,[7] who computed new constants for ammonia, in the equation of state of the form already applied by him to steam, and used by Ledoux for ammonia. He also constructed empirical formulas for the inner and outer latent heat, and the specific heat of the liquid, and by use of these formulas computed tables which appeared in the earlier editions of his thermodynamics.[8] In the later editions[9] the values thus calculated have still been used for temperatures below 32° F., but results of later experimental determinations have been used above that temperature. These tables for ammonia vapor are given in both English and French units.

*Numerals refer to the bibliography given in Section V.

The tables now used most extensively in refrigeration work and included in most handbooks on refrigeration are those of Wood.[10] These tables are computed from formulas derived by the author and based upon Regnault's experiments. About the same date (1889) that Wood's tables appeared, another set of tables was published by Peabody.[11] These tables were based upon the work of Ledoux and upon the same form of equation of state, namely, that derived by Zeuner and applied to ammonia by Ledoux, but with the constants recomputed.

Since the above mentioned tables were published the properties of ammonia have been made the subject of considerable experimental work, although comparatively little is known even now when its importance in technical work is considered. Beside the experimental work enumerated above we have the following determinations: Pressure-temperature relation by Blümcke,[12] Pictet,[13] Faraday,[14] Davies,[15] and Brill;[16] critical temperature and pressure by Dewar,[17] Vincent and Chappuis,[18] Jaquerod,[70] and Scheffer;[71] latent heat of vaporization by von Strombeck,[19] Franklin and Kraus,[20] Estreicher and Schnerr,[21] and Denton and Jacobus;[22] specific heat of the liquid by von Strombeck,[19] Ludeking and Starr,[23] Elleau and Ennis,[24] and Dieterici;[25] volume (or density) of the liquid by D'Andréeff,[26] Lange,[27] and Dieterici;[25] volume of saturated vapor by Dieterici;[25] volume of superheated vapor by Leduc,[28] Guye,[29] and Perman and Davies;[30] specific heat of the superheated vapor by Keutel, Voller, Wiedemann and Nernst, the existing data on the latter property being summed up by Nernst.[31] In addition to the above direct experimental determinations of the various properties of ammonia there is available a set of throttling experiments performed by Wobsa.[59]

It will be seen that a great deal of the recent experimental work has been done by Dieterici. He has computed a table of the properties of ammonia,[25] but this table is available in metric units only, and for reasons which will appear later it may be improved upon. In 1907 Wobsa published tables for ammonia vapor based upon his own experimental work and that of Dieterici. These tables appear in the 1908 edition of Hütte and have been converted into English units and extended for every degree Fahrenheit by Macintire.[60] In 1908 Wobsa published a revised table[61] based upon a new form of equation of state. Among other tables of the properties of ammonia which have appeared in metric units are those of Mollier[62] and of Hýbl.[63]

Very recently Dr. Chas. E. Lucke has published a new table of the properties of ammonia in his treatise on thermodynamics. Lack of

time has precluded a critical examination of the experimental basis of the table.

2. *Purpose of this Investigation.*—The present investigation has been undertaken with the object of collecting and correlating the various scattered experimental data on the subject of the properties of ammonia. An attempt has been made to reconcile these data by means of well-known thermodynamic laws and principles so that the results may be consistent with each other; and to express the various properties by means of formulas from which tables and charts in English units may be prepared for use in solving the many problems connected with refrigeration work.

It is fully realized that the experimental evidence is not as complete as could be desired and that more accurate experimental work may render necessary a slight revision of the value of the constants in the equations here presented. Hence the tabular values resulting from the equations are not to be considered as final, but as best representing our present knowledge of the properties of ammonia.

3. *Acknowledgment.*—The assistance of Mr. G. W. Philleo in establishing the constants in the characteristic equation of the superheated vapor is hereby gratefully acknowledged. To Mr. Ray B. Ponder credit is due for work on the specific heat of liquid ammonia.

4. *Notation*

J = Joule's equivalent.
A = Reciprocal of same.
t = Temperature on the F. or the C. scale.
T = Absolute temperature.
p = Pressure.
v = Specific volume.
γ = Specific weight.
u = Intrinsic energy.
i = Heat content at constant pressure.
s = Entropy.
r = Latent heat of vaporization.
ρ = Internal latent heat.
Ψ = External latent heat $= Ap(v''-v')$.
c = Specific heat.
c_p = Specific heat at constant pressure.
x = Quality of vapor mixture.
Subscript $_k$ indicates critical data.
$(')$ indicates properties of liquid.
$('')$ indicates properties of saturated vapor.

II. Tables

5. *Description of Tables.*—Of the four tables given, the first and second are for the liquid and the saturated vapor of ammonia and give explicitly all of the properties that are ordinarily needed for use. Table 3 is primarily for superheated ammonia vapor, but includes the saturated vapor as the special case of zero superheat. Table 4 gives the thermal properties of liquid ammonia.

In Tables 1 and 4 the argument is the temperature. Since it is often convenient to be able to work roughly at very high and very low temperatures, these tables have been extended on the one hand to the critical point itself and on the other hand to −110° F. However, since the data upon which the tables are based are not well known at these extreme temperatures, the values above about 160° and below about −40° should not be used with too much confidence.

In Tables 2 and 3 the argument is the pressure. The values above 500 lb. per sq. in. and those below 10 lb. per sq. in. are less certain than those lying between these pressures.

TABLE 1.

Saturated Ammonia Vapor: Temperature Table

Temp. Fahr.	Pressure, lb. per sq. in.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	Heat Content of Liquid	Latent Heat of Evap.	Heat Content of Vapor	Internal Energy B. t. u.		Entropy			Temp. Fahr.
							Evap.	Vapor	Liquid	Evap.	Vapor	
t	p	v''	$1/v''$	i'	r	i''	ρ	u''	s'	r/T	s''	
−110°	0.758	291.	0.00344	....	648.5	....	607.7	....	....	1.8548	....	−110°
−105°	0.947	236.	0.00424	....	645.4	....	604.0	....	....	1.8198	....	−105°
−100°	1.176	192.	0.00520	....	642.2	....	600.3	....	....	1.7857	....	−100°
− 95°	1.450	158.	0.00633	....	639.0	....	596.6	....	....	1.7524	....	− 95°
− 90°	1.778	130.	0.00767	....	635.8	....	592.9	....	....	1.7200	....	− 90°
− 85°	2.167	108.	0.00924	....	632.5	....	589.1	....	....	1.6884	....	− 85°
− 80°	2.626	90.3	0.01107	....	629.3	....	585.3	....	....	1.6575	....	− 80°
− 75°	3.164	75.8	0.01320	....	625.9	....	581.5	....	....	1.6273	....	− 75°
− 70°	3.791	64.0	0.01564	....	622.6	....	577.7	....	....	1.5979	....	− 70°
− 65°	4.518	54.2	0.01845	....	619.2	....	573.9	....	....	1.5691	....	− 65°
− 60°	5.358	46.2	0.02165	....	615.8	....	570.0	....	....	1.5410	....	− 60°
− 55°	6.324	39.6	0.02530	....	612.4	....	566.1	....	....	1.5134	....	− 55°
− 50°	7.43	34.01	0.02940	−85.7	608.9	523.3	562.2	476.5	−0.1901	1.4865	1.2964	− 50°
− 49°	7.67	32.98	0.03032	−84.7	608.2	523.6	561.4	476.7	−0.1875	1.4811	1.2936	− 49°
− 48°	7.92	32.00	0.03125	−83.6	607.5	523.9	560.6	476.9	−0.1850	1.4758	1.2908	− 48°
− 47°	8.17	31.07	0.03218	−82.6	606.8	524.3	559.8	477.2	−0.1825	1.4705	1.2880	− 47°
− 46°	8.43	30.19	0.03312	−81.5	606.1	524.6	559.0	477.4	−0.1801	1.4653	1.2853	− 46°
− 45°	8.69	29.34	0.03408	−80.5	605.4	524.9	558.2	477.6	−0.1776	1.4601	1.2825	− 45°
− 44°	8.96	28.52	0.03506	−79.5	604.7	525.3	557.4	477.9	−0.1751	1.4549	1.2797	− 44°
− 43°	9.24	27.72	0.03607	−78.4	604.0	525.6	556.6	478.2	−0.1727	1.4497	1.2770	− 43°
− 42°	9.53	26.94	0.03712	−77.4	603.3	525.9	555.8	478.4	−0.1702	1.4445	1.2743	− 42°
− 41°	9.82	26.18	0.03820	−76.4	602.6	526.2	555.0	478.6	−0.1678	1.4394	1.2716	− 41°
− 40°	10.12	25.45	0.03930	−75.3	601.9	526.6	554.2	478.9	−0.1653	1.4343	1.2690	− 40°
− 39°	10.43	24.74	0.04042	−74.3	601.2	526.9	553.4	479.1	−0.1628	1.4292	1.2663	− 39°
− 38°	10.75	24.06	0.04156	−73.3	600.5	527.2	552.6	479.3	−0.1604	1.4241	1.2637	− 38°
− 37°	11.07	23.40	0.04273	−72.2	599.7	527.5	551.8	479.6	−0.1580	1.4190	1.2611	− 37°
− 36°	11.40	22.76	0.04393	−71.2	599.0	527.9	551.0	479.8	−0.1555	1.4140	1.2585	− 36°
− 35°	11.74	22.14	0.04516	−70.2	598.3	528.2	550.2	480.0	−0.1531	1.4090	1.2559	− 35°
− 34°	12.09	21.55	0.04641	−69.1	597.6	528.5	549.4	480.3	−0.1507	1.4040	1.2533	− 34°
− 33°	12.45	20.97	0.04769	−68.1	596.9	528.8	548.6	480.5	−0.1483	1.3990	1.2507	− 33°
− 32°	12.81	20.41	0.04900	−67.1	596.1	529.1	547.8	480.7	−0.1458	1.3940	1.2482	− 32°
− 31°	13.18	19.87	0.05033	−66.0	595.4	529.4	547.0	481.0	−0.1434	1.3891	1.2457	− 31°
− 30°	13.56	19.35	0.05168	−65.0	594.7	529.8	546.2	481.2	−0.1410	1.3842	1.2432	− 30°
− 29°	13.95	18.84	0.05306	−63.9	594.0	530.1	545.4	481.4	−0.1386	1.3793	1.2407	− 29°
− 28°	14.35	18.35	0.05449	−62.9	593.2	530.4	544.6	481.6	−0.1362	1.3744	1.2382	− 28°
− 27°	14.76	17.87	0.05596	−61.8	592.5	530.7	543.7	481.9	−0.1338	1.3696	1.2358	− 27°
− 26°	15.18	17.40	0.05747	−60.8	591.8	531.0	542.9	482.1	−0.1314	1.3647	1.2333	− 26°
− 25°	15.61	16.95	0.0590	−59.8	591.1	531.3	542.1	482.3	−0.1290	1.3598	1.2309	− 25°
− 24°	16.05	16.51	0.0606	−58.7	590.3	531.6	541.3	482.5	−0.1266	1.3550	1.2285	− 24°
− 23°	16.50	16.09	0.0622	−57.7	589.6	531.9	540.5	482.8	−0.1242	1.3502	1.2260	− 23°
− 22°	16.96	15.68	0.0638	−56.6	588.8	532.2	539.7	483.0	−0.1218	1.3454	1.2236	− 22°
− 21°	17.43	15.28	0.0654	−55.6	588.1	532.5	538.9	483.2	−0.1195	1.3407	1.2213	− 21°
− 20°	17.91	14.89	0.0671	−54.6	587.4	532.8	538.0	483.4	−0.1171	1.3360	1.2189	− 20°
− 19°	18.40	14.52	0.0689	−53.5	586.6	533.1	537.2	483.6	−0.1147	1.3313	1.2165	− 19°
− 18°	18.90	14.16	0.0706	−52.5	585.9	533.4	536.4	483.8	−0.1124	1.3266	1.2142	− 18°
− 17°	19.41	13.81	0.0724	−51.4	585.1	533.7	535.6	484.0	−0.1100	1.3219	1.2118	− 17°
− 16°	19.93	13.48	0.0742	−50.4	584.4	534.0	534.8	484.3	−0.1077	1.3172	1.2095	− 16°

TABLE 1—TEMPERATURE

Temp. Fahr.	Pressure, lb. per sq. in.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	Heat Content of Liquid	Latent Heat of Evap.	Heat Content of Vapor	Internal Energy B. t. u.		Entropy			Temp. Fahr.
							Evap.	Vapor	Liquid	Evap.	Vapor	
t	p	v''	$1/v''$	i'	r	i''	ρ	u''	s'	r/T	s''	
−15°	20.46	13.15	0.0760	−49.4	583.6	534.3	533.9	484.5	−0.1054	1.3126	1.2072	−15°
−14°	21.00	12.83	0.0779	−48.3	582.9	534.6	533.1	484.7	−0.1031	1.3080	1.2049	−14°
−13°	21.56	12.51	0.0799	−47.3	582.1	534.8	532.3	484.9	−0.1007	1.3033	1.2026	−13°
−12°	22.13	12.21	0.0819	−46.2	581.4	535.1	531.4	485.1	−0.0984	1.2987	1.2003	−12°
−11°	22.71	11.92	0.0839	−45.2	580.6	535.4	530.6	485.3	−0.0961	1.2941	1.1980	−11°
−10°	23.30	11.63	0.0860	−44.2	579.9	535.7	529.8	485.5	−0.0938	1.2896	1.1958	−10°
− 9°	23.90	11.35	0.0881	−43.1	579.1	536.0	528.9	485.7	−0.0915	1.2851	1.1935	− 9°
− 8°	24.52	11.08	0.0903	−42.1	578.4	536.3	528.1	485.9	−0.0893	1.2806	1.1913	− 8°
− 7°	25.15	10.82	0.0924	−41.0	577.6	536.6	527.3	486.1	−0.0870	1.2761	1.1891	− 7°
− 6°	25.80	10.57	0.0946	−40.0	576.8	536.9	526.4	486.3	−0.0847	1.2716	1.1869	− 6°
− 5°	26.46	10.32	0.0969	−38.9	576.1	537.1	525.6	486.6	−0.0824	1.2671	1.1847	− 5°
− 4°	27.13	10.08	0.0992	−37.9	575.3	537.4	524.7	486.8	−0.0801	1.2626	1.1825	− 4°
− 3°	27.82	9.85	0.1015	−36.8	574.6	537.7	523.9	487.0	−0.0778	1.2582	1.1804	− 3°
− 2°	28.52	9.62	0.1039	−35.8	573.8	538.0	523.1	487.2	−0.0755	1.2537	1.1782	− 2°
− 1°	29.23	9.40	0.1064	−34.7	573.0	538.2	522.2	487.4	−0.0732	1.2493	1.1761	− 1°
0°	29.95	9.19	0.1089	−33.7	572.2	538.5	521 4	487.6	−0.0709	1.2449	1.1740	0°
1°	30.69	8.98	0.1114	−32.6	571.4	538.8	520.5	487.8	−0.0686	1.2405	1.1719	1°
2°	31.44	8.78	0.1139	−31.6	570.7	539.1	519.7	488.0	−0.0663	1.2362	1.1699	2°
3°	32.21	8.58	0.1165	−30.5	569.9	539.3	518.8	488.2	−0.0640	1.2318	1.1678	3°
4°	32.99	8.39	0.1192	−29.5	569.1	539.6	518.0	488.4	−0.0618	1.2275	1.1657	4°
5°	33.79	8.20	0.1219	−28.4	568.3	539.9	517.1	488.6	−0.0595	1.2231	1.1636	5°
6°	34.60	8.02	0.1247	−27.4	567.5	540.1	516.3	488.7	−0.0572	1.2188	1.1615	6°
7°	35.43	7.84	0.1275	−26.3	566.7	540.4	515.4	488.9	−0.0550	1.2145	1.1595	7°
8°	36.28	7.67	0.1304	−25.3	565.9	540.7	514.6	489.1	−0.0527	1.2102	1.1575	8°
9°	37.14	7.50	0.1333	−24.2	565.2	540.9	513.7	489.3	−0.0505	1.2059	1.1554	9°
10°	38.02	7.34	0.1363	−23.2	564.4	541.2	512.9	489.5	−0.0483	1.2017	1.1534	10°
11°	38.93	7.18	0.1393	−22.1	563.6	541.4	512.0	489.7	−0.0461	1.1975	1.1514	11°
12°	39.84	7.02	0.1424	−21.1	562.8	541.7	511.2	489.9	−0.0438	1.1932	1.1494	12°
13°	40.77	6.87	0.1455	−20.0	562.0	542.0	510.3	490.1	−0.0416	1.1890	1.1474	13°
14°	41.71	6.72	0.1487	−19.0	561.2	542.2	509.4	490.3	−0.0394	1.1848	1.1454	14°
15°	42.67	6.583	0.1519	−17.9	560.4	542.5	508.6	490.5	−0.0372	1.1806	1.1434	15°
16°	43.65	6.444	0.1552	−16.8	559.6	542.7	507.7	490.6	−0.0350	1.1765	1.1415	16°
17°	44.65	6.308	0.1585	−15.8	558.8	543.0	506.8	490.8	−0.0328	1.1723	1.1395	17°
18°	45.67	6.176	0.1619	−14.7	558.0	543.2	506.0	491.0	−0.0306	1.1682	1.1376	18°
19°	46.70	6.047	0.1654	−13.6	557.1	543.5	505.1	491.2	−0.0284	1.1640	1.1356	19°
20°	47.75	5.920	0.1689	−12.6	556.3	543.7	504.2	491.4	−0.0262	1.1599	1.1337	20°
21°	48.82	5.796	0.1725	−11.5	555.5	544.0	503.3	491.6	−0.0240	1.1558	1.1318	21°
22°	49.91	5.676	0.1762	−10.4	554.7	544.2	502.4	491.8	−0.0218	1.1516	1.1298	22°
23°	51.02	5.560	0.1799	− 9.4	553.9	544.5	501.6	491.9	−0.0196	1.1475	1.1279	23°
24°	52.15	5.447	0.1836	− 8.3	553.1	544.7	500.7	492.1	−0.0174	1.1435	1.1260	24°
25°	53.30	5.336	0.1874	− 7.3	552.2	545.0	499.8	492.3	−0.0153	1.1395	1.1242	25°
26°	54.47	5.228	0.1913	− 6.2	551.4	545.2	498.9	492.5	−0.0131	1.1354	1.1223	26°
27°	55.66	5.122	0.1953	− 5.1	550.6	545.5	498.1	492.7	−0.0109	1.1314	1.1205	27°
28°	56.87	5.019	0.1993	− 4.1	549.8	545.7	497.2	492.9	−0.0087	1.1274	1.1187	28°
29°	58.10	4.918	0.2034	− 3.0	549.0	546.0	496.3	493.0	−0.0066	1.1234	1.1168	29°
30°	59.35	4.820	0.2075	− 1.9	548.1	546.2	495.4	493.2	−0.0044	1.1194	1.1150	30°
31°	60.62	4.724	0.2117	− 0.8	547.3	546.4	494.5	493.4	−0.0022	1.1154	1.1132	31°
32°	61.91	4.631	0.2159	+ 0.3	546.5	546.7	493.6	493.6	0.0000	1.1114	1.1114	32°
33°	63.22	4.540	0.2203	+ 1.3	545.6	546.9	492.8	493.8	+0.0021	1.1075	1.1097	33°
34°	64.55	4.451	0.2247	+ 2.4	544.8	547.1	491.9	493.9	+0.0043	1.1036	1.1079	34°

TABLE 1—TEMPERATURE

Temp. Fahr.	Pressure, lb. per sq. in.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	Heat Content of Liquid	Latent Heat of Evap.	Heat Content of Vapor	Internal Energy B. t. u. Evap.	Internal Energy B. t. u. Vapor	Entropy Liquid	Entropy Evap.	Entropy Vapor	Temp. Fahr.
t	p	v''	$1/v''$	i'	r	i''	ρ	u''	s'	r/T	s''	
35°	65.91	4.364	0.2292	3.5	543.9	547.4	491.0	494.1	0.0065	1.0996	1.1061	35°
36°	67.29	4.279	0.2337	4.6	543.1	547.6	490.1	494.3	0.0087	1.0957	1.1043	36°
37°	68.69	4.196	0.2384	5.6	542.2	547.8	489.2	494.5	0.0108	1.0918	1.1026	37°
38°	70.11	4.115	0.2431	6.7	541.4	548.1	488.3	494.6	0.0130	1.0879	1.1009	38°
39°	71.56	4.036	0.2478	7.8	540.5	548.3	487.4	494.8	0.0151	1.0840	1.0991	39°
40°	73.03	3.959	0.2526	8.9	539.7	548.5	486.5	495.0	0.0173	1.0801	1.0974	40°
41°	74.53	3.884	0.2575	10.0	538.8	548.8	485.5	495.2	0.0194	1.0763	1.0957	41°
42°	76.05	3.810	0.2625	11.1	537.9	549.0	484.6	495.3	0.0216	1.0724	1.0940	42°
43°	77.59	3.738	0.2675	12.2	537.1	549.2	483.7	495.5	0.0237	1.0686	1.0923	43°
44°	79.16	3.668	0.2727	13.3	536.2	549.4	482.8	495.7	0.0259	1.0647	1.0906	44°
45°	80.75	3.599	0.2779	14.3	535.3	549.7	481.9	495.9	0.0280	1.0609	1.0889	45°
46°	82.37	3.532	0.2832	15.4	534.5	549.9	481.0	496.0	0.0301	1.0571	1.0872	46°
47°	84.01	3.466	0.2885	16.5	533.6	550.1	480.1	496.2	0.0323	1.0533	1.0856	47°
48°	85.68	3.402	0.2940	17.6	532.7	550.3	479.2	496.4	0.0344	1.0495	1.0839	48°
49°	87.37	3.339	0.2995	18.7	531.8	550.6	478.3	496.5	0.0366	1.0457	1.0822	49°
50°	89.09	3.278	0.3051	19.8	531.0	550.8	477.3	496.7	0.0387	1.0419	1.0806	50°
51°	90.83	3.219	0.3107	20.9	530.1	551.0	476.4	496.9	0.0408	1.0382	1.0790	51°
52°	92.59	3.161	0.3164	22.0	529.2	551.2	475.5	497.0	0.0430	1.0344	1.0774	52°
53°	94.38	3.104	0.3222	23.1	528.3	551.4	474.6	497.2	0.0451	1.0306	1.0757	53°
54°	96.19	3.048	0.3281	24.2	527.4	551.6	473.6	497.4	0.0473	1.0268	1.0741	54°
55°	98.03	2.992	0.3342	25.3	526.5	551.9	472.7	497.5	0.0494	1.0231	1.0725	55°
56°	99.90	2.938	0.3404	26.4	525.6	552.1	471.8	497.7	0.0516	1.0194	1.0710	56°
57°	101.8	2.885	0.3467	27.5	524.7	552.3	470.8	497.9	0.0537	1.0157	1.0694	57°
58°	103.7	2.833	0.3530	28.7	523.8	552.5	469.9	498.0	0.0559	1.0119	1.0678	58°
59°	105.7	2.783	0.3594	29.8	522.9	552.7	469.0	498.2	0.0580	1.0083	1.0663	59°
60°	107.7	2.734	0.3658	30.9	522.0	522.9	468.0	498.4	0.0601	1.0046	1.0647	60°
61°	109.7	2.686	0.3723	32.0	521.1	553.1	467.1	498.5	0.0623	1.0009	1.0632	61°
62°	111.7	2.639	0.3790	33.1	520.2	553.3	466.1	498.7	0.0644	0.9973	1.0617	62°
63°	113.8	2.592	0.3858	34.2	519.3	553.5	465.2	498.9	0.0665	0.9937	1.0602	63°
64°	115.9	2.547	0.3927	35.3	518.4	553.7	464.2	499.0	0.0687	0.9900	1.0587	64°
65°	118.1	2.503	0.3996	36.5	517.5	554.0	463.3	499.2	0.0708	0.9863	1.0571	65°
66°	120.3	2.460	0.4066	37.6	516.5	554.2	462.3	499.4	0.0729	0.9827	1.0556	65°
67°	122.5	2.418	0.4136	38.7	515.6	554.4	461.4	499.5	0.0750	0.9791	1.0541	67°
68°	124.7	2.377	0.4207	39.9	514.7	554.6	460.4	499.7	0.0771	0.9755	1.0526	68°
69°	126.9	2.336	0.4280	41.0	513.7	554.8	459.5	499.9	0.0792	0.9719	1.0511	69°
70°	129.2	2.296	0.4354	42.1	512.8	555.0	458.5	500.0	0.0813	0.9683	1.0496	70°
71°	131.5	2.257	0.4430	43.3	511.9	555.2	457.6	500.2	0.0834	0.9647	1.0481	71°
72°	133.9	2.219	0.4506	44.4	511.0	555.4	456.6	500.4	0.0855	0.9612	1.0467	72°
73°	136.3	2.182	0.4583	45.5	510.0	555.6	455.7	500.5	0.0876	0.9576	1.0452	73°
74°	138.7	2.145	0.4662	46.7	509.1	555.8	454.7	500.7	0.0898	0.9540	1.0438	74°
75°	141.1	2.109	0.4742	47.8	508.1	556.0	453.7	500.9	0.0919	0.9504	1.0423	75°
76°	143.6	2.074	0.4823	49.0	507.2	556.2	452.7	501.0	0.0940	0.9469	1.0409	76°
77°	146.1	2.039	0.4905	50.1	506.2	556.4	451.7	501.2	0.0961	0.9434	1.0395	77°
78°	148.7	2.005	0.4988	51.3	505.3	556.6	450.8	501.3	0.0983	0.9398	1.0381	78°
79°	151.3	1.972	0.5071	52.4	504.3	556.8	449.8	501.5	0.1004	0.9363	1.0367	79°
80°	153.9	1.940	0.5155	53.6	503.4	557.0	448.8	501.7	0.1025	0.9328	1.0353	80°
81°	156.5	1.908	0.5241	54.8	502.4	557.1	447.8	501.8	0.1047	0.9293	1.0339	81°
82°	159.2	1.877	0.5328	55.9	501.4	557.3	446.9	502.0	0.1068	0.9258	1.0326	82°
83°	161.9	1.847	0.5416	57.1	500.5	557.5	445.9	502.2	0.1090	0.9222	1.0312	83°
84°	164.6	1.817	0.5504	58.3	499.5	557.7	444.9	502.3	0.1111	0.9187	1.0298	84°

TABLE 1—TEMPERATURE

Temp. Fahr. t	Pressure, lb. per sq. in. p	Sp. Vol. cu. ft. per lb. v''	Density lb. per cu. ft. $1/v''$	Heat Content of Liquid i'	Latent Heat of Evap. r	Heat Content of Vapor i''	Internal Energy B. t. u. Evap. ρ	Internal Energy B. t. u. Vapor u''	Entropy Liquid s'	Entropy Evap. r/T	Entropy Vapor s''	Tem Fah
85°	167.4	1.788	0.5594	59.4	498.5	557.9	443.9	502.5	0.1132	0.9153	1.0285	85°
86°	170.2	1.759	0.5685	60.6	497.5	558.1	442.9	502.7	0.1153	0.9118	1.0271	86°
87°	173.0	1.731	0.5777	61.8	496.5	558.3	441.9	502.8	0.1175	0.9083	1.0258	87°
88°	175.9	1.704	0.5869	63.0	495.5	558.5	440.9	503.0	0.1196	0.9049	1.0245	88°
89°	178.8	1.677	0.5964	64.2	494.5	558.7	439.9	503.1	0.1217	0.9014	1.0231	89°
90°	181.8	1.650	0.6060	65.3	493.5	558.9	438.9	503.3	0.1238	0.8980	1.0218	90°
91°	184.8	1.624	0.6158	66.5	492.5	559.1	437.9	503.5	0.1259	0.8945	1.0205	91°
92°	187.8	1.598	0.6258	67.7	491.5	559.2	436.9	503.6	0.1281	0.8910	1.0191	92°
93°	190.9	1.573	0.6358	68.9	490.5	559.4	435.9	503.8	0.1302	0.8876	1.0178	93°
94°	194.1	1.548	0.6460	70.1	489.5	559.6	434.9	504.0	0.1323	0.8842	1.0165	94°
95°	197.3	1.524	0.656	71.3	488.5	559.8	433.9	504.1	0.1344	0.8808	1.0152	95°
96°	200.5	1.500	0.667	72.5	487.5	560.0	432.8	504.3	0.1365	0.8774	1.0139	96°
97°	203.8	1.477	0.677	73.7	486.5	560.2	431.8	504.5	0.1387	0.8740	1.0127	97°
98°	207.1	1.454	0.688	74.9	485.4	560.3	430.8	504.6	0.1408	0.8706	1.0114	98°
99°	210.4	1.431	0.699	76.1	484.4	560.5	429.8	504.8	0.1429	0.8672	1.0101	99°
100°	213.8	1.408	0.710	77.3	483.4	560.7	428.7	504.9	0.1450	0.8638	1.0088	100°
101°	217.2	1.386	0.721	78.5	482.3	560.9	427.7	505.1	0.1471	0.8604	1.0076	101°
102°	220.7	1.365	0.732	79.7	481.3	561.1	426.7	505.2	0.1493	0.8570	1.0063	102°
103°	224.2	1.345	0.743	80.9	480.3	561.2	425.6	505.4	0.1514	0.8537	1.0051	103°
104°	227.7	1.325	0.755	82.2	479.2	561.4	424.6	505.6	0.1535	0.8503	1.0039	104°
105°	231.2	1.305	0.766	83.4	478.2	561.6	423.5	505.7	0.1557	0.8469	1.0026	105°
106°	234.8	1.285	0.778	84.6	477.1	561.8	422.5	505.9	0.1578	0.8436	1.0014	106°
107°	238.4	1.266	0.790	85.8	476.1	561.9	421.4	506.1	0.1599	0.8403	1.0002	107°
108°	242.1	1.247	0.802	87.1	475.0	562.1	420.4	506.2	0.1621	0.8369	0.9990	108°
109°	245.8	1.228	0.814	88.3	474.0	562.3	419.3	506.4	0.1642	0.8336	0.9978	109°
110°	249.6	1.210	0.826	89.6	472.9	562.5	418.3	506.6	0.1664	0.8302	0.9966	110°
111°	253.4	1.192	0.839	90.8	471.8	562.6	417.2	506.7	0.1686	0.8268	0.9954	111°
112°	257.3	1.174	0.852	92.1	470.7	562.8	416.1	506.9	0.1707	0.8235	0.9942	112°
113°	261.2	1.156	0.865	93.3	469.6	563.0	415.1	507.1	0.1729	0.8202	0.9930	113°
114°	265.2	1.139	0.878	94.6	468.5	563.1	414.0	507.2	0.1750	0.8169	0.9919	114°
115°	269.2	1.122	0.891	95.9	467.4	563.3	412.9	507.4	0.1772	0.8135	0.9907	115°
116°	273.3	1.105	0.905	97.1	466.3	563.5	411.9	507.6	0.1794	0.8102	0.9896	116°
117°	277.4	1.088	0.919	98.4	465.2	563.7	410.8	507.7	0.1816	0.8068	0.9884	117°
118°	281.5	1.072	0.933	99.7	464.1	563.8	409.7	507.9	0.1838	0.8035	0.9873	118°
119°	285.7	1.057	0.946	101.0	463.0	564.0	408.6	508.1	0.1859	0.8002	0.9861	119°
120°	289.9	1.042	0.960	102.2	461.9	564.2	407.5	508.2	0.1881	0.7969	0.9850	120°
121°	294.2	1.027	0.974	103.5	460.8	464.4	406.4	508.4	0.1903	0.7936	0.9839	121°
122°	298.5	1.012	0.988	104.8	459.7	564.5	405.3	508.6	0.1925	0.7903	0.9828	122°
123°	302.8	0.998	1.002	106.1	458.6	564.7	404.2	508.8	0.1947	0.7870	0.9817	123°
124°	307.2	0.984	1.016	107.4	457.4	564.9	403.1	508.9	0.1968	0.7838	0.9806	124°
125°	311.6	0.970	1.031	108.7	456.3	565.0	402.0	509.1	0.1990	0.7805	0.9795	125°
126°	316.1	0.956	1.046	110.0	455.2	565.2	400.9	509.3	0.2012	0.7772	0.9784	126°
127°	320.6	0.942	1.061	111.3	454.0	565.4	399.8	509.4	0.2034	0.7739	0.9773	127°
128°	325.2	0.929	1.076	112.6	452.8	565.5	398.7	509.6	0.2056	0.7707	0.9763	128°
129°	329.9	0.916	1.092	114.0	451.7	565.7	397.6	509.8	0.2078	0.7674	0.9752	129°
130°	334.6	0.903	1.108	115.3	450.6	565.9	396.4	510.0	0.2100	0.7641	0.9741	130°
131°	339.4	0.890	1.124	116.6	449.4	566.0	395.3	510.2	0.2122	0.7609	0.9731	131°
132°	344.2	0.877	1.140	118.0	448.2	566.2	394.2	510.3	0.2145	0.7576	0.9720	132°
133°	349.0	0.865	1.156	119.3	447.0	566.4	393.0	510.5	0.2167	0.7543	0.9710	133°
134°	353.9	0.853	1.172	120.7	445.8	566.5	391.9	510.7	0.2189	0.7511	0.9700	134°

TABLE 1—TEMPERATURE

mp. hr. t	Pressure, lb. per sq. in. p	Sp. Vol. cu. ft. per lb. v''	Density lb. per cu. ft. $1/v''$	Heat Content of Liquid i'	Latent Heat of Evap. r	Heat Content of Vapor i''	Internal Energy B. t. u. Evap. ρ	Internal Energy B. t. u. Vapor u''	Entropy Liquid s'	Entropy Evap. r/T	Entropy Vapor s''	Temp. Fahr.
5°	358.8	0.841	1.189	122.0	444.7	566.7	390.7	510.8	0.2211	0.7478	0.9689	135°
6°	363.8	0.829	1.206	123.4	443.5	566.8	389.6	511.0	0.2234	0.7445	0.9679	136°
7°	368.9	0.817	1.224	124.7	442.3	567.0	388.4	511.2	0.2256	0.7413	0.9669	137°
8°	374.0	0.806	1.241	126.1	441.1	567.2	387.3	511.3	0.2278	0.7380	0.9658	138°
9°	379.2	0.795	1.258	127.4	439.9	567.3	386.1	511.5	0.2301	0.7347	0.9648	139°
0°	384.4	0.784	1.275	128.8	438.6	567.5	384.9	511.7	0.2323	0.7315	0.9638	140°
1°	389.7	0.773	1.293	130.2	437.4	567.6	383.8	511.8	0.2346	0.7282	0.9628	141°
12°	395.0	0.762	1.312	131.6	436.2	567.8	382.6	512.0	0.2368	0.7250	0.9618	142°
13°	400.4	0.751	1.331	133.0	435.0	567.9	381.4	512.2	0.2391	0.7217	0.9608	143°
14°	405.8	0.741	1.349	134.4	433.7	568.1	380.2	512.4	0.2413	0.7185	0.9598	144°
15°	411.3	0.731	1.368	135.8	432.5	568.3	379.0	512.6	0.2436	0.7153	0.9588	145°
16°	416.8	0.721	1.387	137.2	431.2	568.4	377.8	512.7	0.2459	0.7120	0.9578	146°
47°	422.4	0.711	1.406	138.6	430.0	568.6	376.6	512.9	0.2481	0.7088	0.9569	147°
48°	428.0	0.701	1.425	140.0	428.7	568.7	375.4	513.1	0.2504	0.7055	0.9559	148°
49°	433.7	0.692	1.445	141.5	427.4	568.9	374.2	513.3	0.2527	0.7023	0.9550	149°
50°	439.5	0.683	1.464	142.9	426.2	569.0	373.0	513.4	0.2550	0.6990	0.9540	150°
55°	469.1	0.638	1.567	150.1	419.7	569.8	366.9	514.4	0.2666	0.6828	0.9494	155°
60°	500.1	0.597	1.676	157.5	413.0	570.5	360.6	515.3	0.2784	0.6665	0.9449	160°
65°	532.6	0.558	1.792	165.1	406.2	571.3	354.2	516.2	0.2903	0.6502	0.9405	165°
70°	566.6	0.522	1.915	172.9	399.1	572.0	347.6	517.2	0.3023	0.6339	0.9362	170°
75°	602.2	0.489	2.045	180.9	391.8	572.7	340.8	518.2	0.3146	0.6174	0.9320	175°
80°	639.5	0.458	2.183	189.0	384.3	573.4	333.9	519.2	0.3271	0.6009	0.9280	180°
85°	678.4	0.429	2.330	197.5	376.6	574.0	326.8	520.2	0.3399	0.5842	0.9241	185°
90°	719.0	0.402	2.488	206.2	368.5	574.7	319.4	521.2	0.3530	0.5673	0.9203	190°
95°	761.4	0.376	2.660	215.2	360.2	575.4	311.8	522.3	0.3664	0.5502	0.9166	195°
200°	805.6	0.352	2.84	....	351.5	....	303.9	....	....	0.5328	....	200°
205°	851.7	0.330	3.03	....	342.4	....	295.7	....	....	0.5152	....	205°
210°	899.7	0.309	3.24	....	332.9	....	287.2	....	....	0.4971	....	210°
215°	949.6	0.289	3.46	....	322.9	....	278.2	....	....	0.4786	....	215°
220°	1001.4	0.270	3.70	....	312.3	....	268.8	....	....	0.4595	....	220°
225°	1055.3	0.252	3.97	....	301.1	....	258.9	....	....	0.4398	....	225°
230°	1111.3	0.235	4.26	....	289.2	....	248.3	....	....	0.4193	....	230°
235°	1169.5	0.219	4.57	....	276.3	....	237.0	....	....	0.3977	....	235°
240°	1229.9	0.203	4.93	....	262.3	....	224.8	....	....	0.3749	....	240°
245°	1292.5	0.188	5.32	....	246.9	....	211.4	....	....	0.3504	....	245°
250°	1357.4	0.173	5.8	....	229.7	....	196.4	....	....	0.3237	....	250°
255°	1424.7	0.158	6.3	....	210.0	....	179.4	....	....	0.2938	....	255°
260°	1494.4	0.142	7.0	....	186.4	....	149.1	....	....	0.2590	....	260°
265°	1566.6	0.125	8.0	....	156.2	....	133.2	....	....	0.2156	....	265°
270°	1641.3	0.104	9.6	....	110.1	....	93.9	....	....	0.1510	....	270°
273.2°	1690.0	0.068	14.75	....	000.0	....	00.0	....	....	0.0000	....	273.2°

TABLE 2

Saturated Ammonia Vapor: Pressure Table

Pressure, lb.	Temp. Fahr.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	Heat Content of Liquid	Latent Heat of Evap.	Heat Content of Vapor	Internal Energy B. t. u.		Entropy			Pre sur lb
							Evap.	Vapor	Liquid	Evap.	Vapor	
p	t	v''	$1/v''$	i'	r	i''	ρ	u''	s'	r/T	s''	p
1	−103.7	225.	0.0044	....	644.6	....	603.0	....	....	1.8106	....	
2	− 87.1	117.	0.0085	....	633.9	....	590.7	....	....	1.7016	....	
3	− 76.5	80.	0.0125	....	626.9	....	582.7	....	....	1.6362	....	
4	− 68.5	61.	0.0164	....	621.6	....	576.5	....	....	1.5890	....	
5	− 62.0	49.3	0.0203	−98.1	617.2	519.1	571.5	473.4	−0.2207	1.5522	1.3315	
6	− 56.6	41.6	0.0241	−92.5	613.5	521.1	567.3	474.9	−0.2070	1.5222	1.3152	
7	− 51.9	35.9	0.0279	−87.6	610.2	522.7	563.7	476.2	−0.1947	1.4964	1.3017	
8	− 47.6	31.6	0.0316	−83.2	607.3	524.1	560.4	477.1	−0.1840	1.4739	1.2899	
9	− 43.9	28.3	0.0353	−79.3	604.7	525.3	557.4	478.0	−0.1747	1.4540	1.2793	
10	− 40.4	25.75	0.0388	−75.7	602.2	526.4	554.6	478.8	−0.1661	1.4362	1.2701	10
11	− 37.2	23.60	0.0424	−72.4	599.9	527.5	552.0	479.5	−0.1584	1.4201	1.2617	11
12	− 34.3	21.75	0.0460	−69.4	597.8	528.4	549.6	480.2	−0.1513	1.4053	1.2540	12
13	− 31.5	20.16	0.0496	−66.5	595.8	529.3	547.4	480.9	−0.1446	1.3916	1.2470	13
14	− 28.9	18.79	0.0532	−63.8	593.9	530.1	545.4	481.5	−0.1384	1.3788	1.2404	14
15	− 26.4	17.60	0.0568	−61.2	592.1	530.9	543.3	482.0	−0.1324	1.3668	1.2344	15
16	− 24.1	16.56	0.0604	−58.8	590.4	531.6	541.4	482.5	−0.1268	1.3556	1.2288	16
17	− 21.9	15.64	0.0639	−56.5	588.8	532.2	539.6	483.0	−0.1215	1.3450	1.2235	17
18	− 19.8	14.82	0.0675	−54.4	587.2	532.8	537.9	483.4	−0.1165	1.3350	1.2185	18
19	− 17.8	14.09	0.0710	−52.3	585.7	533.4	536.3	483.9	−0.1119	1.3256	1.2137	19
20	− 15.9	13.45	0.0744	−50.3	584.3	534.0	534.7	484.3	−0.1075	1.3167	1.2092	20
21	− 14.0	12.82	0.0780	−48.4	582.9	534.6	533.1	484.7	−0.1032	1.3081	1.2049	21
22	− 12.2	12.27	0.0815	−46.5	581.5	535.1	531.6	485.1	−0.0990	1.2998	1.2008	22
23	− 10.5	11.77	0.0850	−44.7	580.2	535.6	530.2	485.4	−0.0950	1.2919	1.1969	23
24	− 8.8	11.30	0.0885	−42.9	579.0	536.1	528.8	485.8	−0.0912	1.2843	1.1931	24
25	− 7.2	10.88	0.0919	−41.3	577.8	536.5	527.4	486.1	−0.0874	1.2770	1.1896	25
26	− 5.7	10.50	0.0953	−39.7	576.6	536.9	526.1	486.4	−0.0840	1.2700	1.1861	26
27	− 4.2	10.13	0.0987	−38.1	575.4	537.4	524.9	486.7	−0.0805	1.2633	1.1828	27
28	− 2.7	9.78	0.1022	−36.5	574.3	537.8	523.7	487.0	−0.0771	1.2568	1.1797	28
29	− 1.3	9.47	0.1056	−35.0	573.2	538.2	522.5	487.3	−0.0739	1.2506	1.1767	29
30	+ 0.1	9.17	0.1090	−33.6	572.1	538.5	521.3	487.6	−0.0708	1.2446	1.1738	30
31	1.4	8.90	0.1124	−32.2	571.1	538.9	520.2	487.8	−0.0677	1.2388	1.1711	31
32	2.7	8.64	0.1158	−30.8	570.1	539.3	519.1	488.1	−0.0648	1.2331	1.1683	32
33	4.0	8.39	0.1192	−29.5	569.1	539.6	518.0	488.4	−0.0617	1.2274	1.1657	33
34	5.3	8.15	0.1226	−28.2	568.1	540.0	516.9	488.6	−0.0589	1.2219	1.1630	34
35	6.5	7.93	0.1260	−26.9	567.1	540.3	515.8	488.8	−0.0561	1.2167	1.1606	35
36	7.7	7.73	0.1294	−25.6	566.2	540.6	514.8	489.1	−0.0534	1.2115	1.1580	36
37	8.9	7.52	0.1328	−24.4	565.3	540.9	513.8	489.3	−0.0508	1.2065	1.1557	37
38	10.0	7.84	0.1362	−23.2	564.4	541.2	512.8	489.5	−0.0483	1.2017	1.1534	38
39	11.1	7.16	0.1397	−22.0	563.5	541.5	511.9	489.7	−0.0458	1.1970	1.1512	39
40	12.2	6.989	0.1431	−20.8	562.6	541.8	511.0	489.9	−0.0433	1.1923	1.1490	40
41	13.3	6.824	0.1465	−19.7	561.7	542.0	510.0	490.1	−0.0409	1.1877	1.1468	41
42	14.4	6.668	0.1500	−18.6	560.9	542.3	509.1	490.3	−0.0386	1.1832	1.1447	42
43	15.4	6.520	0.1534	−17.5	560.0	542.6	508.2	490.5	−0.0363	1.1789	1.1426	43
44	16.4	6.380	0.1567	−16.4	559.2	542.8	507.3	490.7	−0.0341	1.1747	1.1406	44

TABLE 2—PRESSURE

Pressure, lb. p	Temp. Fahr. t	Sp. Vol. cu. ft. per lb. v''	Density lb. per cu. ft. $1/v''$	Heat Content of Liquid i'	Latent Heat of Evap. r	Heat Content of Vapor i''	Internal Energy B. t. u. Evap. ρ	Internal Energy B. t. u. Vapor u''	Entropy Liquid s'	Entropy Evap. r/T	Entropy Vapor s''	Pressure, lb. p
45	17.4	6.247	0.1601	−15.3	558.4	543.1	506.4	490.9	−0.0319	1.1706	1.1387	45
46	18.4	6.120	0.1634	−14.3	557.6	543.3	505.6	491.1	−0.0296	1.1665	1.1369	46
47	19.4	5.999	0.1667	−13.3	556.8	543.6	504.7	491.3	−0.0276	1.1625	1.1349	47
48	20.3	5.883	0.1700	−12.3	556.1	543.8	503.9	491.4	−0.0255	1.1586	1.1331	48
49	21.2	5.772	0.1733	−11.3	555.3	544.0	503.1	491.6	−0.0235	1.1558	1.1313	49
50	22.1	5.665	0.1765	−10.3	554.6	544.3	502.3	491.8	−0.0216	1.1511	1.1296	50
51	23.0	5.562	0.1798	− 9.3	553.9	544.5	501.5	491.9	−0.0196	1.1475	1.1279	51
52	23.9	5.463	0.1831	− 8.4	553.1	544.7	500.8	492.1	−0.0177	1.1440	1.1263	52
53	24.8	5.367	0.1863	− 7.5	552.4	544.9	500.0	492.3	−0.0158	1.1405	1.1246	53
54	25.6	5.274	0.1896	− 6.6	551.7	545.1	499.3	492.4	−0.0140	1.1371	1.1230	54
55	26.4	5.184	0.1929	− 5.7	551.1	545.3	498.6	492.6	−0.0122	1.1337	1.1215	55
56	27.3	5.097	0.1962	− 4.8	550.4	545.5	497.8	492.7	−0.0103	1.1303	1.1200	56
57	28.1	5.012	0.1996	− 3.9	549.7	545.7	497.1	492.9	−0.0085	1.1270	1.1185	57
58	28.9	4.929	0.2029	− 3.0	549.0	545.9	496.4	493.0	−0.0068	1.1238	1.1170	58
59	29.7	4.848	0.2063	− 2.2	548.4	546.1	495.7	493.2	−0.0050	1.1206	1.1156	59
60	30.5	4.770	0.2097	− 1.3	547.7	546.3	495.0	493.3	−0.0033	1.1174	1.1141	60
61	31.3	4.695	0.2130	− 0.5	547.0	546.5	494.3	493.5	−0.0016	1.1143	1.1127	61
62	32.1	4.623	0.2163	+ 0.3	546.4	546.7	493.6	493.6	+0.0001	1.1112	1.1113	62
63	32.8	4.554	0.2196	1.1	545.8	546.9	492.9	493.7	0.0018	1.1082	1.1100	63
64	33.6	4.487	0.2229	1.9	545.1	547.1	492.2	493.9	0.0035	1.1052	1.1086	64
65	34.3	4.422	0.2262	2.7	544.5	547.2	491.6	494.0	0.0051	1.1022	1.1073	65
66	35.1	4.359	0.2294	3.5	543.8	547.4	490.9	494.1	0.0067	1.0993	1.1060	66
67	35.8	4.298	0.2327	4.3	543.2	547.6	490.3	494.3	0.0082	1.0965	1.1047	67
68	36.5	4.238	0.2360	5.1	542.6	547.7	489.6	494.4	0.0097	1.0938	1.1035	68
69	37.2	4.179	0.2394	5.8	542.0	547.9	489.0	494.5	0.0113	1.0910	1.1023	69
70	37.9	4.121	0.2427	6.6	541.4	548.1	488.4	494.6	0.0128	1.0882	1.1010	70
71	38.6	4.065	0.2461	7.4	540.8	548.2	487.7	494.8	0.0143	1.0855	1.0998	71
72	39.3	4.011	0.2494	8.1	540.2	548.4	487.1	494.9	0.0158	1.0828	1.0986	72
73	40.0	3.958	0.2527	8.9	539.6	548.5	486.5	495.0	0.0173	1.0801	1.0974	73
74	40.7	3.907	0.2560	9.6	539.0	548.7	485.9	495.1	0.0187	1.0775	1.0962	74
75	41.3	3.858	0.2592	10.3	538.5	548.8	485.3	495.2	0.0201	1.0750	1.0951	75
76	42.0	3.811	0.2624	11.0	537.9	549.0	484.7	495.3	0.0215	1.0725	1.0940	76
77	42.6	3.765	0.2656	11.7	537.4	549.1	484.1	495.4	0.0229	1.0700	1.0929	77
78	43.3	3.720	0.2688	12.4	536.8	549.3	483.5	495.6	0.0243	1.0675	1.0918	78
79	43.9	3.676	0.2721	13.1	536.3	549.4	482.9	495.7	0.0257	1.0650	1.0907	79
80	44.5	3.633	0.2753	13.8	535.8	549.5	482.3	495.8	0.0271	1.0626	1.0897	80
81	45.1	3.591	0.2785	14.5	535.2	549.7	481.8	495.9	0.0284	1.0602	1.0887	81
82	45.8	3.549	0.2818	15.2	534.6	549.8	481.2	496.0	0.0297	1.0579	1.0876	82
83	46.4	3.508	0.2851	15.8	534.1	550.0	480.6	496.1	0.0310	1.0556	1.0866	83
84	47.0	3.468	0.2884	16.5	533.6	550.1	480.1	496.2	0.0323	1.0533	1.0856	84
85	47.6	3.429	0.2917	17.2	533.1	550.2	479.5	496.3	0.0336	1.0510	1.0846	85
86	48.2	3.390	0.2950	17.8	532.5	550.4	479.0	496.4	0.0349	1.0487	1.0836	86
87	48.8	3.352	0.2983	18.5	532.0	550.5	478.4	496.5	0.0362	1.0464	1.0826	87
88	49.4	3.315	0.3017	19.1	531.5	550.6	477.9	496.6	0.0374	1.0442	1.0816	88
89	50.0	3.278	0.3051	19.8	531.0	550.8	477.3	496.7	0.0386	1.0420	1.0806	89
90	50.5	3.252	0.3075	20.4	530.5	550.9	476.8	496.8	0.0398	1.0399	1.0797	90
91	51.1	3.216	0.3110	21.0	530.0	551.0	476.3	496.9	0.0410	1.0378	1.0788	91
92	51.7	3.181	0.3144	21.7	529.5	551.1	475.8	497.0	0.0422	1.0357	1.0779	92
93	52.2	3.147	0.3178	22.3	529.0	551.2	475.3	497.1	0.0434	1.0336	1.0770	93
94	52.8	3.114	0.3212	22.9	528.5	551.4	474.8	497.2	0.0446	1.0315	1.0761	94

TABLE 2—PRESSURE

Pressure, lb. p	Temp. Fahr. t	Sp. Vol. cu. ft. per lb. v''	Density lb. per cu. ft. $1/v''$	Heat Content of Liquid i'	Latent Heat of Evap. r	Heat Content of Vapor i''	Internal Energy B. t. u. Evap. ρ	Internal Energy B. t. u. Vapor u''	Entropy Liquid s'	Entropy Evap. r/T	Entropy Vapor s''
95	53.3	3.082	0.3245	23.5	528.0	551.5	474.3	497.3	0.0458	1.0294	1.0752
96	53.9	3.051	0.3278	24.1	527.5	551.6	473.8	497.4	0.0470	1.0273	1.0743
97	54.4	3.021	0.3310	24.7	527.0	551.7	473.3	497.4	0.0482	1.0253	1.0735
98	55.0	2.992	0.3342	25.3	526.5	551.9	472.8	497.5	0.0494	1.0233	1.0726
99	55.5	2.964	0.3374	25.9	526.1	552.0	472.3	497.6	0.0505	1.0213	1.0718
100	56.0	2.936	0.3406	26.5	525.6	552.1	471.8	497.7	0.0516	1.0194	1.0709
101	56.6	2.909	0.3438	27.1	525.1	552.2	471.3	497.8	0.0527	1.0174	1.0701
102	57.1	2.882	0.3470	27.7	524.6	552.3	470.8	497.9	0.0539	1.0154	1.0693
103	57.6	2.855	0.3503	28.2	524.2	552.4	470.3	498.0	0.0550	1.0135	1.0685
104	58.1	2.829	0.3535	28.8	523.7	552.5	469.8	498.1	0.0561	1.0116	1.0677
105	58.6	2.803	0.3568	29.3	523.3	552.6	469.3	498.1	0.0572	1.0097	1.0669
106	59.1	2.777	0.3601	29.9	522.8	552.7	468.9	498.2	0.0583	1.0078	1.0661
107	59.6	2.752	0.3634	30.4	522.4	552.8	468.4	498.3	0.0594	1.0060	1.0653
108	60.1	2.727	0.3667	31.0	521.9	552.9	467.9	498.4	0.0604	1.0042	1.0646
109	60.6	2.703	0.3700	31.5	521.5	553.0	467.5	498.5	0.0614	1.0024	1.0638
110	61.1	2.679	0.3733	32.1	521.0	553.1	467.0	498.6	0.0625	1.0005	1.0630
111	61.6	2.656	0.3765	32.6	520.6	553.2	466.5	498.6	0.0636	0.9987	1.0623
112	62.1	2.633	0.3798	33.2	520.1	553.3	466.1	498.7	0.0646	0.9969	1.0615
113	62.6	2.611	0.3831	33.7	519.7	553.4	465.6	498.8	0.0657	0.9950	1.0607
114	63.1	2.589	0.3863	34.3	519.2	553.5	465.1	498.9	0.0668	0.9932	1.0600
115	63.6	2.568	0.3895	34.8	518.8	553.6	464.6	499.0	0.0678	0.9915	1.0593
116	64.0	2.547	0.3927	35.4	518.4	553.7	464.2	499.0	0.0688	0.9898	1.0586
117	64.5	2.526	0.3959	35.9	517.9	553.8	463.8	499.1	0.0697	0.9882	1.0579
118	64.9	2.506	0.3991	36.4	517.5	553.9	463.4	499.2	0.0706	0.9866	1.0572
119	65.4	2.486	0.4023	36.9	517.1	554.0	462.9	499.3	0.0716	0.9849	1.0565
120	65.8	2.466	0.4056	37.4	516.7	554.1	462.5	499.4	0.0725	0.9833	1.0557
121	66.3	2.446	0.4089	37.9	516.3	554.2	462.1	499.4	0.0735	0.9816	1.0551
122	66.8	2.426	0.4121	38.5	515.8	554.3	461.6	499.5	0.0745	0.9799	1.0544
123	67.2	2.409	0.4153	39.0	515.4	554.4	461.2	499.6	0.0754	0.9783	1.0538
124	67.7	2.390	0.4185	39.5	515.0	554.5	460.8	499.7	0.0764	0.9767	1.0531
125	68.1	2.371	0.4218	40.0	514.6	554.6	460.4	499.7	0.0773	0.9751	1.0524
126	68.6	2.353	0.4250	40.5	514.1	554.7	459.9	499.8	0.0783	0.9735	1.0518
127	69.0	2.335	0.4283	41.0	513.7	554.8	459.5	499.9	0.0792	0.9719	1.0511
128	69.5	2.317	0.4316	41.5	513.3	554.9	459.0	500.0	0.0802	0.9703	1.0504
129	69.9	2.300	0.4348	42.0	512.9	555.0	458.6	500.0	0.0811	0.9687	1.0498
130	70.4	2.283	0.4381	42.5	512.5	555.0	458.2	500.1	0.0820	0.9671	1.0492
131	70.8	2.266	0.4414	43.0	512.1	555.1	457.8	500.2	0.0829	0.9655	1.0484
132	71.2	2.249	0.4447	43.5	511.7	555.2	457.4	500.2	0.0838	0.9640	1.0478
133	71.6	2.233	0.4479	44.0	511.3	555.3	457.0	500.3	0.0847	0.9625	1.0472
134	72.0	2.217	0.4511	44.5	510.9	555.4	456.6	500.4	0.0856	0.9610	1.0466
135	72.5	2.201	0.4544	45.0	510.5	555.5	456.2	500.5	0.0865	0.9595	1.0460
136	72.9	2.185	0.4577	45.5	510.1	555.6	455.8	500.5	0.0874	0.9580	1.0454
137	73.3	2.169	0.4610	46.0	509.7	555.6	455.4	500.6	0.0883	0.9565	1.0448
138	73.7	2.154	0.4643	46.4	509.4	555.7	455.0	500.7	0.0892	0.9550	1.0442
139	74.1	2.139	0.4675	46.9	509.0	555.8	454.6	500.7	0.0901	0.9535	1.0436
140	74.5	2.124	0.4708	47.3	508.6	555.9	454.2	500.8	0.0910	0.9520	1.0430
141	75.0	2.109	0.4741	47.8	508.2	556.0	453.8	500.9	0.0919	0.9505	1.0424
142	75.4	2.095	0.4773	48.3	507.8	556.1	453.4	500.9	0.0928	0.9490	1.0418
143	75.8	2.082	0.4804	48.8	507.4	556.1	453.0	501.0	0.0936	0.9476	1.0412
144	76.2	2.070	0.4835	49.2	507.0	556.2	452.6	501.1	0.0944	0.9462	1.0406

TABLE 2—PRESSURE

Pressure, lb. p	Temp. Fahr. t	Sp. Vol. cu. ft. per lb. v''	Density lb. per cu. ft. $1/v''$	Heat Content of Liquid i'	Latent Heat of Evap. r	Heat Content of Vapor i''	Internal Energy B. t. u. Evap. ρ	Internal Energy B. t. u. Vapor u''	Entropy Liquid s'	Entropy Evap. r/T	Entropy Vapor s''	Pressure, lb. p
145	76.5	2.057	0.4867	49.6	506.7	556.3	452.2	501.1	0.0952	0.9449	1.0401	145
146	76.9	2.043	0.4899	50.0	506.3	556.4	451.8	501.2	0.0960	0.9435	1.0395	146
147	77.3	2.029	0.4931	50.5	506.0	556.4	451.4	501.2	0.0968	0.9422	1.0390	147
148	77.7	2.015	0.4963	50.9	505.6	556.5	451.0	501.3	0.0976	0.9409	1.0385	148
149	78.1	2.002	0.4995	51.4	505.2	556.6	450.7	501.4	0.0985	0.9395	1.0379	149
150	78.5	1.989	0.5028	51.8	504.8	556.7	450.3	501.4	0.0993	0.9381	1.0374	150
151	78.9	1.976	0.5060	52.3	504.4	556.7	449.9	501.5	0.1002	0.9367	1.0368	151
152	79.3	1.964	0.5092	52.7	504.0	556.8	449.5	501.6	0.1010	0.9353	1.0363	152
153	79.6	1.952	0.5123	53.1	503.7	556.9	449.2	501.6	0.1018	0.9340	1.0358	153
154	80.0	1.940	0.5155	53.6	503.3	557.0	448.8	501.7	0.1026	0.9327	1.0353	154
155	80.4	1.928	0.5187	54.0	503.0	557.0	448.4	501.7	0.1034	0.9313	1.0347	155
156	80.8	1.916	0.5220	54.5	502.6	557.1	448.0	501.8	0.1042	0.9300	1.0342	156
157	81.2	1.904	0.5253	54.9	502.2	447.2	447.6	501.8	0.1050	0.9287	1.0337	157
158	81.5	1.892	0.5286	55.3	501.9	557.2	447.3	501.9	0.1058	0.9274	1.0332	158
159	81.9	1.880	0.5320	55.8	501.5	557.3	446.9	502.0	0.1066	0.9261	1.0327	159
160	82.3	1.868	0.5353	56.2	501.1	557.4	446.6	502.1	0.1074	0.9248	1.0322	160
161	82.7	1.857	0.5386	56.7	500.7	557.5	446.2	502.1	0.1082	0.9235	1.0317	161
162	83.0	1.846	0.5418	57.1	500.4	557.5	445.9	502.2	0.1090	0.9222	1.0312	162
163	83.4	1.835	0.5450	57.5	500.0	557.6	445.5	502.2	0.1098	0.9209	1.0307	163
164	83.8	1.824	0.5483	58.0	499.7	557.7	445.1	502.3	0.1106	0.9196	1.0302	164
165	84.1	1.814	0.5515	58.4	499.4	557.7	444.8	502.3	0.1114	0.9183	1.0297	165
166	84.5	1.803	0.5547	58.8	499.0	557.8	444.4	502.4	0.1122	0.9170	1.0292	166
167	84.9	1.793	0.5578	59.3	498.6	557.9	444.0	502.5	0.1130	0.9157	1.0287	167
168	85.2	1.783	0.5609	59.7	498.3	558.0	443.7	502.5	0.1137	0.9145	1.0282	168
169	85.6	1.773	0.5641	60.1	497.9	558.0	443.3	502.6	0.1145	0.9132	1.0277	169
170	85.9	1.763	0.5673	60.5	497.6	558.1	443.0	502.7	0.1152	0.9120	1.0272	170
171	86.3	1.753	0.5705	61.0	497.2	558.2	442.6	502.7	0.1160	0.9107	1.0267	171
172	86.6	1.743	0.5738	61.4	496.9	558.2	442.3	502.8	0.1167	0.9095	1.0263	172
173	87.0	1.733	0.5771	61.8	496.5	558.3	441.9	502.8	0.1175	0.9083	1.0258	173
174	87.3	1.723	0.5804	62.2	496.2	558.4	441.6	502.9	0.1182	0.9071	1.0253	174
175	87.7	1.713	0.5836	62.6	495.8	558.4	441.2	502.9	0.1190	0.9059	1.0249	175
176	88.0	1.704	0.5869	63.0	495.5	558.5	440.9	503.0	0.1197	0.9047	1.0244	176
177	88.4	1.694	0.5902	63.4	495.1	558.6	440.5	503.0	0.1204	0.9036	1.0240	177
178	88.7	1.685	0.5935	63.8	494.8	558.6	440.2	503.1	0.1211	0.9024	1.0235	178
179	89.0	1.676	0.5967	64.2	494.5	558.7	439.9	503.1	0.1218	0.9012	1.0231	179
180	89.4	1.666	0.6000	64.6	494.1	558.8	439.5	503.2	0.1226	0.9000	1.0226	180
181	89.7	1.656	0.6034	65.0	493.8	558.8	439.2	503.3	0.1233	0.8988	1.0222	181
182	90.1	1.647	0.6068	65.4	493.4	558.9	438.8	503.3	0.1241	0.8976	1.0217	182
183	90.4	1.639	0.6102	65.8	493.1	558.9	438.5	503.4	0.1248	0.8965	1.0213	183
184	90.7	1.630	0.6135	66.2	492.8	559.0	438.2	503.4	0.1254	0.8954	1.0209	184
185	91.1	1.621	0.6168	66.6	492.4	559.1	437.8	503.5	0.1261	0.8943	1.0204	185
186	91.4	1.613	0.6200	67.0	492.1	559.1	437.5	503.5	0.1268	0.8932	1.0200	186
187	91.7	1.605	0.6233	67.4	491.8	559.2	437.2	503.6	0.1274	0.8920	1.0195	187
188	92.1	1.596	0.6266	67.8	491.5	559.3	436.8	503.6	0.1283	0.8908	1.0191	188
189	92.4	1.588	0.6298	68.2	491.2	559.3	436.5	503.7	0.1289	0.8897	1.0186	189
190	92.7	1.580	0.6330	68.6	490.9	559.4	436.2	503.7	0.1296	0.8886	1.0182	190
191	93.0	1.572	0.6362	68.9	490.5	559.4	435.9	503.8	0.1303	0.8875	1.0178	191
192	93.4	1.563	0.6395	69.3	490.1	559.5	435.5	503.9	0.1310	0.8864	1.0173	192
193	93.7	1.555	0.6428	69.7	489.8	559.6	435.2	503.9	0.1317	0.8853	1.0169	193
194	94.0	1.548	0.6460	70.1	489.5	559.6	434.9	504.0	0.1323	0.8842	1.0165	194

TABLE 2—PRESSURE

Pressure, lb.	Temp. Fahr.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	Heat Content of Liquid	Latent Heat of Evap.	Heat Content of Vapor	Internal Energy B. t. u.		Entropy			Pressure, lb.
							Evap.	Vapor	Liquid	Evap.	Vapor	
p	t	v''	$1/v''$	i'	r	i''	ρ	u''	s'	r/T	s''	p
195	94.3	1.541	0.6492	70.5	489.2	559.7	434.5	504.0	0.1329	0.8832	1.0161	19
196	94.6	1.533	0.6524	70.8	488.9	559.7	434.2	504.1	0.1336	0.8821	1.0157	19
197	94.9	1.526	0.6554	71.2	488.6	559.8	433.9	504.1	0.1342	0.8811	1.0153	19
198	95.2	1.519	0.6584	71.6	488.3	559.8	433.6	504.2	0.1349	0.8800	1.0149	19
199	95.5	1.512	0.6614	71.9	488.0	559.9	433.3	504.2	0.1356	0.8789	1.0145	19
200	95.9	1.504	0.665	72.3	487.6	560.0	433.0	504.3	0.1363	0.8778	1.0141	20
202	96.5	1.489	0.672	73.1	487.0	560.1	432.3	504.4	0.1376	0.8757	1.0133	20
204	97.1	1.474	0.679	73.8	486.4	560.2	431.7	504.5	0.1389	0.8736	1.0125	20
206	97.7	1.460	0.685	74.6	485.8	560.3	431.1	504.6	0.1402	0.8715	1.0117	20
208	98.3	1.447	0.691	75.3	485.1	560.4	430.5	504.7	0.1414	0.8696	1.0110	20
210	98.9	1.433	0.698	76.0	484.5	560.5	429.8	504.8	0.1427	0.8675	1.0102	21
212	99.5	1.419	0.705	76.7	483.9	560.6	429.2	504.9	0.1440	0.8655	1.0094	21
214	100.1	1.406	0.711	77.4	483.3	560.7	428.6	505.0	0.1452	0.8635	1.0087	21
216	100.7	1.394	0.717	78.1	482.7	560.8	428.0	505.0	0.1464	0.8615	1.0079	21
218	101.2	1.382	0.724	78.8	482.1	560.9	427.4	505.1	0.1476	0.8596	1.0072	21
220	101.8	1.370	0.730	79.5	481.5	561.0	426.8	505.2	0.1488	0.8577	1.0065	22
222	102.4	1.358	0.736	80.2	480.9	561.1	426.2	505.3	0.1500	0.8558	1.0058	22
224	103.0	1.346	0.743	80.9	480.3	561.2	425.6	505.4	0.1512	0.8539	1.0051	22
226	103.5	1.335	0.749	81.6	479.7	561.3	425.1	505.5	0.1524	0.8520	1.0044	22
228	104.1	1.323	0.756	82.3	479.1	561.4	424.5	505.6	0.1537	0.8500	1.0037	22
230	104.7	1.312	0.762	83.0	478.5	561.5	423.9	505.7	0.1549	0.8481	1.0030	230
232	105.2	1.301	0.769	83.7	477.9	561.6	423.3	505.8	0.1561	0.8462	1.0023	232
234	105.8	1.290	0.775	84.4	477.3	561.7	422.7	505.9	0.1573	0.8443	1.0016	234
236	106.3	1.279	0.782	85.0	476.8	561.8	422.2	505.9	0.1585	0.8425	1.0010	236
238	106.9	1.268	0.789	85.7	476.2	561.9	421.6	506.0	0.1597	0.8406	1.0003	238
240	107.4	1.258	0.795	86.4	475.6	562.0	421.0	506.1	0.1609	0.8388	0.9997	240
242	108.0	1.248	0.801	87.1	475.0	562.1	420.4	506.2	0.1621	0.8370	0.9990	242
244	108.5	1.238	0.808	87.7	474.5	562.2	419.8	506.3	0.1632	0.8352	0.9984	244
246	109.0	1.228	0.814	88.4	473.9	562.3	419.3	506.4	0.1643	0.8334	0.9978	246
248	109.6	1.218	0.821	89.1	473.3	562.4	418.7	506.5	0.1655	0.8316	0.9971	248
250	110.1	1.208	0.828	89.7	472.8	562.5	418.1	506.6	0.1666	0.8299	0.9965	250
252	110.6	1.199	0.834	90.4	472.2	562.6	417.6	506.6	0.1677	0.8282	0.9959	252
254	111.1	1.189	0.841	91.0	471.6	562.6	417.1	506.7	0.1688	0.8265	0.9953	254
256	111.7	1.179	0.848	91.7	471.0	562.7	416.5	506.8	0.1700	0.8247	0.9946	256
258	112.2	1.170	0.855	92.3	470.5	562.8	415.9	506.9	0.1711	0.8229	0.9940	258
260	112.7	1.161	0.861	93.0	470.0	562.9	415.4	507.0	0.1722	0.8212	0.9934	260
262	113.2	1.153	0.867	93.6	469.4	563.0	414.8	507.1	0.1733	0.8195	0.9928	262
264	113.7	1.144	0.874	94.2	468.9	563.1	414.3	507.2	0.1744	0.8178	0.9922	264
266	114.2	1.136	0.880	94.8	468.3	563.2	413.8	507.2	0.1755	0.8161	0.9916	266
268	114.7	1.127	0.887	95.5	467.8	563.3	413.2	507.3	0.1766	0.8144	0.9910	268
270	115.2	1.119	0.894	96.1	467.2	563.4	412.7	507.4	0.1777	0.8128	0.9905	270
272	115.7	1.110	0.901	96.7	466.7	563.4	412.2	507.5	0.1787	0.8112	0.9899	272
274	116.2	1.102	0.908	97.4	466.1	563.5	411.7	507.6	0.1798	0.8095	0.9893	274
276	116.7	1.094	0.914	98.0	465.6	563.6	411.2	507.7	0.1809	0.8079	0.9887	276
278	117.1	1.087	0.920	98.6	465.1	563.7	410.7	507.7	0.1819	0.8063	0.9882	278
280	117.6	1.079	0.927	99.2	464.6	563.8	410.2	507.8	0.1829	0.8047	0.9877	280
282	118.1	1.071	0.934	99.8	464.0	563.9	409.6	507.9	0.1840	0.8031	0.9871	282
284	118.6	1.063	0.941	100.4	463.5	563.9	409.1	508.0	0.1850	0.8015	0.9866	284
286	119.1	1.056	0.947	101.1	462.9	564.0	408.5	508.1	0.1861	0.7999	0.9860	286
288	119.6	1.049	0.953	101.7	462.4	564.1	408.0	508.2	0.1872	0.7983	0.9855	288

TABLE 2—PRESSURE

Pressure, lb. p	Temp. Fahr. t	Sp. Vol. cu. ft. per lb. v''	Density lb. per cu. ft. $1/v''$	Heat Content of Liquid i'	Latent Heat of Evap. r	Heat Content of Vapor i''	Internal Energy B. t. u. Evap. ρ	Internal Energy B. t. u. Vapor u''	Entropy Liquid s'	Entropy Evap. r/T	Entropy Vapor s''	Pressure, lb. p
290	120.0	1.042	0.960	102.3	461.9	564.2	407.5	508.2	0.1882	0.7968	0.9850	290
292	120.5	1.035	0.966	102.9	461.4	564.3	407.0	508.3	0.1892	0.7953	0.9845	292
294	120.9	1.028	0.973	103.5	460.9	564.3	406.5	508.4	0.1902	0.7938	0.9840	294
296	121.4	1.021	0.980	104.1	460.4	564.4	406.0	508.5	0.1912	0.7923	0.9835	296
298	121.9	1.014	0.986	104.7	459.8	564.5	405.5	508.6	0.1922	0.7907	0.9829	298
300	122.4	1.007	0.993	105.3	459.3	564.6	405.0	508.7	0.1932	0.7892	0.9824	300
310	124.6	0.975	1.026	108.2	456.8	565.0	402.5	509.0	0.1981	0.7819	0.9800	310
320	126.8	0.945	1.059	111.1	454.3	565.3	400.0	509.4	0.2030	0.7746	0.9775	320
330	129.0	0.916	1.092	114.0	451.8	565.7	397.6	509.8	0.2078	0.7675	0.9753	330
340	131.1	0.889	1.125	116.8	449.3	566.1	395.2	510.1	0.2125	0.7605	0.9730	340
350	133.2	0.863	1.159	119.6	446.8	566.4	392.8	510.5	0.2171	0.7537	0.9708	350
360	135.2	0.838	1.193	122.3	444.4	566.7	390.5	510.8	0.2216	0.7471	0.9687	360
370	137.2	0.815	1.227	125.0	442.0	567.0	388.2	511.2	0.2261	0.7406	0.9667	370
380	139.2	0.793	1.261	127.7	439.6	567.3	385.9	511.5	0.2305	0.7342	0.9647	380
390	141.1	0.772	1.295	130.3	437.3	567.6	383.7	511.9	0.2348	0.7280	0.9628	390
400	142.9	0.752	1.330	132.9	435.0	567.9	381.5	512.2	0.2390	0.7219	0.9608	400
410	144.8	0.733	1.364	135.5	432.7	568.2	379.3	512.5	0.2431	0.7160	0.9591	410
420	146.6	0.715	1.399	138.1	430.4	568.5	377.2	512.8	0.2472	0.7101	0.9573	420
430	148.4	0.698	1.434	140.6	428.2	568.8	375.0	513.2	0.2513	0.7043	0.9556	430
440	150.1	0.681	1.469	143.1	426.0	569.0	372.9	513.5	0.2553	0.6986	0.9539	440
450	151.9	0.665	1.504	145.6	423.8	569.3	370.8	513.8	0.2593	0.6930	0.9523	450
460	153.5	0.650	1.539	148.0	421.6	569.6	368.7	514.1	0.2632	0.6875	0.9507	460
470	155.2	0.636	1.574	150.4	419.4	569.8	366.6	514.4	0.2671	0.6821	0.9492	470
480	156.9	0.622	1.608	152.8	417.2	570.1	364.5	514.7	0.2710	0.6767	0.9477	480
490	158.5	0.609	1.642	155.2	415.0	570.3	362.5	515.0	0.2749	0.6714	0.9463	490
500	160.0	0.597	1.675	157.5	413.0	570.5	360.5	515.3	0.2786	0.6663	0.9449	500
525	163.9	0.566	1.765	163.4	407.7	571.1	355.6	516.0	0.2876	0.6539	0.9415	525
550	167.6	0.539	1.855	169.2	402.5	571.7	350.8	516.7	0.2965	0.6418	0.9383	550
575	171.2	0.514	1.946	174.8	397.4	572.2	346.0	517.4	0.3052	0.6300	0.9352	575
600	174.7	0.491	2.038	180.4	392.3	572.7	341.3	518.1	0.3138	0.6185	0.9323	600
625	178.1	0.469	2.132	185.9	387.2	573.1	336.6	518.8	0.3223	0.6072	0.9295	625
650	181.4	0.449	2.227	191.4	382.2	573.6	332.0	519.5	0.3307	0.5962	0.9269	650
675	184.6	0.431	2.321	196.8	377.2	574.0	327.4	520.1	0.3389	0.5855	0.9244	675
700	187.7	0.414	2.416	202.1	372.2	574.4	322.8	520.7	0.3469	0.5751	0.9220	700

TABLE 3—SATURATED AN

Pressure lb.	Liquid	Sat. Vapor	Degrees of Superheat								
			10	20	30	40	50	60	70	80	90
5 t	−62.0		−52.0	−42.0	−32.0	−22.0	−12.0	−2.0	+8.0	18.0	28.0
v	0.023	49.3	50.7	52.1	53.4	54.7	56.1	57.4	58.7	60.0	61.3
i	−98.1	519.1	524.1	529.1	534.1	539.0	543.9	548.8	553.6	558.4	563.3
s	−0.2207	1.3315	1.3440	1.3560	1.3677	1.3791	1.3901	1.4008	1.4113	1.4216	1.4317
6 t	−56.6		−46.6	−36.6	−26.6	−16.6	−6.6	+3.4	13.4	23.4	33.4
v	0.023	41.6	42.8	43.9	45.0	46.1	47.2	48.3	49.4	50.5	51.6
i	−92.5	521.0	526.1	531.1	536.1	541.0	546.0	550.9	555.8	560.7	565.6
s	−0.2070	1.3152	1.3277	1.3398	1.3515	1.3628	1.3738	1.3845	1.3949	1.4051	1.4151
7 t	−51.9		−41.9	−31.9	−21.9	−11.9	−1.9	+8.1	18.1	28.1	38.1
v	0.023	35.9	37.0	38.9	38.9	39.9	40.8	41.8	42.7	43.7	44.6
i	−87.6	522.7	527.8	532.9	537.9	542.9	547.9	552.8	557.7	562.6	567.5
s	−0.1947	1.3017	1.3141	1.3261	1.3378	1.3491	1.3600	1.3706	1.3810	1.3912	1.4012
8 t	−47.6		−37.6	−27.6	−17.6	−7.6	+2.4	12.4	22.4	32.4	42.4
v	0.023	31.6	32.6	33.5	34.4	35.2	36.0	36.9	37.7	38.5	39.3
i	−83.2	524.1	529.3	534.4	539.5	544.6	549.6	554.5	559.5	564.4	569.3
s	−0.1840	1.2899	1.3023	1.3144	1.3261	1.3374	1.3483	1.3589	1.3693	1.3794	1.3894
9 t	−43.9		−33.9	−23.9	−13.9	−3.9	+6.1	16.1	26.1	36.1	46.1
v	0.023	28.3	29.2	30.0	30.8	31.5	32.3	33.0	33.7	34.5	35.2
i	−79.3	525.3	530.5	535.7	540.9	546.0	551.0	556.0	561.0	566.0	570.9
s	−0.1747	1.2793	1.2918	1.3039	1.3155	1.3268	1.3377	1.3483	1.3587	1.3688	1.3788
10 t	−40.4		−30.4	−20.4	−10.4	−0.4	+9.6	19.6	29.6	39.6	49.6
v	0.023	25.8	26.5	27.2	27.9	28.5	29.2	29.9	30.5	31.2	31.9
i	−75.7	526.4	531.7	537.0	542.2	547.3	552.4	557.4	562.4	567.4	572.3
s	−0.1661	1.2701	1.2826	1.2947	1.3063	1.3176	1.3285	1.3391	1.3494	1.3595	1.3694
11 t	−37.2		−27.2	−17.2	−7.2	+2.8	12.8	22.8	32.8	42.8	52.8
v	0.023	23.6	24.2	24.9	25.5	26.1	26.7	27.3	27.9	28.5	29.1
i	−72.4	527.5	532.8	538.1	543.3	548.4	553.5	558.6	563.6	568.6	573.6
s	−0.1584	1.2617	1.2742	1.2863	1.2979	1.3092	1.3201	1.3307	1.3410	1.3511	1.3609
12 t	−34.3		−24.3	−14.3	−4.3	+5.7	15.7	25.7	35.7	45.7	55.7
v	0.024	21.7	22.3	22.9	23.5	24.1	24.6	25.2	25.7	26.3	26.8
i	−69.4	528.4	533.8	539.1	544.3	549.5	554.6	559.7	564.8	569.8	574.8
s	−0.1513	1.2540	1.2665	1.2786	1.2902	1.3015	1.3124	1.3229	1.3332	1.3433	1.3531
13 t	−31.5		−21.5	−11.5	−1.5	+8.5	18.5	28.5	38.5	48.5	58.5
v	0.024	20.2	20.7	21.3	21.8	22.3	22.8	23.4	23.9	24.4	24.9
i	−66.5	529.3	534.7	540.1	545.4	550.6	555.7	560.8	565.9	570.9	576.0
s	−0.1446	1.2470	1.2595	1.2716	1.2832	1.2945	1.3054	1.3160	1.3263	1.3364	1.3462
14 t	−28.9		−18.9	−8.9	+1.1	11.1	21.1	31.1	41.1	51.1	61.1
v	0.024	18.8	19.3	19.8	20.3	20.8	21.3	21.8	22.2	22.7	23.2
i	−63.8	530.1	535.6	541.0	546.3	551.5	556.7	561.8	566.9	571.0	577.0
s	−0.1384	1.2404	1.2530	1.2651	1.2767	1.2880	1.2989	1.3095	1.3198	1.3298	1.3396
15 t	−26.4		−16.4	−6.4	+3.6	13.6	23.6	33.6	43.6	53.6	63.6
v	0.024	17.6	18.1	18.6	19.1	19.5	20.0	20.4	20.9	21.3	21.7
i	−61.2	530.9	536.4	541.8	547.1	552.4	557.6	562.7	567.8	572.9	578.0
s	−0.1324	1.2344	1.2470	1.2591	1.2707	1.2820	1.2929	1.3035	1.3138	1.3238	1.3335
16 t	−24.1		−14.1	−4.1	+5.9	15.9	25.9	35.9	45.9	55.9	65.9
v	0.024	16.6	17.1	17.5	17.9	18.4	18.8	19.2	19.6	20.0	20.4
i	−58.8	531.6	537.1	542.5	547.9	553.2	558.5	563.7	568.8	573.9	579.0
s	0.1268	1.2288	1.2414	1.2535	1.2651	1.2763	1.2872	1.2978	1.3081	1.3181	1.3278

UPERHEATED AMMONIA VAPOR

Degrees of Superheat											Pressure lb.
100	110	120	130	140	150	160	180	200	250	300	
38.0	48.0	58.0	68.0	78.0	88.0	98.0	118.0	138.0	188.0	238.0	t 5
62.6	63.8	65.1	66.4	67.7	69.0	70.3	72.8	75.3	81.7	88.1	v
568.1	572.9	577.7	582.6	587.5	592.3	597.2	606.9	616.7	641.4	666.5	i
1.4415	1.4511	1.4605	1.4697	1.4788	1.4878	1.4966	1.5138	1.5304	1.5701	1.6074	s
43.4	53.4	63.4	73.4	83.4	93.4	103.4	123.4	143.4	193.4	243.4	t 6
52.7	53.7	54.8	55.9	56.9	58.0	59.1	61.2	63.3	68.6	73.9	v
570.4	575.3	580.1	585.0	589.9	594.7	599.6	609.4	619.2	644.0	669.1	i
1.4249	1.4344	1.4438	1.4530	1.4621	1.4710	1.4797	1.4968	1.5133	1.5528	1.5899	s
48.1	58.1	68.1	78.1	88.1	98.1	108.1	128.1	148.1	198.1	248.1	t 7
45.5	46.4	47.4	48.3	49.2	50.1	51.0	52.9	54.7	59.2	63.7	v
572.4	577.3	582.2	587.1	592.0	596.8	601.7	611.6	621.4	646.3	671.4	i
1.4109	1.4204	1.4297	1.4389	1.4479	1.4568	1.4655	1.4826	1.4990	1.5383	1.5752	s
52.4	62.4	72.4	82.4	92.4	102.4	112.4	132.4	152.4	202.4	252.4	t 8
40.1	40.9	41.8	42.6	42.4	44.2	45.0	46.6	48.2	52.2	56.1	v
574.2	579.1	584.1	589.0	593.9	598.8	603.7	613.6	623.4	648.3	673.5	i
1.3990	1.4085	1.4178	1.4269	1.4359	1.4447	1.4533	1.4703	1.4867	1.5259	1.5626	s
56.1	66.1	76.1	86.1	96.1	106.1	116.1	136.1	156.1	206.1	256.1	t 9
35.9	36.6	37.3	38.1	38.8	39.5	40.2	41.6	43.1	46.6	50.1	v
575.8	580.7	585.7	590.6	595.5	600.4	605.4	615.3	625.1	650.1	675.3	i
1.3884	1.3978	1.4071	1.4162	1.4251	1.4339	1.4425	1.4594	1.4758	1.5148	1.5514	s
59.6	69.6	79.6	89.6	99.6	109.6	119.6	139.6	159.6	209.6	259.6	t 10
32.5	33.2	33.8	34.5	35.1	35.8	36.4	37.7	39.0	42.1	45.3	v
577.2	582.2	587.2	592.1	597.1	602.0	606.9	616.8	626.7	651.8	677.1	i
1.3790	1.3884	1.3977	1.4067	1.4156	1.4243	1.4330	1.4498	1.4661	1.5049	1.5414	s
62.8	72.8	82.8	92.8	102.8	112.8	122.8	142.8	162.8	212.8	262.8	t 11
29.7	30.3	30.9	31.5	32.1	32.7	33.3	34.4	35.6	38.5	41.4	v
578.6	583.6	588.5	593.4	598.4	603.4	608.4	618.3	628.2	653.3	678.6	i
1.3705	1.3799	1.3892	1.3982	1.4071	1.4158	1.4244	1.4411	1.4574	1.4961	1.5325	s
65.7	75.7	85.7	95.7	105.7	115.7	125.7	145.7	165.7	215.7	265.7	t 12
27.4	27.9	28.5	29.0	29.5	30.1	30.6	31.7	32.8	35.4	38.1	v
579.8	584.8	589.8	594.8	599.7	604.7	609.7	619.6	629.5	654.7	680.1	i
1.3627	1.3721	1.3813	1.3903	1.3992	1.4079	1.4165	1.4332	1.4494	1.4880	1.5243	s
68.5	78.5	88.5	98.5	108.5	118.5	128.5	148.5	168.5	218.5	268.5	t 13
25.4	25.9	26.4	26.9	27.4	27.9	28.4	29.4	30.4	32.8	35.3	v
581.0	586.0	591.0	596.0	600.9	605.9	610.9	620.8	630.8	656.0	681.5	i
1.3557	1.3651	1.3743	1.3833	1.3921	1.4008	1.4093	1.4260	1.4422	1.4807	1.5169	s
71.1	81.1	91.1	101.1	111.1	121.1	131.1	151.1	171.1	221.1	271.1	t 14
23.7	24.1	24.6	25.1	25.5	26.0	26.4	27.4	28.3	30.5	32.8	v
582.1	587.1	592.1	597.1	602.1	607.1	612.1	622.0	632.0	657.3	682.8	i
1.3491	1.3585	1.3677	1.3767	1.3855	1.3942	1.4027	1.4194	1.4355	1.4739	1.5100	s
73.6	83.6	93.6	103.6	113.6	123.6	133.6	153.6	173.6	223.6	273.6	t 15
22.2	22.6	23.1	23.5	23.9	24.4	24.8	25.7	26.5	28.6	30.8	v
583.1	588.1	593.2	598.2	603.2	608.2	613.2	623.2	633.2	658.5	684.0	i
1.3430	1.3524	1.3616	1.3706	1.3794	1.3880	1.3965	1.4131	1.4292	1.4676	1.5036	s
75.9	85.9	95.9	105.9	115.9	125.9	135.9	155.9	175.9	225.9	275.9	t 16
20.8	21.3	21.7	22.1	22.5	22.9	23.3	24.1	24.9	26.9	28.9	v
584.0	589.1	594.1	599.1	604.2	609.2	614.2	624.2	634.2	659.5	685.1	i
1.3373	1.3467	1.3558	1.3648	1.3736	1.3822	1.3907	1.4073	1.4233	1.4617	1.4976	s

TABLE 3—SUPERHEATED VAPOR

Pressure lb.		Liquid	Sat. Vapor	Degrees of Superheat								
				10	20	30	40	50	60	70	80	90
17	t	−21.9		−11.9	−1.9	+8.1	18.1	28.1	38.1	48.1	58.1	68.1
	v	0.024	15.6	16.1	16.5	16.9	17.3	17.7	18.1	18.5	18.9	19.3
	i	−56.5	532.2	537.8	543.3	548.7	554.0	559.3	564.5	569.6	574.7	579.8
	s	−0.1215	1.2235	1.2361	1.2482	1.2598	1.2710	1.2819	1.2925	1.3028	1.3128	1.322
18	t	−19.8		−9.8	+0.2	10.2	20.2	30.2	40.2	50.2	60.2	70.2
	v	0.024	14.8	15.3	15.7	16.1	16.4	16.8	17.2	17.6	17.9	18.3
	i	−54.4	532.8	538.5	544.0	549.4	554.8	560.1	565.3	570.4	575.6	580.7
	s	−0.1165	1.2185	1.2311	1.2432	1.2548	1.2661	1.2770	1.2876	1.2978	1.3078	1.317
19	t	−17.8		−7.8	+2.2	12.2	22.2	32.2	42.2	52.2	62.2	72.2
	v	0.024	14.1	14.5	14.9	15.3	15.6	16.0	16.4	16.7	17.1	17.4
	i	−52.3	533.4	539.1	544.6	550.0	555.4	560.7	566.0	571.2	576.4	581.5
	s	−0.1119	1.2137	1.2263	1.2384	1.2501	1.2614	1.2723	1.2829	1.2931	1.3031	1.312
20	t	−15.9		−5.9	+4.1	14.1	24.1	34.1	44.1	54.1	64.1	74.1
	v	0.024	13.4	13.8	14.2	14.5	14.9	15.2	15.6	15.9	16.3	16.6
	i	−50.3	534.0	539.7	545.2	550.7	556.1	561.4	566.7	571.9	577.1	582.3
	s	−0.1075	1.2092	1.2218	1.2339	1.2456	1.2569	1.2678	1.2784	1.2886	1.2986	1.308
21	t	−14.0		−4.0	+6.0	16.0	26.0	36.0	46.0	56.0	66.0	76.0
	v	0.024	12.8	13.2	13.6	13.9	14.2	14.6	14.9	15.2	15.5	15.8
	i	−48.3	534.6	540.3	545.9	551.4	556.8	562.1	567.4	572.6	577.8	583.0
	s	−0.1032	1.2049	1.2176	1.2298	1.2415	1.2527	1.2636	1.2742	1.2844	1.2944	1.3041
22	t	−12.2		−2.2	+7.8	17.8	27.8	37.8	47.8	57.8	67.8	77.8
	v	0.024	12.3	13.7	13.0	13.3	13.6	13.9	14.2	14.5	14.9	15.2
	i	−46.5	535.1	540.9	546.5	552.0	557.4	562.8	568.1	573.3	578.5	583.7
	s	−0.0990	1.2008	1.2135	1.2257	1.2375	1.2487	1.2596	1.2702	1.2804	1.2904	1.3001
23	t	−10.5		−0.5	+9.5	19.5	29.5	39.5	49.5	59.5	69.5	79.5
	v	0.024	11.8	12.1	12.4	12.8	13.1	13.4	13.7	14.0	14.3	14.5
	i	−44.7	535.6	541.4	547.0	552.5	558.0	563.4	568.7	574.0	579.2	584.4
	s	−0.0950	1.1969	1.2096	1.2218	1.2336	1.2449	1.2558	1.2664	1.2766	1.2865	1.2962
24	t	−8.8		+1.2	11.2	21.2	31.2	41.2	51.2	61.2	71.2	81.2
	v	0.024	11.3	11.7	12.0	12.3	12.6	12.8	13.1	13.4	13.7	14.0
	i	−42.9	536.1	541.9	547.6	553.1	558.6	564.0	569.3	574.6	579.8	585.0
	s	−0.0912	1.1931	1.2159	1.2182	1.2300	1.2412	1.2521	1.2627	1.2729	1.2828	1.2925
25	t	−7.2		+2.8	12.8	22.8	32.8	42.8	52.8	62.8	72.8	82.8
	v	0.024	10.9	11.2	11.5	11.8	12.1	12.4	12.7	12.9	13.2	13.5
	i	−41.3	536.5	542.4	548.1	553.6	559.1	564.5	569.9	575.2	580.4	585.6
	s	−0.0874	1.1896	1.2024	1.2147	1.2264	1.2377	1.2486	1.2591	1.2693	1.2793	1.2890
26	t	−5.7		+4.3	14.3	24.3	34.3	44.3	54.3	64.3	74.3	84.3
	v	0.024	10.5	10.8	11.1	11.4	11.6	11.9	12.2	12.4	12.7	13.0
	i	−39.7	536.9	542.8	548.5	554.1	559.6	565.0	570.4	575.7	581.0	586.2
	s	−0.0840	1.1861	1.1990	1.2113	1.2230	1.2342	1.2451	1.2557	1.2659	1.2759	1.2856
27	t	−4.2		+5.8	15.8	25.8	35.8	45.8	55.8	65.8	75.8	85.8
	v	0.024	10.1	10.4	10.7	11.0	11.2	11.5	11.7	12.0	12.3	12.5
	i	−38.1	537.4	543.2	549.0	554.6	560.1	565.6	571.0	576.3	581.6	586.8
	s	−0.0805	1.1828	1.1957	1.2080	1.2197	1.2310	1.2418	1.2524	1.2626	1.2726	1.2823
28	t	−2.7		+7.3	17.3	27.3	37.3	47.3	57.3	67.3	77.3	87.3
	v	0.024	9.78	10.08	10.35	10.60	10.85	11.10	11.35	11.60	11.85	12.09
	i	−36.5	537.8	543.7	549.5	555.1	560.6	566.1	571.5	576.8	582.1	587.4
	s	−0.0771	1.1797	1.1926	1.2050	1.2167	1.2279	1.2388	1.2494	1.2596	1.2695	1.2792

TABLE 3—SUPERHEATED VAPOR

Degrees of Superheat											Pressure lb.
100	110	120	130	140	150	160	180	200	250	300	
78.1	88.1	98.1	108.1	118.1	128.1	138.1	158.1	178.1	228.1	278.1	t 17
19.7	20.1	20.5	20.9	21.3	21.6	22.0	22.8	23.5	25.4	27.3	v
584.9	590.0	595.1	600.1	605.1	610.1	615.1	625.2	635.3	660.6	686.2	i
1.3321	1.3414	1.3505	1.3595	1.3683	1.3769	1.3853	1.4019	1.4179	1.4562	1.4921	s
80.2	90.2	100.2	110.2	120.2	130.2	140.2	160.2	180.2	230.2	280.2	t 18
18.7	19.1	19.4	19.8	20.1	20.5	20.9	21.6	22.3	24.1	25.8	v
585.8	590.9	595.9	601.0	606.0	611.0	616.1	626.1	636.2	661.6	687.2	i
1.3271	1.3364	1.3455	1.3543	1.3632	1.3718	1.3802	1.3968	1.4128	1.4510	1.4868	s
82.2	92.2	102.2	112.2	122.2	132.2	142.2	162.2	182.2	232.2	282.2	t 19
17.8	18.1	18.5	18.8	19.1	19.5	19.8	20.5	21.2	22.9	24.5	v
586.6	591.7	596.8	601.8	606.9	611.9	616.9	627.0	637.1	662.5	688.1	i
1.3224	1.3316	1.3407	1.3496	1.3584	1.3670	1.3754	1.3920	1.4079	1.4460	1.4818	s
84.1	94.1	104.1	114.1	124.1	134.1	144.1	164.1	184.1	234.1	284.1	t 20
16.9	17.3	17.6	17.9	18.2	18.6	18.9	19.5	20.2	21.8	23.4	v
587.4	592.5	597.5	602.6	607.7	612.7	617.8	627.9	638.0	663.4	689.1	i
1.3178	1.3271	1.3362	1.3451	1.3539	1.3625	1.3709	1.3874	1.4033	1.4413	1.4771	s
86.0	96.0	106.0	116.0	126.0	136.0	146.0	166.0	186.0	236.0	286.0	t 21
16.2	16.5	16.8	17.1	17.4	17.7	18.0	18.7	19.3	20.8	22.3	v
588.1	593.3	598.4	603.4	608.5	613.5	618.6	628.7	638.9	664.3	690.0	i
1.3136	1.3229	1.3319	1.3408	1.3496	1.3582	1.3666	1.3831	1.3990	1.4370	1.4727	s
87.8	97.8	107.8	117.8	127.8	137.8	147.8	167.8	187.8	237.8	287.8	t 22
15.5	15.8	16.1	16.4	16.7	17.0	17.3	17.8	18.4	19.9	21.3	v
588.8	594.0	599.1	604.2	609.3	614.3	619.4	629.5	639.7	665.2	690.9	i
1.3096	1.3189	1.3279	1.3368	1.3455	1.3541	1.3625	1.3789	1.3948	1.4327	1.4684	s
89.5	99.5	109.5	119.5	129.5	139.5	149.5	169.5	189.5	239.5	289.5	t 23
14.8	15.1	15.4	15.7	16.0	16.3	16.5	17.1	17.7	19.1	20.5	v
589.5	594.7	599.8	604.9	610.0	615.0	620.1	630.3	640.5	666.0	691.7	i
1.3057	1.3150	1.3240	1.3329	1.3416	1.3502	1.3586	1.3750	1.3909	1.4287	1.4643	s
91.2	101.2	111.2	121.2	131.2	141.2	151.2	171.2	191.2	241.2	291.2	t 24
14.3	14.5	14.8	15.1	15.4	15.6	15.9	16.4	17.0	18.3	19.6	v
590.2	595.3	600.5	605.6	610.7	615.8	620.9	631.0	641.2	666.7	692.5	i
1.3020	1.3113	1.3203	1.3292	1.3379	1.3465	1.3549	1.3713	1.3872	1.4249	1.4605	s
92.8	102.8	112.8	122.8	132.8	142.8	152.8	172.8	192.8	242.8	292.8	t 25
13.7	14.0	14.2	14.5	14.8	15.0	15.3	15.8	16.3	17.6	18.9	v
590.8	596.0	601.2	606.3	611.4	616.5	621.6	631.7	641.9	667.5	693.3	i
1.2985	1.3078	1.3168	1.3257	1.3344	1.3429	1.3513	1.3677	1.3835	1.4212	1.4567	s
94.3	104.3	114.3	124.3	134.3	144.3	154.3	174.3	194.3	244.3	294.3	t 26
13.2	13.5	13.7	14.0	14.2	14.5	14.7	15.2	15.7	17.0	18.2	v
591.4	596.6	601.8	606.9	612.0	617.1	622.2	632.4	642.6	668.2	694.0	i
1.2950	1.3043	1.3133	1.3222	1.3309	1.3394	1.3479	1.3642	1.3800	1.4176	1.4531	s
95.8	105.8	115.8	125.8	135.8	145.8	155.8	175.8	195.8	245.8	295.8	t 27
12.8	13.0	13.3	13.5	13.7	14.0	14.2	14.7	15.2	16.4	17.6	v
592.0	597.2	602.4	607.5	612.6	617.7	622.8	633.0	643.3	668.9	694.7	i
1.2917	1.3010	1.3100	1.3189	1.3276	1.3361	1.3445	1.3608	1.3766	1.4142	1.4497	s
97.3	107.3	117.3	127.3	137.3	147.3	157.3	177.3	197.3	247.3	297.3	t 28
12.33	12.56	12.80	13.04	13.27	13.50	13.74	14.20	14.67	15.81	16.96	v
592.6	597.8	603.0	608.1	613.2	618.4	623.5	633.7	644.0	669.6	695.5	i
1.2887	1.2979	1.3069	1.3157	1.3244	1.3330	1.3414	1.3577	1.3735	1.4110	1.4464	s

TABLE 3—SUPERHEATED VAPOR

Pressure lb.		Liquid	Sat. Vapor	Degrees of Superheat 10	20	30	40	50	60	70	80	90
30	t	+0.1		10.1	20.1	30.1	40.1	50.1	60.1	70.1	80.1	90.1
	v	0.024	9.17	9.45	9.70	9.94	10.18	10.41	10.64	10.87	11.10	11.32
	i	−33.6	538.5	544.5	550.3	556.0	561.6	567.1	572.5	577.9	583.2	558.5
	s	−0.0708	1.1738	1.1867	1.1990	1.2107	1.2220	1.2329	1.2435	1.2537	1.2636	1.2733
32	t	2.7		12.7	22.7	32.7	42.7	52.7	62.7	72.7	82.7	92.7
	v	0.024	8.64	8.89	9.12	9.35	95.8	9.80	10.01	10.23	10.44	10.66
	i	−30.8	539.3	545.2	551.1	556.8	567.9	567.9	573.4	578.8	584.1	589.4
	s	−0.0648	1.1683	1.1811	1.1933	1.2050	1.2164	1.2273	1.2378	1.2480	1.2580	1.2677
34	t	5.3		15.3	25.3	35.3	45.3	55.3	65.3	75.3	85.3	95.3
	v	0.025	8.15	8.40	8.62	8.83	9.04	9.25	9.46	9.66	9.86	10.06
	i	−28.2	540.0	546.0	551.9	557.7	563.3	568.8	574.3	579.7	585.1	590.4
	s	−0.0589	1.1630	1.1759	1.1881	1.1999	1.2113	1.2222	1.2327	1.2429	1.2528	1.2625
36	t	7.7		17.7	27.7	37.7	47.7	57.7	67.7	77.7	87.7	97.7
	v	0.025	7.73	7.96	8.17	8.37	8.57	8.77	8.96	9.16	9.35	9.54
	i	−25.6	540.6	546.7	552.6	558.4	564.1	569.7	575.2	580.6	586.0	591.4
	s	−0.0534	1.1580	1.1709	1.1831	1.1948	1.2062	1.2171	1.2276	1.2378	1.2478	1.2576
38	t	10.0		20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
	v	0.025	7.34	7.56	7.76	7.96	8.15	8.33	8.52	8.70	8.88	9.06
	i	−23.2	541.2	547.3	553.3	559.1	564.8	570.5	576.0	581.4	586.8	592.2
	s	−0.0483	1.1534	1.1764	1.1786	1.1904	1.2018	1.2127	1.2232	1.2334	1.2433	1.2530
40	t	12.2		22.2	32.2	42.2	52.2	52.2	72.2	82.2	92.2	102.2
	v	0.025	6.99	7.21	7.40	7.58	7.76	7.94	8.12	8.29	8.46	8.63
	i	−20.8	541.8	547.9	553.9	559.8	565.5	571.2	576.8	582.3	587.7	593.0
	s	−0.0433	1.1490	1.1619	1.1742	1.1861	1.1974	1.2082	1.2188	1.2291	1.2390	1.2487
42	t	14.4		24.4	34.4	44.4	54.4	64.4	74.4	84.4	94.4	104.4
	v	0.025	6.67	6.88	7.07	7.24	7.41	7.58	7.75	7.92	8.09	8.25
	i	−18.6	542.3	548.5	554.6	560.5	566.2	571.9	577.5	583.0	588.5	593.8
	s	−0.0386	1.1447	1.1578	1.1701	1.1820	1.1933	1.2042	1.2147	1.2250	1.2349	1.2446
44	t	16.4		26.4	36.4	46.4	56.4	66.4	76.4	86.4	96.4	106.4
	v	0.025	6.38	6.58	6.76	6.93	7.10	7.26	7.42	7.58	7.74	7.89
	i	−16.4	542.8	549.1	555.2	561.1	566.9	572.6	578.2	583.7	589.2	594.6
	s	−0.0341	1.1406	1.1537	1.1661	1.1779	1.1892	1.2001	1.2107	1.2210	1.2309	1.2406
46	t	18.4		28.4	38.4	48.4	58.4	68.4	78.4	88.4	98.4	108.4
	v	0.025	6.12	6.30	6.48	6.65	6.81	6.96	7.12	7.27	7.42	7.57
	i	−14.3	543.3	549.7	555.8	561.7	567.5	573.2	578.9	584.4	589.9	595.4
	s	−0.0296	1.1369	1.1500	1.1624	1.1742	1.1855	1.1964	1.2070	1.2172	1.2271	1.2368
48	t	20.3		30.3	40.3	50.3	60.3	70.3	80.3	90.3	100.3	110.3
	v	0.025	5.88	6.06	6.23	6.39	6.54	6.69	6.84	6.98	7.13	7.27
	i	−12.3	543.8	550.2	556.3	562.2	568.1	573.8	579.5	585.1	590.6	596.1
	s	−0.0255	1.1331	1.1463	1.1587	1.1706	1.1819	1.1929	1.2035	1.2137	1.2236	1.2333
50	t	22.1		32.1	42.1	52.1	62.1	72.1	82.1	92.1	102.1	112.1
	v	0.025	5.67	5.83	5.99	6.14	6.29	6.44	6.58	6.72	6.86	7.00
	i	−10.3	544.3	550.7	556.8	562.7	568.6	574.4	580.1	585.7	591.2	596.7
	s	−0.0216	1.1296	1.1427	1.1552	1.1671	1.1784	1.1894	1.2000	1.2102	1.2201	1.2298
55	t	26.4		36.4	46.4	56.4	66.4	76.4	86.4	96.4	106.4	116.4
	v	0.025	5.18	5.33	5.47	5.61	5.75	5.88	6.01	6.14	6.27	6.40
	i	−5.7	545.3	551.8	558.0	564.0	569.9	575.8	581.5	587.1	592.7	598.2
	s	−0.0122	1.1215	1.1346	1.1470	1.1589	1.1702	1.1813	1.1919	1.2021	1.2120	1.2217

TABLE 3—SUPERHEATED VAPOR

Degrees of Superheat											Pressure lb.
00	110	120	130	140	150	160	180	200	250	300	
.1	110.1	120.1	130.1	140.1	150.1	160.1	180.1	200.1	250.1	300.1	t 30
1.55	11.77	11.99	12.21	12.43	12.65	12.87	13.30	13.74	14.81	15.88	v
3.7	598.9	604.1	609.3	614.4	619.5	624.7	635.0	645.2	670.9	696.9	i
2827	1.2919	1.3010	1.3098	1.3185	1.3270	1.3384	1.3516	1.3674	1.4049	1.4402	s
2.7	112.7	122.7	132.7	142.7	152.7	162.7	182.7	202.7	252.7	302.7	t 32
.87	11.08	11.28	11.49	11.70	11.90	12.11	12.52	12.92	13.93	14.93	v
4.7	599.9	605.2	610.4	615.5	620.6	625.8	636.1	646.4	672.1	698.1	i
2771	1.2863	1.2954	1.3042	1.3129	1.3214	1.3297	1.3459	1.3617	1.3992	1.4344	s
5.3	115.3	125.3	135.3	145.3	155.3	165.3	185.3	205.3	255.3	305.3	t 34
0.26	10.46	10.66	10.85	11.05	11.24	11.43	11.82	12.20	13.15	14.09	v
5.7	601.0	606.2	611.4	616.6	621.7	626.9	637.2	647.5	673.3	699.3	i
2720	1.2812	1.2902	1.2990	1.3077	1.3162	1.3245	1.3407	1.3565	1.3939	1.4290	s
7.7	117.7	127.7	137.7	147.7	157.7	167.7	187.7	207.7	257.7	307.7	t 36
9.73	9.91	10.10	10.28	10.47	10.65	10.83	11.20	11.56	12.45	13.34	v
6.7	602.0	607.2	612.4	617.6	622.7	627.9	638.3	648.6	674.4	700.5	i
2671	1.2763	1.2853	1.2941	1.3027	1.3112	1.3195	1.3357	1.3514	1.3888	1.4239	s
0.0	120.0	130.0	140.0	150.0	160.0	170.0	190.0	210.0	260.0	310.0	t 38
9.24	9.42	9.60	9.77	9.95	10.12	10.29	10.64	10.98	11.83	12.67	v
7.5	602.8	608.1	613.3	618.5	623.7	628.9	639.3	649.6	675.5	701.6	i
2624	1.2716	1.2806	1.2895	1.2981	1.3065	1.3148	1.3310	1.3467	1.3840	1.4190	s
2.2	122.2	132.2	142.2	152.2	162.2	172.2	192.2	212.2	262.2	312.2	t 40
8.80	8.97	9.14	9.31	9.47	9.64	9.80	10.13	10.46	11.26	12.07	v
8.3	603.6	608.9	614.2	619.4	624.6	629.8	640.2	650.6	676.6	702.7	i
2581	1.2673	1.2763	1.2851	1.2937	1.3021	1.3104	1.3265	1.3422	1.3795	1.4145	s
4.4	124.4	134.4	144.4	154.4	164.4	174.4	194.4	214.4	264.4	314.4	t 42
8.41	8.57	8.73	8.89	9.05	9.21	9.37	9.68	9.99	10.76	11.52	v
9.2	604.5	609.8	615.1	620.3	625.5	630.7	641.1	651.5	677.6	703.7	i
2540	1.2632	1.2721	1.2809	1.2896	1.2980	1.3063	1.3224	1.3380	1.3752	1.4102	s
16.4	126.4	136.4	146.4	156.4	166.4	176.4	196.4	216.4	266.4	316.4	t 44
8.05	8.20	8.36	8.51	8.66	8.81	8.96	9.26	9.56	10.29	11.02	v
0.0	605.3	610.6	615.9	621.1	626.4	631.6	642.0	652.4	678.5	704.7	i
2500	1.2592	1.2681	1.2769	1.2856	1.2940	1.3023	1.3184	1.3340	1.3711	1.4060	s
18.4	128.4	138.4	148.4	158.4	168.4	178.4	198.4	218.4	268.4	318.4	t 46
7.72	7.87	8.01	8.16	8.30	8.45	8.59	8.88	9.16	9.86	10.56	v
0.8	606.1	611.4	616.7	621.9	627.2	632.5	642.9	653.3	679.4	705.6	i
2463	1.2555	1.2644	1.2732	1.2819	1.2903	1.2985	1.3146	1.3302	1.3673	1.4021	s
20.3	130.3	140.3	150.3	160.3	170.3	180.3	200.3	220.3	270.3	320.3	t 48
7.42	7.56	7.70	7.84	7.98	8.12	8.25	8.53	8.80	9.47	10.14	v
01.5	606.8	612.1	617.4	622.7	628.0	633.3	643.7	654.2	680.3	706.5	i
.2427	1.2519	1.2608	1.2696	1.2782	1.2866	1.2948	1.3109	1.3265	1.3635	1.3983	s
22.1	132.1	142.1	152.1	162.1	172.1	182.1	202.1	222.1	272.1	322.1	t 50
7.14	7.27	7.41	7.54	7.68	7.81	7.94	8.20	8.46	9.11	9.76	v
02.1	607.5	612.8	618.1	623.4	628.7	634.0	644.4	655.0	681.1	707.4	i
.2392	1.2484	1.2573	1.2661	1.2747	1.2831	1.2913	1.3074	1.3230	1.3600	1.3947	s
26.4	136.4	146.4	156.4	166.4	176.4	186.4	206.4	226.4	276.4	326.4	t 55
6.52	6.64	6.77	6.89	7.01	7.13	7.25	7.49	7.73	8.32	8.91	v
03.7	609.1	614.5	619.8	625.1	630.4	635.8	646.3	656.8	683.0	709.4	i
.2311	1.2403	1.2492	1.2580	1.2666	1.2750	1.2832	1.2993	1.3148	1.3517	1.3863	s

TABLE 3—SUPERHEATED VAPOR

Pressure lb.		Liquid	Sat. Vapor	Degrees of Superheat 10	20	30	40	50	60	70	80	9
60	t	30.5		40.5	50.5	60.5	70.5	80.5	90.5	100.5	110.5	120
	v	0.025	4.77	4.91	5.04	5.17	5.30	5.42	5.54	5.66	5.77	5
	i	−1.3	546.3	552.8	559.1	565.2	571.2	577.1	582.9	588.6	594.2	599
	s	−0.0033	1.1141	1.1272	1.1396	1.1515	1.1629	1.1740	1.1846	1.1948	1.2047	1.2
65	t	34.3		44.3	54.3	64.3	74.3	84.3	94.3	104.3	114.3	124
	v	0.025	4.42	4.55	4.68	4.80	4.91	5.02	5.13	5.24	5.35	5
	i	2.7	547.2	553.7	560.1	566.3	572.4	578.3	584.1	589.8	595.5	601
	s	0.0051	1.1073	1.1205	1.1329	1.1448	1.1562	1.1673	1.1779	1.1881	1.1980	1.2
70	t	37.9		47.9	57.9	67.9	77.9	87.9	97.9	107.9	117.9	127
	v	0.025	4.12	4.24	4.36	4.47	4.58	4.68	4.79	4.89	4.99	5.
	i	6.6	548.1	554.6	561.1	567.3	573.4	579.4	585.3	591.0	596.7	602
	s	0.0128	1.1010	1.1141	1.1266	1.1385	1.1500	1.1611	1.1717	1.1814	1.1918	1.2
75	t	41.3		51.3	61.3	71.3	81.3	91.3	101.3	111.3	121.3	131.
	v	0.025	3.86	3.97	4.08	4.19	4.29	4.39	4.49	4.58	4.68	4.
	i	10.3	548.8	555.5	562.0	568.3	574.4	580.4	586.4	592.2	597.9	603.
	s	0.0201	1.0951	1.1083	1.1208	1.1328	1.1443	1.1554	1.1660	1.1763	1.1862	1.1
80	t	44.5		54.5	64.5	74.5	84.5	94.5	104.5	114.5	124.5	134.
	v	0.026	3.63	3.74	3.84	3.94	4.04	4.13	4.22	4.31	4.40	4.
	i	13.8	549.5	556.3	562.8	569.1	574.3	581.4	587.4	593.2	599.0	604.
	s	0.0271	1.0897	1.1028	1.1154	1.1274	1.1389	1.1500	1.1606	1.1709	1.1808	1.19
85	t	47.6		57.6	67.6	77.6	97.6	97.6	107.6	117.6	127.6	137.
	v	0.026	3.43	3.53	3.63	3.72	3.81	3.90	3.98	4.07	4.15	4.
	i	17.2	550.2	557.0	563.6	570.0	576.3	582.4	588.3	594.2	600.0	605.
	s	0.0336	1.0846	1.0978	1.1104	1.1224	1.1339	1.1450	1.1556	1.1659	1.1758	1.18
90	t	50.5		60.5	70.5	80.5	90.5	100.5	110.5	120.5	130.5	140.
	v	0.026	3.25	3.34	3.43	3.52	3.61	3.69	3.77	3.85	3.93	4.
	i	20.4	550.9	557.7	564.3	570.8	577.1	583.2	589.2	595.2	601.0	606.
	s	0.0398	1.0797	1.0929	1.1056	1.1177	1.1292	1.1402	1.1509	1.1612	1.1711	1.18
95	t	53.3		63.3	73.3	83.3	93.3	103.3	113.3	123.3	133.3	143.
	v	0.026	3.08	3.17	3.26	3.34	3.43	3.51	3.58	3.66	3.74	3.
	i	23.5	551.5	558.4	565.0	571.5	577.9	584.0	590.1	596.1	601.9	607.
	s	0.0458	1.0752	1.0884	1.1011	1.1132	1.1247	1.1358	1.1465	1.1568	1.1667	1.17
100	t	56.0		66.0	76.0	86.0	96.0	106.0	116.0	126.0	136.0	146.
	v	0.026	2.94	3.02	3.10	3.18	3.26	3.34	3.41	3.49	3.56	3.
	i	26.5	552.1	559.0	565.7	572.2	578.6	584.8	590.9	596.9	602.8	608.
	s	0.0516	1.0709	1.0841	1.0968	1.1089	1.1204	1.1315	1.1422	1.1525	1.1625	1.17
105	t	58.6		68.6	78.6	88.6	98.6	108.6	118.6	128.6	138.6	148.
	v	0.026	2.80	2.88	2.96	3.04	3.12	3.19	3.26	3.33	3.40	3.
	i	29.3	552.6	559.6	566.3	572.9	579.3	585.6	591.7	597.7	603.6	609.
	s	0.0572	1.0669	1.0801	1.0928	1.1049	1.1165	1.1276	1.1383	1.1486	1.1586	1.16
110	t	61.1		71.1	81.1	91.1	101.1	111.1	121.1	131.1	141.1	151.
	v	0.026	2.68	2.76	2.84	2.91	2.98	3.05	3.12	3.18	3.25	3.
	i	32.1	553.1	560.2	573.0	573.6	580.0	586.3	592.5	598.5	604.4	610.
	s	0.0625	1.0630	1.0762	1.0890	1.1011	1.1126	1.1237	1.1345	1.1448	1.1548	1.16
115	t	63.6		73.6	83.6	93.6	103.6	113.6	123.6	133.6	143.6	153.
	v	0.026	2.57	2.65	2.72	2.79	2.86	2.92	2.99	3.05	3.12	3.1
	i	34.8	553.6	560.8	567.6	574.3	580.7	587.0	593.2	599.3	605.2	611.1
	s	0.0678	1.0593	1.0726	1.0854	1.0975	1.1091	1.1203	1.1310	1.1413	1.1513	1.16

TABLE 3—SUPERHEATED VAPOR

Degrees of Superheat											Pressure lb.
0	110	120	130	140	150	160	180	200	250	300	
.5	140.5	150.5	160.5	170.5	180.5	190.5	210.5	230.5	280.5	330.5	**t 60**
.01	6.12	6.23	6.35	6.46	6.57	6.68	6.90	7.12	7.66	8.20	v
.2	610.7	616.1	621.5	626.8	632.1	637.5	648.0	658.6	684.9	711.3	i
238	1.2330	1.2419	1.2506	1.2592	1.2676	1.2758	1.2919	1.3074	1.3442	1.3787	s
.3	144.3	154.3	164.3	174.3	184.3	194.3	214.3	234.3	284.3	334.3	**t 65**
.57	5.67	5.78	5.88	5.99	6.09	6.19	6.40	6.60	7.10	7.60	v
.6	612.1	617.5	622.9	628.3	633.7	639.0	649.6	660.2	686.6	713.1	i
171	1.2263	1.2352	1.2439	1.2524	1.2608	1.2690	1.2851	1.3006	1.3374	1.3718	s
.9	147.9	157.9	167.9	177.9	187.9	197.9	217.9	237.9	287.9	337.9	**t 70**
.19	5.29	5.39	5.49	5.58	5.68	5.77	5.96	6.15	6.62	7.08	v
.9	613.4	618.9	624.3	629.7	635.1	640.5	651.1	661.7	688.2	714.8	i
109	1.2201	1.2291	1.2378	1.2463	1.2547	1.2629	1.2789	1.2943	1.3310	1.3653	s
.3	151.3	161.3	171.3	181.3	191.3	201.3	221.3	241.3	291.3	341.3	**t 75**
.86	4.96	5.05	5.14	5.23	5.32	5.41	5.59	5.76	6.20	6.63	v
.2	614.7	620.2	625.6	631.1	636.5	641.9	652.6	663.2	689.8	716.4	i
2053	1.2145	1.2234	1.2320	1.2405	1.2489	1.2571	1.2731	1.2885	1.3251	1.3594	s
.5	154.5	164.5	174.5	184.5	194.5	204.5	224.5	244.5	294.5	344.5	**t 80**
.57	4.66	4.75	4.83	4.92	5.00	5.08	5.25	5.42	5.83	6.23	v
.3	615.8	621.3	626.8	632.3	637.7	643.1	653.9	664.6	691.2	717.9	i
1999	1.2091	1.2180	1.2267	1.2352	1.2436	1.2518	1.2676	1.2830	1.3196	1.3538	s
7.6	157.6	167.6	177.6	187.6	197.6	207.6	227.6	247.6	297.6	347.6	**t 85**
4.32	4.40	4.48	4.56	4.64	4.72	4.80	4.96	5.11	5.50	5.88	v
1.4	617.0	622.5	628.0	633.5	639.0	644.4	655.2	665.8	692.5	719.3	i
1949	1.2041	1.2130	1.2217	1.2302	1.2385	1.2467	1.2626	1.2780	1.3145	1.3487	s
0.5	160.5	170.5	180.5	190.5	200.5	210.5	230.5	250.5	300.5	350.5	**t 90**
4.09	4.17	4.24	4.32	4.40	4.47	4.55	4.70	4.84	5.21	5.57	v
2.4	618.0	623.6	629.1	634.6	640.1	645.6	656.4	667.0	693.8	720.6	i
1902	1.1994	1.2083	1.2170	1.2255	1.2338	1.2420	1.2579	1.2732	1.3096	1.3437	s
3.3	163.3	173.3	183.3	193.3	203.3	213.3	233.3	253.3	303.3	353.3	**t 95**
3.89	3.96	4.03	4.10	4.18	4.25	4.32	4.46	4.60	4.95	5.29	v
3.4	619.0	624.6	630.2	635.7	641.2	646.7	657.5	668.2	695.1	721.9	i
1858	1.1949	1.2038	1.2125	1.2210	1.2293	1.2375	1.2534	1.2687	1.3051	1.3391	s
6.0	166.0	176.0	186.0	196.0	206.0	216.0	236.0	256.0	306.0	356.0	**t 100**
3.70	3.77	3.84	3.91	3.98	4.05	4.12	4.25	4.38	4.71	5.03	v
4.3	619.9	625.6	631.2	636.7	642.2	647.7	658.6	669.4	696.3	723.2	i
1815	1.1907	1.1996	1.2083	1.2168	1.2251	1.2333	1.2492	1.2645	1.3008	1.3348	s
8.6	168.6	178.6	188.6	198.6	208.6	218.6	238.6	258.6	308.6	358.6	**t 105**
3.53	3.60	3.67	3.73	3.80	3.86	3.93	4.06	4.18	4.49	4.80	v
5.2	620.8	626.5	632.1	637.6	643.2	648.7	659.6	670.5	697.4	724.3	i
1776	1.1867	1.1956	1.2043	1.2138	1.2211	1.2293	1.2451	1.2604	1.2967	1.3307	s
1.1	171.1	181.1	191.1	201.1	211.1	221.1	241.1	261.1	311.1	361.1	**t 110**
3.38	3.44	3.51	3.57	3.63	3.70	3.76	3.88	4.00	4.30	4.60	v
16.1	621.8	627.5	633.1	638.6	644.2	649.7	660.6	671.5	698.5	725.5	i
1738	1.1829	1.1918	1.2005	1.2090	1.2173	1.2255	1.2413	1.2566	1.2928	1.3268	s
3.6	173.6	183.6	193.6	203.6	213.6	223.6	243.6	263.6	313.6	363.6	**t 115**
3.24	3.30	3.36	3.42	3.48	3.54	3.60	3.72	3.84	4.12	4.40	v
16.9	622.7	628.4	634.0	639.6	645.2	650.7	661.6	672.5	699.6	726.6	i
.1703	1.1794	1.1883	1.1970	1.2055	1.2138	1.2220	1.2378	1.2530	1.2892	1.3231	s

TABLE 3—SUPERHEATED VAPOR

Pressure lb.		Liquid	Sat. Vapor	Degrees of Superheat 10	20	30	40	50	60	70	80	9
120	t	65.8		75.8	85.8	95.8	105.8	115.8	125.8	135.8	145.8	155
	v	0.026	2.47	2.54	2.61	2.68	2.74	2.80	2.87	2.93	2.99	3
	i	37.4	554.1	561.3	568.2	574.9	581.4	587.7	593.9	600.0	605.9	611
	s	0.0725	1.0557	1.0690	1.0818	1.0939	1.1055	1.1167	1.1274	1.1377	1.1477	1.1
125	t	68.1		78.1	88.1	98.1	108.1	118.1	128.1	138.1	148.1	158
	v	0.026	2.37	2.44	2.51	2.57	2.64	2.70	2.76	2.82	2.88	2.
	i	40.0	554.6	561.8	568.7	575.4	582.0	588.3	594.6	600.7	606.7	612
	s	0.0773	1.0524	1.0657	1.0785	1.0906	1.1022	1.1134	1.1241	1.1344	1.1444	1.1
130	t	70.4		80.4	90.4	100.4	110.4	120.4	130.4	140.4	150.4	160
	v	0.026	2.28	2.35	2.42	2.48	2.54	2.60	2.66	2.72	2.77	2
	i	42.5	555.0	562.3	569.2	576.0	582.6	589.0	595.2	601.3	607.4	613
	s	0.0820	1.0492	1.0626	1.0754	1.0875	1.0992	1.1104	1.1211	1.1314	1.1414	1.1
135	t	72.5		82.5	92.5	102.5	112.5	122.5	132.5	142.5	152.5	162
	v	0.027	2.20	2.27	2.33	2.39	2.45	2.51	2.56	2.62	2.67	2
	i	45.0	555.5	562.7	569.7	576.5	583.1	589.6	595.8	602.0	608.1	614
	s	0.0865	1.0460	1.0595	1.0723	1.0844	1.0961	1.1073	1.1180	1.1283	1.1383	1.1
140	t	74.5		84.5	94.5	104.5	114.5	124.5	134.5	144.5	154.5	164
	v	0.027	2.12	2.19	2.25	2.31	2.37	2.42	2.48	2.53	2.58	2
	i	47.3	555.9	563.1	570.2	577.0	583.6	590.1	596.4	602.6	608.7	614
	s	0.0910	1.0430	1.0565	1.0693	1.0814	1.0931	1.1043	1.1150	1.1254	1.1354	1.1
145	t	76.5		86.5	96.5	106.5	116.5	126.5	136.5	146.5	156.5	166
	v	0.027	2.06	2.12	2.18	2.23	2.28	2.34	2.39	2.44	2.50	2
	i	49.6	556.3	563.6	570.7	577.5	584.2	590.7	597.0	603.2	609.3	615
	s	0.0952	1.0401	1.0536	1.0664	1.0786	1.0903	1.1015	1.1122	1.1225	1.1325	1.1
150	t	78.5		88.5	98.5	108.5	118.5	128.5	138.5	148.5	158.5	168
	v	0.027	1.99	2.05	2.11	2.16	2.21	2.26	2.32	2.37	2.42	2
	i	51.8	556.7	564.0	571.1	578.0	584.7	591.2	597.5	603.8	609.9	615
	s	0.0993	1.0374	1.0509	1.0637	1.0759	1.0876	1.0988	1.1095	1.1198	1.1298	1.1
155	t	80.4		90.4	100.4	110.4	120.4	130.4	140.4	150.4	160.4	170
	v	0.027	1.93	1.99	2.04	2.09	2.14	2.19	2.24	2.29	2.34	2.
	i	54.0	557.0	564.4	571.6	578.5	585.2	591.7	598.0	604.3	610.5	616.
	s	0.1034	1.0347	1.0482	1.0610	1.0732	1.0849	1.0961	1.1068	1.1172	1.1272	1.1
160	t	82.3		92.3	102.3	112.3	122.3	132.3	142.3	152.3	162.3	172.
	v	0.027	1.87	1.93	1.98	2.03	2.08	2.13	2.18	2.22	2.27	2.
	i	56.2	557.4	564.8	572.0	578.9	585.7	592.2	598.6	604.9	611.1	617.
	s	0.1074	1.0322	1.0457	1.0585	1.0707	1.0824	1.0936	1.1043	1.1147	1.1247	1.1
165	t	84.1		94.1	104.1	114.1	124.1	134.1	144.1	154.1	164.1	174.
	v	0.027	1.81	1.87	1.92	1.97	2.02	2.07	2.11	2.16	2.20	2.
	i	58.4	557.7	565.2	572.4	579.3	586.1	592.7	599.1	605.4	611.6	617.
	s	0.1114	1.0297	1.0432	1.0560	1.0682	1.0799	1.0911	1.1018	1.1123	1.1223	1.1
170	t	85.9		95.9	105.9	115.9	125.9	135.9	145.9	155.9	165.9	175.
	v	0.027	1.76	1.81	1.86	1.91	1.96	2.01	2.06	2.11	2.15	2.
	i	60.5	558.1	565.5	572.8	579.8	586.6	593.2	599.6	606.0	612.2	618.
	s	0.1152	1.0272	1.0407	1.0536	1.0658	1.0775	1.0878	1.0995	1.1099	1.1199	1.12
180	t	89.4		99.4	109.4	119.4	129.4	139.4	149.4	159.4	169.4	179.
	v	0.027	1.67	1.72	1.77	1.81	1.86	1.90	1.94	1.99	2.03	2.
	i	64.6	558.8	566.3	573.6	580.6	587.5	594.1	600.6	607.0	613.2	619.
	s	0.1226	1.0226	1.0362	1.0491	1.0614	1.0731	1.0843	1.0950	1.1054	1.1154	1.12

TABLE 3 — SUPERHEATED VAPOR

	Degrees of Superheat										Pressure lb.
0	110	120	130	140	150	160	180	200	250	300	
1.8	175.8	185.8	195.8	205.8	215.8	225.8	245.8	265.8	315.8	365.8	t 120
.11	3.17	3.23	3.29	3.34	3.40	3.46	3.57	3.68	3.96	4.23	v
6.6	623.4	629.2	634.8	640.4	646.0	651.5	662.5	673.4	700.6	727.6	i
1667	1.1759	1.1848	1.1935	1.2020	1.2103	1.2184	1.2342	1.2494	1.2856	1.3195	s
1.1	178.1	188.1	198.1	208.1	218.1	228.1	248.1	268.1	318.1	368.1	t 125
.00	3.05	3.10	3.16	3.21	3.27	3.33	3.44	3.54	3.81	4.07	v
6.4	624.2	630.0	635.6	641.3	646.9	652.4	663.4	674.4	701.6	728.7	i
1634	1.1726	1.1815	1.1902	1.1987	1.2070	1.2151	1.2309	1.2461	1.2823	1.3161	s
1.4	180.4	190.4	200.4	210.4	220.4	230.4	250.4	270.4	320.4	370.4	t 130
.88	2.94	2.99	3.05	3.10	3.15	3.20	3.31	3.41	3.67	3.92	v
6.2	625.0	630.8	636.4	642.1	647.7	653.3	664.3	675.3	702.6	729.7	i
1604	1.1696	1.1785	1.1872	1.1957	1.2040	1.2121	1.2278	1.2430	1.2791	1.3129	s
1.5	182.5	192.5	202.5	212.5	222.5	232.5	252.5	272.5	322.5	372.5	t 135
.78	2.83	2.89	2.94	2.99	3.04	3.09	3.19	3.29	3.54	3.78	v
6.9	625.7	631.5	637.2	642.9	648.5	654.1	665.1	676.1	703.5	730.6	i
1574	1.1665	1.1754	1.1841	1.1926	1.2009	1.2090	1.2248	1.2400	1.2760	1.3098	s
.5	184.5	194.5	204.5	214.5	224.5	234.5	254.5	274.5	324.5	374.5	t 140
.69	2.74	2.79	2.84	2.89	2.94	2.99	3.08	3.17	3.41	3.65	v
.6	626.4	632.2	637.9	643.6	649.3	654.9	665.9	676.9	704.3	731.5	i
544	1.1635	1.1724	1.1811	1.1896	1.1979	1.2060	1.2218	1.2370	1.2730	1.3067	s
.5	186.5	196.5	206.5	216.5	226.5	236.5	256.5	296.5	326.5	376.5	t 145
.60	2.65	2.70	2.74	2.79	2.84	2.89	2.98	3.07	3.30	3.53	v
.2	627.1	632.9	638.7	644.4	650.0	655.6	666.7	677.8	705.2	732.4	i
516	1.1607	1.1696	1.1783	1.1868	1.1951	1.2032	1.2190	1.2342	1.2701	1.3038	s
.5	188.5	198.5	208.5	218.5	228.5	238.5	258.5	278.5	328.5	378.5	t 150
.51	2.56	2.61	2.66	2.70	2.75	2.80	2.89	2.97	3.19	3.41	v
.9	627.8	633.6	639.4	645.1	650.7	656.3	667.5	678.6	706.0	733.3	i
489	1.1580	1.1669	1.1757	1.1841	1.1924	1.2005	1.2163	1.2315	1.2674	1.3010	s
.4	190.4	200.4	210.4	220.4	230.4	240.4	260.4	280.4	330.4	380.4	t 155
.44	2.48	2.53	2.57	2.62	2.66	2.71	2.80	2.88	3.10	3.31	v
.5	628.4	634.2	640.0	645.7	651.4	657.0	668.2	679.3	706.8	734.2	i
463	1.1554	1.1643	1.1730	1.1814	1.1897	1.1978	1.2136	1.2288	1.2647	1.2983	s
.3	192.3	202.3	212.3	222.3	232.3	242.3	262.3	282.3	332.3	382.3	t 160
.36	2.41	2.45	2.50	2.54	2.58	2.63	2.71	2.79	3.00	3.21	v
.2	629.1	634.9	641.7	646.4	652.1	657.7	669.0	680.1	707.6	735.0	i
438	1.1529	1.1618	1.1705	1.1789	1.1872	1.1954	1.2112	1.2263	1.2622	1.2957	s
1.1	194.1	204.1	214.1	224.1	234.1	244.1	264.1	284.1	334.1	384.1	t 165
.29	2.34	2.38	2.42	2.47	2.51	2.55	2.63	2.71	2.91	3.10	v
3.7	629.7	635.5	641.3	647.1	652.8	658.4	669.7	680.8	708.4	735.8	i
414	1.1505	1.1594	1.1681	1.1765	1.1848	1.1929	1.2086	1.2237	1.2596	1.2931	s
5.9	195.9	205.9	215.9	225.9	235.9	245.9	265.9	285.9	335.9	385.9	t 170
2.23	2.27	2.31	2.36	2.40	2.44	2.48	2.56	2.64	2.83	3.02	v
4.3	630.3	636.1	641.9	647.7	653.4	659.0	670.3	681.5	709.1	736.6	i
1390	1.1481	1.1570	1.1657	1.1741	1.1824	1.1905	1.2062	1.2213	1.2572	1.2907	s
9.4	199.4	209.4	219.4	229.4	239.4	249.4	269.4	289.4	339.4	389.4	t 180
2.11	2.15	2.19	2.23	2.27	2.31	2.35	2.42	2.49	2.68	2.86	v
5.4	631.4	637.3	643.1	648.9	654.7	660.3	671.6	682.8	710.5	738.1	i
1346	1.1436	1.1525	1.1612	1.1696	1.1779	1.1860	1.2017	1.2168	1.2527	1.2861	s

TABLE 3—SUPERHEATED VAPOR

Pressure lb.	Liquid	Sat. Vapor	Degrees of Superheat								
			10	20	30	40	50	60	70	80	9
190 t	92.7		102.7	112.7	122.7	132.7	142.7	152.7	162.7	172.7	182.
v	0.027	1.58	1.63	1.68	1.72	1.76	1.80	1.84	1.88	1.92	1.
i	68.6	559.4	567.0	574.3	581.4	588.3	595.0	601.5	607.9	614.2	620.
s	0.1296	1.0182	1.0319	1.0448	1.0571	1.0688	1.0800	1.0908	1.1012	1.1112	1.1
200 t	95.9		105.9	115.9	125.9	135.9	145.9	155.9	165.9	175.9	185.
v	0.027	1.50	1.55	1.59	1.63	1.67	1.71	1.75	1.79	1.83	1.
i	72.3	560.0	567.6	575.0	582.2	589.1	595.8	602.4	608.8	615.2	621.
s	0.1363	1.0141	1.0278	1.0407	1.0530	1.0648	1.0760	1.0868	1.0972	1.1072	1.11
210 t	98.9		108.9	118.9	128.9	138.9	148.9	158.9	168.9	178.9	188.
v	0.027	1.43	1.48	1.52	1.56	1.60	1.64	1.67	1.71	1.75	1.
i	76.0	560.5	568.2	575.7	582.9	589.8	596.6	603.2	609.7	616.1	622.
s	0.1427	1.0102	1.0239	1.0368	1.0491	1.0609	1.0721	1.0829	1.0933	1.1033	1.11
220 t	101.8		111.8	121.8	131.8	141.8	151.8	161.8	171.8	181.8	191.
v	0.028	1.37	1.41	1.45	1.49	1.53	1.56	1.60	1.64	1.67	1.
i	79.5	561.0	568.8	576.3	583.5	590.5	597.3	603.9	610.5	616.9	623.
s	0.1488	1.0065	1.0202	1.0332	1.0455	1.0573	1.0685	1.0793	1.0897	1.0997	1.10
230 t	104.7		114.7	124.7	134.7	144.7	154.7	164.7	174.7	184.7	194.
v	0.028	1.31	1.35	1.39	1.43	1.46	1.50	1.53	1.57	1.60	1.
i	83.0	561.5	569.4	576.9	584.1	591.2	598.0	604.7	611.3	617.7	624.
s	0.1549	1.0030	1.0167	1.0297	1.0421	1.0539	1.0651	1.0759	1.0863	1.0963	1.10
240 t	107.4		117.4	127.4	137.4	147.4	157.4	167.4	177.4	187.4	197.
v	0.028	1.26	1.30	1.33	1.37	1.40	1.44	1.47	1.50	1.53	1.
i	86.4	562.0	569.9	577.4	584.7	591.8	598.7	605.4	612.0	618.5	624.
s	0.1609	0.9997	1.0134	1.0264	1.0387	1.0505	1.0617	1.0725	1.0829	1.0930	1.10
250 t	110.1		120.1	130.1	140.1	150.1	160.1	170.1	180.1	190.1	200.
v	0.028	1.21	1.25	1.28	1.31	1.35	1.38	1.41	1.44	1.47	1.
i	89.7	562.5	570.4	578.0	585.3	592.4	599.4	606.1	612.8	619.3	625.
s	0.1666	0.9965	1.0102	1.0232	1.0356	1.0474	1.0587	1.0695	1.0799	1.0899	1.09
260 t	112.7		122.7	132.7	142.7	152.7	162.7	172.7	182.7	192.7	202.
v	0.028	1.16	1.20	1.23	1.26	1.30	1.33	1.36	1.39	1.42	1.
i	93.0	562.9	570.9	578.5	585.9	593.0	600.0	606.8	613.5	620.0	626.
s	0.1722	0.9934	1.0072	1.0202	1.0326	1.0444	1.0556	1.0664	1.0768	1.0869	1.09
270 t	115.2		125.2	135.2	145.2	155.2	165.2	175.2	185.2	195.2	205.
v	0.028	1.12	1.15	1.18	1.22	1.25	1.28	1.31	1.34	1.37	1.
i	96.1	563.4	571.4	579.0	586.4	593.6	600.6	607.4	614.1	620.7	627.
s	0.1777	0.9905	1.0043	1.0173	1.0297	1.0415	1.0527	1.0635	1.0739	1.0840	1.09
280 t	117.6		127.6	137.6	147.6	157.6	167.6	177.6	187.6	197.6	207.
v	0.028	1.08	1.11	1.14	1.17	1.20	1.23	1.26	1.29	1.32	1.
i	99.2	563.8	571.8	579.4	586.9	594.2	601.2	608.0	614.7	621.3	627.
s	0.1829	0.9877	1.0015	1.0144	1.0268	1.0386	1.0498	1.0606	1.0711	1.0812	1.09
290 t	120.0		130.0	140.0	150.0	160.0	170.0	180.0	190.0	200.0	210.
v	0.028	1.04	1.07	1.10	1.13	1.16	1.19	1.22	1.25	1.27	1.
i	102.3	564.2	572.2	579.9	587.4	594.7	601.8	608.7	615.4	622.0	628.
s	0.1882	0.9850	0.9987	1.0117	1.0241	1.0359	1.0472	1.0580	1.0685	1.0786	1.08
300 t	122.4		132.4	142.4	152.4	162.4	172.4	182.4	192.4	202.4	212.
v	0.029	1.01	1.04	1.07	1.10	1.13	1.15	1.18	1.21	1.23	1.
i	105.3	564.6	572.7	580.5	588.0	595.3	602.4	609.3	616.1	622.7	629.
s	0.1932	0.9824	0.9962	1.0093	1.0217	1.0335	1.0448	1.0556	1.0660	1.0761	1.08

TABLE 3—SUPERHEATED VAPOR

Degrees of Superheat											Pressure lb.
00	110	120	130	140	150	160	180	200	250	300	
.7	202.7	212.7	222.7	232.7	242.7	252.7	272.7	292.7	342.7	392.7	t 190
.00	2.04	2.08	2.12	2.15	2.19	2.23	2.30	2.37	2.55	2.72	v
.5	632.5	638.4	644.3	650.1	655.9	661.6	672.9	684.1	711.9	739.6	i
1303	1.1394	1.1483	1.1570	1.1654	1.1737	1.1818	1.1975	1.2126	1.2483	1.2817	s
.9	205.9	215.9	225.9	235.9	245.9	255.9	275.9	295.9	345.9	395.9	t 200
1.90	1.94	1.97	2.01	2.04	2.08	2.11	2.18	2.25	2.41	2.57	v
.5	633.5	639.5	645.4	651.2	657.0	662.7	674.1	685.4	713.3	741.0	i
1263	1.1354	1.1443	1.1530	1.1614	1.1697	1.1778	1.1935	1.2085	1.2442	1.2776	s
.9	208.9	218.9	228.9	238.9	248.9	258.9	278.9	298.9	348.9	398.9	t 210
1.82	1.85	1.89	1.92	1.96	1.99	2.02	2.09	2.15	2.31	2.47	v
.4	634.5	640.5	646.4	652.3	658.1	663.8	675.2	686.6	714.5	742.2	i
1224	1.1316	1.1405	1.1492	1.1576	1.1659	1.1740	1.1897	1.2047	1.2403	1.2736	s
1.8	211.8	221.8	231.8	241.8	251.8	261.8	281.8	301.8	351.8	401.8	t 220
1.74	1.77	1.80	1.84	1.87	1.90	1.93	2.00	2.06	2.21	2.36	v
.3	635.4	641.4	647.4	653.3	659.1	664.9	676.3	687.7	715.7	743.4	v
1188	1.1280	1.1369	1.1455	1.1539	1.1622	1.1703	1.1860	1.2010	1.2366	1.2699	s
1.7	214.7	224.7	234.7	244.7	254.7	264.7	284.7	304.7	354.7	404.7	t 230
1.67	1.70	1.73	1.76	1.79	1.82	1.85	1.91	1.97	2.12	2.26	v
.2	636.3	642.3	648.3	654.2	660.1	665.9	677.4	688.8	716.8	744.6	i
1154	1.1245	1.1334	1.1421	1.1505	1.1588	1.1669	1.1826	1.1976	1.2331	1.2664	s
.4	217.4	227.4	237.4	247.4	257.4	267.4	287.4	307.4	357.4	407.4	t 240
1.60	1.63	1.66	1.69	1.72	1.75	1.78	1.84	1.89	2.03	2.17	v
1.1	637.2	643.2	649.2	655.1	661.0	666.9	678.4	689.8	717.9	745.8	i
1121	1.1212	1.1301	1.1388	1.1472	1.1555	1.1635	1.1792	1.1943	1.2298	1.2630	s
.1	220.1	230.1	240.1	250.1	260.1	270.1	290.1	310.1	360.1	410.1	t 250
1.53	1.56	1.59	1.62	1.65	1.68	1.71	1.76	1.81	1.95	2.08	v
1.9	638.0	644.1	650.1	656.0	661.9	667.8	679.4	690.8	719.0	747.0	i
1090	1.1182	1.1271	1.1358	1.1442	1.1524	1.1604	1.1761	1.1911	1.2266	1.2598	s
.7	222.7	232.7	242.7	252.7	262.7	272.7	292.7	312.7	362.7	412.7	t 260
1.48	1.51	1.53	1.56	1.59	1.62	1.65	1.70	1.75	1.88	2.01	v
.7	638.8	644.9	651.0	656.9	662.8	668.7	680.3	691.8	720.1	748.1	i
1060	1.1151	1.1240	1.1327	1.1411	1.1493	1.1574	1.1731	1.1881	1.2236	1.2567	s
.2	225.2	235.2	245.2	255.2	265.2	275.2	295.2	315.2	365.2	415.2	t 270
1.42	1.45	1.48	1.50	1.53	1.56	1.58	1.64	1.69	1.81	1.93	v
.4	639.6	645.7	651.8	657.7	663.7	669.6	681.2	692.7	721.1	749.1	i
1032	1.1123	1.1211	1.1298	1.1382	1.1464	1.1545	1.1702	1.1852	1.2207	1.2537	s
.6	227.6	237.6	247.6	257.6	267.6	277.6	297.6	317.6	367.6	417.6	t 280
1.37	1.40	1.43	1.45	1.48	1.50	1.53	1.58	1.63	1.75	1.87	v
1.1	640.3	646.5	652.5	658.5	664.5	670.4	682.1	693.6	722.0	750.1	i
1004	1.1095	1.1184	1.1271	1.1355	1.1437	1.1518	1.1674	1.1824	1.2178	1.2508	s
.0	230.0	240.0	250.0	260.0	270.0	280.0	300.0	320.0	370.0	420.0	t 290
1.33	1.35	1.38	1.40	1.43	1.45	1.48	1.53	1.57	1.69	1.80	v
1.8	641.1	647.2	653.3	659.3	665.3	671.2	682.9	694.5	722.9	751.1	i
0977	1.1068	1.1157	1.1244	1.1328	1.1411	1.1492	1.1647	1.1797	1.2151	1.2481	s
.4	232.4	242.2	252.4	262.4	272.4	282.4	302.4	322.4	372.4	422.4	t 300
1.28	1.31	1.33	1.36	1.38	1.41	1.43	1.48	1.52	1.63	1.74	v
.5	641.8	648.0	654.1	660.1	666.1	672.0	683.8	695.4	723.9	752.1	i
0953	1.1044	1.1132	1.1219	1.1303	1.1386	1.1467	1.1622	1.1772	1.2125	1.2455	s

TABLE 3—SUPERHEATED VAPOR

Pressure lb.		Liquid	Sat. Vapor	Degrees of Superheat								
				10	20	30	40	50	60	70	80	9
320	t	126.8		136.8	146.8	156.8	166.8	176.8	186.8	196.8	206.8	216
	v	0.029	0.94	0.97	1.00	1.03	1.06	1.08	1.11	1.13	1.16	1
	i	111.1	565 3	573.4	581.3	588.9	596.2	603.4	610.4	617.2	623.8	630
	s	0.2030	0.9775	0.9912	1.0043	1.0167	1.0285	1.0398	1.0506	1.0611	1.0712	1.0
340	t	131.1		141.1	151.1	161.1	171.1	181.1	191.1	201.1	211.1	221
	v	0.029	0.89	0.92	0.94	0.97	0.99	1.02	1.40	1.06	1.09	1
	i	116.8	566.1	574.2	582.1	589.8	597.2	604.4	611.4	618.3	625.0	631
	s	0.2125	0.9730	0.9867	0.9998	1.0121	1.0239	1.0352	1.0461	1.0566	1.0667	1.0
360	t	135.2		145.2	155.2	165.2	175.2	185.2	195.2	205.2	215.2	225
	v	0.029	0.84	0.86	0.89	0.91	0.94	0.96	0.98	1.01	1.03	1
	i	122.3	566.7	574.9	582.9	590.6	598.1	605.4	612.4	619.3	626.1	632
	s	0.2216	0.9687	0.9824	0.9955	1.0079	1.0197	1.0311	1.0419	1.0524	1.0625	1.0
380	t	139.2		149.2	159.2	169.2	179.2	189.2	199.2	209.2	219.2	229
	v	0.029	0.79	0.82	0.84	0.87	0.89	0.91	0.93	0.95	0.97	0
	i	127.7	567.3	575.6	583.7	591.4	599.0	606.3	613.3	620.3	627.1	633
	s	0.2305	0.9647	0.9785	0.9916	1.0040	1.0158	1.0271	1.0380	1.0485	1.0586	1.0
400	t	142.9		152.9	162.9	172.9	182.9	192.9	202.9	212.9	222.9	232
	v	0.029	0.75	0.78	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0
	i	132.9	567.9	576.3	584.4	592.2	599.8	607.1	614.2	621.2	628.0	634
	s	0.2390	0.9608	0.9746	0.9877	1.0001	1.0119	1.0232	1.0341	1.0446	1.0547	1.0
425	t	147.5		157.5	167.5	177.5	187.5	197.5	207.5	217.5	227.5	237
	v	0.030	0.71	0.73	0.75	0.77	0.79	0.81	0.83	0.85	0.87	0
	i	139.8	568.6	577.1	585.2	593.1	600.7	608.1	615.3	622.3	629.2	636
	s	0.2492	0.9565	0.9703	0.9834	0.9958	1.0076	1.0190	1.0299	1.0404	1.0505	1.0
450	t	151.9		161.9	171.9	181.9	191.9	201.9	211.9	221.9	231.9	241
	v	0.030	0.67	0.69	0.71	0.73	0.75	0.77	0.79	0.80	0.82	0
	i	145.6	569.3	577.8	586.0	594.0	601.6	609.1	616.3	623.3	630.3	637
	s	0.2593	0.9523	0.9662	0.9793	0.9917	1.0035	1.0149	1.0258	1.0363	1.0464	1.0
475	t	156.1		166.1	176.1	186.1	196.1	206.1	216.1	226.1	236.1	246
	v	0.030	0.63	0.65	0.67	0.69	0.71	0.72	0.74	0.76	0.78	0
	i	151.6	569.9	578.5	586.8	594.8	602.5	610.0	617.3	624.3	631.3	638
	s	0.2690	0.9485	0.9624	0.9755	0.9880	0.9998	1.0112	1.0220	1.0325	1.0426	1.0
500	t	160.0		170.0	180.0	190.0	200.0	210.0	220.0	230.0	240.0	250
	v	0.031	0.60	0.62	0.64	0.65	0.67	0.69	0.70	0.72	0.74	0
	i	157.5	570.5	579.2	587.5	595.5	603.3	610.8	618.2	625.3	632.2	639
	s	0.2786	0.9449	0.9587	0.9718	0.9842	0.9960	1.0074	1.0183	1.0287	1.0388	1.0
550	t	167.6		177.6	187.6	197.6	207.6	217.6	227.6	237.6	247.6	257
	v	0.031	0.54	0.56	0.57	0.59	0.61	0.62	0.64	0.65	0.67	0
	i	169.2	571.7	580.4	588.8	596.9	604.8	612.4	619.8	627.0	634.0	641
	s	0.2965	0.9383	0.9521	0.9651	0.9775	0.9893	1.0007	1.0116	1.0221	1.0322	1.0
600	t	174.7		184.7	194.7	204.7	214.7	224.7	234.7	244.7	254.7	264
	v	0.032	0.49	0.51	0.52	0.54	0.55	0.57	0.58	0.60	0.61	0
	i	180.4	572.7	581.5	590.0	598.2	606.1	613.8	621.2	628.5	635.7	642
	s	0.3138	0.9323	0.9460	0.9591	0.9715	0.9833	0.9947	1.0056	1.0161	1.0263	1.0

TABLE 3—SUPERHEATED VAPOR

Degrees of Superheat											Pres-sure lb.
100	110	120	130	140	150	160	180	200	250	300	
26.8	236.8	246.8	256.8	266.8	276.8	286.8	306.8	326.8	376.8	426.8	t 320
1.20	1.23	1.25	1.27	1.30	1.32	1.34	1.39	1.43	1.54	1.64	v
36.8	643.1	649.3	655.4	661.5	667.5	673.5	685.3	696.9	725.6	753.9	i
.0903	1.0994	1.1083	1.1170	1.1254	1.1337	1.1417	1.1572	1.1722	1.2075	1.2404	s
31.1	241.1	251.1	261.1	271.1	281.1	291.1	311.1	331.1	381.1	431.1	t 340
1.13	1.16	1.18	1.20	1.22	1.24	1.26	1.30	1.34	1.44	1.54	v
38.0	644.3	650.6	656.8	662.9	668.9	674.9	686.8	698.4	727.2	755.6	i
.0859	1.0950	1.1039	1.1125	1.1209	1.1292	1.1372	1.1527	1.1677	1.2030	1.2358	s
35.2	245.2	255.2	265.2	275.2	285.2	295.2	315.2	335.2	385.2	435.2	t 360
1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.23	1.27	1.37	1.46	v
39.2	645.5	651.8	658.1	664.2	670.3	676.3	688.2	699.9	728.8	757.3	i
.0816	1.0907	1.0996	1.1082	1.1166	1.1249	1.1329	1.1484	1.1634	1.1987	1.2314	s
39.2	249.2	259.2	269.2	279.2	289.2	299.2	319.2	339.2	389.2	439.2	t 380
1.01	1.03	1.05	1.07	1.09	1.11	1.13	1.17	1.21	1.30	1.38	v
40.3	646.7	653.0	659.3	665.4	671.5	677.6	689.5	701.3	730.3	758.9	i
.0777	1.0868	1.0957	1.1043	1.1127	1.1210	1.1290	1.1445	1.1595	1.1947	1.2274	s
42.9	252.9	262.9	272.9	282.9	292.9	302.9	322.9	342.9	392.9	442.9	t 400
0.96	0.98	1.00	1.02	1.04	1.06	1.07	1.11	1.14	1.23	1.31	v
41.3	647.7	654.1	660.4	666.6	672.7	678.8	690.7	702.6	731.7	760.4	i
.0739	1.0830	1.0919	1.1005	1.1089	1.1171	1.1251	1.1406	1.1556	1.1907	1.2234	s
47.5	257.5	267.5	277.5	287.5	297.5	307.5	327.5	347.5	397.5	447.5	t 425
0.91	0.93	0.94	0.96	0.98	0.99	1.01	1.05	1.08	1.16	1.24	v
42.5	649.0	655.4	661.8	668.0	674.2	680.3	692.3	704.2	733.4	762.2	i
.0696	1.0787	1.0875	1.0961	1.1045	1.1128	1.1208	1.1363	1.1511	1.1862	1.2189	s
51.9	261.9	271.9	281.9	291.9	301.9	311.9	331.9	351.9	401.9	451.9	t 450
0.85	0.87	0.89	0.91	0.92	0.94	0.95	0.99	1.02	1.10	1.17	v
43.7	650.2	656.7	663.1	669.3	675.5	681.7	693.8	705.7	735.0	763.8	i
.0656	1.0747	1.0835	1.0921	1.1005	1.1087	1.1167	1.1322	1.1470	1.1821	1.2147	s
56.1	266.1	276.1	286.1	296.1	306.1	316.1	336.1	356.1	406.1	456.1	t 475
0.81	0.83	0.84	0.86	0.87	0.89	0.90	0.93	0.96	1.04	1.11	v
44.8	651.4	657.8	664.2	670.5	676.8	682.9	695.1	707.1	736.5	765.4	i
.0617	1.0708	1.0796	1.0883	1.0967	1.1048	1.1127	1.1282	1.1430	1.1780	1.2106	s
60.0	270.0	280.0	290.0	300.0	310.0	320.0	340.0	360.0	410.0	460.0	t 500
0.77	0.78	0.80	0.81	0.83	0.84	0.86	0.89	0.92	0.99	1.05	v
45.8	652.5	658.9	665.3	671.7	678.0	684.1	696.3	708.4	737.9	766.9	i
.0579	1.0671	1.0759	1.0845	1.0929	1.1011	1.1090	1.1245	1.1393	1.1743	1.2068	s
67.6	277.6	287.6	297.6	307.6	317.6	327.6	347.6	367.6	417.6	467.6	t 550
0.70	0.71	0.73	0.74	0.75	0.77	0.78	0.81	0.83	0.89	0.95	v
47.8	654.5	661.0	667.5	673.9	680.2	686.4	698.7	710.9	740.6	769.8	i
.0513	1.0604	1.0692	1.0778	1.0862	1.0944	1.1023	1.1178	1.1326	1.1675	1.1999	s
74.7	284.7	294.7	304.7	314.7	324.7	334.7	354.7	374.7	424.7	474.7	t 600
0.64	0.65	0.66	0.68	0.69	0.70	0.71	0.74	0.76	0.82	0.87	v
49.5	656.3	662.9	669.4	675.9	682.2	688.5	700.9	713.1	743.0	772.3	i
.0452	1.0543	1.0631	1.0717	1.0801	1.0883	1.0962	1.1116	1.1264	1.1612	1.1936	s

TABLE 4

Thermal Properties of Liquid Ammonia at Saturation Pressur

Temp. Fahr.	Pressure lb.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	144 Apv′	Heat Content	Temp. Fahr.
−110°	0.758	0.02202	45.42	0.003	......	−110°
−105°	0.947	0.02211	45.23	0.004	......	−105°
−100°	1.176	0.02220	45.04	0.005	......	−100°
− 95°	1.450	0.02229	44.85	0.006	......	− 95°
− 90°	1.778	0.02239	44.66	0.007	......	− 90°
− 85°	2.167	0.02248	44.47	0.009	......	− 85°
− 80°	2.626	0.02258	44.28	0.011	......	− 80°
− 75°	3.164	0.02268	44.09	0.013	......	− 75°
− 70°	3.791	0.02278	43.89	0.016	......	− 70°
− 65°	4.518	0.02288	43.70	0.019	......	− 65°
− 60°	5.358	0.02299	43.51	0.023	......	− 60°
− 55°	6.324	0.02309	43.31	0.027	......	− 55°
− 50°	7.43	0.02320	43.11	0.032	−85.67	− 50°
− 45°	8.69	0.02331	42.91	0.038	−80.49	− 45°
− 40°	10.12	0.02342	42.71	0.044	−75.31	− 40°
− 35°	11.74	0.02353	42.50	0.051	−70.13	− 35°
− 30°	13.56	0.02364	42.30	0.059	−64.94	− 30°
− 25°	15.61	0.02376	42.09	0.069	−59.75	− 25°
− 20°	17.91	0.02388	41.88	0.079	−54.56	− 20°
− 15°	20.46	0.02400	41.67	0.091	−49.36	− 15°
− 10°	23.30	0.02412	41.46	0.104	−44.15	− 10°
− 5°	26.46	0.02424	41.25	0.119	−38.92	− 5°
0°	29.95	0.02437	41.04	0.135	−33.68	0°
5°	33.79	0.02450	40.83	0.153	−28.43	5°
10°	38.02	0.02463	40.61	0.173	−23.17	10°
15°	42.67	0.02476	40.39	0.196	−17.89	15°
20°	47.75	0.02490	40.17	0.220	−12.59	20°
25°	53.30	0.02503	39.95	0.247	− 7.26	25°
30°	59.35	0.02518	39.72	0.277	− 1.90	30°
35°	65.91	0.02531	39.50	0.309	+ 3.48	35°
40°	73.03	0.02547	39.27	0.344	+ 8.88	40°
45°	80.75	0.02562	39.04	0.383	+14.32	45°
50°	89.1	0.02577	38.81	0.425	19.80	50°
55°	98.0	0.02593	38.57	0.471	25.31	55°
60°	107.7	0.02609	38.33	0.520	30.87	60°
65°	118.1	0.02626	38.09	0.574	36.48	65°
70°	129.2	0.02642	37.85	0.632	42.14	70°
75°	141.1	0.02660	37.60	0.70	47.84	75°
80°	153.9	0.02678	37.35	0.76	53.60	80°
85°	167.4	0.02696	37.10	0.84	59.43	85°
90°	181.8	0.02714	36.84	0.92	65.32	90°
95°	197.3	0.02734	36.58	1.00	71.28	95°
100°	213.8	0.02753	36.32	1.09	77.30	100°
105°	231.2	0.02774	36.06	1.19	83.39	105°
110°	249.6	0.02795	35.79	1.29	89.57	110°
115°	269.2	0.02816	35.51	1.40	95.85	115°
120°	289.9	0.02839	35.23	1.52	102.24	120°

TABLE 4—LIQUID

Temp. Fahr.	Pressure lb.	Sp. Vol. cu. ft. per lb.	Density lb. per cu. ft.	144 Apv′	Heat Content	Temp. Fahr.
125°	311.6	0.02862	34.95	1.65	108.73	125°
130°	334.6	0.02886	34.66	1.79	115.32	130°
135°	358.8	0.02910	34.36	1.93	122.02	135°
140°	384.4	0.02936	34.06	2.09	128.83	140°
145°	411.3	0.02963	33.76	2.26	135.78	145°
150°	439.5	0.0299	33.45	2.43	142.87	150°
155°	469.1	0.0302	33.13	2.62	150.11	155°
160°	500.1	0.0305	32.80	2.82	157.52	160°
165°	532.6	0.0308	32.47	3.04	165.11	165°
170°	566.6	0.0312	32.03	3.27	172.88	170°
175°	602.2	0.0315	31.8	3.51	180.85	175°
180°	639.5	0.0318	31.5	3.77	189.03	180°
185°	678.4	0.0322	31.1	4.05	197.45	185°
190°	719.0	0.0326	30.7	4.34	206.16	190°
195°	761.4	0.0330	30.3	4.65	215.18	195°
200°	805.6	0.0335	29.9	4.99	......	200°
205°	851.7	0.0340	29.4	5.36	......	205°
210°	899.7	0.0345	29.0	5.75	......	210°
215°	949.6	0.0350	28.6	6.16	......	215°
220°	1001.4	0.0355	28.2	6.59	......	220°
225°	1055.3	0.0361	27.7	7.1	......	225°
230°	1111.3	0.0368	27.2	7.6	......	230°
235°	1169.5	0.0376	26.6	8.1	......	235°
240°	1229.9	0.0384	26.0	8.7	......	240°
245°	1292.5	0.0393	25.4	9.4	......	245°
250°	1357.4	0.0404	24.8	10.2	......	250°
255°	1424.7	0.0417	24.0	11.0	......	255°
260°	1494.4	0.0435	23.0	12.0	......	260°
265°	1566.6	0.0457	21.8	13.3	......	265°
270°	1641.3	0.0500	20.0	15.2	......	270°
273.2°	1690.0	0.0678	14.75	21.2	......	273.2°

III. Mollier Diagram for Ammonia

7. *Description of Diagram.*—In order to facilitate the solution of refrigeration problems, a *Heat Content-Entropy Diagram* has been prepared to accompany this bulletin, and is inserted in an envelope attached to the back cover. This diagram has two families of curves: (a) curves of constant pressure, and (b) curves of constant quality in the saturated region and constant temperature in the region of superheat. The ordinates are heat contents; the abscissas are entropies.

In order to use as large scales as possible and at the same time to bring the diagram into reasonable compass oblique coördinates have been used. The axis of ordinates instead of being vertical is inclined to the left at an angle of 30 degrees with the axis of abscissas. Taking the values of entropy along the horizontal axis, values of heat content may be taken either along the inclined axis or along a vertical axis. Since vertical distances are equal to distances along the oblique axis multiplied by the sine of 30 degrees, the scale for measuring heat contents along the oblique lines is such that a distance measured along the 30 degree lines represents sine 30° (or ½) times as many units of heat content as are represented by the same distance measured vertically.

The 30° lines are lines of constant entropy and show the change of condition of the ammonia during adiabatic compression or expansion. Measurements along these lines give the change in heat content during adiabatic compression or expansion. Horizontal lines are lines of constant heat content; they show the change in condition of the ammonia which results from a throttling process, such as the passage through the expansion valve of a refrigerating machine.

The lines of constant pressure after meeting the liquid line should be extended beyond this line to the left to represent the cooling of the liquid at constant pressure below the temperature of saturation for that pressure. The constant pressure lines are so nearly tangent to the liquid line that the several lines could not be distinguished with the scale employed. The process of cooling the liquid below saturation temperature is therefore represented for all pressures by the liquid line itself.

9. *Examples of the Use of the Diagram.*—The following examples will illustrate the method of solution of some of the more commonly occurring problems.

WORK DONE IN THE COMPRESSOR ASSUMING THE RANKINE IDEAL CYCLE

In the Rankine cycle, the ammonia is admitted at a constant pressure, is compressed adiabatically to the discharge pressure and is dis-

charged against this constant pressure. The compressor is supposed to have no clearance. The cycle is ideal with no internal friction, no heat losses and no free or imperfectly resisted expansion. In such a cycle the work done per pound of ammonia is the difference in heat content at the beginning and end of compression, that is $i_1 - i_2$. As the only compression that takes place is adiabatic, the ammonia leaving has the same entropy as that entering.

To find the work done upon one pound of ammonia, in a Rankine cycle admitting ammonia at a known quality and pressure, and compressing it to a known discharge pressure, locate the point representing the quality and pressure of the suction ammonia, and measure the distance upward to the left along a 30° line to the discharge pressure line. If the quality and pressure at discharge and the suction pressure only are known, locate the point representing the quality and pressure of the discharge ammonia, and measure the distance downward to the right along a 30° line to the suction pressure line. In either case the distance represents the work done in B.t.u.

Example 1. In a dry compression system, ammonia, initially dry and saturated at a pressure of 25 lb. per sq. in. absolute, goes through a Rankine cycle in which it is compressed to a pressure of 200 lb. per sq. in. absolute. How much work is done on each pound of ammonia?

The heat content of dry and saturated ammonia at a pressure of 25 lb. is seen from the diagram to be 536 B.t.u. Ammonia of the same entropy at 200 lb. pressure has a temperature of 271° F. and has a heat content of 671 B.t.u. The work of the Rankine cycle is

$$536 - 671 = -135 \text{ B.t.u.},$$

the minus sign signifying work done on the ammonia.

The same value is obtained with twice as great accuracy by measuring the distance along the 30° line between the points representing the initial and final states of the ammonia.

Example 2. In a wet compression system, ammonia, after having gone through a Rankine cycle with a suction pressure of 25 lb., is discharged dry and saturated at a pressure of 200 lb. How much work is done on each pound of ammonia?

The heat content of dry and saturated ammonia at a pressure of 200 lb. is seen to be 560 B.t.u. Ammonia of the same entropy at 25 lb. pressure has a quality of 0.86 and has a heat content of 457 B.t.u. The work of the Rankine cycle is

$$457 - 560 = -103 \text{ B.t.u.}$$

HEAT REJECTED BY AMMONIA IN CONDENSER AND COOLER

In any constant pressure process the change of i measures the heat taken in or given out. To show this take the general equation:

$$dQ = du + Apdv$$

Then

$$\begin{aligned} dQ &= (du + Apdv + Avdp) - Avdp \\ &= [du + Ad(pv)] - Avdp \\ &= di - Avdp \end{aligned}$$

Hence, when p is constant during the change,

$$dQ = di$$

and $Q = i_2 - i_1$ for any two states.

Therefore the heat rejected to the cooling water in the condenser and cooler is directly measured by the decrease in i which the ammonia undergoes after adiabatic compression up to the expansion valve.

To find the heat rejected per pound of ammonia in the condenser and cooler, locate the point representing the state of the ammonia after compression, and pass down a constant pressure curve to the point representing the state of the ammonia before the expansion valve. The vertical distance passed through, that is the difference between the heat contents at the initial and final states, gives the heat rejected to the cooling water per pound of ammonia. Part of this heat is taken out as the superheat, if any, is removed; the greater part is taken out as the ammonia is condensed; and the remainder is taken out as the liquid is cooled from the temperature of saturation at the given pressure to the temperature before the expansion valve; this latter part usually takes place in a cooler separate from the condenser.

Example 3. Ammonia, after being compressed as in example 1 to a pressure of 200 lb. and a temperature of 271° F., is passed through the cooler and condenser and when it reaches the expansion valve has a temperature of 80° F. How much heat is rejected to the cooling water?

The heat content after compression was found in example 1 to be 671 B.t.u. Passing down the constant pressure line the superheat is removed, the ammonia condensed, and the liquid cooled from saturation temperature of 96° F. to 80° F. At the latter point the heat content is seen from the diagram to be 54 B.t.u. The heat rejected to the cooling water is

$$671 - 54 = 617 \text{ B.t.u.}$$

Example 4. Find the heat rejected to the cooling water if the ammonia compressed as in example 2 to a dry and saturated state at 200 lb. pressure is cooled to a temperature of 80° F. before the expansion valve.

By a process similar to that given in example 3, the heat rejected to the cooling water is

$$560-54=506 \text{ B.t.u.}$$

THROTTLING THROUGH THE EXPANSION VALVE

If vapor is allowed to expand through a small orifice, such as the expansion valve, without the addition or abstraction of heat, and is finally brought to its initial velocity, its heat content will be unchanged. The proof of this statement is based on the thermodynamic theory of flow of fluids and may be found in any standard text. Horizontal lines on the diagram are lines of constant heat content and consequently show the changes in the condition of the ammonia which result from throttling in the expansion valve.

Example 5. Liquid ammonia in front of the expansion valve has a temperature of 80° F., and after passing through the valve the pressure of the mixture of vapor and liquid is 25 lb. What is the quality of the mixture?

The horizontal line which intersects the liquid line at a temperature of 80° F., intersects the 25 lb. pressure line at a point which is found by interpolation to correspond to a quality of 0.16. This horizontal line represents a constant heat content of 54 B.t.u.

REFRIGERATING EFFECT

The refrigerating effect is measured directly by the change in i which the ammonia undergoes during evaporation, this being a constant pressure process in which, as shown above, the increase in i is equal to the heat absorbed.

To find the refrigerating effect per pound of ammonia, locate the point representing the state of the ammonia after its passage through the expansion valve, and pass up along a constant pressure line to the point representing the state of the ammonia at the beginning of compression. The vertical distance passed through, that is the difference between the heat contents at the initial and final states, gives the refrigerating effect per pound of ammonia.

Example 6. After being throttled through the expansion valve to a pressure of 25 lb. and a quality of 0.16, the ammonia evaporates in the refrigerating coils until it is just dry and saturated at this pressure. What is the refrigerating effect per pound of ammonia circulated?

From the diagram the change in i, or the heat absorbed by the ammonia is

$$536-54=482 \text{ B.t.u.}$$

Example 7. Find the refrigerating effect per pound of ammonia, when the condition after the expansion valve is the same as in example 6, but evaporation takes place as in the wet compression system of example 2 only until a quality of 0.86 is reached.

From the chart the refrigerating effect is

$$457-54=403 \text{ B.t.u.}$$

THEORETICAL COEFFICIENT OF PERFORMANCE AND REFRIGERATING EFFECT PER HORSE-POWER HOUR

By definition the coefficient of performance is equal to the ratio of the heat absorbed by the refrigerating medium to the work done upon it. It has been shown that both of these quantities can be read from the diagram, and therefore the coefficient of performance can be calculated for any assumed conditions of operation.

Example 8. What is the coefficient of performance of the dry compression process of examples 1, 3, 5, and 6?

From example 6 the heat absorbed is 482 B.t.u., and from example 1 the work done on the ammonia is 135 B.t.u. The coefficient of performance is therefore =

$$482 \div 135 = 3.57.$$

Since one horse-power hour is equal to 2546 B.t.u., the refrigerating effect per horse-power hour is

$$2546 \times 3.57 = 9090 \text{ B.t.u.}$$

Example 9. What is the coefficient of performance of the wet compression system of examples 2, 4, 5, and 7?

From example 7 the heat absorbed is 403 B.t.u., and from example 2 the work done on the ammonia is 103 B.t.u. The coefficient of performance is therefore =

$$403 \div 103 = 3.91.$$

The refrigerating effect per horse-power hour is

$$2546 \times 3.91 = 9960 \text{ B.t.u.}$$

From the above examples it is seen that the theoretical loss with dry compression as compared with wet compression under these conditions is

$$\frac{9960-9090}{9960} = 8.7\%$$

SAVING POSSIBLE BY USE OF EXPANSION CYLINDER

If instead of the expansion valve an expansion cylinder were used the irreversible throttling process would be replaced by a reversible adiabatic expansion. In that case the reduction of pressure instead of being represented on the diagram by a horizontal line extending from

the point representing the state before the expansion valve to the refrigerator pressure line, would be represented by an adiabatic. The length of this adiabatic would measure in heat units the work recovered by the use of the expansion cylinder and the gain in refrigerating effect would be measured by the change in i along the adiabatic.

Example 10. Find the per cent. gain due to substituting an expansion cylinder for the expansion valve in the wet compression system described in examples 2, 4, 5, and 7.

The work of compression would be the same as in example 2, or 103 B.t.u. The heat rejected to cooling water would be the same as in example 4, or 506 B.t.u. The expansion in the expansion cylinder would be represented by a 30° adiabatic extending from the point representing liquid at a temperature of 80° F. downward to the 25 lb. pressure line. The length of this line gives the work obtained in the expansion cylinder, assuming a Rankine cycle, or 9 B.t.u. The refrigerating effect would be increased by the same amount, or

$$54-45=9 \text{ B.t.u.}$$

The coefficient of performance would therefore be

$$\frac{403+9}{103-9}=4.38$$

The per cent gain due to using the expansion cylinder is therefore

$$\frac{4.38-3.91}{3.91}=12\%$$

IV. Basis of the Tables

The tables given in the preceding pages are based upon the most reliable experimental data available at the present time. The purpose of the following pages is to present this body of data together with the derivation therefrom of the formulas from which the tables were calculated.

10. *Absolute Temperature.*—In the computation of tables of the properties of a vapor, absolute temperatures must be used; hence an evaluation of the absolute thermodynamic scale is necessary. In this evaluation two problems must be solved. The first is the determination of the absolute temperature of melting ice; and the second is the determination, degree by degree, of the difference between the absolute scale and that of the nitrogen-in-glass thermometer which is the usual standard in scientific work. For the present purpose the second of these problems need not be considered, for the variation of the nitrogen thermometer from the absolute scale is nowhere greater than one twentieth of a degree Fahrenheit between 0° and 400°. Even con-

siderably larger errors than this would be within the limit of error of the other experimental data.

The determination of the absolute zero has been satisfactorily accomplished and the results are summed up by Marks and Davis[33]. The following table is taken from their "Steam Tables" and gives the determinations of Berthelot[34], Buckingham[35], and Rose-Innes[36].

TABLE 5. DETERMINATIONS OF ICE-POINT ON ABSOLUTE SCALE

Author	Year	Method	Gases	Final Value for Ice-point
Berthelot	1903	Extrapolation to p=o..............	H_2 and N_2	491.54°F
Berthelot	1903	Joule-Thomson effeet..............	H_2, N_2, CO_2, air	491.63
Buckingham	1907	Joule-Thomson effect..............	H_2, N_2, CO_2, air	491.71
Rose-Innes	1908	Joule-Thomson effect and other data.	H_2 and N_2	491.64

The determination of Rose-Innes is probably the best and incidentally it agrees almost exactly with the mean of the other three. Since the temperature of melting ice on the ordinary Fahrenheit scale is 32°, the number that must be added to temperatures on the ordinary scale to reduce them to absolute temperatures is

$$491.64-32=459.64$$

11. *Mechanical Equivalent of Heat.*—In several of the equations used in the computation of vapor tables the mechanical equivalent of heat, J, or its reciprocal, A, appears. This equivalent has been experimentally determined by two methods: by transforming mechanical work into heat directly, and by transforming electrical energy into heat. The first method has been used by Rowland,[37] and Reynolds and Moorby;[38] the latter by Griffiths,[39] Schuster and Gannon,[40] and Barnes.[41] These various investigations have been discussed by Smith,[42] who accepts as most trustworthy the mean of the values of Barnes and of Reynolds and Moorby. Smith gives as this mean,

1 mean calorie $=4.1834\times10^7$ ergs. and this is the value used in the Marks and Davis Steam Tables.

Barnes[43] later points out that his work must be considered as leading to the value 4.1849. Since the work done by Reynolds and Moorby leads, according to Smith, to the value 4.1836, the mean of these determinations is 4.1842.

Griffiths in his discussion of the most trustworthy experiments then available (1893) decided that the most probable value of J is 4.184. That is, taking the 17½° gram-calorie, or the mean calorie which is practically equal to it,

1 mean calorie $=4.184\times10^7$ ergs. To change this into English units, one needs the conversion factor[44]

1 kg. = 2.204622 lb.

and a value of the gravitation constant, for which

$g = 980.655$ cm. per sec.^{2}

has been adopted by international agreement.[45] The result is

1 mean B.t.u. = 777.64 standard ft. lb.

This value has been used in the present investigation.

It may be mentioned here that all factors for conversion from metric into English units have been taken from the tables given in the Marks and Davis Steam Tables.

12. *Relation between Pressure and Temperature of Saturated Vapor.*—To express the relation between the pressure and temperature of saturated vapors scores of formulas have been proposed. Some are of more or less rational form involving empirical constants, but the greater number are purely empirical. A few of the formulas that have been applied to ammonia, together with the constants giving p in lb. per sq. in. when temperatures are given in degrees Fahrenheit, are here given.

(a) Biot's equation as used by Regnault has the form

$$\log p = a - b\alpha^n + c\beta^n, \text{ where } n = t - k$$

The constants used by Regnault are;[1]

$$\begin{aligned} a &= 9.790830 \\ b &= 7.450352 \\ c &= 0.949967 \\ \log \alpha &= \bar{1}.999779 \\ \log \beta &= \bar{1}.996652 \\ n &= t + 7.6 \end{aligned}$$

Limits, −8° F. to 180° F.

The same formula was used in the computation of Peabody's[11] tables.

(b) Zeuner[8] used the Biot formula, but omitted the third term and expressed the second term as a function of t:

$$\log p = a - b\alpha^\tau$$

where $a = 3.844672$

$$\log b\alpha^\tau = 0.312482 - 0.0019457\,(t - 32)$$

This form was used also by Dieterici,[25] who applied it at temperatures above the limit of the range for which Zeuner's empirical equation for $\log b\alpha^\tau$ holds. He thus obtained pressures far too low at high temperatures, also low values of $\frac{dp}{dt}$ and consequently low values of r when the Clapeyron-Clausius equation is used.

(c) Wood[10] used Rankine's formula in the form

$$\log p = a - \frac{b}{T}$$

with

$$a = 6.2495$$
$$b = 2196$$

Limits, $-20°$ F. to $100°$ F.

(d) Goodenough[32] used the Bertrand formula in the form

$$\log p = \log k - n \log \frac{T}{T-b}$$

with

$$\log k = 5.87395$$
$$n = 50$$
$$b = 84.3$$

It does not seem to be possible to find constants which will make equations of the above forms apply over more than a limited range of temperature, either with steam or with other vapors. However, an equation for steam has recently been proposed by Marks,[46] which is remarkable in that it gives pressures agreeing closely with experimental values throughout the range from 32° F. to 706.1° F., the latter point being the critical temperature. This equation is based on the equation used by van der Waals:

$$\log \frac{p_k}{p} = a\left(\frac{T_k}{T} - 1\right)$$

where p_k and T_k denote respectively the critical pressure and critical temperature. Instead of being a constant as used by van der Waals, a varies for different substances and for different temperatures with the same substance. Prof. Marks found values of a at different temperatures for steam and then found an expression for its value in terms of the temperature and the critical temperature. By substituting this expression for a and the proper values for p_k and T_k in the equation $\log \frac{p_k}{p} = a\left(\frac{T_k}{T} - 1\right)$, he arrived at the following relation between the pressure and temperature of saturated steam:

$$\log p = a - \frac{b}{T} - cT + eT^2 \qquad (1)$$

where

$$a = 10.515354$$
$$b = 4873.71$$
$$c = 0.00405096$$
$$e = 0.000001392964$$

In the present investigation an attempt was made to use the above method to determine the pressure-temperature relation of ammonia. Several factors rendered the use of this method undesirable. They are: the sensitiveness of the method to changes in the critical data and the

uncertainty of these data for ammonia; the lack of experimental data at temperatures near the critical point; and the large discrepancies existing in the data throughout the entire range. As will be seen later, however, this equation plays an important part in the present investigation.

Physicists have repeatedly attempted to find a relation between the pressures of different vapors at the same temperature such that a determination of the function $p=f(t)$ for one vapor would serve to determine this function for all other vapors. Ramsey and Young[47] proposed the law

$$R=R'+k(T-T')$$

Where R and R' are the ratios of the temperatures of two saturated vapors at two different pressures, and T and T' are the temperatures of one of the vapors corresponding to these pressures. They show that the law holds very closely for some 22 different substances arranged in 23 different pairs. In their paper they use the T and T' in the term $k(T-T')$ as being indiscriminately the numerators or the denominators of the ratios R and R'; i. e. they use the two equations

$$\frac{T_b}{T_a}=\frac{T_b'}{T_a'}+k'(T_a-T_a') \qquad \text{(a)}$$

$$\frac{T_a}{T_b}=\frac{T_a'}{T_b'}+k(T_a-T_a') \qquad \text{(b)}$$

without making any mention of the change from one form to the other and without recognizing that the two forms lead to different results.

This law has been corroborated by the work of Richardson[48] and by the work of Ramsey and Travers[49] on crypton, argon and xenon.

Ayrton and Perry,[50] and Everett[51] have each remarked upon the lack of symmetry of equation (a) and have shown that equation (b) is symmetrical. This is most clearly demonstrated by Moss,[52] who shows that (a) may be thrown into the form

$$\frac{T_b}{T_a}=\left[\frac{T_b'}{T_a'}-k'T_a'\right]+k'T_a$$

and (b) may be thrown into the form

$$\frac{T_a}{T_b}=\left[\frac{T_a'}{T_b'}-kT_a'\right]+kT_a$$

Since T_a' and T_b' are the temperatures corresponding to some particular vapor pressure, these equations may be written respectively

$$\frac{T_b}{T_a}=k'T_a+c' \qquad \text{(c)}$$

and

$$\frac{T_a}{T_b}=kT_a+c \qquad \text{(d)}$$

Now to test the symmetry of these equations they may be written

$$\frac{T_a}{T_b}=\left[-\frac{k'}{c'}\right]\frac{T_a^2}{T_b}+\frac{1}{c'} \tag{e}$$

and

$$\frac{T_b}{T_a}=\left[-\frac{k}{c}\right]T_b+\frac{1}{c} \tag{f}$$

It is seen that if the left hand member of (c) is a linear function of the denominator, its reciprocal will not be a linear function of the new denominator. On the other hand, if the left hand member of (d) is a linear function of the numerator, its reciprocal will also be a linear function of the new numerator.

Moss replotted the data used by Ramsey and Young in their original paper but used in all cases the symmetrical form (d). He found that in all cases where they had used the unsymmetrical form the symmetrical form fitted just as well and in most cases much better, in addition to its superiority in being reversible.

Equation (d) may be written in the simple form

$$\frac{1}{T_b}=c\frac{1}{T_a}+k \tag{g}$$

$\frac{1}{T_b}$ and $\frac{1}{T_a}$ thus being linear functions of each other. Suppose there are available two values of saturation temperatures corresponding to known vapor pressures for some substance. If water vapor or some other substance whose vapor pressures are known is taken as the other substance, then from the two temperatures the constants c and k may be determined. The temperatures for all vapor pressures for the substance in question may then be readily computed.

Since equation (g) may be written

$A-\frac{B}{T_b}=A'-\frac{B'}{T_a}$, it follows that any equation to be applicable to all vapors in the same form with only its constants changed, and at the same time to be consistent with the temperature ratio law as stated above, must satisfy the condition that $p=f(A-\frac{B}{T})$, all other constants remaining the same for all vapors. Now it happens that the equations which may be thrown into this form, such as the Rankine short form, the Roche equation, etc., do not satisfy the experimental data for steam throughout its range with the necessary degree of accuracy. On the other hand the Marks equation, which does satisfy these data with remarkable accuracy throughout its complete range, cannot be thrown into the required form. In view of the various considerations mentioned it has been decided in the present investigation to accept

the temperature ratio law as expressed in equation (g), to use water as the standard substance, to use the Marks equation, equation (1), as representing the pressure-temperature relation for water, and to make a step by step solution of the pressure-temperature relation for ammonia by means of these equations.

The method used in applying the temperature ratio law and in determining the value of the constants c and k in equation (g), is that used by Moss[52], who has applied it to 17 different vapors. A description of this method will now be given.

Using formula (g) to obtain the saturation temperatures of ammonia from those of water,

$$\frac{1}{T_a}=c\frac{1}{T_w}+k$$

Put

$$-\frac{1}{T_w}=y \text{ and } -\frac{1}{T_a}=x;$$

then

$$-x=-cy+k$$

Since y is a function of the water vapor temperature corresponding to pressure p, y is also a function of p. The value of y for a given value of p may be found by making use of the tables of water vapor pressure. The quantity x is a function of the temperature of the ammonia corresponding to this same vapor pressure p. The curve giving x as a function of y is evidently a straight line.

Since y is a function of the pressure p and x a function of the corresponding temperature, we may label the values of x and y with the respective values of the pressure and temperature which they represent. Then we may read off directly from the diagram the corresponding values of saturation temperature and vapor pressure.

Fig. 1 was constructed in this manner. Integral values of temperature were assumed and the corresponding values of x were computed according to the relation $x=-\frac{1}{T}$. These values of x were then laid off to a convenient scale and each labeled with the temperature to which the value of x corresponds. Integral values of pressure were assumed, the corresponding values of temperature found from steam tables, and the corresponding values of y computed according to the relation $y=-\frac{1}{T}$. These values of y were laid off to a convenient scale and each labeled with the pressure to which the value of y corresponds.

The value of vapor pressure at a given temperature as taken from the Marks and Davis Steam Tables was found to agree with the value

given by the new Marks formula with sufficient accuracy up to a temperature of 400° F.; above this temperature the values differed materially. In the construction of the chart, therefore, the Steam Tables were used to find the pressures corresponding to temperatures below 400° F. and the newer formula was used to find the pressures corresponding to temperatures above that point.

The following tables show the calculations for a few of the values of Fig. 1.

TABLE 6. CALCULATIONS FOR VALUES OF FIG. 1

ABSCISSAS

Fahrenheit Temperature	Absolute Temperature	Reciprocal of Absolute Temp.	Corresponding value of x in inches referred to the point -10 as origin $= -(8000 \times \frac{1}{T} - 10)$
−100	359.64	0.0027806	12.244
−50	409.64	0.0024412	9.529
0	459.64	0.0021756	7.405
+50	509.64	0.0019622	5.697
100	559.64	0.0017869	4.295
150	609.64	0.0016403	3.122
200	659.64	0.0015160	2.128
250	709.64	0.0014092	1.273

ORDINATES

Pressure lb. per sq. in.	Corresponding Saturation Temperature of Water Vapor in degrees F.	Absolute Temperature	Reciprocal of Absolute Temperature	Corresponding value of y in inches referred to the point -18 as origin $= -(20000 \times \frac{1}{T} - 18)$
1	101.83	561.47	0.0017810	17.621
5	162.28	621.92	0.0016079	14.158
10	193.22	652.86	0.0015317	12.634
50	281.0	740.64	0.0013502	9.004
100	327.8	787.44	0.0012699	7.399
500	467.2	926.84	0.0010789	3.579
1000	544.9	1004.54	0.0009955	1.910
1700	613.4	1073.04	0.0009319	0.639

In order to settle upon the most probable location of the straight line representing the pressure-temperature relation of saturated ammonia upon this chart the experimental data must be plotted. The data obtained by Regnault[1] are given in Table 7 and are shown in Fig. 1. by the open circles. The data obtained by Blümcke[12] are given in Table 8, and are plotted in Fig. 1 as crosses. The data obtained by Brill[16] are given in Table 9 and are indicated in Fig. 1 by the solid circles. The data obtained by Davies[15] are given in Table 10 and are represented in Fig. 1 by means of solid squares. The pressures observed by Pictet[13] over a temperature range of −22° F. to 122° F. agree almost exactly with those observed by Regnault; they are neither tabulated nor plotted. Faraday's[14] results, which are given in Table 11, are, on the other hand, quite inconsistent with those of Regnault. They are neither plotted in Fig. 1 nor given weight in the determination of the constants of the equation.

The two points in the upper right hand corner of Fig. 1 represent the experiments at the critical point. The open square shows the

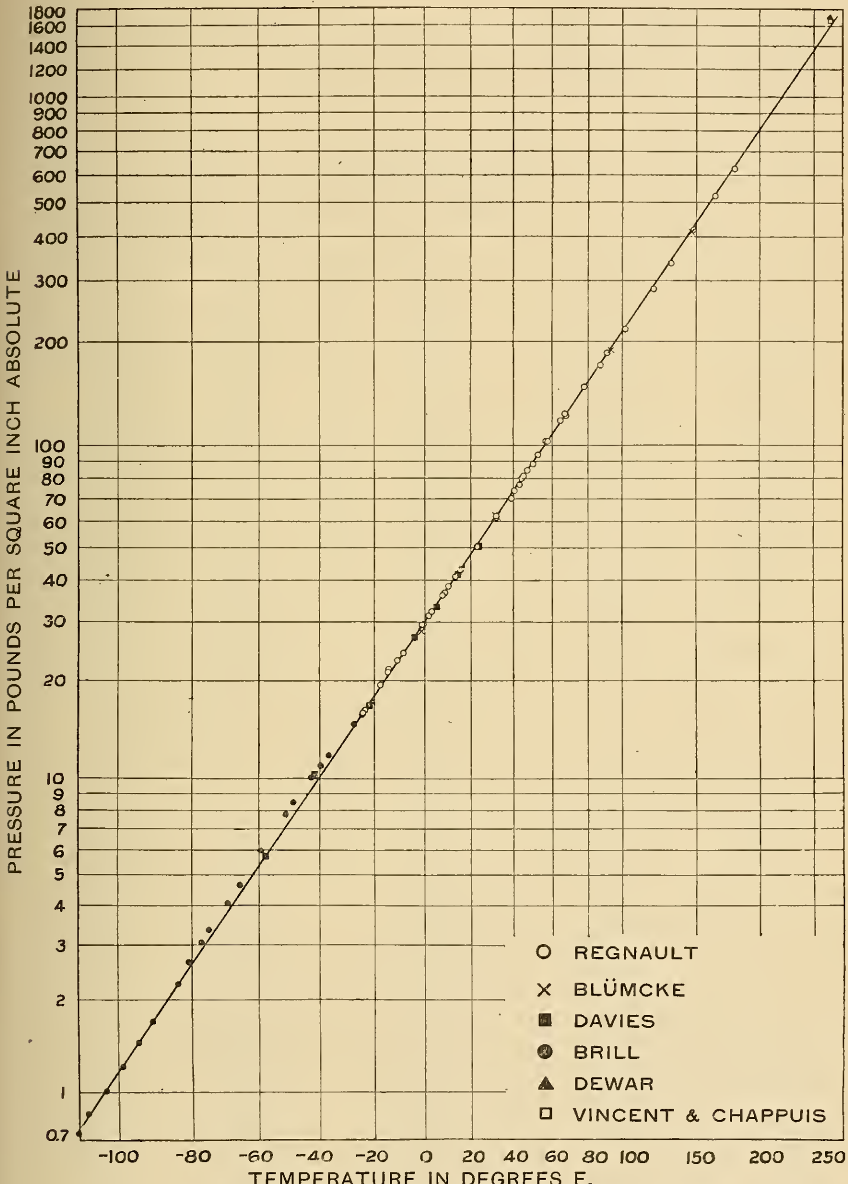

FIG. 1. SHOWING THE AGREEMENT OF PRESSURES FOUND BY THE TEMPERATURE RATIO LAW WITH THOSE FOUND BY EXPERIMENT.

values given by Vincent and Chappuis[18] and the solid triangle the values given by Dewar.[17] The values from which these points were plotted are given in Table 12.

After plotting the points representing the experimental data the straight line shown in Fig. 1 was drawn in such a manner as to represent all the points in the best possible manner.

TABLE 7. SUMMARY OF PRESSURE–TEMPERATURE DETERMINATIONS OF REGNAULT

Series	Temperature °C.	Pressure in mm. of Mercury	Temperature °F.	Pressure in lb. per sq. in.
2	−31.48	815.09	−24.66	15.77
2	−31.37	823.59	−24.47	15.93
2	−30.96	838.41	−23.73	16.22
2	−27.47	1002.91	−17.45	19.40
2	−27.36	1006.71	−17.25	19.47
3	−25.84	1097.21	−14.51	21.22
3	−25.70	1117.74	−14.26	21.62
3	−23.92	1187.30	−11.06	22.97
2	−22.71	1244.47	− 8.88	24.07
2	−22.60	1251.71	− 8.68	24.21
2	−18.37	1518.19	− 1.07	29.37
2	−18.35	1513.53	− 1.03	29.28
3	−18.31	1518.70	− 0.96	29.38
3	−18.10	1529.90	− 0.58	29.59
1	−18.03	1524.92	− 0.45	29.50
1	−16.79	1614.86	+ 1.78	31.24
1	−16.58	1630.47	+ 2.16	31.54
3	−16.17	1669.95	+ 2.89	32.20
3	−13.55	1859.93	+ 7.61	35.98
3	−13.51	1867.93	+ 7.68	36.13
1	−13.09	1900.37	+ 8.44	36.76
1	−12.15	1980.87	+10.13	38.32
2	−10.52	2133.78	+13.06	41.27
2	−10.42	2119.44	+13.24	41.00
3	− 9.21	2222.35	+15.42	42.99
3	− 5.03	2599.55	+22.95	50.28
3	− 0.10	3159.07	+31.82	61.11
2	0	3203.66		
2	0	3206.68		
1	0	3207.70		
1	0	3212.66	+32.00	62.04
1	0	3207.48		
1	0	3214.73		
1	0	3198.44		
1	+ 3.97	3615.65	+39.15	69.94
1	+ 4.72	3812.09	+40.50	73.74
3	+ 6.24	3963.67	+43.23	76.67
2	+ 6.93	4139.96	+44.47	80.08
2	+ 7.32	4199.74	+45.18	81.24
2	+ 7.34	4198.12	+45.21	81.21
2	+ 8.45	4368.87	+47.21	84.51
3	+ 9.98	4550.44	+49.96	88.02
2	+11.42	4835.82	+52.57	93.54
2	+13.52	5335.26	+56.34	103.20
3	+14.38	5302.37	+57.88	102.57
2	+18.15	6134.92	+64.67	118.67
2	+19.29	6399.12	+66.72	123.62
2	+19.29	6382.70		
3	+19.70	6317.97	+67.46	122.21
2	+25.35	7676.26	+77.63	148.48
3	+30.49	8802.88	+86.88	170.28
2	+32.70	9569.56	+90.86	185.11
3	+38.90	11236.07	+102.02	217.34
3	+48.93	14669.97	+120.07	283.77
3	+55.47	17333.92	+131.85	335.30
3	+64.35	21619.67	+147.83	418.20
3	+73.32	26766.40	+163.98	517.75
3	+81.72	32171.03	+179.10	622.30

NOTE — In this table all pressures, besides being multiplied by the proper factor to convert mm. of mercury into lb. per sq. in., were multiplied by the number 1.00033, this being the constant given by Landolt and Börnstein to reduce pressures at Paris where Regnault worked to pressures at 45° latitude and at sea-level.

TABLE 8. SUMMARY OF PRESSURE–TEMPERATURE DETERMINATIONS OF BLÜMCKE

Temperature °F.	Pressure in lb. per sq. in.
+ 1.30	28.08
+ 32.00	62.05
+ 93.20	188.2
+146.30	412.2

TABLE 9. SUMMARY OF PRESSURE–TEMPERATURE DETERMINATIONS OF BRILL

Temperature °C.	Pressure in mm. of Mercury	Temperature °F.	Pressure in lb. per sq. in.
−79.0	38.0	−110.20	0.735
−77.6	44.1	−107.68	0.853
−75.1	51.8	−103.18	1.002
−72.7	62.5	− 98.86	1.209
−70.4	74.9	− 94.72	1.448
−68.3	87.5	− 90.94	1.692
−64.4	116.0	− 83.92	2.243
−62.8	136.0	− 81.04	2.630
−60.8	157.6	− 77.44	3.048
−59.5	172.4	− 75.10	3.334
−56.5	210.0	− 69.70	4.061
−54.4	239.5	− 65.92	4.631
−50.7	309.3	− 59.26	5.981
−46.2	403.5	− 51.16	7.802
−45.0	437.1	− 49.00	8.452
−41.5	521.9	− 42.70	10.09
−39.8	568.2	− 39.64	10.99
−38.2	610.4	− 36.76	11.80
−33.0	761.0	− 27.40	14.72

TABLE 10. SUMMARY OF PRESSURE–TEMPERATURE DETERMINATIONS OF DAVIES

Temperature °C.	Pressure in mm. of Mercury	Temperature °F.	Pressure in lb. per sq. in.
−49.8	297.95	−57.64	5.761
−41.0	530.95	−41.80	10.27
−30.0	866.95	−22.00	16.76
−20.0	1392.9	− 4.00	26.93
−15.0	1726.2	+ 5.00	33.38
−10.0	2145.9	+14.00	41.50
− 5.0	2616.9	+23.00	50.60

TABLE 11. SUMMARY OF PRESSURE–TEMPERATURE DETERMINATIONS OF FARADAY

Temp. °F.	Pressure in Atmos. of 30″ of Mercury	Pressure in lb. per sq. in.	Temp. °F.	Pressure in Atmos. of 30″ of Mercury	Pressure in lb. per sq. in.
0	2.48	36.54	49.0	5.83	85.90
9.3	3.00	44.20	51.4	6.00	88.41
18.0	3.50	51.57	52.0	6.10	89.88
21.0	3.72	54.81	55.0	6.38	94.01
26.0	4.04	59.53	56.5	6.50	95.78
32.0	4.44	65.42	60.0	6.90	101.67
33.0	4.50	66.31	61.3	7.00	103.14
41.0	5.10	75.15	65.6	7.50	110.51
44.0	5.36	78.98	67.0	7.63	112.43
45.0	5.45	80.30	83.0	10.00	147.35

TABLE 12. SUMMARY OF DETERMINATIONS OF CRITICAL DATA FOR AMMONIA

Investigator	Date	Temperature° Centigrade	Pressure in Atmospheres	Temperature° Fahrenheit	Pressure in lb. per sq. in.
Dewar	1884	130.0	115	266.0	1690.0
Vincent and Chappuis	1886	131.0	113	267.8	1660.8
Jaquerod	1908	132.3	109.6	270.1	1610.7
Sheffer	1910	132.1	111.3	269.8	1635.7

The effectiveness of any method that enables one to throw given data into such a form that they may be represented by straight lines lies in the fact that straight lines immediately and strikingly disclose any departure from the general trend and unerringly reveal points that depart from this trend. It enables one to give various observations their proper weight and it affords a much safer basis of extrapolation than can be obtained from curves. The case in hand illustrates this principle. If the points are plotted on the regular *pt* plane it is easily seen that no smooth curve such as represents a law of nature, could, if passed through Regnault's lower temperature points and Brill's lower points, at the same time pass through Brill's points in the region −80° F. to −30° F. Furthermore the chart discloses nothing, as an infinite number of curves could be drawn, giving different weights to the different points, or all points could be given equal weight and the equation could then be determined by least squares. An inspection of Fig. 1 shows, however, that the straight line which best represents the whole range represents very accurately Regnault's lower points, Brill's highest point and all of his points from −80° F. to −110° F. In the range −80° F. to −30° F. all of Brill's points lie above the line, as does Davies' −41.8° F. point. It is significant, however, that Davies' −57.64° F. point lies exactly on the line at precisely the point where Brill's points lie farthest from it.

Through the range −30° F. to +100° F. the line represents the experimental points very accurately; above 100° F. Regnault's points lie slightly below the line. Now by referring to Table 7 it is seen that all of Regnault's points above 100° F. belong to his third series of experiments. In all of the higher pressure points of this series Regnault used a closed manometer and extensive corrections had to be applied to the height of mercury observed. We may now compare the experiments in the range 40° F. to 90° F. where the second and third series overlap; it is seen that invariably the points determined in the third series by the use of the closed manometer lie below those determined in the second series by the use of the open manometer. The conclusion is evident that there was probably an error in the correction applied by Regnault to his readings with the closed manometer; the fact that

these points lie below the line should therefore not be taken as evidence of the incorrectness of the line.

Now as to the critical data. It is seen that the points of both Dewar, and Vincent and Chappuis lie above the line. If the line be taken as correct this indicates either higher temperature, lower pressure, or both at the critical point. It will be instructive in this connection to examine the history of the determinations of the critical data for water. Table 13, which is for the most part taken from Marks' paper[46] referred to above, gives several of these determinations.

TABLE 13. SUMMARY OF DETERMINATIONS OF CRITICAL DATA FOR WATER

Investigator	Date	Critical Temperature,° Fahrenheit	Critical Pressure, lb. per sq. in.
Nadejdine	1885	676.6	
Battelli	1890	687.7	2859
Cailletet and Colardeau	1891	689.0	2994
Strauss	1892	698.0	2873
Traube and Teichner	1904	705.2	
Holborn and Bauman	1910	706.3	3200

This table shows that as methods have been improved and greater precautions taken in the experimental work, the value found for the critical temperature has risen steadily until the latest determination, that of Holborn and Bauman, is 30° higher than that found by Nadejdine, 25 years earlier. The only available determinations of the critical data of ammonia were made at about the same time and with about the same methods as the earliest determination for water as shown in the table. It is reasonable to assume that the same errors may have been made in the ammonia determinations that were made in the earlier water determinations and that the true critical temperature of ammonia is higher than that found by either Dewar, or Vincent and Chappuis. The table shows also that a higher value for the critical pressure of water has been found in the later determinations. In the present investigation the critical data of ammonia are not of prime importance and the purely arbitrary assumption has been made, on the basis of the above reasoning and for want of better data, that the higher of the critical pressure determinations, that of Dewar, is correct. The corresponding temperature as determined by the temperature ratio law has been taken as the critical temperature. The resulting values are:

$$p_k = 1690 \text{ lb. per sq. in.}$$

$$t_k = 273.2° \text{ F.}$$

Since the above conclusions were drawn the articles giving the determinations of Jaquerod[70] and Scheffer[71] have been found. If

plotted in Fig. 1 the point representing Jaquerod's determination falls below the straight line and that representing Scheffer's value falls almost exactly upon it. If Scheffer's value for the critical temperature is substituted in the temperature ratio law the resulting pressure is 1638.6 lb. per sq. in. while his experimental value is 1635.7 lb. per sq. in., the difference being about $1/6$ of one per cent. These results offer corroboration of the correctness of the location of the straight line in Fig. 1, although the arbitrary values chosen for the critical temperature and pressure may be too high.

The equation of the straight line in Fig. 1 is

$$\frac{1}{T_a}=1.70356\,\frac{1}{T_w}-0.0002242 \qquad (2)$$

In constructing tables with pressure as the argument the temperature of saturated steam at any pressure is found from steam tables. The corresponding saturation temperature of ammonia is then found by using equation (2) in the form

$$T_a=\frac{1}{\dfrac{1.70356}{T_w}-0.0002242}$$

If temperature is taken as the argument the temperature at which steam will be saturated under the same pressure may be found by using equation (2) in the form

$$T_w=\frac{1}{\dfrac{0.587006}{T_a}+0.0001316}$$

and the corresponding pressure is found from steam tables.

13. *Specific Volume of the Liquid.*—There are available three sets of experimental data from which the specific volume of the liquid may be determined. Lange[27] determined the density of liquid ammonia over the temperature range of $-56°$ F. to $+208°$ F.; his results are given in Table 14. Dieterici,[25] working by Young's method, obtained simultaneously the specific volumes of the liquid and of the saturated vapor over a temperature range of 32° F. to 222° F.; his results for the former are given in Table 15. In addition to these we have the specific gravity determinations of D'Andréeff,[26] from which the specific volume may be calculated; these experiments cover the range of 14° F. to 68° F. and are given in Table 16.

In Fig. 2 the experimental values are plotted with specific volumes as ordinates and temperatures as abscissas. The form of equation used to express volumes of the liquid in terms of the temperature is that used by Avenarius,[53] or

$$v'=a-b\log\,(t_k-t) \qquad (3)$$

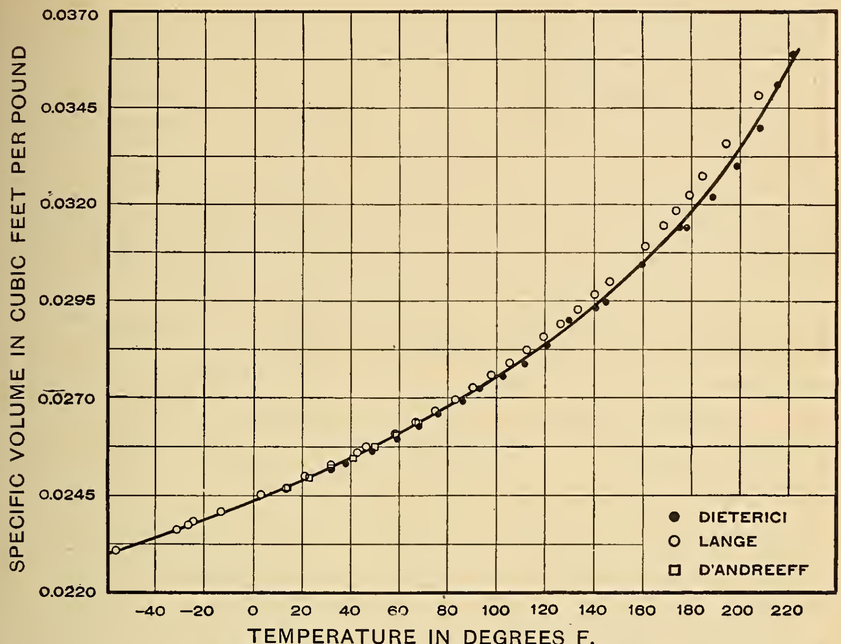

FIG. 2. SHOWING THE AGREEMENT OF VOLUMES OF THE LIQUID AS CALCULATED WITH THOSE FOUND BY EXPERIMENT.

TABLE 14. SUMMARY OF DETERMINATIONS OF VOLUME OF THE LIQUID BY LANGE.

Temp. °C.	Sp. Density of Liquid in gr. per c.c.	Temp. °F.	Sp. Density of Liquid in lb. per cu. ft.	Sp. Vol. of Liquid in cu. ft. per lb.
−49.0	0.6943	− 56.20	43.344	0.023071
−35.0	0.6781	− 31.00	42.333	0.023622
−32.5	0.6742	− 26.50	42.089	0.023759
−31.2	0.6720	− 24.16	41.952	0.023837
−25.0	0.6650	− 13.00	41.515	0.024088
−16.2	0.6534	+ 2.84	40.791	0.024515
−10.2	0.6488	13.64	40.503	0.024690
− 6.0	0.6408	21.20	40.004	0.024998
0.0	0.6341	32.00	39.586	0.025261
+ 6.0	0.6256	42.80	39.055	0.025605
7.9	0.6215	46.22	38.799	0.025774
14.7	0.6141	58.46	38.337	0.026084
19.4	0.6072	66.92	37.906	0.026381
23.9	0.6007	75.02	37.501	0.026666
28.4	0.5939	83.12	37.076	0.026972
32.6	0.5875	90.68	36.677	0.027265
36.7	0.5807	98.06	36.252	0.027585
40.8	0.5739	105.44	35.828	0.027911
44.7	0.5672	112.46	35.409	0.028241
48.6	0.5606	119.48	34.997	0.028574
52.5	0.5540	126.50	34.585	0.028914
56.4	0.5472	133.52	34.161	0.029273
60.2	0.5404	140.36	33.736	0.029642
63.6	0.5340	146.48	33.337	0.029997
71.6	0.5181	160.88	32.344	0.030918
75.8	0.5096	168.44	31.813	0.031434
78.8	0.5033	173.84	31.420	0.031827
81.8	0.4966	179.24	31.002	0.032256
84.7	0.4899	184.46	30.584	0.032697
90.2	0.4771	194.36	29.785	0.033574
97.8	0.4576	208.04	28.567	0.034805

The curve shown in Fig. 2, up to 160° F. represents this equation with the constants as follows

$$a = 0.06335$$
$$b = 0.016$$
$$t_k = 273.2$$

In determining these constants the work of Dieterici was given the most weight, both because the methods he employed are superior to the others, and because the volumes of the vapor to be used with these values are those determined by Dieterici by the same method in the same series of experiments.

The part of the curve above 160° F. represents the values found by the method discussed in Article (16).

TABLE 15. SUMMARY OF DETERMINATIONS OF VOLUME OF THE LIQUID BY DIETERICI.

Temp. °C.	Sp. Vol. of Liquid in c. c. per gm.	Temp. °F.	Sp. Vol. of Liquid in cu. ft. per lb.	Sp. Density of Liquid in lb. per cu. ft.
0.0	1.5656	32.00	0.025078	39.876
3.4	1.5769	38.12	0.025259	39.590
9.5	1.5977	49.10	0.025593	39.073
15.0	1.6161	59.00	0.025887	38.629
20.1	1.6358	68.18	0.026203	38.164
24.65	1.6565	76.37	0.026534	37.688
30.3	1.6778	86.54	0.026876	37.208
34.3	1.6983	93.74	0.027204	36.759
39.3	1.7221	102.74	0.027585	36.252
44.4	1.7448	111.92	0.027949	35.779
49.6	1.7695	121.28	0.028345	35.280
55.4	1.8090	129.92	0.028977	34.510
60.4	1.8289	140.72	0.029296	34.134
60.9	1.8309	141.62	0.029328	34.097
62.8	1.8403	145.04	0.029479	33.922
71.1	1.8957	159.98	0.030366	32.932
79.8	1.9619	175.64	0.031426	31.821
81.2	1.9618	178.16	0.031425	31.822
87.2	2.0119	188.96	0.032227	31.030
92.6	2.0624	198.68	0.033026	30.279
98.1	2.1211	208.58	0.033977	29.432
102.1	2.1878	215.78	0.035045	28.535
105.6	2.2425	222.08	0.035921	27.839

TABLE 16. SUMMARY OF DETERMINATIONS OF VOLUME OF THE LIQUID BY D'ANDRÉEFF.

Temp. °F.	Sp. gravity, referred to Water as Unity	Density in lb. per cu. ft.	Sp. Vol. in cu. ft. per lb.
14	0.6492	40.481	0.02470
23	0.6429	40.088	0.02495
32	0.6364	39.683	0.02520
41	0.6298	39.271	0.02546
50	0.6230	38.847	0.02574
59	0.6160	38.411	0.02603
68	0.6089	37.968	0.02634

14. *Latent Heat of Vaporization.*—The experimental information regarding the latent heat of vaporization of ammonia is too meager to be used as a basis for the determination of the relation existing

between it and the temperature of vaporization. The latent heat may, however, be determined indirectly by thermodynamic relations, and the available data will serve as a check on the method.

Let equation (g) be differentiated with respect to the pressure Then

$$\frac{-\frac{dT_a}{dp}}{T_a^2}=\frac{-c\frac{dT_w}{dp}}{T_w^2},$$

which may be written

$$\frac{T_a\frac{dp}{dT_a}}{T_w\frac{dp}{dT_w}}=\frac{1}{c}\,\frac{T_w}{T_a},$$

or since

$$\frac{T_w}{T_a}=c+kT_w$$

there results

$$\frac{T_a\frac{dp}{dT_a}}{T_w\frac{dp}{dT_w}}=1+\left(\frac{k}{c}\right)T_w \tag{h}$$

Now by the well-known Clapeyron relation connecting the latent heat and the absolute temperature of vaporization with the change of volume during vaporization we have

$$\frac{(v''-v')}{r}=\frac{1}{144\,AT\left(\frac{dp}{dT}\right)_{sat.}}$$

Substituting in equation (h)

$$\frac{(v''-v')_w/r_w}{(v''-v')_a/r_a}=1+\left(\frac{k}{c}\right)T_w \tag{i}$$

or with the proper values of the constants introduced

$$\frac{(v''-v')_w/r_w}{(v''-v')_a/r_a}=1-0.0001316\ T_w \tag{4}$$

The quantity $\frac{(v''-v')_w}{r_w}$ may be found from steam tables or as follows:

From the Clapeyron relation

$$\frac{(v''-v')_w}{r_w}=\frac{1}{144\ AT\left[\frac{dp}{dT_w}\right]_{sat.}}$$

By differentiating equation (1)

$$\left[\frac{dp}{dT_w}\right]_{sat.}=p\left[\frac{11222.13}{T_w^2}+0.00000641484\ T_w-0.00932768\ T_w^2\right]$$

Substituting this value in the Clapeyron relation we have

$$\frac{(v''-v')_w}{r_w}=\frac{1}{p\left[\frac{2078.07}{T_w}-0.00172726\,T_w+0.00000118787\,T_w^2\right]} \tag{5}$$

By the use of equation (4) and equation (5), $\frac{(v''-v')_a}{r_a}$ may be calculated and if either one is known the other may be found. Fortunately we have the experimental determinations of Dieterici of the specific volumes of the liquid and saturated vapor; the former were given in the last section and the latter will be discussed in the next section. From these determinations various values of $(v''-v')$ were found and values of r calculated as described above.

According to the generally accepted ideas concerning the critical point, at that point $r=0$ and $\frac{dr}{dt}=-\infty$. These facts led Thiesen[54] to suggest as an empirical formula

$$r=C(t_k-t)^n$$

This form of equation has been used for water by Thiesen, Henning, and Marks and Davis, and for ammonia by Dieterici.[25] When plotted on logarithmic cross-section paper this form of equation is represented by a straight line, the slope of the line being equal to n.

The values of r found by the above method were plotted to the corresponding values of (t_k-t) on logarithmic cross-section paper and were found to lie almost exactly on a straight line, indicating that the Thiesen formula might be used to express the desired relation. The formula may be written in the form

$$\log r=\log C+n\log(t_k-t)$$

and it was found that the values of the constants giving the best agreement with the plotted points were

$$\log C=1.856064$$

and

$$n=0.37$$

Therefore the final equation is

$$\log r=1.856064+0.37\log(273.2-t) \tag{6}$$

The available data concerning the latent heat of vaporization are given in Table 17 and the various points are plotted in Fig. 3, the full line in this figure representing equation (6). Of the determinations of r made by Regnault, the greater number were lost in the reign of the Commune, but twelve were saved and later published.[5] The results of these experiments do not give r directly and have been variously interpreted by different writers. The table contains the interpretation of Jacobus[55] and the three values of r as quoted from Regnault by

TABLE 17. SUMMARY OF VARIOUS DETERMINATIONS OF THE LATENT HEAT OF VAPORIZATION.

Temp. °C.	Cal. per Kg.	Temp. °F.	B. t. u. per lb.	Authority
10.90	287.0	51.62	516.6	
15.53	285.2	59.95	513.3	
16.00	290.5	60.80	522.8	
12.94	283.8	55.29	510.8	
11.90	285.8	53.42	514.4	
10.73	288.1	51.31	518.5	Regnault (Jacobus)
11.04	292.5	51.87	526.4	
10.15	292.4	50.27	526.2	
9.52	295.0	49.14	531.0	
10.99	293.3	51.78	527.9	
12.60	291.6	54.68	524.8	
7.80	291.8	46.04	525.2	
7.80	294.2	46.04	529.5	
11.00	291.3	51.80	524.3	Regnault (Dieterici)
16.00	297.4	60.80	535.3	
19.53	296.5	67.15	533.7	von Strombeck
17.00	296.8	62.60	534.2	
−33.64	337.0	−28.55	606.6	Franklin and Kraus
−33.40	321.3	−28.12	578.3	Estreicher and Schnerr
		84.6	524.8	
		82.7	525.7	
		87.7	512.4	
		−10.7	569.2	Denton and Jacobus
		− 3.2	603.5	
		+14.5	570.4	

Dieterici.[25] Two values for r are given by von Strombeck,[19] one being an average of twelve experiments, the other of eight. The value given by Franklin and Kraus[20] is an average of three determinations at the normal boiling point, and the value obtained is exactly the value deduced by the same writers from the absolute boiling point and the molecular elevation. A different value was found at the normal boiling point by Estreicher and Schnerr.[21] The values given by Denton and Jacobus,[22] represented in Fig. 3 by crosses, were calculated from readings taken during a test of an ammonia compressor and hence cannot be considered of much weight; because of the scarcity of scientific data, however, these points are included in the table and chart.

In Fig. 3 are also plotted the curves that represent various equations used in calculating the latent heat of vaporization, as given in the various existing tables. These are plotted exactly as given and it should be remembered that there is some variation in the value of the heat unit used by different writers; consequently the various curves are not exactly comparable and are given simply to show the variation in the values of latent heat as given in the tables now in use.

The equations of the curves shown in Fig. 3 are

Ledoux[6]: $r = 583.33 - 0.5499t - 0.0001173t^2$

Wood[10]: $r = 555.5 - 0.613t - 0.000219t^2$

Peabody[11]: $r = 540 - 0.8\ (t - 32)$

Dieterici[25]: $\log r = 1.56141 + 0.5 \log\ (266.9 - t)$

Goodenough[32]: $\log r = 1.7920 + 0.4 \log\ (266 - t)$

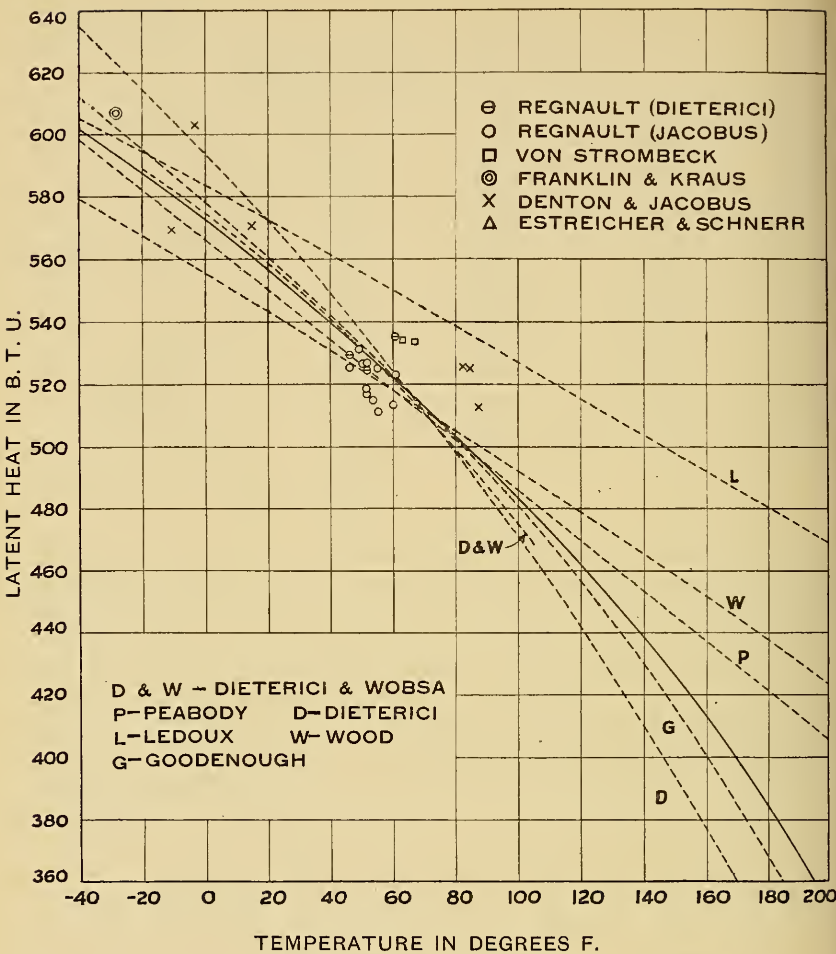

Fig. 3. Showing Comparison Between Experimental Values for Latent Heat of Vaporization and Values as Calculated by Various Writers.

No comment is necessary regarding the curves for r according to Ledoux, Wood, and Peabody, since their equations are not of such form as to give correct results at high temperatures. Moreover they were derived before we had experimental determinations of the volume of the saturated vapor, from which latent heats could be calculated.

Dieterici, as mentioned in Section 12, used Zeuner's empirical expression for the second term of Regnault's pressure-temperature equation, and omitted the third term entirely. This approximation is quite accurate for low temperatures but as the temperature increases

the pressures thus calculated become too small and this error increases rapidly at high temperatures. For instance, Regnault extrapolates his curve and gives for 212° F. a pressure of 901.6 lb. per sq. in., while the value calculated by Dieterici for this temperature is 847.1 lb. per sq. in. or 6% lower. As the error is increasing, the values of $\left[\frac{dp}{dt}\right]_{sat.}$ are too small and values of r calculated by means of the Clapeyron equation are too small at high temperatures. Since the relation employed between r and (t_k-t) is a straight line on logarithmic paper and the values of $\frac{dp}{dt}$, consequently of r, agree at some medium temperature, the values given by the equation at low temperatures will be too high. This is shown in Fig. 3, where it is seen that Dieterici's curve passes above even the highest of the actual experimental determinations of r at $-28°$ F.

The equation used by Goodenough fits the actual determinations and is consistent with the Clausius relation, using Dieterici's values for volumes and Goodenough's constants in Bertrand's pressure-temperature equation. This latter equation, however, only applies over a small temperature range. The variation between Goodenough's curve for r and the present one is due to the different pressure-temperature relation used and the higher value assumed for the critical temperature.

It is believed that, due to the form of equation (6) and the accuracy with which it represents the derived values of r where known, the equation may be safely extrapolated as far as required in this investigation.

The comparison between the values of r given by equation (6), the corresponding values of v'' from the Clausius relation, and the experimental values for v'' properly belongs in the next section and is there shown.

15. *Specific Volume of Saturated Vapor.*—The only available experimental determinations of the specific volume of the saturated vapor are those of Dieterici.[25] These experiments extend up to a temperature of 222° F., but unfortunately were not carried below a temperature of 32° F. Owing to the form of the curve representing the volume-temperature relation, any extended extrapolation of the curve or of any empirical equation to represent this relation would be very unsafe below 32° F.; it is seen that below this point the volume is increasing very rapidly.

As shown in the preceding section, however, equation (6) is believed to be of such form that extrapolation can be carried to very low tem-

peratures. Therefore in the present investigation the volumes of the saturated vapor have been calculated as follows:

By the use of equation (4) and equation (5) values of $\frac{(v''-v')_a}{r_a}$ were calculated. These values multiplied by the corresponding values of r found from equation (6) gave the values of $(v''-v')$. The addition of the proper values of v' found from equation (3) gave the values of v'', the quantity desired.

The values of v'' as found by Dieterici are given in Table 18, and the agreement of the values calculated by the above method is shown in Fig. 4, the curve in this figure representing the calculated values and the points the observed values.

TABLE 18. SUMMARY OF DETERMINATIONS OF VOLUME OF THE SATURATED VAPOR BY DIETERICI.

Temp.° C.	Sp. Vol. of Sat. Vapor in c. c. per gm.	Temp.° F.	Sp. Vol. of Sat. Vapor in cu. ft. per lb.	Sp. Density of Sat. Vapor in lb. per cu. ft.
3.4	257.2	38.12	4.120	0.2427
9.5	208.2	49.10	3.335	0.2999
15.0	173.9	59.00	2.786	0.3589
20.1	147.5	68.18	2.363	0.4232
24.65	125.2	76.37	2.006	0.4985
30.3	106.6	86.54	1.708	0.5855
34.3	93.8	93.74	1.503	0.6653
39.3	82.5	102.74	1.322	0.7564
44.4	74.3	111.92	1.190	0.8403
49.6	63.2	121.28	1.012	0.9881
55.4	52.8	129.92	0.8458	1.1823
60.4	48.1	140.72	0.7705	1.2979
60.9	47.2	141.62	0.7561	1.3226
62.8	45.4	145.04	0.7272	1.3751
71.1	37.22	159.98	0.5962	1.6773
79.8	30.00	175.64	0.4806	2.0807
81.2	29.92	178.16	0.4793	2.0864
87.2	26.23	188.96	0.4202	2.3798
92.6	23.10	198.68	0.3700	2.7027
98.1	20.11	208.58	0.3221	3.1046
102.1	17.88	215.78	0.2864	3.4916
105.6	16.11	222.08	0.2581	3.8745

16. *Specific Volume of Liquid and Saturated Vapor at High Temperatures.*—The "Law of the Straight Diameter" was first proposed by Cailletet and Mathias[56] in 1886. This law, as originally given, stated that if the densities of a liquid and its saturated vapor are plotted as abscissas, against the corresponding temperature as ordinates to form a dome, the mid-points of the horizontal chords of this dome will lie in a straight line nearly parallel to the axis of temperatures.

This law has been frequently tested and applied to many different substances. References to the law may be found in an article by Davis,[57] who applies the law to water as a basis for the calculation of volumes of saturated steam at high temperatures.

It was found by Young[58] that the diameter is actually straight in the case of but few substances, normal pentane being an example of

this class. In the case of most substances, however, the diameter can be represented accurately by a second degree equation; some substances, such as alcohols, require a third degree equation.

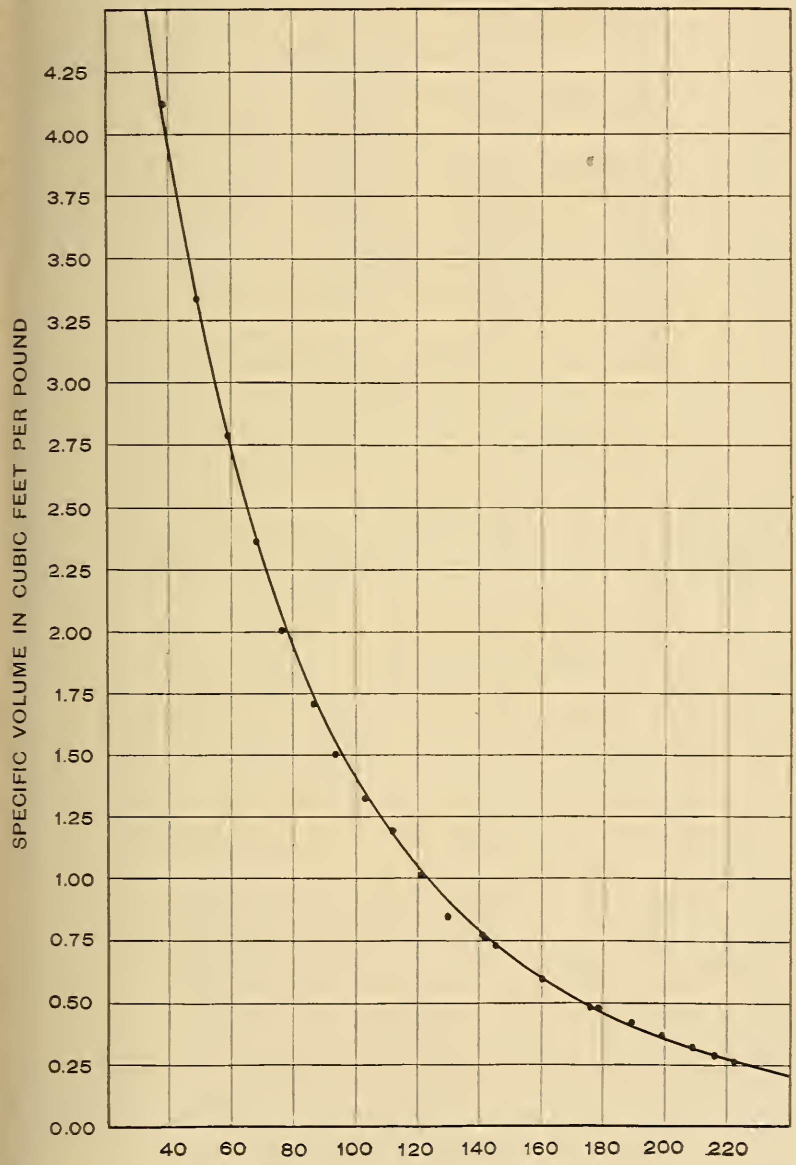

FIG. 4. SHOWING AGREEMENT OF VOLUMES OF THE SATURATED VAPOR AS CALCULATED WITH THOSE FOUND EXPERIMENTALLY BY DIETERICI.

TABLE 19. THE LAW OF THE STRAIGHT DIAMETER FOR AMMONIA.

Temp.° F.	Density of Vapor	Density of Liquid	Mean Density	Mean Density by Formula	Difference
− 40	0.0393	42.704	21.372	21.372	±0.000
− 30	0.0517	42.296	21.174	21.174	±0.000
− 20	0.0671	41.883	20.975	20.974	−0.001
− 10	0.0860	41.461	20.774	20.774	±0.000
0	0.1088	41.039	20.574	20.572	−0.002
+ 10	0.1362	40.608	20.372	20.370	−0.002
20	0.1689	40.169	20.169	20.167	−0.002
30	0.2075	39.722	19.965	19.964	−0.001
40	0.2525	39.267	19.760	19.759	−0.001
50	0.3051	38.803	19.554	19.554	±0.000
60	0.3657	38.329	19.347	19.347	±0.000
70	0.4355	37.844	19.140	19.140	±0.000
80	0.5152	37.348	18.932	18.932	±0.000
90	0.6061	36.841	18.724	18.724	±0.000
100	0.7099	36.319	18.514	18.514	±0.000
110	0.8265	35.782	18.304	18.303	−0.001
120	0.9590	35.228	18.093	18.092	−0.001
130	1.1062	34.655	17.881	17.880	−0.001
140	1.2756	34.061	17.668	17.667	−0.001
150	1.4643	33.444	17.454	17.453	−0.001
160	1.6763	32.798	17.237	17.238	+0.001

In order to find the equation of the "straight diameter" for ammonia the values of the densities of the liquid and of the saturated vapor were found at 10° intervals. The values for the liquid were calculated by the use of equation (3), and those for the vapor by the method described in Section 15. At each of the temperatures the mean density was calculated. The results are given in Table 19 and plotted in Fig. 5. In this figure the points for the liquid and vapor are shown as large

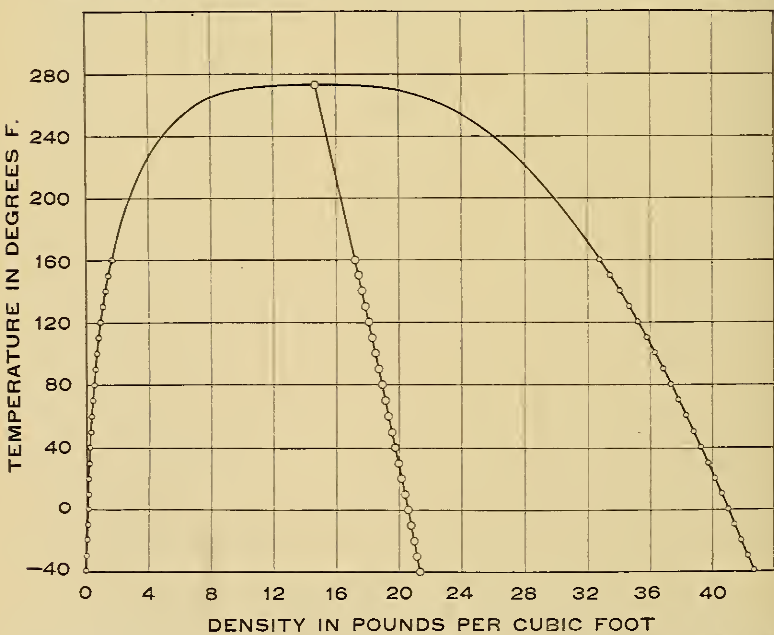

FIG. 5. SHOWING THE DOME ON THE TEMPERATURE-DENSITY PLANE AND THE "STRAIGHT DIAMETER" OF CAILLETET AND MATHIAS.

circles and the mean points as small circles. The diameter is seen to be slightly curved but it may be accurately represented by the second pegree equation:

$$\gamma = 19.3473 - 0.02067\ (t-60) - 0.0000042\ (t-60)^2 \tag{7}$$

The table shows that this equation represents the mean densities as calculated by the other method with a maximum error of about 0.01 of 1% over a temperature range of $-40°$ F. to $+160°$ F.

The method used at ordinary temperatures for finding the change of volume on vaporization or $(v''-v')$, may be employed at temperatures up to the critical point if we assume that the values for r found by equation (6) are correct up to that point. Equation (7) gives values for

$$\gamma = \frac{\gamma'' + \gamma'}{2} = \frac{1}{2}\left[\frac{1}{v''} + \frac{1}{v'}\right]$$

A simple algebraic manipulation gives

$$v'' = \frac{\gamma(v''-v') + 1 + \sqrt{\gamma^2(v''-v')^2 + 1}}{2\gamma}$$

Having values for v'' and $(v''-v')$, the values for v' may be easily found. This method enables the values of the volumes of the liquid and the saturated vapor to be found at least qualitatively up to the critical point itself.

It was found that above 160° F. the values found for v' by this method were materially lower than those found by equation (3) and that the difference increased as the temperature increased above this point. An inspection of equation (3) shows that this equation would give a value of $+\infty$ for the volume of the liquid at the critical point, whereas we know that at this point the liquid has a finite volume. In the case of other vapors it has been shown that the form of equation used, that of Avenarius, gives accurate values at temperatures somewhat removed from the critical point, but that it does not hold near this point. Therefore, in the present case equation (3) has been used only up to 160° F., and the values above this temperature have been calculated by the law of the straight diameter.

Since the diameter is nearly parallel to the axis of temperatures a considerable error in the value of the critical temperature will cause but a small error in the resulting value of the critical density. The substitution in equation (7) of the value of t_k assumed in Section 12, or 273.2° F., gives for the critical density and volume respectively

$$\gamma_k = 14.75 \text{ lb. per cu. ft.}$$
$$v_k = 0.0678 \text{ cu. ft. per lb.}$$

17. *Specific Volume of the Superheated Vapor.*—The attempt has often been made to deduce rationally an equation of state which with suitable change of constants will represent the relation $f(p, v, T)=0$ for various fluids in all states from the gaseous condition above the critical temperature to the liquid condition. Such equations are constructed with special reference to the behavior of fluids in the neighborhood of the critical state and apply more particularly to fluids the critical temperature of which is within the range encountered in practice. In the case of a fluid such as ammonia, however, the critical temperature of which is far above the working range, purely empirical equations of simpler form give better results throughout the smaller range covered in practice and in addition lend themselves much more readily to the formation of the various heat equations. A few of these empirical equations which have been applied to ammonia are here given.

(a) Zeuner[9] and Ledoux[6] both used the form

$$pv=BT-Cp^n$$

The constants used by Zeuner with the variables expressed in metric units are $B=52.642$ $C=29.783$ $n=0.3655$

This form was also used by Peabody.[11] In English units with p in lb. per sq. ft., his values for the constants are

$$B=99 \qquad C=710 \qquad n=0.25$$

(b) Wood[10] used the form proposed by Rankine

$$pv=BT-\frac{C}{v^n}$$

In English units with p in lb. per sq. ft., his values of the constants are

$$B=91 \qquad C=16920 \qquad n=0.97$$

(c) Wobsa[59] in his first paper used the form of equation proposed by Linde

$$v=\frac{BT}{p}-(1+ap)\left[\frac{C(273)^n}{T}-D\right]$$

In metric units his values for the constants are

$$B=49.8 \qquad a=0.0000014$$

$$C\times 273^n=2250 \qquad n=2 \qquad D=0.01$$

(d) In his second paper Wobsa[61] used a new form of equation

$$v-a=\frac{BT}{p}-\frac{C}{T^n}+\frac{b}{p}$$

with $B=49.736$ $C=2450$ $b=80$
$a=0.0075$ $n=2$

The equations of Ledoux, Zeuner, Peabody, and Wood were derived before experimental determinations of the volume of either the saturated or the superheated vapor were available, and they were based upon various doubtful assumptions. Wobsa's first equation gives values

that do not agree closely with experimental results, but his second equation is quite satisfactory in this respect.

In choosing a characteristic equation several points must be considered.

(1) The equation must represent with fair accuracy the available reliable experimental data on the relations of p, v, and T.

(2) The equation should be of such form as to give as simple forms as possible to the various thermodynamic relations that are derived from it.

(3) These derived equations must represent accurately the experimental data.

Wobsa's second equation fulfils the first of these conditions admirably. It is, however, somewhat defective with respect to the other requirements. The good results obtained from Goodenough's equation[64] for superheated steam, namely,

$$v+c=\frac{BT}{p}-(1+ap)\frac{m}{T^n} \tag{j}$$

suggested the adoption of the same form of equation for supherheated ammonia in the preliminary investigation. This equation satisfies the second condition, in that it gives derived relations of comparatively simple form. A trial with various sets of constants showed that it could be made to represent the volume measurements with substantially the same accuracy as Wobsa's second equation, and thereby satisfy the first condition. Having established the fact that the proposed equation is permissible, the next step was the determination of the constants. In connection with this process emphasis must be placed on the third consideration heretofore mentioned. From the characteristic equation are derived expressions for (a) the specific heat at constant pressure; (b) the heat content of the superheated and also of the saturated vapor; (c) the Joule-Thomson coefficient. Hence the constants must not be chosen with reference to volume alone. While the volume measurements must be satisfied, three other derived relations must equally well conform to the experimental data in the respective fields.

With due consideration of all the conditions the following values were finally assumed for the constants:

$B=0.6321$, p in lb. per sq. in.

$\log m=12.900000$

$c=0.100$

$n=5$

$a=0$

The final equation with constants inserted is therefore

$$v+0.100=0.6321\frac{T}{p}-\frac{79433\times10^8}{T^5} \tag{8}$$

It will be seen that the constant a is taken as zero. In view of the fact that the available data consist only of values along the saturation curve and in most cases but one point on each isotherm in the superheated region, no information is available regarding the shape or curvature of these isotherms on the $pv-p$ plane; moreover the derived equations demand an exceedingly small value of a. The use of straight lines for these isotherms therefore seems to be as well justified as the use of parabolas; hence a was made equal to zero.

Table 20. Summary of Various Determinations of Volume of the Superheated Vapor.

Authority	Pressure in Atmos.	Temp. °C.	Volume in Liters per gm.	Temp. °F.	Volume in Cu. ft. per lb.
Perman	½	0	2.6096	32	41.8006
Perman	1	— 20	1.19575	— 4	19.1535
Perman	1	0	1.2973	32	20.7802
Perman	1	50	1.5473	122	24.7847
Perman	1	100	1.7964	212	28.7747
Leduc	1	0	1.2955	32	20.7513
Guye	1	0	1.2974	32	20.7818

Table 21. Comparison of Values Obtained from Various Equations for Volume of the Superheated Vapor with Experimental Values.

Pressure in Atmos.	Temp. °F.	Experimental Values		Computed from (8)		Computed by Wobsa I		Computed by Wobsa II	
		Authority	Value	Value	% Diff. from Exp.	Value	% Diff. from Exp.	Value	% Diff. from Exp.
1	−4	Perman	19.154	19.093	−0.32	19.121	−0.17	19.140	−0.07
1	32	Perman	20.780	20.769	−0.05	20.745	−0.17	20.767	−0.06
1	32	Guye	20.782	20.769	−0.06	20.745	−0.18	20.767	−0.07
1	32	Leduc	20.751	20.769	+0.09	20.745	−0.03	20.767	+0.08
1	122	Perman	24.785	24.798	+0.05	24.746	−0.16	24.775	−0.04
1	212	Perman	28.775	28.730	−0.16	28.695	−0.28	28.725	−0.17
½	32	Perman	41.801	41.916	+0.28	41.821	+0.05	41.939	+0.33

A summary of the various direct determinations of the specific volume of the superheated vapor according to Perman and Davies,[30] Leduc[28] and Guye[29] is given in Table 20. In Table 21 the values given by equation (8) and by Wobsa's first and second equations are compared with these experimental determinations. It is seen that both equation (8) and Wobsa's second equation give results agreeing better with experiment at the one atmosphere points that those obtained from Wobsa's first equation. At the 32°, 122°, and 212° points at one atmosphere pressure equation (8) and Wobsa's second equation give practically the same percentage deviation from the experimental values. At −4° Wobsa's second equation gives a better agreement with the

experimental value than does equation (8); this point, however, is considerably below the range found for superheated ammonia in practice, and it is believed that the considerations appearing in the discussion of the heat equations derived from the characteristic equation justify the use of equation (8) in preference to Wobsa's equations.

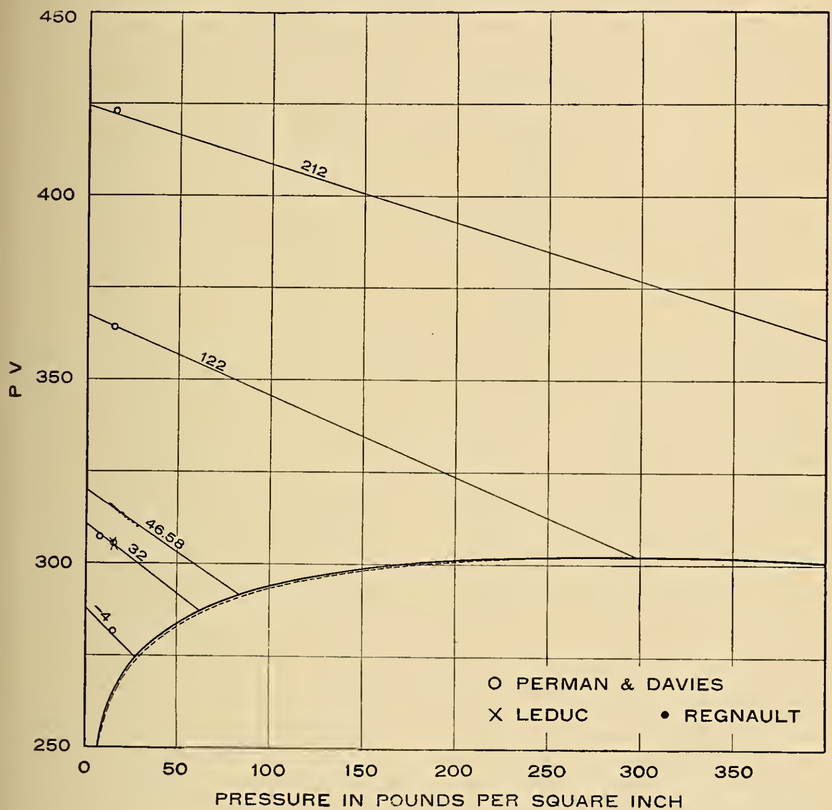

FIG. 6. COMPARISON OF ISOTHERMS DEDUCED FROM EQUATION (8) WITH THE POINTS REPRESENTING EXPERIMENTAL DETERMINATION ON THE PV-P PLANE.

In Fig. 6 the isotherms deduced from equation (8) are drawn on the $pv-p$ plane and the various experimental points are plotted in order to show the agreement. From this chart it is seen that if both of the determinations of Perman and Davies at 32° are correct, the one at one atmosphere and the other at one-half atmosphere, then either the saturation curve should lie considerably higher or the 32° isotherm should be sharply curved. Since the one atmosphere determination agrees closely with the determinations of Leduc and Guye, since there

is good authority for the location of the saturation curve, and since the flatness of the isotherms is fairly well established, it would seem that the half atmosphere point of Perman and Davies is probably in error.

Table 22. Comparison of Values Obtained from Equation (8) with Values Obtained from Equation (6) and the Clapeyron–Clausius Formula.

Temp. °F	Calculated from (6) and the Clapeyron-Clausius formula	Calculated from (8)	Per cent. difference
− 40	25.452	25.501	+0.19
− 20	14.895	14.933	0.25
0	9.187	9.214	0.29
20	5.920	5.936	0.27
40	3.9595	3.9695	0.25
60	2.7344	2.7402	0.21
80	1.9400	1.9428	0.14
100	1.4087	1.4099	0.09
120	1.0424	1.0425	0.01
140	0.7840	0.7835	−0.06
160	0.5966	0.5962	−0.07
180	0.4577	0.4580	+0.07
200	0.3525	0.3539	0.40

In Table 22 the values of the specific volume along the saturation curve as calculated by equation (8) are compared with those found by the use of equation (6) and the Clapeyron-Clausius formula. In Fig. 6 the full line curve represents values resulting from equation (8) and the dotted line curve represents the values resulting from equation (6). A comparison of Wobsa's equation is not given because the constants in those equations were determined on the assumption of a different pressure-temperature relation along the saturation curve; the substitution in Wobsa's equations of the values of pressure corresponding to given temperatures as used in this investigation therefore would not give values of volume comparable with those deduced from equation (8). It may be stated, however, that Wobsa's equations give very fair agreement with the values found experimentally by Dieterici. The maximum deviation of the second equation from the values given in Dieterici's table being a little over 2%.

In addition to the preceeding there are available the results of a series of experiments performed by Regnault, which may be used for the purpose of checking the characteristic equation. In these experiments measurements were made of the relative volumes occupied by a quantity of ammonia gas at different pressures along the isotherm corresponding to a temperature of 8.1° C. or 46.58° F. As the weight of ammonia used was not recorded, the experiments can be used only to obtain the relative values of the specific volume or of the product pv along this isotherm. In making use of these values the products pv were plotted against the corresponding values of p, both quantities

being measured in the units employed by Regnault. A straight line was next passed through these points by the method of least squares. It was found that the pressure equivalent to 20 lb. per sq. in. came at about the center of the group of points, and the value of the specific volume was calculated by equation (8) for this pressure and a temperature of 46.58° F.; this value multiplied by the pressure gave a value of the product pv in English units. The value of this product in the units used by Regnault was then found at the point where the straight line determined by least squares crossed the pressure co-ordinate equivalent to a pressure of 20 lb. per sq. in. From these two values for pv the conversion factor 0.00561 was found, this being the number by which Regnault's values of pv must be multiplied in order that the isotherm calculated from equation (8) shall pass through the center of Regnault's group of points. All of his values of pv were multiplied by

TABLE 23. SUMMARY OF REGNAULT'S *PV* MEASUREMENTS ALONG THE ISOTHERM FOR 46.58 DEGREES F.

Mark	Volume (Relative)	Pressure in. mm. Hg.	$pv/10$	New pv	Pressure in lb. per sq. in.
72	841.95	668.93	56325	315.98	12.94
68	800.00	703.53	56282	315.74	13.61
64	758.56	741.23	56227	315.43	14.34
60	717.26	783.18	56174	315.14	15.15
56	675.84	829.98	56094	314.69	16.06
52	634.46	882.98	56022	314.28	17.08
48	592.83	943.18	55915	313.68	18.25
44	551.40	1013.63	55892	313.55	19.61
40	509.98	1092.53	55715	312.56	21.13
36	468.37	1186.38	55568	311.74	22.95
32	426.85	1299.11	55452	311.09	25.13
28	384.89	1435.33	55243	309.91	27.77

this factor and the resulting values appear in Table 23. The points representing these values of pv are plotted in Fig. 6 as small black dots. The only information they afford is a check on the slope of the isotherm for 46.58° F. on the $pv-p$ plane. The points seem to indicate that the isotherms, near this temperature at least, are very flat curves or even straight lines.

The data on coefficients of dilatation, of compressibility, and similar coefficients afford a further check on the characteristic equation. In the experiments just discussed Regnault found for 46.58° F. and for the range from 1 to 2 atmospheres, $\frac{p_1v_1}{p_2v_2}=1.01881$. Calculating this same ratio for the same pressure limits from the characteristic equation, there results $\frac{p_1v_1}{p_2v_2}=1.01606$.

At 9.7° C., or 49.46° F., Lord Rayleigh[72] found

$$\frac{pv \text{ at } \tfrac{1}{2} \text{ atmosphere}}{pv \text{ at } 1 \text{ atmosphere}}=1.00632.$$

From the characteristic equation there results

$$\frac{pv \text{ at } \frac{1}{2} \text{ atmosphere}}{pv \text{ at 1 atmosphere}}=1.00770$$

It is seen that the deviations of the values resulting from the equation from the experimental values are in opposite directions at these two temperatures which are but 3° F. apart and it seems probable that one or the other of the experimental values is in error.

For the coefficient of dilatation, $\frac{1}{v_0}\cdot\frac{\Delta v}{\Delta t}$, from 32° F. to 212° F. at atmospheric pressure Perman and Davies[30] found 0.003847 and Leduc[73] found 0.003797. The value found from the characteristic equation is 0.003833, which agrees well with the mean of the experimental values.

The values of $\frac{1}{v}\cdot\frac{dv}{dt}$ at 32° F. and one atmosphere as found experimentally by Leduc[73] and from the characteristic equation are respectively 0.003857 and 0.003932. The corresponding values of $\frac{1}{p}\cdot\frac{dp}{dt}$ at the same point are respectively 0.003801 and 0.003710.

It will be seen that the characteristic equation gives results which do not always agree closely with the experimental values of the various coefficients. The other proposed characteristic equations, however, give results which do not agree any better with the whole body of experimental data, and considering the inconsistencies in the experimental data it is probable that equation (8) is as good an equation as can be devised on the basis of the existing information.

18. *Specific Heat of the Superheated Vapor.*—The first determination of the specific heat of superheated ammonia was made by Regnault,[3] who found a value of 0.50836 at atmospheric pressure and over a temperature range of 75° F. to 420° F. Later Wiedemann[65] performed two series of experiments at a pressure of about 16 lb. per sq. in.; he found a value of 0.5202 between 77° F. and 212° F., and a value of 0.5356 between 77° F. and 392° F. He proposed the equation

$$c_p=0.4949+0.000172t$$

Thus far the history is much the same as in the case of superheated steam, the specific heat of which was long supposed to be independent of both temperature and pressure and equal to 0.48 as determined by Regnault; later the investigations of Mallard and LeChatelier and of Lange agreed in making it a linear function of the temperature. In the case of steam, however, the experiments of Knoblauch and Jakob, Thomas, and Knoblauch and Mollier have shown that the specific heat of superheated steam depends upon the pressure. While no

experiments have been made to investigate the variation of the specific heat of superheated ammonia with pressure, it is very probable that there is such a variation and its rate may be determined by means of Clausius' thermodynamic relation, providing a sufficiently accurate characteristic equation has been determined. The process of obtaining an explicit expression for c_p in terms of the variables p and T has inherent difficulties and is rendered more difficult in this case by the meagerness of the data available. The success with which the method has been applied to superheated steam by Goodenough, however, seems to warrant its use in the present investigation.

The Clausius relation is expressed by the equation

$$\left[\frac{\partial c_p}{\partial p}\right]_T = -AT\left[\frac{\partial^2 v}{\partial T^2}\right]_p$$

The derivative in the right hand member is determined from the characteristic equation. Thus from equation (8)

$$\frac{\partial v}{\partial T}=\frac{B}{p}+\frac{mn}{T^{n+1}}$$

and

$$\frac{\partial^2 v}{\partial T^2}=\frac{-mn(n+1)}{T^{n+2}}$$

Substituting the second derivative in the Clausius relation the result is

$$\left[\frac{\partial c_p}{\partial p}\right]_T = \frac{Amn(n+1)}{T^{n+1}}$$

Taking T as a constant and integrating with respect to p as the independent variable, the result is

$$c_p=\frac{Amn(n+1)}{T^{n+1}}\,p + \text{const. of integration.}$$

The constant of integration may be a function of T since T was held constant during the integration; hence

$$c_p=\phi(T)+\frac{Amn(n+1)}{T^{n+1}}\,p$$

If the arbitrary function $\phi(T)$ can be determined, there will result an explicit expression for c_p in terms of the variables p and T. Wobsa has used the same method of analysis starting with his own characteristic equations and has obtained equations for c_p similar to the present one.

The determination of the proper function ϕ (T) is the critical point of the investigation. In evaluating this function Wobsa equated his expression for c_p at atmospheric pressure to Wiedemann's linear equation for c_p at atmospheric pressure and thus found the $\phi(T)$ to be of the form

$$\phi(T)=a+bT-\frac{c}{T^3}$$

He shows that the term in the third power of T, which is the "correction term" for atmospheric pressure, is vanishingly small above 212° F.

and that above this temperature c_p at infinitely small pressures will not differ from c_p at atmospheric pressure, or $\phi(T)=0.4949+0.000172t$. Below 212° F. where the correction term becomes appreciable, Wobsa substitutes the linear function $\phi(T)=0.4712+0.000278t$.

For the determination of $\phi(T)$ in the present investigation there are available the recent determinations of c_p at high temperatures by Nernst. In addition to the results of his own work, Nernst[31] gives a summary of the results of previous investigations; See Table 24. In

TABLE 24. SUMMARY OF DETERMINATIONS OF SPECIFIC HEAT AT CONSTANT PRESSURE AS GIVEN BY NERNST.

Authority	Temp. °C	Molecular c_p	Temp. °F	c_p
Keutel	20	8.64	68	0.506
Voller	20	8.62	68	0.505
E. Wiedemann	25–100	8.84	77–212	0.518
E. Wiedemann	25–200	9.11	77–392	0.534
Regnault	24–216	8.71	75–421	0.510
Nernst	365–567	10.4	689–1053	0.609
Nernst	480–680	11.2	896–1256	0.656

this table the values are given as molecular specific heats and these have been converted into specific heats per unit of weight by dividing by the molecular weight of ammonia or 17.064. It is seen that the values given for Wiedemann are lower than those quoted earlier in this section from the original; while the value given for Regnault is higher than the original. As the original papers of Keutel and Voller are not at hand the values given in this table for their results are assumed to be correct, but the values used for Weidemann and Regnault are those taken from the original.

If in the equation for c_p, $p=0$, the equation reduces to $c_p=\phi(T)$. Therefore if we have values for c_p at zero pressure the $\phi(T)$ may be determined. To obtain these values the proper correction terms were calculated and subtracted from the values of c_p. The resulting values of $(c_p)_0$ seemed to justify the assumption that $(c_p)_0$ is a linear function of the temperature. This linear relation may not, and probably does not, hold for the entire range of superheat, but it may be assumed as a close approximation, and the only assumption justified by the experimental data available. The function therefore takes the simple form

$$\phi(T)=\alpha+\beta T$$

and the equation for c_p becomes

$$c_p=\alpha+\beta T+\frac{Amn(n+1)}{T^{n+1}}\,p \tag{k}$$

The constants m and n are those heretofore given in connection with the characteristic equation, and the values finally chosen for α and β

are: $\alpha = 0.382$, $\beta = 0.000174$. Replacing $Amn(n+1)$ by a single constant C, and substituting the proper value for the various constants the formula for the specific heat becomes:

$$c_p = 0.382 + 0.000174T + p\frac{C}{T^6} \tag{9}$$

where $\log C = 13.644705$.

Values of c_p have been calculated for various pressures up to 400 lb. per sq. in. and for temperatures up to 1100° F.; these values are plotted against temperatures in Fig. 7. The experimental determina-

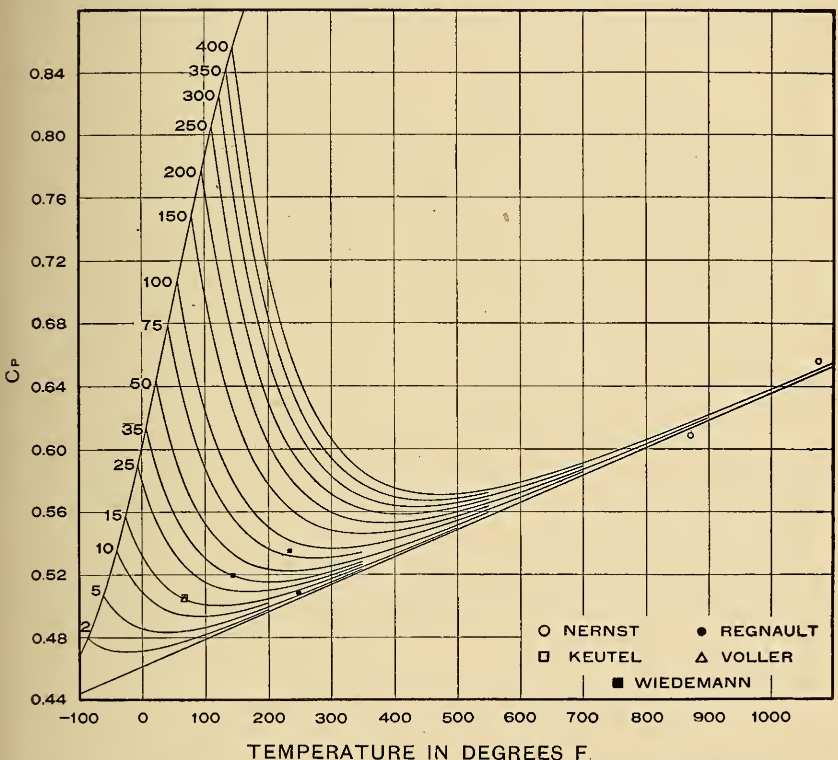

FIG. 7. CURVES SHOWING VALUES OF C_P AT DIFFERENT PRESSURES AND TEMPERATURES.

tions are also plotted to show the agreement. The determinations of Regnault, Nernst, Keutel, and Voller were made at atmospheric pressure, while those of Wiedemann were at a pressure of about 16 lb. per sq. in. absolute. The agreement between the curve derived from equation (9) for a pressure of 15 lb. per sq. in. and the points representing the determinations of Regnault, Nernst, Keutel, and Voller is seen

to be very good, while the points representing Wiedemann's determinations lie considerably above the curve.

19. *Heat Content of Saturated and Superheated Vapor.*—Having an explicit formula for the specific heat at constant pressure in terms of the variables p and T, an expression for the heat content in terms of these variables may be easily derived. For this purpose the general equation

$$dq = c_p dT - AT\left[\frac{\partial v}{\partial T}\right]_p dp \tag{l}$$

is most convenient. Since the heat content is defined by the relation

$$i = A(u+pv)$$

then

$$di = A(du + d(pv))$$

or

$$di = dq + Avdp$$

Hence by substitution in equation (l) there results

$$di = c_p dT - A\left[T\frac{\partial v}{\partial T} - v\right] dp \tag{m}$$

From the characteristic equation

$$\frac{\partial v}{\partial T} = \frac{B}{p} + \frac{nm}{T^{n+1}}\ .$$

whence

$$T\frac{\partial v}{\partial T} - v = (n+1)\frac{m}{T^n}$$

Substituting this and the general expression for c_p in equation (m) the result is

$$di = (\alpha + \beta T)dT + Amn(n+1)p\frac{dT}{T^{n+1}} - \frac{Am(n+1)}{T^n}dp - Acdp \tag{n}$$

Since i depends upon the state of the substance only, the second member of equation (n) must be an exact differential. The integral is readily found to be

$$i = \alpha T + \frac{\beta}{2}T^2 - A(n+1)p\frac{m}{T^n} - Acp + i_o \tag{o}$$

The constant of integration i_o is determined by passing to the saturation limit. Since the constants α, β, m, n, and c are known, the value of the right hand member of equation (o), exclusive of i_o may be found for any given pressure and temperature. If the value of i is known at this point i_o may be determined by subtraction. Now for the saturated vapor

$$i'' = i' + r = u + Apv' + r$$

If the energy u be taken equal to zero at 32° F., then at this temperature

$$i''_{32} = 0 + Ap_{32}v'_{32} + r_{32}$$

Substituting the proper values as determined by the formulas for the saturated vapor there results

$$i''_{32}=0.3+546.4=546.7$$

By the substitution of the proper values of p and T in equation (o) there results

$$i-i_o=188.7$$

By subtraction

$$i_o=358.0$$

Introducing known constants equation (o) becomes, with pressure in lb. per sq. in.

$$i=0.382T+0.000087T^2-p\frac{C}{T^5}-0.0185p+358.0, \qquad (10)$$

where $\log C=12.945735$.

There are no direct measurements of the total heat or heat content of the superheated or saturated vapor by which equation (10) may be checked. A check on the accuracy of the equation at the saturation limit is afforded by a comparison of the values of the heat content of the liquid, obtained by subtracting values of r calculated by equation (6) from values of i'' calculated by equation (10), with the experimental determinations of Dieterici. This comparison is shown in the following section. A check on the accuracy of the formula in the superheated region may be obtained by a comparison, by means of the law of corresponding states, of the values for the Joule-Thomson coefficient resulting from equation (10) with the values of this coefficient for other vapors. There is also available a set of throttling experiments performed by Wobsa.[59]

Since i is constant in a throttling process, the Joule-Thomson coefficient μ may be defined as the derivative

$$\left[\frac{\partial T}{\partial p}\right]_i$$

From calculus

$$\left[\frac{\partial T}{\partial p}\right]_i=-\frac{\dfrac{\partial i}{\partial p}}{\dfrac{\partial i}{\partial T}}$$

and from the definition of heat content i,

$$\frac{\partial i}{\partial T}=c_p$$

Hence

$$\mu=\left[\frac{\partial T}{\partial p}\right]_i=-\frac{1}{c_p}\frac{\partial i}{\partial p}$$

or

$$\mu=\frac{A}{c_p}\left[\frac{m(n+1)}{T^n}+c\right] \qquad (p)$$

From equation (p) it is seen that μ varies with the pressure; as the temperature rises, however, the influence of pressure decreases. Joule and Kelvin, working with gases far removed from the saturation limit, found that μ varies inversely as the square of the absolute temperature. In reviewing the available data on the Joule-Thomson coefficient for hydrogen, nitrogen, oxygen, air, and carbon dioxide, and applying the law of corresponding states, Buckingham[66] discusses the variation of the "reduced" values of μ with the "reduced" temperature, but does not take into account the effect of pressure. Davis,[67] by making use of the throttling experiments of Grindley, Griessmann, Peake, and Dodge on superheated steam, has shown that carbon dioxide and water obey the law of corresponding states. The experimental values of μ for steam expressed in pounds per square inch and degrees Fahrenheit were reduced by multiplying by 2.56, a factor which is the ratio of the critical pressure of water (2947 lb. per sq. in.) to the critical temperature (1149° F.), these critical values being the ones determined by Cailletet and Colardeau. Davis gives a curve which he says represents the experimental values for water in the best possible manner; the values in this curve are those translated back to ordinary units. Later determination of the critical constants for water by Holborn and Bauman give 3200 lb. per sq. in. for the critical pressure and 1166° F. for the critical temperature. If then the values of μ on the Davis curve are multiplied by $\frac{3200}{1166}$, or 2.745, and the temperatures are divided by 1166, there results a curve expressing the variation of μ with temperature, both expressed in reduced units, according to the best available experimental data. This curve is shown in Fig. 8 as the heavy full line curve. The values resulting from equation (p) and those resulting from Wobsa's equation for ammonia may now be compared with the values for water. Wobsa's throttling experiments were performed at pressures of from 2 to 10 atmospheres and at temperatures of 42.8°, 60.8°, 86°, and 104° F. In his paper he gives a table comparing the values of μ resulting from his equation with his experimental values. The values of μ at the same pressures and temperatures have been calculated from equation (p). In Table 20 the first line at each temperature gives the averages of Wobsa's experimental determinations, the second line the values resulting from Wobsa's equation, and the third line the values given by equation (p). The values in the second and third lines have been converted into reduced units and are plotted in Fig. 8, the dotted curves representing the values from Wobsa's equation and the full line curves representing values

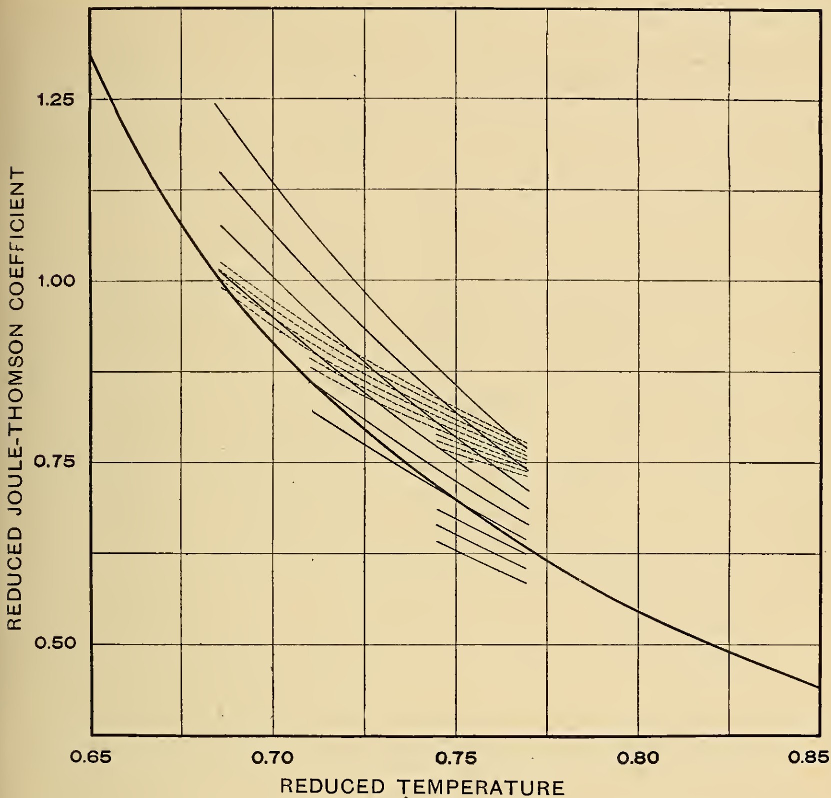

FIG. 8. COMPARISON OF VALUES OF REDUCED JOULE-THOMSON COEFFICIENT FOR AMMONIA ACCORDING TO WOBSA AND THOSE RESULTING FROM EQUATION (p) WITH VALUES FOR WATER AS GIVEN BY DAVIS.

from equation (p). It is seen that while the full line curves form a rather strongly diverging brush yet the Davis line passes through the center of the brush and it appears that the brush would converge into the Davis curve at high temperatures. On the other hand the brush of dotted curves has an entirely different trend and would not merge into the Davis curve.

In his discussion Davis states that he has investigated the data for steam in search of a systematic variation of the value of μ with the pressure, but that none could be found. He therefore concludes that if there is such a variation it is within the error of the experimental observations. It will be interesting, however, to compare the pair of lines for each pressure as shown in Fig. 8 with the points according to Grindley, Griessmann, Peake, and Dodge for the same reduced pressure.

TABLE 25. COMPARISON OF VALUES OF JOULE–THOMSON COEFFICIENT RESULTING FROM PRESENT EQUATIONS AND THOSE RESULTING FROM WOBSA'S EQUATIONS WITH WOBSA'S EXPERIMENTAL VALUES.

Pres. in lb. sq. in.	29.39	44.09	58.78	73.48	88.18	102.87	117.57	132.26	146.96
42.8°F.	0.434	0.439	0.435	0.436					
	0.445	0.440	0.435	0.430					
	0.535	0.498	0.466	0.438					
60.8°F.	0.393	0.393	0.392	0.390	0.387	0.382			
	0.407	0.402	0.397	0.392	0.387	0.377			
	0.464	0.438	0.414	0.393	0.374	0.356			
86.0°F.	0.361	0.356	0.348	0.343	0.343	0.347	0.342	0.339	
	0.364	0.360	0.356	0.353	0.349	0.345	0.342	0.338	
	0.382	0.365	0.349	0.335	0.322	0.310	0.298	0.288	
104.0°F.		0.332	0.331	0.327		0.322	0.317		0.314
	0.337	0.334	0.332	0.329	0.327	0.325	0.322	0.320	0.317
	0.334	0.321	0.310	0.299	0.289	0.279	0.271	0.262	0.255

To do this the reduced values as given by Davis have been multiplied by the proper factor to express the quantities in terms of the latest determination of the critical data for water; the reduced temperatures were multiplied by $\frac{1149}{1166}=0.9854$, the reduced pressures by $\frac{2947}{3200}=0.921$, and the reduced coefficients by $\frac{2.745}{2.56}=1.072$. The values were then

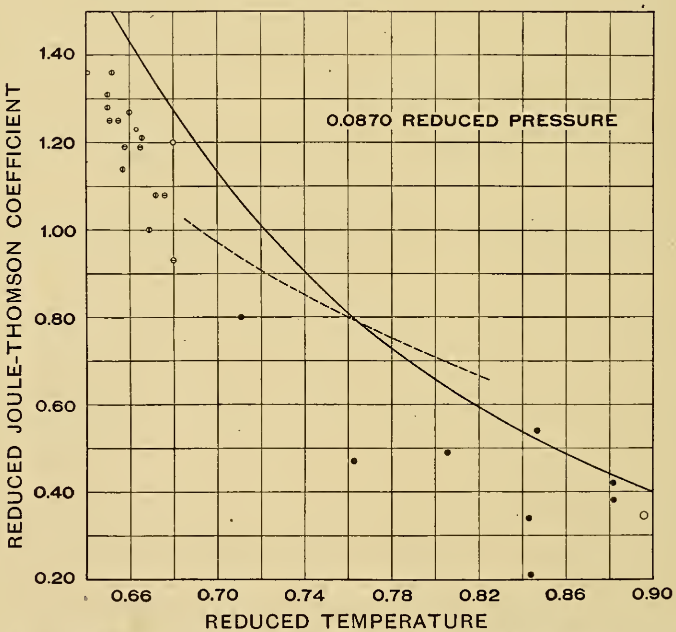

FIG. 9 (a) COMPARISON OF VALUES OF REDUCED JOULE-THOMSON COEFFICIENT FOR AMMONIA ACCORDING TO WOBSA AND THOSE RESULTING FROM EQUATION (p) WITH THE DETERMINATIONS FOR WATER BY GRINDLEY, GRIESSMANN, PEAKE, AND DODGE.

divided into nine groups so that each group included the determinations at reduced pressures nearest to the reduced pressures for ammonia corresponding to 2 to 10 atmospheres. The data divided into such groups are given in Table 26. In Fig. 9 the points in each of the groups are plotted together with the proper curve calculated from equation (p) again shown as full line and the dotted line curve representing Wobsa's

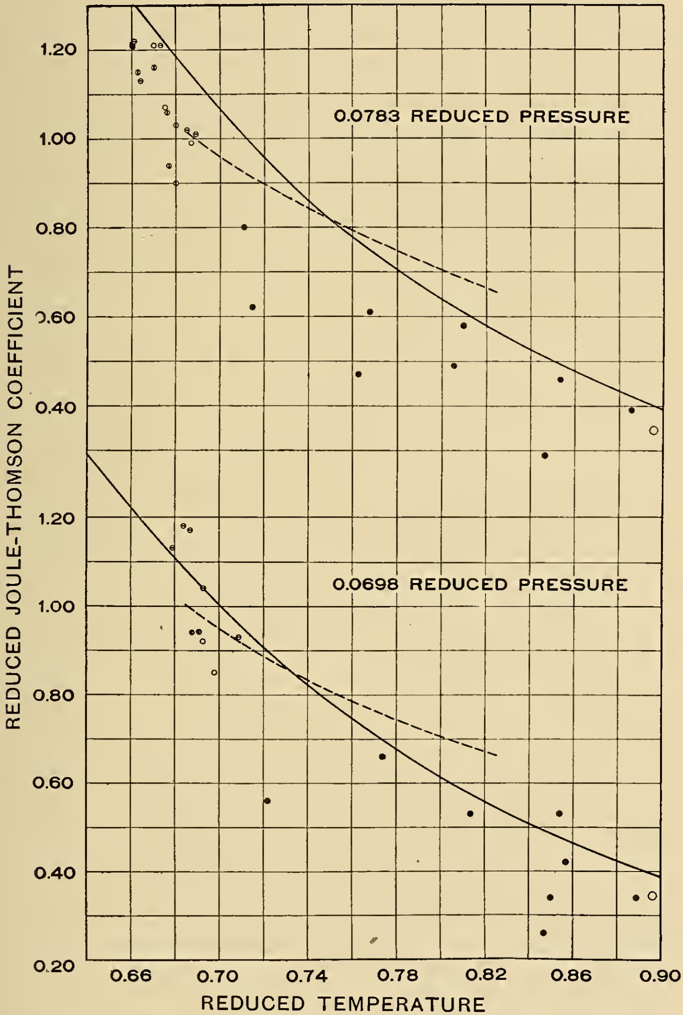

FIG. 9 (b) COMPARISON OF VALUES OF REDUCED JOULE-THOMSON COEFFICIENT FOR AMMONIA ACCORDING TO WOBSA AND THOSE RESULTING FROM EQUATION (p) WITH THE DETERMINATIONS FOR WATER BY GRINDLEY, GRIESSMANN, PEAKE, AND DODGE.

equation. Grindley's values are plotted as open circles, Griessmann's as circles with vertical crossbars, and Peake's as circles with horizontal crossbars; Dodge's values given in Table IV of Davis' paper are plotted as large black dots and his other values as corrected by Davis and given in Table V of his paper appear as small black dots. The large open circle in the lower right hand corner of each curve represents the value found for carbon dioxide by Kester at the lowest temperature investigated. It will be seen that the full line curves give excellent agreement with the experimental values for pressures of 4, 5, 7, and 9 atmospheres although Fig. 8 shows that the 4 and 5 atmosphere curves lie above the Davis curve, the 9 atmosphere curve lies below it and the 7 atmosphere curve cuts across it. The 2 and 3 atmosphere curves lie above the experimental values and the 10 atmosphere curve lies below

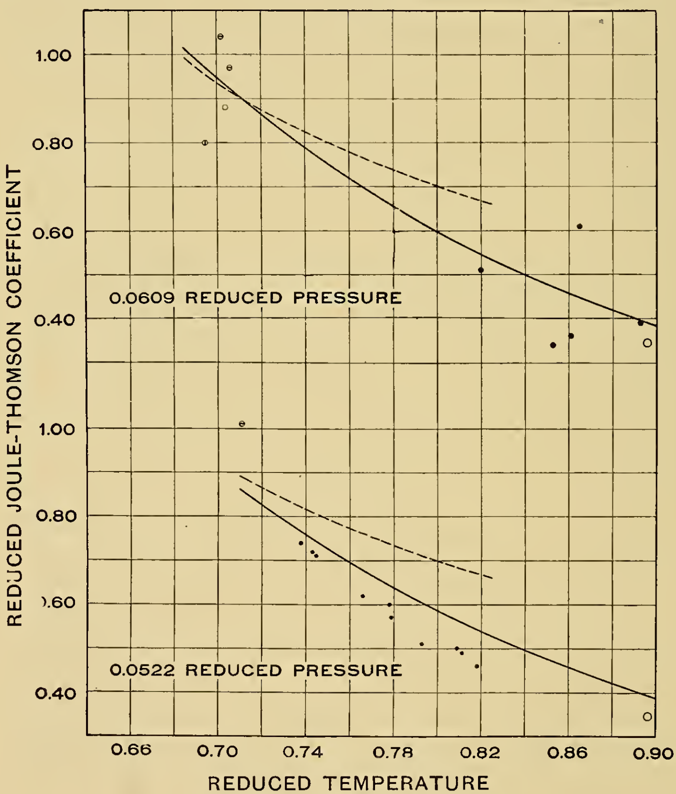

FIG. 9 (c) COMPARISON OF VALUES OF REDUCED JOULE-THOMSON COEFFICIENT FOR AMMONIA ACCORDING TO WOBSA AND THOSE RESULTING FROM EQUATION (p) WITH THE DETERMINATIONS FOR WATER BY GRINDLEY, GRIESSMANN, PEAKE, AND DODGE.

them but in all cases the curves follow the trend of the points. The agreement is not good in the case of the 6 atmosphere curve and a sufficient number of points do not lie in the region of the 8 atmosphere curve to show the trend. The Wobsa curves in all cases show a trend distinctly different from that of the points and in most cases the curves lie considerably above the points. The values in the first line at each temperature in Table 25 show that the values obtained experimentally by Wobsa are quite erratic and in several cases show no regular variation whatever in the value of μ for different pressures at the same temperature. This fact in connection with the evidence offered by the use of the law of corresponding states tends to discredit the accuracy of the experimental work of Wobsa and his resulting equation.

It should also be mentioned that in calculating values of i and s Wobsa makes the assumption that the specific heat at zero pressure is constant and equal to 0.49 although he has previously assumed it to be equal to $0.4949+0.000172\ t$ up to 212° F. and equal to $0.4712+0.000278\ t$ above that temperature.

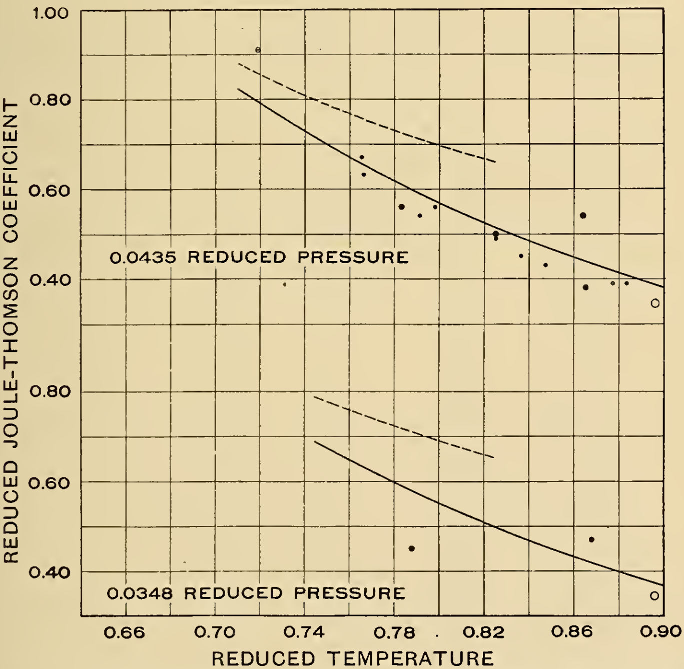

FIG. 9 (d) COMPARISON OF VALUES OF REDUCED JOULE-THOMSON COEFFICIENT FOR AMMONIA ACCORDING TO WOBSA AND THOSE RESULTING FROM EQUATION (p) WITH THE DETERMINATIONS FOR WATER BY GRINDLEY, GRIESSMANN, PEAKE, AND DODGE.

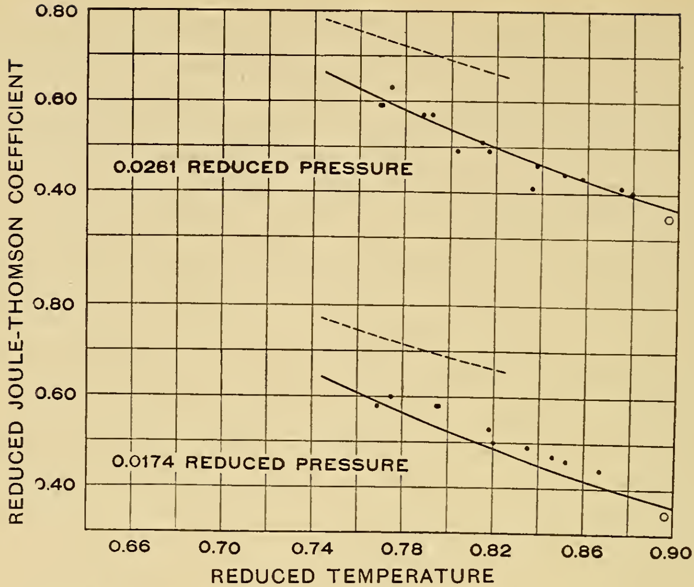

Fig. 9 (e) Comparison of Values of Reduced Joule-Thomson Coefficient for Ammonia According to Wobsa and Those Resulting from Equation (p) With the Determinations for Water by Grindley, Griessmann, Peake, and Dodge.

Table 26. Reduced Joule–Thomson Coefficients for Water Recalculated from Data Given by Davis.

FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.01739

Authority	Reduced quantities based on critical data of Cailletet and Colardeau			Reduced quantities based on critical data of Holborn and Bauman		
	Reduc. Press.	Reduc. Temp.	Reduc. μ	Reduc. Press.	Reduc. Temp.	Reduc. μ
Grindley.....	0.0207	0.690	1.12	0.0191	0.680	1.20
	0.0197	0.673	1.15	0.0181	0.663	1.23
	0.0172	0.651	1.27	0.0158	0.641	1.36
Griessmann...	0.0188	0.682	1.01	0.0173	0.672	1.08
	0.0219	0.679	0.93	0.0202	0.669	1.00
	0.0180	0.676	1.13	0.0166	0.666	1.21
	0.0176	0.670	1.18	0.0162	0.660	1.27
	0.0202	0.667	1.06	0.0186	0.657	1.14
	0.0199	0.662	1.27	0.0183	0.652	1.36
	0.0144	0.660	1.22	0.0133	0.650	1.31
	0.0210	0.660	1.99	0.0193	0.650	1.28
	0.0144	0.646	1.26	0.0133	0.637	1.35
Peake........	0.0213	0.690	0.87	0.0196	0.680	0.93
	0.0221	0.686	1.01	0.0204	0.676	1.08
	0.0199	0.675	1.11	0.0183	0.665	1.19
	0.0233	0.668	1.11	0.0215	0.658	1.19
	0.0200	0.665	1.17	0.0184	0.655	1.25
	0.0165	0.661	1.17	0.0152	0.651	1.25
Dodge I......	0.0196	0.895	0.39	0.0181	0.882	0.42
	0.0195	0.895	0.35	0.0180	0.882	0.38
	0.0196	0.860	0.50	0.0181	0.847	0.54
	0.0195	0.857	0.20	0.0180	0.844	0.21
	0.0183	0.856	0.32	0.0169	0.843	0.34
	0.0184	0.818	0.46	0.0169	0.806	0.49
	0.0178	0.774	0.44	0.0164	0.763	0.47
	0.0195	0.722	0.75	0.0180	0.711	0.80

FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.02609

Authority	Reduced quantities based on critical data of Cailletet and Colardeau			Reduced quantities based on critical data of Holborn and Bauman		
	Reduc. Press.	Reduc. Temp.	Reduc. μ	Reduc. Press.	Reduc. Temp.	Reduc. μ
Grindley.....	0.0277	0.697	0.92	0.0255	0.687	0.99
	0.0296	0.685	1.00	0.0273	0.675	1.07
	0.0253	0.680	1.13	0.0233	0.670	1.21
Griessmann...	0.0313	0.690	0.84	0.0288	0.680	0.90
	0.0268	0.690	0.96	0.0247	0.680	1.03
	0.0308	0.687	0.88	0.0284	0.677	0.94
	0.0243	0.686	0.99	0.0224	0.676	1.06
	0.0237	0.680	1.08	0.0218	0.670	1.16
	0.0260	0.673	1.07	0.0239	0.663	1.15
Peake........	0.0304	0.699	0.94	0.0280	0.689	1.01
	0.0312	0.695	0.95	0.0287	0.685	1.02
	0.0274	0.683	1.13	0.0252	0.673	1.21
	0.0277	0.674	1.05	0.0255	0.664	1.13
	0.0260	0.671	1.14	0.0239	0.661	1.22
Dodge I......	0.0289	0.899	0.36	0.0266	0.886	0.39
	0.0308	0.867	0.43	0.0284	0.854	0.46
	0.0246	0.860	0.27	0.0227	0.847	0.29
	0.0306	0.822	0.54	0.0282	0.810	0.58
	0.0248	0.818	0.46	0.0228	0.806	0.49
	0.0306	0.779	0.57	0.0282	0.768	0.61
	0.0288	0.774	0.44	0.0265	0.763	0.47
	0.0288	0.726	0.58	0.0265	0.715	0.62
	0.0286	0.722	0.75	0.0263	0.711	0.80

FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.03478

Grindley.....	0.0412	0.708	0.79	0.0379	0.698	0.85
	0.0344	0.703	0.86	0.0317	0.693	0.92
Griessmann ...	0.0398	0.701	0.88	0.0367	0.691	0.94
	0.0347	0.698	0.88	0.0320	0.688	0.94
Peake........	0.0413	0.709	0.87	0.0380	0.699	0.93
	0.0403	0.703	0.97	0.0371	0.693	1.04
	0.0405	0.697	1.09	0.0373	0.687	1.17
	0.0379	0.694	1.10	0.0349	0.684	1.18
	0.0337	0.689	1.05	0.0310	0.679	1.13
Dodge I......	0.0356	0.902	0.32	0.0328	0.889	0.34
	0.0409	0.870	0.39	0.0377	0.857	0.42
	0.0356	0.867	0.49	0.0328	0.854	0.53
	0.0347	0.863	0.32	0.0320	0.850	0.34
	0.0343	0.860	0.24	0.0316	0.847	0.26
	0.0409	0.826	0.49	0.0377	0.814	0.53
	0.0409	0.785	0.62	0.0377	0.774	0.66
	0.0389	0.733	0.52	0.0358	0.722	0.56

FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.04348

Grindley.....	0.0480	0.714	0.82	0.0442	0.704	0.88
Griessmann...	0.0438	0.705	0.75	0.0403	0.695	0.80
Peake........	0.0502	0.716	0.90	0.0460	0.706	0.97
	0.0499	0.712	0.97	0.0462	0.702	1.04
Dodge I......	0.0482	0.906	0.36	0.0444	0.893	0.39
	0.0482	0.878	0.57	0.0444	0.865	0.61
	0.0516	0.874	0.34	0.0475	0.861	0.36
	0.0432	0.866	0.32	0.0398	0.853	0.34
	0.0516	0.832	0.48	0.0475	0.820	0.51
	0.0516	0.790	0.54	0.0475	0.778	0.58

Authority	Reduced quantities based on critical data of Cailletet and Colardeau			Reduced quantities based on critical data of Holborn and Bauman		
	Reduc. Press.	Reduc. Temp.	Reduc. μ	Reduc. Press.	Reduc. Temp.	Reduc. μ
FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.05218						
Peake........	0.0569	0.722	0.94	0.0524	0.711	1.01
Dodge II.....	0.057	0.830	0.43	0.0525	0.818	0.46
	0.057	0.805	0.48	0.0525	0.793	0.51
	0.057	0.777	0.58	0.0525	0.766	0.62
	0.057	0.749	0.69	0.0525	0.738	0.74
	0.056	0.823	0.46	0.0516	0.811	0.49
	0.056	0.791	0.53	0.0516	0.779	0.57
	0.056	0.756	0.66	0.0516	0.745	0.71
	0.055	0.821	0.47	0.0507	0.809	0.50
	0.055	0.790	0.56	0.0507	0.778	0.60
	0.055	0.754	0.67	0.0507	0.743	0.72
FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.06087						
Peake........	0.0639	0.730	0.85	0.0588	0.719	0.91
Dodge I......	0.0681	0.877	0.50	0.0627	0.864	0.54
	0.0626	0.878	0.35	0.0577	0.865	0.38
	0.0625	0.837	0.47	0.0576	0.825	0.50
	0.0625	0.795	0.52	0.0576	0.783	0.56
Dodge II	0.070	0.890	0.36	0.0645	0.877	0.39
	0.070	0.860	0.40	0.0645	0.847	0.43
	0.070	0.803	0.50	0.0645	0.791	0.54
	0.070	0.777	0.59	0.0645	0.766	0.63
	0.069	0.837	0.46	0.0635	0.825	0.49
	0.069	0.810	0.52	0.0635	0.798	0.56
	0.069	0.776	0.62	0.0635	0.765	0.67
	0.068	0.896	0.36	0.0626	0.883	0.39
	0.068	0.848	0.42	0.0626	0.836	0.45
FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.06975						
Dodge I......	0.0765	0.881	0.44	0.0705	0.868	0.47
	0.0724	0.800	0.42	0.0667	0.788	0.45
FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.07826						
Dodge II.....	0.087	0.893	0.37	0.0801	0.880	0.40
	0.087	0.871	0.40	0.0801	0.858	0.43
	0.087	0.850	0.43	0.0801	0.838	0.46
	0.087	0.829	0.46	0.0801	0.817	0.49
	0.087	0.800	0.53	0.0801	0.788	0.57
	0.087	0.781	0.55	0.0801	0.770	0.59
	0.087	0.848	0.38	0.0801	0.836	0.41
	0.087	0.815	0.46	0.0801	0.803	0.49
	0.087	0.780	0.55	0.0801	0.769	0.59
	0.086	0.888	0.38	0.0792	0.875	0.41
	0.086	0.863	0.41	0.0792	0.850	0.44
	0.086	0.826	0.48	0.0792	0.814	0.51
	0.086	0.785	0.59	0.0792	0.774	0.63
	0.086	0.804	0.53	0.0792	0.792	0.57
FOR NH_3 CURVE AT REDUCED PRESSURE OF 0.08696						
Dodge II.....	0.103	0.880	0.41	0.0949	0.867	0.44
	0.103	0.865	0.43	0.0949	0.852	0.46
	0.103	0.847	0.46	0.0949	0.835	0.49
	0.103	0.832	0.47	0.0949	0.820	0.50
	0.103	0.808	0.54	0.0949	0.796	0.58
	0.103	0.786	0.56	0.0949	0.775	0.60
	0.103	0.880	0.41	0.0949	0.867	0.44
	0.103	0.859	0.44	0.0949	0.846	0.47
	0.103	0.830	0.49	0.0949	0.818	0.53
	0.103	0.807	0.54	0.0949	0.795	0.58
	0.103	0.780	0.54	0.0949	0.769	0.58

20. *Heat Content of the Liquid.*—The values of heat content of the liquid may be found by subtracting the values of r found by equation (6) from the values of i'' found by equation (10). The most important

check on these values is derived from a series of experiments performed by Dieterici[25] and by Drewes[68] working in Dieterici's laboratory. In these experiments the quantity measured was the amount of heat given up by a certain weight of a mixture of saturated and liquid ammonia cooling at constant volume in a sealed tube. This quantity was corrected for the heat due to the latent heat of the portion of vapor condensed and the remaining amount of heat was divided by the change in temperature to obtain the mean "inner" specific heat of the liquid over that temperature range. In his paper Dieterici gives only the formula which he chose as best representing the results of his experiments, which states that the mean "inner" specific heat between t and 32° F. $=1.118+0.000578(t-32)$. Since this is a linear function of the temperature it follows that the instantaneous "inner" specific heat $=1.118+0.001176(t-32)$. The original data of the experiments are not given, but Professor Dieterici has kindly furnished the values which he actually obtained for the mean "inner" specific heat and also a copy of Mr. Drewes' dissertation containing the values obtained by him. If each of these values of the mean "inner" specific heat is multiplied by the temperature range covered the result will be approximately equal to the internal energy u above 32° F. at the temperature t. Since $i'=u'+Apv'$, in order to obtain values of i' the product Apv' corresponding to the temperature t must be added to the value of u'

TABLE 27. THE HEAT CONTENT OF THE LIQUID AS DEDUCED FROM THE EXPERIMENTS OF DIETERICI AND DREWES.

Authority	$(C_i')_m$ (t-0°C)	u' above 0°C. Cal. of 4.222 Joules	u' above 0°C. Cal. of 4.184 Joules	Temp. °C.	Temp. °F.	u' above 32° F. B. t. u.	$i'=u'+Apv'$
Dieterici	1.148	10.65	10.75	9.28	48.70	19.35	19.77
	1.162	10.93	11.03	9.41	48.94	19.85	20.27
	1.141	24.25	24.47	21.25	70.25	44.04	44.67
	1.140	24.57	24.79	21.55	70.79	44.62	45.26
	1.139	38.67	39.02	33.95	93.11	70.23	71.21
	1.147	39.27	39.62	34.24	93.63	71.31	72.30
	1.200	42.03	42.41	35.02	95.03	76.34	77.35
Drewes,—reliable according to Dieterici	1.172	48.17	48.60	41.1	105.96	87.46	88.68
	1.158	48.35	48.78	41.75	107.15	87.78	89.02
	1.186	61.44	62.00	51.8	125.22	111.60	113.25
	1.163	60.42	60.97	51.95	125.46	109.75	111.41
	1.187	70.74	71.38	59.6	139.3	128.50	130.57
	1.164	72.75	73.41	62.5	144.5	132.15	134.40
	1.179	82.76	83.50	70.2	158.4	150.30	153.07
	1.205	87.48	88.27	72.6	162.7	158.90	161.84
Drewes,—questioned by Dieterici	0.974	11.50	11.60	11.8	53.24	20.89	21.35
	1.098	21.62	21.81	19.7	67.46	39.26	39.86
	1.069	22.82	23.02	21.35	70.43	41.44	42.08
	1.140	34.61	34.92	30.36	86.64	62.85	63.72
	1.148	35.83	36.16	31.21	88.17	65.08	65.97
	0.7065	−10.49	−10.59	−14.85	5.27	−19.06	−18.91

obtained as described. The results of these calculations appear in Table 27. Since Dieterici reported his results in terms of a mean calorie equal to 4.222 joules while in the present investigation a mean calorie of 4.184 joules is used, all of Dieterici's heat quantities must be multiplied by 1.009. The table includes the determinations made by Dieterici himself and all of the determinations made by Drewes: part of the latter are called reliable by Dieterici, while the rest are marked "falsch" in the copy of Drewes' dissertation loaned by Dieterici. The ground for this statement is not given. In Fig. 10

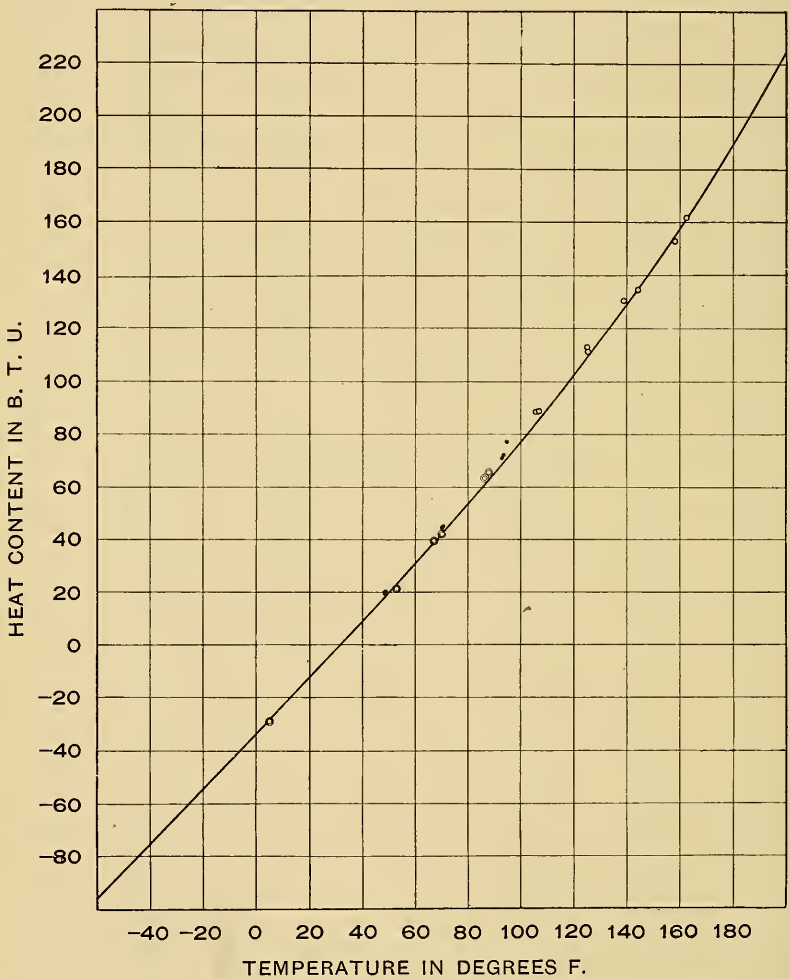

FIG. 10. COMPARISON OF CALCULATED VALUES OF HEAT CONTENT OF THE LIQUID WITH VALUES DEDUCED FROM THE EXPERIMENTAL WORK OF DIETERICI AND DREWES.

Dieterici's values are plotted as black dots, and those of Drewes as circles; the values questioned by Dieterici are indicated by means of a double circle. The curve represents the values found by the use of equations (6) and (10). It is seen that in several cases the curve passes between the Dieterici points and the doubtful Drewes points.

Since the quotient obtained by dividing the change in i between two temperatures by the difference in temperature is very nearly equal to the specific heat, the other determinations which have been made of the specific heat may be used to check the calculated values of i'.

TABLE 28. SUMMARY OF DETERMINATIONS OF THE SPECIFIC HEAT OF LIQUID AMMONIA.

Authority	Temp.°C.	Temp.°F.	Specific Heat of Liquid
von Strombeck	62–30	144–86	1.22876
Elleau and Ennis	20–0	68–32	1.02
Ludeking and Starr	77.7	139.9	0.886
A. J. Wood	20–16	68–61	1.094

Table 28 contains a summary of the determinations of the specific heat of the liquid in addition to the work of Dieterici and Drewes already given; this includes the work of Elleau and Ennis,[24] von Strombeck,[19] Ludeking and Starr,[23] and A. J. Wood.[69] In Fig. 11 these values

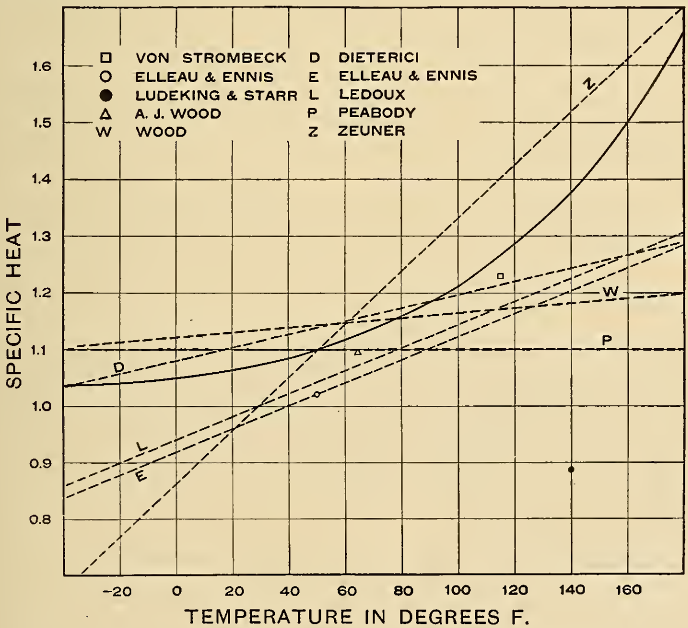

FIG. 11. COMPARISON OF VARIOUS DETERMINATIONS OF SPECIFIC HEAT OF THE LIQUID AND VARIOUS EQUATIONS FOR THE SAME.

are plotted, together with the curves representing the following equations used in the computation of various tables:

Dieterici:[25]	$c' = 1.118 + 0.001156\ (t-32)$
Zeuner:[9]	$c' = 1.01235 + 0.00468\ (t-32)$
Wood:[10]	$c' = 1.12136 + 0.000438\ t$
Ledoux:[6]	$c' = 1.0058 + 0.0020322\ (t-32)$
Elleau and Ennis:[24]	$c' = 0.9834 + 0.0020322\ (t-32)$
Peabody:[11]	$c' = 1.1$ constant.

The full line curve represents the values of the tangents to the heat content curve of the present investigation. Since according to modern ideas of the critical point the specific heat of the liquid there becomes equal to plus infinity, the full line curve is of a more rational form than any linear relation.

21. *Entropy of the Saturated and Superheated Vapor.*—An expression for entropy is readily found from equation (l) which is

$$dq = c_p dT - AT\left[\frac{\partial v}{\partial T}\right]_p dp \tag{l}$$

Thus
$$ds = \frac{dq}{T} = c_p\frac{dT}{T} - A\left[\frac{\partial v}{\partial T}\right]_p dp \tag{q}$$

Introducing in equation (q) the expressions previously derived for c_p and $\left[\frac{\partial v}{\partial T}\right]_p$, the result is

$$ds = \left[\frac{\alpha}{T} + \beta\right]dT + Amn(n+1)p\frac{dT}{T^{n+2}} - AB\frac{dp}{p} - \frac{Amn}{T^{n+1}}dp \tag{r}$$

The integral of equation (r) is

$$s = \alpha \log_e T + \beta T - AB \log_e p - Anp\frac{m}{T^{n+1}} + s_o \tag{s}$$

The constant s_o is determined by passing to the saturation limit at 32° F., where $s' = 0$ and therefore $s'' = \frac{r}{T}$. Substituting this value for s in the left hand member of equation (s) and the proper values of p and T in the right hand member, s_o is found to be equal to -0.82656. Substituting this value and the known values of the other constants, and passing to common logarithms, equation (s) becomes

$$s = 0.8796 \log T + 0.000174T - 0.2695 \log p - \frac{p}{T^6}C - 0.82656 \tag{11}$$

where $\log C = 12.866554$

22. *Computation of Tables.*—The methods by which the various tabular values were obtained from the experimental data are in many cases obvious. In Tables 1 and 2, the first seven columns either depend directly on the data selected or their derivation has been explained in

the previous paragraphs. The eighth column, giving the internal energy of evaporation, comes from the formula

$$\rho = r - 144\, Ap\, (v'' - v');$$

and the ninth column, giving the internal energy of the saturated vapor, from the corresponding formula

$$u'' = i'' - 144\, Apv''$$

The entropy of the saturated vapor in column twelve was calculated by means of equation (11). The entropy of evaporation in column eleven is r/T. Column ten is the difference between columns twelve and eleven.

In all these cases the required quantities were computed, usually to one extra significant figure, for each of a series of standard temperatures. These values were then plotted to a large scale and smooth curves drawn through the points. Intermediate values were read from these curves and checked by the method of differences.

In the computation of Table 3 the process was as follows: The expression for heat content is

$$i = 0.382T + 0.000087T^2 - p\frac{C}{T^5} - 0.0185p + 358.0$$

where $\log C = 12.945735$.
This expression was separated into two parts, a temperature function $(0.382T + 0.000087T^2 + 358.0)$, and a function of both pressure and temperature $p\left(0.0185 + \frac{C}{T^5}\right)$. The value of the temperature function was calculated at integral values throughout the necessary range of temperature and plotted to a large scale; a smooth curve was then drawn through the points. Similarly a curve was drawn to represent the part of the second function which appears in the parenthesis, or $\left(0.0185 + \frac{C}{T^5}\right)$. To find the value of i at any desired pressure and temperature the value of this parenthesis was read from the curve and multiplied by p; this value subtracted from the proper value of the temperature function as read from its curve gave the required value of i.

A similar method was used in the calculation of values of entropy and specific volume. All final values were checked by the method of differences.

V. Bibliography

1. Regnault. Mém. de l'Inst. de France, 26, p. 598 (1847). (Relation des Expériences, 2.)
2. Regnault. Ibid. p. 252.
3. Regnault. Ibid. p. 162.
4. Regnault. Mém. de l'Inst. de France, 37, p. 183 (1870). (Relation des Expériences, 3.)
5. Regnault. Annales de Chimie et de Physique, (4) 24, p. 375 (1871).
6. Ledoux. Théorie des machines à froid, Paris (1878). Extrait des Annales des Mines, livraison de juillet-août (1878).
7. Zeuner. Zivilingenieur, p. 449 (1881).
8. Zeuner. Thermodynamik, v. 2, p. 14.
9. Zeuner. Technical Thermodynamics (Klein) 2, p. 252 (1907).
10. Wood. Thermodynamics, p. 325 (1889).
    Trans. Am. Soc. Mech. Eng., 10, p. 627 (1889).
11. Peabody. Tables of Properties of Saturated Steam and Other Vapors (1890).
    Thermodynamics, p. 456 (1889).
12. Blümcke. Wiedemann Annalen, 34, p. 10 (1888).
13. Pictet. Landolt and Börnstein Tabellen.
14. Faraday. Phil. Trans., 135, p. 170 (1845).
15. Davies. Pro. Roy. Soc., 78A, p. 41 (1906).
16. Brill. Annalen der Physik, 326, p. 170 (1906).
17. Dewar. Phil. Mag., (5) 18, p. 210 (1884).
18. Vincent and Chappuis. Jour de Physique, (2) 5, p. 58 (1886).
19. von Strombeck. Jour. Frank. Inst., Dec. (1890) and Jan. (1891).
20. Franklin and Kraus. Jour. Phys. Chem., 11, p. 553 (1907).
21. Estreicher and Schnerr. Acad. Sci. Cracovie, Bulletin 7A, p. 345 (1910).
22. Denton and Jacobus. Trans. Am. Soc. Mech. Eng., 13, p. 507 (1892).
23. Ludeking and Starr. Am. Jour. Science, (3) 45, p. 200 (1893).
24. Elleau and Ennis. Jour. Frank. Inst., Apr. (1898).
25. Dieterici. Zeit. f. d. ges. Kälte-Industrie, p. 21 (1904).
26. D'Andréeff. Annalen der Physik, (3) 56, p. 317.
    Smithsonian Miss. Collec., 32 (1888).
27. Lange. Zeit. f. d. ges. Kälte-Industrie (1898).
28. Leduc. Comptes Rendus, 125, p. 571 (1897).
29. Guye. Comptes Rendus, 141, p. 51 (1905).

30. Perman and Davies. Pro. Roy. Soc., 78A, p. 28 (1906).
31. Nernst. Zeit. f. Elektrochemie, 16, p. 96 (1910).
32. Goodenough. Thermodynamics, p. 321 (1911).
33. Marks and Davis. Steam Tables, p. 88 (1909).
34. Berthelot. Trav. et Mém. Bur. Int., 13 (1903).
35. Buckingham. Bull. of the Bur. of Stan., 3, p. 237 (1907).
36. Rose-Innes. Phil. Mag., (6) 15, p. 301 (1908).
37. Rowland. Pro. Am. Acad., 15, p. 75 (1880).
38. Reynolds and Moorby. Phil. Trans., 190A, p. 301 (1908).
39. Griffiths. Phil. Trans., 184A, p. 361 (1893).
40. Schuster and Gannon. Phil. Trans., 186A, p. 415 (1895).
41. Barnes. Phil. Trans., 199A, p. 149 (1902).
42. Smith. Monthly Weather Review, October (1907).
43. Barnes. Pro. Roy. Soc., 82A., p. 390 (1909).
44. Fischer. Bull. of the Bur. of Stan., 1, p. 380 (1904–5).
45. Troisieme Conf. gen. des poids et mes., p. 66 (1901).
46. Marks. Jour. Am. Soc. Mech. Eng., 33, p. 563 (1911).
47. Ramsey and Young. Phil. Mag., (5) 21, p. 33 (1886), and 22, p. 37 (1886).
48. Richardson. Jour. Chem. Soc., 49, p. 761 (1886).
49. Ramsey and Travers. Phil. Trans., 197A, p. 67 (1901).
50. Ayrton and Perry. Phil. Mag., (5) 21, p. 259 (1886).
51. Everett. Phil. Mag., (6) 4, p. 335 (1902).
52. Moss. Physical Review, 16, p. 356 (1903), and Physical Review, 25, p. 453 (1907).
53. Avenarius. Kiewsche Universitätsnachrichten, Heft 6, p. 249 (1884).
54. Thiesen. Verh. Phys. Ges. zu Berlin, 16, p. 80 (1897).
55. Jacobus. Trans. Am. Soc. Mech. Eng., 12, p. 307 (1891).
56. Cailletet and Mathias. Comptes Rendus, 102, p. 1202 (1886), and Comptes Rendus, 104, p. 1563 (1887).
57. Davis. Proc. Am. Acad. of Arts and Sciences, 45, p. 305 (1910).
58. Young. Jour. Chem. Soc., 63, p. 1237 (1893).
59. Wobsa. Dresden Dissertation, and Zeit. f. d. ges. Kälte-Industrie, p. 61 (1907).
60. Macintire. Ice and Refrigeration, 41, p. 44 (1911).
61. Wobsa. Zeit. f. kom. u. flüs. Gase, 11, p. 5 (1908).
Zeit. f. d. ges. Kälte-Industrie, p. 11 (1908).
62. Mollier. Zeit. f. d. ges. Kälte-Industrie, p. 91 (1895).
63. Hÿbl. Zeit. f. d. ges. Kälte-Industrie, p. 161 (1911).
64. Goodenough. Jour. Am. Soc. Mech. Eng., 34, p. 465 (1912).

65. Wiedemann. Phil. Mag., (5) 2, p. 81 (1876).
66. Buckingham. Bull. Bur. Stand., Vol. 3, No. 2 (1908).
67. Davis. Pro. Am. Acad. Arts & Sciences, 45, p. 243 (1910).
68. Drewes. Hannover Dissertation, Über die wichtigsten thermischen Daten des Ammoniaks (1903).
69. A. J. Wood. Ice and Refrigeration, April (1912).
70. Jaquerod. Mém. Soc. de Phys. et d'Hist. Nat. de Genève, 35, p. 547 (1908).
71. Scheffer. Zeit. f. Phys. Chem., 71, p. 671 (1910).
72. Rayleigh. Zeit. f. Phys. Chem., 52, p. 705 (1905).
73. Leduc. Comp. Rend. 148, p. 1173 (1909).

References not made use of in the Bulletin

74. de Forcrand. Ann. de Chem. et Phys., 7 (28), p. 537 (1903). Ann. der Phys., 24, p. 367 (1907).
75. Gibbs. Jour. Am. Chem. Soc., 27, p. 851 (1905).
76. Franklin. Jour. Phys. Chem., 11, p. 553 (1907).
77. Holborn and Wien. Wied. Ann., 59, p. 213 (1896).
78. Ladenburg and Krugel. Ber. d. deutsch. chem. Ges., 33, p. 637 (1900).
79. Dickerson. Cp. O'Connor Sloane, Liquid air and Liquefaction of gases; London (1900).
80. Thomson. Gmelin-Watts' Translation, p. 280.
81. Biot and Arago. Gmelin-Watts' Translation, p. 280.
82. v. Fehling. Dictionary of Chemistry (1871).
83. Roth. Wied. Ann., 11, p. 1 (1880).
84. Lunge. Ann. Chem. u. Pharm., 110, p. 1, (1859).
85. Lange and Hertz. Zeit. f. angewand. Chem., p. 227 (1897).
86. Jolly. Gmelin, Handbuch der Chemie (1861).

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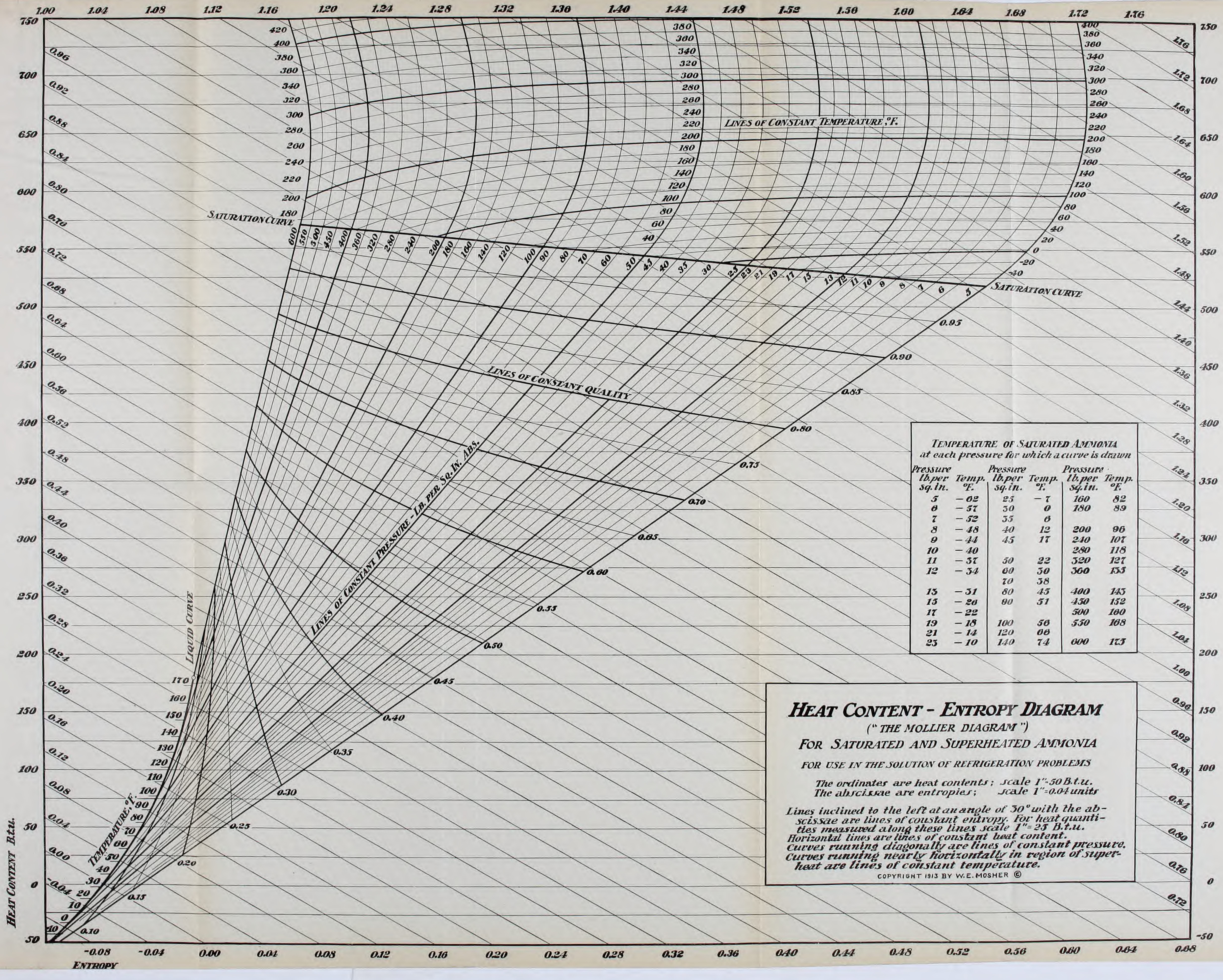

TEMPERATURE OF SATURATED AMMONIA
at each pressure for which a curve is drawn

Pressure lb. per sq. in.	Temp. °F.	Pressure lb. per sq. in.	Temp. °F.	Pressure lb. per sq. in.	Temp. °F.
5	−62	25	−7	160	82
6	−57	30	0	180	89
7	−52	35	6		
8	−48	40	12	200	96
9	−44	45	17	240	107
10	−40			280	118
11	−37	50	22	320	127
12	−34	60	30	360	135
		70	38		
13	−31	80	45	400	143
15	−26	90	51	450	152
17	−22			500	160
19	−18	100	56	550	168
21	−14	120	66		
23	−10	140	74	600	175

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BULLETIN NO. 67

REINFORCED CONCRETE WALL FOOTINGS AND COLUMN FOOTINGS

BY

ARTHUR N. TALBOT

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 67 MARCH, 1913

REINFORCED CONCRETE WALL FOOTINGS AND COLUMN FOOTINGS

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS

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REINFORCED CONCRETE WALL FOOTINGS AND COLUMN FOOTINGS.

I. Introduction.

1. *Preliminary.*—Footings form an important element in the design of masonry structures. The two forms of footing most commonly used may be named the wall footing and the column footing, the former projecting laterally on the two sides of a longitudinal wall and the latter extending in four directions from the base of a column or pedestal block. It is usually assumed in the design of foundations that the earth conditions are such as to make the upward pressure on the footing uniform over its surface. Wide differences exist in methods of designing, due to differences in the assumptions made with reference to the structural action of the footing. It is not strange that these differences exist, since little or no experimental data are available which apply directly to the conditions of footings. Relatively short and deep beams and slabs under heavy uniform loads, with the supporting pressure largely concentrated at the center of the structure, may not be expected to give the same results as have been obtained in tests with the more slender beams and slabs and with the methods of support and of application of load which have generally been used in tests. With the present extensive application of reinforced concrete to footings, especially in connection with tall buildings carrying very heavy column loads, a more definite knowledge of the structural action of footings has come to be of importance. It is appreciated that the tests herein described are applicable only to a limited field, but they are offered as a contribution on a subject in which little experimentation has been done.

It may seem strange, considering the wide variations in practice, that few failures of footings have been publicly reported. It must be remembered, however, that these structures are out of sight, buried deep in the earth without opportunity for inspection. A failure in a footing may effect a change in the distribution of the load over the bed of the footing, resulting only in increased settlement. Possibly many instances of undue settlement of buildings may be due to failure in the footings. Possibly, in other cases, the earth at the center of the footing may be able to take the increased load under the conditions of side restraint developed. It is also probable that many footings have been made unduly strong.

2. *Acknowledgment.*—The investigations were made in the Laboratory of Applied Mechanics of the University of Illinois as a part of the work of the University of Illinois Engineering Experiment Station. Direct supervision of the work of making both the wall footings and the column footings was given by Mr. D. A. Abrams, Associate in the Engineering Experiment Station. He and Mr. W. A. Slater, First Assistant in the Engineering Experiment Station, directed and assisted in the tests of column footings. The tests of the wall footings were made under the direction of members of the laboratory staff. Acknowledgment is also made to Mr. Slater and to Mr. H. F. Gonnerman, Assistant in the Engineering Experiment Station, for assistance in the preparation of this bulletin. The investigation was undertaken at the suggestion of Dr. N. C. Ricker, Professor of Architecture. Assistance in the work of testing was given by senior students in architectural engineering and civil engineering who used the results in their theses. The following participated in the tests: Wall footings—series of 1908, Herbert Amery Brand, Horace Leland Bushnell, Arch. Eng'g, '08. Wall footings—series of 1909, Charles Emery Bressler, Jr., Nels Reuben Hjort, Civil Eng'g, '09. Wall footings—series of 1911, Charles Aloysius Petry, William Henry Ruskamp, Civil Eng'g, '11, Thomas James Giboney, Civil Eng'g, '12. Column footings—series of 1909, Norman Haden Hill, Edward Forde Zahrobsky, Arch. Eng'g, '09. Column footings—series of 1910, Charles Harris, James Verney Richards, Arch. Eng'g, '10. Column footings—series of 1911, Edward Raylor Kent, Earle Robinson Math, Arch. Eng'g, '11. Column footings—series of 1912, William Howard Farnum, Cyrus Edmund Palmer, Arch. Eng'g, '12.

3. *Scope of Bulletin.*—The tests of 114 wall footings and 83 column footings are described in the bulletin. The wall footings were 12 in. wide, generally 5 ft. in length and 12 in. in depth or 10 in. to the center of the reinforcing bars, with a 12x12x12-in. stem in the middle to represent the wall through which the test load was applied. The wall footing rested on a bed of springs arranged in such a way as to approximate conditions of uniform upward pressure on the bottom surface of the footing. A variety in method of reinforcement was employed to throw light on the development of tensile stress in the steel and on the resistance to bond, diagonal tension, and shear. Tests of brick footings, unreinforced concrete footings, and footings having I-beams encased in concrete were included in the investigation of wall footings. The column footings were 5 ft. square and generally 12 in. thick or 10 in. to the

center of the reinforcing bars, and had a 12x12x12-in. pier built over the middle through which the load was applied. The column footings also were tested on a bed of springs which gave conditions approximating those of uniform upward pressure. Variety was given to the amount and method of reinforcement and to other conditions with a view of determining the structural action with respect to tension, bond, diagonal tension, and shear, and to give information which would bear upon methods of calculation of stresses. It is thought that these are the first experimental tests on column footings, and probably the first on wall footings on a bed of springs. Analyses are given of the stresses in wall footings and column footings and methods of calculation are discussed and compared with the results of the tests.

4. *General Theory.*—In wall footings and pier footings the weight or load is applied vertically through the wall or base block or pier, and the upward bearing pressure of the soil (which may also be called the load, since its amount and distribution determine the stresses) supports this weight from below. The usual assumption on which design of footings is based is that the soil pressure is uniform over the bed of the footing. Before uniformity of pressure on the footing will obtain, the footing must bend to the amount and form which would be caused by a uniformly distributed load. The assumption of uniform pressure is warranted if the earth layer is an elastic compressible soil of considerable thickness and of not too high a modulus of compressibility, as under these conditions the amount of bend of the projection of the footing is slight in comparison with the amount of compression of the earth. Also, in soft soils which flow laterally, as in a so-called floating foundation, the settlement and changes in the soil will produce conditions approximating uniform pressure. Where the bed is rock the pressure will be transmitted more nearly directly from the wall or pier to the rock, and as the projections of the footing have little opportunity for being bent upward this portion of the footing may be expected to take only a small part of the load. This lack of uniformity of distribution of pressure is more likely to be present with reinforced footings than with the less flexible unreinforced footings which would carry the same load.

The principles of beam action are, in general, applicable to wall footings, but not so fully to column footings, which partake more of the nature of slabs. The formulas for calculating stresses in reinforced concrete beams have been treated in Bulletin No. 4, "Tests of Reinforced Concrete Beams: Series of 1905," and in Bulletin No. 29, "Tests of

Reinforced Concrete Beams: Resistance to Web Stresses." The principal formulas for beam action in rectangular beams reinforced for tension only, as used in this bulletin, will be repeated here.

The resisting moment of the reinforced concrete beam is (see Bulletin No. 29, page 6)

$$M = Afd' = Afjd \qquad (13)$$

where A is the area of cross section of longitudinal reinforcement, d is the distance from the compression face to the center of the longitudinal reinforcement, d' is the distance from the center of the reinforcement to the center of gravity of compressive stresses, j is the ratio of d' to d (which, for the beams of this bulletin, may be considered to vary from .82 to .92), and f is the tensile stress per unit of area in the metal reinforcement.

The formula for the maximum vertical shearing unit-stress in the concrete in any vertical section is

$$v = \frac{V}{jbd} = \frac{V}{bd'} \qquad (18)$$

where V is the total vertical shear at the given section (equivalent to the resultant of vertical forces on one side of the section considered), and b is the breadth of the beam. This formula neglects any horizontal tensile stresses in the concrete.

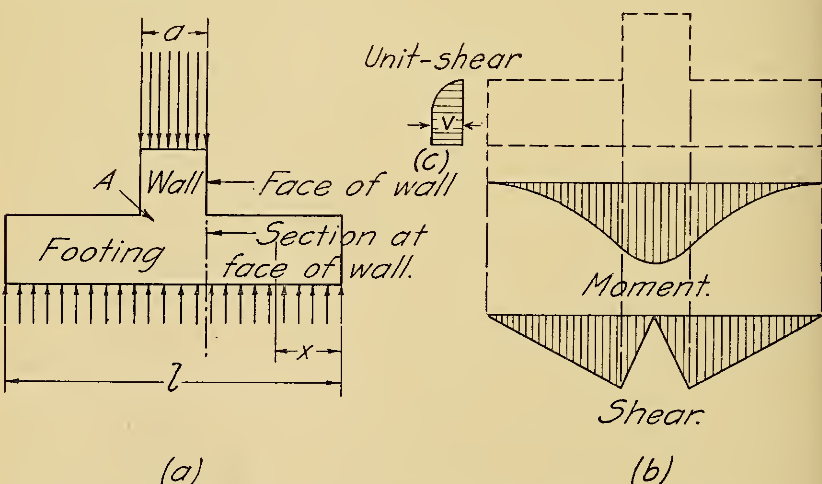

FIG. 1. (a) DISTRIBUTION OF LOAD AND PRESSURE IN WALL FOOTING. (b) MOMENT CURVE AND SHEAR CURVE. (c) DISTRIBUTION OF SHEARING STRESS OVER VERTICAL SECTION.

The formula for bond unit-stress in horizontal reinforcing bars is

$$u=\frac{V}{mod'} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

where o is the periphery of one longitudinal reinforcing bar, m is the number of bars, and the other symbols are as used before. This formula neglects any horizontal tensile stresses in the concrete.

These formulas were derived for certain assumed conditions in the beam. Since it is convenient to use them as a means of comparison for conditions other than those assumed, as, for example, when the bars are bent up at the end, the values obtained from these formulas will sometimes be referred to as nominal vertical shearing stresses and nominal bond stresses.

The value of the maximum diagonal tensile unit-stress in any section when tensile stresses exist is

$$t=\frac{1}{2}s+\sqrt{\frac{1}{4}s^2+v^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

where s is the horizontal tensile unit-stress existing in the concrete and v is the horizontal or vertical shearing unit-stress. The direction and amount of this maximum diagonal tensile stress will vary with the relative values of s and v. In general, it may be said that in the ordinary reinforced concrete beam the value of t probably varies from one to two times v. This applies to the parts where tensile stresses exist in the concrete. Where the tensile strength of the concrete has been exceeded, it is customary to use the same formula.

It is evident that the value of the diagonal tension is generally indeterminate. No working formulas are available. For this reason it is the practice, now becoming nearly universal, in beams without web reinforcement to calculate the value of the vertical shearing unit-stress v, and to use this as the measure or means of comparison of the diagonal tensile stress developed in the beam; with the understanding, of course, that the actual diagonal tension is considerably greater than the vertical shearing stress. It has been found that the value of v developed in beams will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam.

5. *Analysis of Wall Footings.*—Fig. 1(a) shows a wall footing and a typical set of external forces acting upon the footing. In the discussion, the stem or pier above will be called the wall and the remainder the footing proper. The projecting portion of the footing will be called the projection.

The bending moment at a section of the footing x distant from the end (calling w the uniform upward pressure per lineal foot of length of footing for a given width of section) is

$$M = \frac{1}{2} wx^2 \quad \ldots\ldots (22)$$

For a section at the face of the wall, the bending moment will be

$$M = \frac{1}{8} w(l-a)^2 \quad \ldots\ldots (23)$$

where l is the extreme dimension of the footing and a is the thickness of wall. For a section through the middle of the wall, assuming the load to be distributed uniformly over the wall, the bending moment will be

$$M = \frac{1}{8} w\,(l^2 - la) \quad \ldots\ldots (24)$$

The variation of the bending moment along the footing is shown in Fig. 1(b).

Although the maximum bending moment is shown by the above analysis to be at the section which passes through the middle of the wall, the resisting moment of that section will be far greater than that of a section of the projection of the footing in those cases where the wall and footing are poured at the same time or where they are well bonded together. Even with a weak bond the horizontal shearing stress at the junction of wall and footing will, in footings of the ordinary proportions, be so small that the combined section may be expected to act together. Besides, the pressure from the wall, instead of being distributed as shown, will be concentrated to some extent on the footing near the faces of the wall, as at A, Fig. 1(a), and this will act to reduce differences of moments. Altogether, it may be expected that the section at the face of the wall will be the critical section for bending moment and resisting moment and that equation (23) will express the value of the critical bending moment as closely as may be determined by ordinary analysis.

Fig. 1(b) shows also the variation in the external vertical shear V over the length of the footing for uniform loading. The theory of beams gives a distribution of the intensity of the vertical shearing stresses throughout the vertical section which is represented in Fig. 1(c) and is more fully discussed in Bulletin No. 29, page 9. Due to a concentration of pressure near the face of the footing (as at A, Fig. 1(a) and to the transmission of pressure diagonally therefrom in

a manner which is analogous to arch action (as is also to be found in short simple beams), it may be expected that at vertical sections near the wall the vertical shearing stresses will be greater in the compression portion of the vertical section and less below the neutral axis than is given by the beam analysis of Bulletin No. 29. This modification of the distribution of the vertical shearing stresses may be expected to reduce the amount of the diagonal tension stress de-

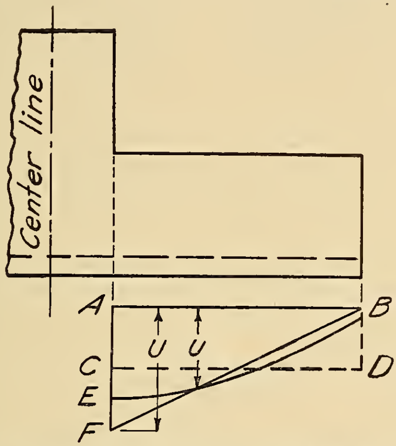

Fig. 2. Distribution of Bond Stress along the Reinforcement in a Wall Footing.

veloped near the wall, and the position of the critical section for diagonal tension failure may be expected to be away from the face of the wall. The values of the vertical shearing stresses given in this bulletin as a means of comparing or measuring the resistance to diagonal tension in the wall footing tests are based upon a section distant d from the wall (a section which is shown to give reasonable values), and the vertical shear V at this section is used in equation (18). A comparison with the values at a section at the face of the wall will also be made.

The bond stress between the surface of the horizontal reinforcement and the concrete will also be affected by variations from true beam action. By equation (17), page 8, the bond stress is a maximum at the face of the wall as represented by the line AF in Fig. 2, and decreases uniformly toward the end of the beam, as shown by ordinates to the line FB, becoming zero at B. Due to the deformations accompanying the stretching of the steel under the wall and to the relative deformations necessary to develop bond between the steel and the con-

crete, as well as to variation from true beam action, the bond stress will not follow the ordinates to the straight line. It seems probable

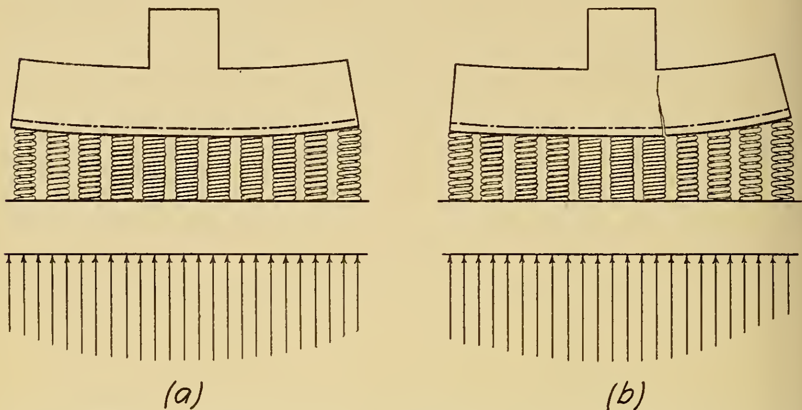

Fig. 3. Effect of Deflection of Wall Footing upon Distribution of Load by Springs.

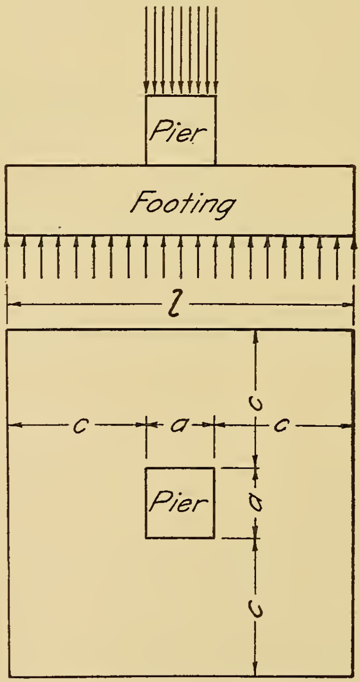

Fig. 4. Distribution of Load and Pressure in Column Footing.

that the bond stresses developed are less at the face of the wall and greater at points farther out from the wall than is indicated by the analysis. It would seem that the bond stress will be expressed by some such line as the curved line EB, of the figure. This distribution is different still from the uniform bond stress indicated by the dotted line, which is based upon length of embedment and total amount of surface, a method assumed by some in such calculations. The distribution of the load may also affect the bond stresses. However, although the true bond stress at a section at a face of the wall may be expected to be less than that given by the ordinary beam analysis, in the absence of a better method it seems best to use equation (17), page 8, for the calculation of bond stresses.

When footings are tested on a bed of springs, the deflection produced in the beam results in compressing the springs at the middle of the footing more than at the ends, and hence the pressure will not be uniformly distributed along the length of the footing. If the compression of the springs at points along the length of the footing is known, and also the deflection of the footing at these points, the distribution of the load may be determined and the resulting bending moment calculated. Fig. 3 illustrates the effect of the deflection of the footing upon the distribution of the load. The bending moment so calculated will be somewhat less than that based on uniform distribution of the load, and the amount of the resulting tensile stress, bond stress, and vertical shearing stress will be less. The amount of the difference will depend upon the stiffness of the springs and the deflection of the footing under load, but within so-called critical loads it will not be large. Of course, in designing footings, our knowledge of the distribution of the pressure by the soil is too imperfect to consider the effect of deflection upon distribution of pressure.

In testing on a bed of springs, the load may not be symmetrically applied, and one end of the footing may receive more load than the other. The stresses in the end in which the springs receive the greater compression will, of course, be larger than values based on uniform distribution of load.

6. *Analysis of Column Footings.*—Fig. 4 represents a column footing of the form used in the tests. The stem representing the bottom of a column or a pedestal block will be termed the pier, and its lateral faces the faces of the pier. The load will be considered as applied uniformly over the top of the pier and the upward pressure as uniformly distributed over the lower surface of the footing. It is seen that the

footing may be considered to be a cantilever slab (rather than cantilever beams) supported at the top over a central area and loaded uniformly by an upward load, and that as the projecting portion of the footing deflects upward its surface will be bowl-shaped, in reality a double-curved surface. The determination of the distribution of the stresses over the various parts of the column footing is a much more difficult problem than is presented in wall footings.

Various methods of calculating the strength of column footings have been proposed. In some cases the offsets have been considered as cantilever beams having the full width of the footing and the full load on this area is considered to be taken by this beam, the critical section being at AB, Fig. 5. Although the load at the corners is counted twice,

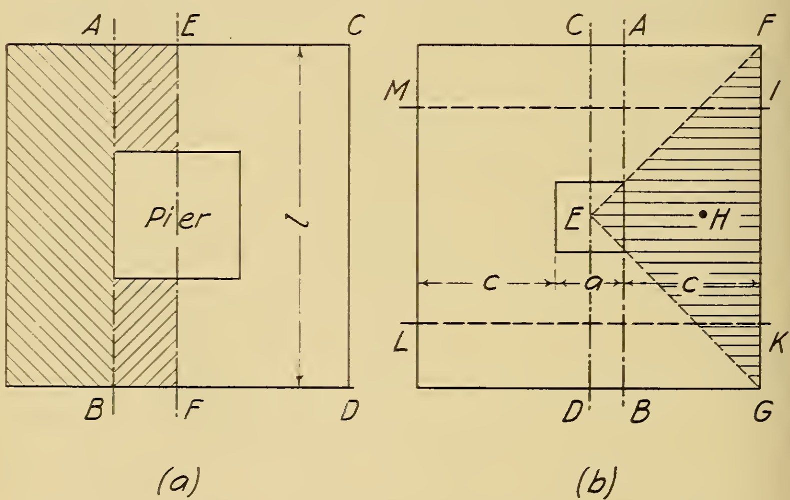

FIG. 5. LOADED AREAS ASSUMED IN DESIGNING COLUMN FOOTINGS.

the error is not great when the offset is small in comparison with the dimensions of the pier. If the dangerous section is considered to pass through the center of the footing, EF, Fig. 5 (a), a greater discrepancy exists. A common method of design is to consider that one-fourth of the total load is applied on the triangle EFG, Fig. 5 (b), and having found the center of pressure (as H) of the part of the load at one side of AB or CD (according to which is used as the dangerous section), to calculate the bending moment as the product of this amount of load by the distance from this center of pressure to AB or CD, the dangerous or critical section.

When the bending moment has been obtained by one of these methods, it is considered to be resisted by a beam IKLM, Fig. 5 (b) of width somewhat greater than the width of the pier, say, the width plus once the depth of the footing, according to the views of the designer. That is to say, the reinforcement in this assumed width is considered to develop stresses which altogether are sufficient in a beam of the depth of the footing to withstand the calculated bending moment. If the cross section of the steel lying within the assumed width is A, the resisting moment will be $M=Afjd$. The steel lying outside the dotted lines is considered to carry load to the beam formed by the reinforcement which lies at right angles to these lines, just as the steel parallel to and near FG carries load to the beam IKLM, and in this method of design no account is taken of it in the main beam. Whether the spacing of the outer bars should be the same as that of the interior or be greater is then left as a matter of judgment. In the determination of both bending moment and resisting moment, then, the practice of engineers varies considerably.

A rational analysis of the stresses would involve a determinate expression for the deflection of the footing at every point of the cantilever slab and also for the radius of curvature in each direction. A full treatment would include a consideration of the effect produced by having stresses act at right angles to each other and of the action of other combined stresses. However, it may be expected that this effect will not be large, as in reinforced concrete footings the presence of stresses in two directions affects principally the amount of the compressive stresses and the compressive stresses will not be the controlling element of strength in footings as ordinarily designed. It is easily seen that an analytical treatment of a cantilever slab of this kind which approached completeness would be very complicated. This and the uncertainty involved in the assumptions made at some steps of the analysis renders the correctness of the results of the mathematical work of such an analysis quite problematical. In view of the complexity of the problem and the uncertainty of portions of the work, it seems futile to attempt to derive thoroughly rational formulas for stresses in column footings. This being so, it seems best to utilize approximate solutions based on other considerations.

A study of the phenomena of the flexure of the column footing may be helpful in judging of the division of the load in the production of bending moment in the two directions and of the development of stress in the different parts. It is apparent that the stresses will be propor-

tional to the deformations developed and that the deformations at any point will depend upon the curvature at that point. It will be recalled that in the analysis of beams in mechanics of materials the stresses developed are found to be inversely proportional to the radius of curvature or directly proportional to the curvature. At corresponding points on similar sections the curvature and hence the stress may be considered to vary somewhat as the change of deflection at these points. With these considerations in mind, we may be able to judge of the effect of the varying curvature in different parts of the footing.

Fig. 6 represents in a general way the form which the footing under load may be expected to take along various sections. The full lines in the lower figure represent the deflections or flexure curves of vertical

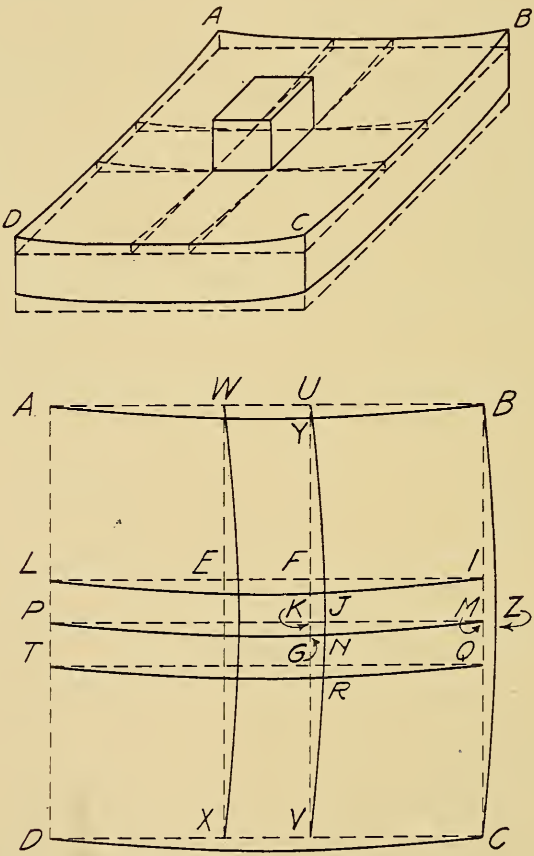

Fig. 6. Form Taken by Column Footings under Load.

sections taken along the dotted lines. The vertical rise at a corner B will be the sum of the deflection at M (KG) and the deflection of the lateral face BC (MZ). The three sections through the faces of the pier and the center of the pier which give the flexure curves LJI, PNM, and TRQ, may be considered to have nearly the same stress. The section at a lateral face of the footing will give a flexure curve AYB which will generally have less deflection and less flexural curvature at Y (and hence less stress) than is to be found in the section at the face of the pier which gives the curve LJI. The amount of the difference between these two curves will depend upon the relation of the cantilever span to thickness of pier and to amount and distribution of reinforcement. For sections between AB and LI the flexure curves and the conditions of curvature will range between those of the limiting curves. If we knew the flexure curves in all parts of the footing we should be able to get at the distribution of stresses.

If, with two-way reinforcement, we consider the load or pressure on the footing to be carried by two beams or sets of beams running parallel to the sides of the footing, the proportion of load or pressure taken by each beam from any elementary area may be considered to depend in some way upon the relative deflection of the beams in the two directions. In Fig. 7 (a), for convenience of description, consider the top of the

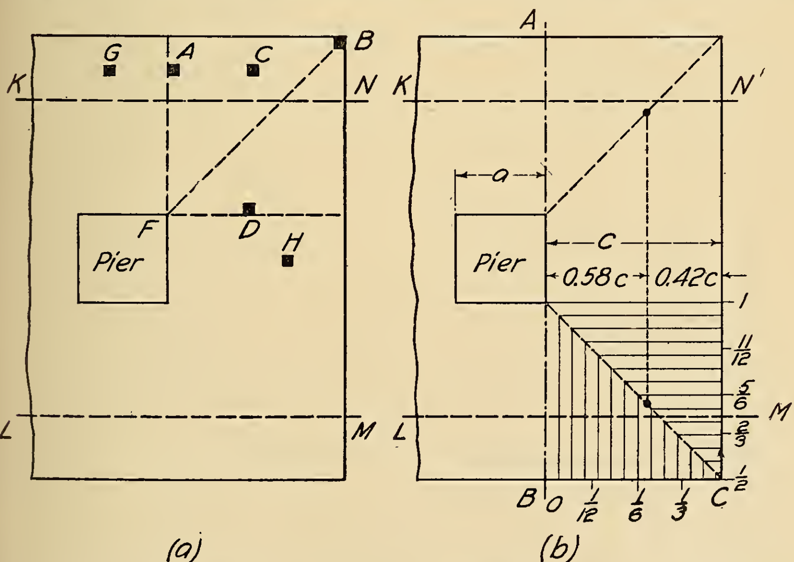

Fig. 7. Diagram Showing Assumed Distribution of Load in Producing Bending Moment.

diagram north, and that the footing is formed of a beam running in the east and west direction, and of another running in the north and south direction. For an element at A the deflection laterally from the north and south beam will be very slight and the total load on this element may, without much error, be considered to produce bending in the beam running in the north and south direction. For the corner area B part of the load may be considered as producing moment in the beam which runs in the north and south direction and part in the beam which runs laterally (east and west). For an element at C the amount of deflection of the footing from C to A will be much less than that from C to D; it seems evident that the proportion of load at C producing moment in the north-and-south beam is much greater than that acting on the east-and-west beam. Similarly, at D a greater proportion acts on the east-and-west beam than on the north-and-south beam. Along the diagonal line BF we may consider that half of the load acts on each beam. At G all acts on the north-and-south beam; at H none of it.

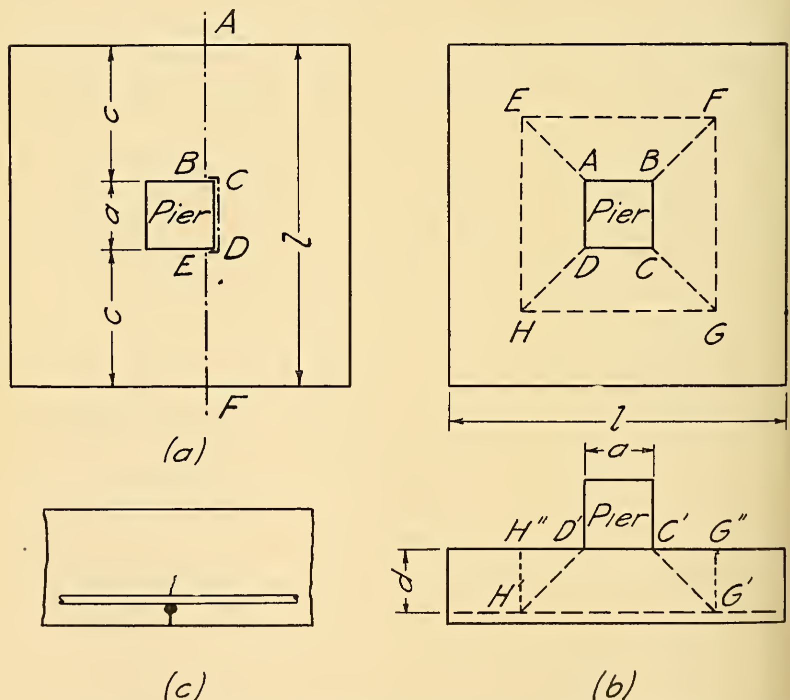

FIG. 8. CRITICAL OR DANGEROUS SECTION FOR RESISTING MOMENT AND POSITION OF SECTION FOR CALCULATING VERTICAL SHEARING STRESSES.

After making a study of the flexure curves obtained on a number of the column footings tested, the fractions given on the diagram in Fig. 7 (b) were taken as roughly representing the proportion of the unit-load at the points indicated which acts upon the east-and-west beam to produce bending moment and curvature. For the variation of the proportions along the lines of the diagram between the limits noted a curvilinear relation was assumed, and a process of approximate integration was applied to the load division problem. Of the part above the diagonal line, approximately two-thirds of the load or pressure upon the triangle was found to go to produce bending moment in the east-and-west beam, and of the part below the diagonal line approximately one-third, the remainder in both cases going to produce bending moment in the beam in the north-and-south direction; and of course altogether one-half of the load on the corner square must be considered to produce bending moment on each beam. By the calculation, under the assumed division of load, the center of pressure of the various parts of the load tributary to the north-and-south beam from a corner square was found to be 0.58 c from a line through the face of the pier. That is to say, this analysis results in considering that the pressure on the corner square affects the bending moment of the north-and-south beam the same as if one-half of the load of this corner square were placed at a point distant 0.58 of the width of the square from a line through the face of the pier, see Fig. 7 (b). As the method of assuming the division of load will not warrant refinement of calculation it seems well to adopt the more convenient and more conservative value of 0.6 c for the position of the center of pressure, and this value will be used in the calculations in this bulletin. It may be added, also, that other methods of attacking the problem locate the center of pressure not far from the position here chosen.

The location of the critical or dangerous section for which the bending moment is to be found is also of importance. For footings made in such a way that the pier and footing are bonded together sufficiently not to permit failure by horizontal shear between them, as were all the column footings described in this bulletin, a section at the face of the pier CD, Fig. 8 (a), will be the critical section for the part of the beam immediately in front of the pier. For the part of the footing on either side of this, the critical section possibly may be somewhat back of the face of the pier. From some of the tests which were made it would seem that a combination section made up of three sections, one coinciding with the face of the pier and the other two slightly back of this, as

shown by AB, CD, and EF in Fig. 8 (a), might represent the critical section. However, after making a study of all the tests, it is concluded that a section through the face of the pier is fairly representative of the tests, and this section will be used in the calculations in this bulletin. For very broad footings a section somewhat back of the pier may properly be assumed. The formula for the critical bending moment may then be expressed as follows:

$$M=\left[\frac{1}{8}a(l-a)^2+\frac{3}{40}(l-a)^3\right]w$$

or

$$M=\left(\frac{1}{2}ac^2+0.6c^3\right)w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

where a is one dimension of the square pier, l one dimension of the square footing, and c is the dimension of the offset of the footing, see Fig. 8 (a).

The bending moment thus obtained goes to produce curvature across the section and may be said to be resisted by the entire section, but the stresses may be expected to be different in different parts of the section, being a maximum under the pier and having the least stress at the edge

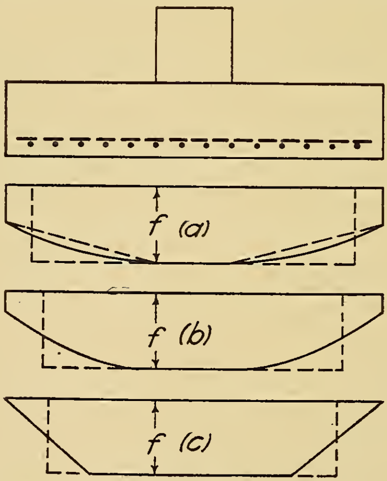

FIG. 9. VARIATION OF STRESS AMONG REINFORCING BARS.

of the section. The range in stresses may be illustrated by Fig. 9, where the stress in the reinforcing bars at right angles to the section considered is represented by the ordinates in the diagram. The stress of the bars lying under the pier may be considered to be uniform and

represented by f. The bars in the projection of the footing which lie near the pier will be stressed nearly as high. The stress in a rod near the edge of the footing will be less, say from $0.25f$ to $0.75f$, depending upon the proportions of the footing and the distribution of the reinforcement. Between the pier and the edge of the footing the stress in the bars will vary by some law, probably a curvilinear relation. The total resisting moment developed in the full width of beam may be made up by using the stresses in the several bars. We may obtain this resisting moment in terms of the *maximum* stress f, by finding the equivalent proportion of bars which when stressed to the maximum stress f will give the same total resisting moment as is developed by all the bars with their varying stresses. If the bars are uniformly spaced, this is the same as taking the bars within a rectangle which will give the same area as is included by the curved line. For the dimensions of footing and pier used in the tests, if the minimum stress be $0.25f$ and a curvilinear variation be assumed, then 80% of the bars stressed to the maximum stress f will produce a resisting moment equivalent to that due to the assumed distribution of stress. If the stress at the edge be $0.50f$ and a curvilinear relation be used, the resisting moment will be equivalent to the use of 87% of the bars; if a rectilinear relation from the pier to the edge of the footing be used, 80% of the bars would give the equivalent resisting moment. As an extreme assumption, if the stress at the edge of the footing be $0.75f$ the use of 93% of the steel will give an equivalent resisting moment. In footings with short thick projections the stress in a bar near the edge will be nearly as great as in a bar under the pier, while in broad thin footings the stress in a bar at the edge of the pier will be considerably less than the maximum.

In connection with this dicussion, it seems well to point out that the ordinary assumption of beams superimposed in two directions presumes that outside the pier and out on the projections bars must act to give lateral stiffness and that these bars have a function as carrying bars to what may be considered the main beam, so that the value of the stress in these outer bars for the purpose in hand must be taken as auxiliary rather than as directly tributary to the main beams. It is uncertain to what extent this action must be considered in determining resisting moments of the section of the footing. If the distribution of stress across the section were known, it would seem that the stress in all the rods should be used in calculating the resisting moment of the section.

The preceding discussion assumes a uniform spacing of bars. If the bars are spaced more closely toward the middle the same methods may be employed and the probable distribution of stress across the section determined. If provision is to be made for lateral stiffness or carrying-stress, a further estimate must be made. If the bars are bunched near the edge of the footings the assumptions would have to be modified.

Another view may be had by assuming two equivalent main beams at right angles across the footing which resist the bending moment already obtained. The width of beam assumed as the equivalent width will be that width for which the calculated stress will agree with the actual stress in the most stressed bar, when only the reinforcing bars within the equivalent width are used in the calculation of resisting moment. It is plain that this width is greater than the width of the pier and less than the full width of the footing. It is evident that the equivalent width will vary with the size of the pier, the thickness of the footing, the dimension of the projecting portion, and the amount and distribution of reinforcement. An expression for the equivalent width of beam for use in calculations, even though empirical and not altogether general, will be useful.

A study of the observations and results of the tests of the footings made in the laboratory indicates that the bars for some distance on either side of the pier have nearly the same stress as those under the pier. As a working basis applicable when the spacing of the bars is uniform or does not vary far from this, the conclusion was reached that the resisting moment of the footing in each of the two directions may be based upon the amount of steel in a width of beam equal to the width of pier plus twice the depth of the footing to the reinforcement, plus one-half the remainder of the width of footing and that the use of this amount of steel will determine the maximum steel stress. Expressed as a formula the equivalent beam width then is

$$b = a + 2d + \frac{1}{2}(l - a - 2d) \quad \text{.............. (25)}$$

where l is the width of the footing. If the width given by the first two terms of the second member is greater than the width of the footing, then the width of the beam may be taken as the full width of the footing.

It may be thought that the concrete along the edges of the footing will of its own strength be sufficient to carry the loads laterally without reinforcement, but the deformation due to flexure along these edges may

be much greater than concrete will stand and reinforcement near the edges serves a useful and necessary purpose, especially in distributing the deformations of the concrete preventing the concentration of elongation at single cracks.

The resisting moment, of course, will be

$$M = Afjd \quad \text{.......................} \quad (13)$$

where A is the area of the reinforcement in the given direction for the equivalent width of beam above specified, f is the unit-stress in the most stressed reinforcing bars, and the other symbols are as given on page 8. If the relative stress in the individual bars across the section is known or assumed, $M = \Sigma Afjd$ will express the total resisting moment developed over the section, f here being a variable denoting the unit-stress in the individual bar and A the area of one bar.

The bond stresses may be based upon the shear at the section at the face of the pier. For this the external vertical shear will be the amount of load used in determining the critical bending moment. At the face of the pier this shear is

$$V = \frac{1}{4}(l^2 - a^2)w = (ac + c^2)w \quad \text{................} \quad (29)$$

The expression for bond stress will be taken to be

$$u = \frac{V}{mojd} \quad \text{.........................} \quad (17)$$

where m is the number of reinforcing bars included within the equivalent width of beam as used in calculating the maximum tensile stress.

The calculated bond stress is greater at this section than it is towards the end of the bar, and hence the bond stress is considerably greater than the average bond stress found by considering that the total stress in the steel at the given section is taken off in bond over the surface of the bar between this section and the end of the bar. The same variation of bond stress from middle to end does not hold for the bars near the edge of the footing, and in these the concentration of bond stress is probably considerably greater towards the end of the bar. Where bars are bent up towards their ends the bond stress is also increased in parts of the bar. It is also apparent that the method of calculating bond stress will not apply when the bars are placed in exterior bands without reinforcement under the pier.

In measuring the resistance to diagonal tension failure we may follow the practice used in beams, and for comparison of resistance to diagonal tension we may use the vertical shearing stress developed. Because the diagonal tension failures in footings tested gave fractures at an angle

of about 45° with the vertical, the frustum running from the faces of the pier and reaching the reinforcement at a distance d (the depth from surface to center of reinforcement) from a section through a face of the pier, it seems reasonable to take as the critical section a vertical section enveloping the base of the frustum indicated by EFGH, Fig. 8(b). This position gives results in agreement with those found for wall footings, and by analogy with the reasoning used in wall footings it may be expected that this is the section which has the distribution of shear giving maximum diagonal tensile stresses. In order to be in agreement with the other formulas for vertical shearing stresses, jd will be used in the formula for shearing stress, thus using the maximum unit-stress of the section instead of the average stress. The external vertical shear V may be considered to be that part of the load on the footing outside of the sections considered. The following formula expresses the amount of the vertical shear by this assumption.

$$V=\left[l^2-(a+2d)^2\right]w \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The expression for the critical vertical shearing stress becomes

$$v=\frac{V}{4(a+2d)jd} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

It will be borne in mind that these values of the vertical shearing stress will be used as a measure of the tendency to produce diagonal tension failure. The shearing stress at sections around the pier (punching shear) may be considered to be that given by the expression $\frac{(l^2-a^2)w}{4ajd}$, and the working stresses for punching shear applied.

II. Materials, Test Pieces, and Method of Testing

7. *Materials.*—The materials used in making the test footings were similar to those used in the reinforced concrete beams described in Bulletin No. 29. The stone and sand were bought in the open market. The Universal portland cement was furnished by the manufacturers. The Chicago AA portland cement and the Lehigh portland cement were bought in the open market. The mild steel rods used for the reinforcement were furnished by the Illinois Steel Company. The corrugated bars were supplied by the Corrugated Bar Company.

Stone. The stone was a good quality of crushed limestone from Kankakee, Illinois, ordered screened through a 1-in. screen and over a

¼-in. screen. It contained from 45% to 50% voids and weighed from 80 to 83 lb. per cu. ft. The mechanical analyses made agree very closely with those given on page 21 of Bulletin No. 29.

Sand. The sand was of good quality, sharp, well graded, and generally clean. It weighed 100 to 105 lb. per cu. ft. and contained about 28% voids. The mechanical analyses for that used in the series of 1908 is the same as for the 1908 sand given on page 21 of Bulletin No. 29, while that for the series of 1909 is nearly the same as that for the 1907 sand given on the same page. The sand used in 1910, 1911, and 1912 had the same general characteristics.

Cement. Tests of the three brands of cement are given in Table 2, Table 1 gives analyses of fineness.

Concrete. Men skilled in mixing concrete and making test pieces were employed in the work. The foreman, a contractor for small con-

TABLE 1.

MECHANICAL ANALYSIS OF CEMENT

Sieve No.	Per cent passing					
	1908		1909		1911	1912
	Universal	Chicago AA	Universal	Chicago AA	Universal	Universal
75	99.4	98.2	98.8	97.5	98.9	....
100	98.3	94.9	96.3	92.8	96.5	97.2
200	89.9	80.0	81.3	74.7	82.5	81.8

crete work, has had the making of test pieces in the laboratory for the past seven years. Care was taken in measuring, mixing, and tamping to secure as uniform a concrete as is possible under working conditions. In 1908 and 1909 all materials were proportioned by loose volume, and weights were taken as a check on the measurement. In the later years, the method of measurement of the sand and stone was the same, but 95 lb. of cement was taken to be a cubic foot. The latter method of proportioning gives a somewhat richer concrete than when the cement is measured loose. Except in 1912, when all the concrete was machine mixed, the mixing was done with shovels by hand. The sand and cement were first mixed dry; the stone, which had previously been thoroughly moistened, was added, and the mass then turned until of a uniform appearance. Water was then added in such proportion as to give a fairly wet mixture. The mass was again turned until thoroughly mixed.

Steel. The steel reinforcing bars consisted of plain round rods and deformed bars. The round rods were open-hearth mild steel. The deformed bars were square and round corrugated bars, types B and C.

TABLE 2.

Tensile Strength of Cement

These tests were made with standard Ottawa sand. Each value is the average of 5 briquettes.

Ref. No.	Ultimate Strength, lb. per sq. in.							
	Chicago AA Cement				Universal Cement			
	Age 7 days		Age 28 days		Age 7 days		Age 28 days	
	Neat	1–3	Neat	1–3	Neat	1–3	Neat	1–3
SERIES OF 1908.								
1...........	559	145	707	247	563	244	764	319
2...........	732	192	857	318	809	248	885	336
3...........	665	175	779	266	728	232	776	285
4...........	811	227	833	307	699	242	754	292
5...........	666	182	792	284	702	229	763	315
6...........	693	191	781	283	...	295*	...	366*
7...........	719	206	767	303	...	...	...	...
	...	248*	...	335*				
Average...	692	188	788	287	700	239	788	309
SERIES OF 1909.								
1...........	742	205	783	270	617	160	853	278
2...........	716	232	807	306	595	179	772	280
		288*		331*				
3...........	725	176	768	254	607	197	732	281
Average...	728	204	786	277	606	179	786	280
SERIES OF 1910.								
1...........	...	...	...	...	629	219	678	328
2...........	...	...	...	...	613	217	649	315
3...........	...	...	...	...	670	190	697	297
Average...	...	...	...	...	637	209	675	313
SERIES OF 1911.								
1...........	719†	248†	805†	329†	589	198	674	278
						265*		323*
2...........	...	...	...	...	684	227	709	283
3...........	...	...	...	...	653	240	731	319
4...........	...	...	...	...	662	214	696	282
Average...	...	...	...	...	647	220	702	290
SERIES OF 1912.								
1...........	...	...	...	...	585	239	685	315
2...........	...	...	...	...	577	225	694	297
3...........	...	...	...	...	691	242	715	306
4...........	...	...	...	...	617	231	792	326
5...........	...	...	...	...	588	246	672	333
6...........	...	...	...	...	612	253	758	323
7...........	...	...	...	...	698	287	884	372
Average...	...	...	...	...	624	246	743	325

*Briquettes made with the same sand as was used in concrete. Not included in average.
† Lehigh Portland cement.

Test pieces were cut from the reinforcing bars. Table 3 gives the average of the results of tension tests for the steel used in the different years.

8. *Wall Footings.*—All the concrete wall footings were 12 in. wide. The stem of the footing (the "wall") was in all cases 12x12x12 in. The length of the footing was 5 ft., except that four of the series of

TABLE 3.

TENSION TESTS OF REINFORCING BARS

Nominal Size inches	Description	Stress at Yield Point lb. per sq. in.	Number of Tests	Maximum Variation from Average per cent
		1908 TESTS		
½	Plain round	41 500	17	7.4
½	Cor. square	50 400	22	12.7
¾	Plain round	41 000	2	1.2
5	I-beam	......	..	....
		1909 TESTS		
¼	Cor. square	33 000	1	....
⅓	Cor. square	31 200	10	10.3
⅜	Plain round	42 600	8	6.2
½	Plain round	41 200	16	13.4
½	Cor. square	46 800	8	8.5
$\frac{9}{16}$	Cor. round	53 500	16	6.0
⅝	Plain round	37 300	9	9.9
¾	Plain round	38 800	8	1.7
¾	Cor. square	50 300	6	6.1
No. 7*	Wire mesh	124 000†	6	8.1
No. 11*	Wire mesh	114 000†	6	1.3
		1910 TESTS		
½	Plain round	34 000	2	5.0
½	Cor. square	52 700	3	9.7
$\frac{9}{16}$	Cor. round	53 500	16	6.0
⅝	Plain round	37 100	2	1.4
¾	Plain round	41 700	4	3.6
		1911 TESTS		
⅜	Plain round	41 200	11	8.8
⅜	Cor. round	44 000	11	10.4
½	Plain round	39 500	10	3.9
½	Cor. square	51 500	3	5.9
⅝	Plain round	40 100	8	2.4
		1912 TESTS		
⅜	Plain round	48 100	14	5.2
⅝	Plain round	35 900	8	2.2
⅝	Cor. round	52 200	10	6.9

*Birmingham or Stubs' gauge.
†Ultimate strength—No yield point could be detected.

1908 were 6 ft. 8 in. long, two of the series of 1909 were 7 ft. long, three of the unreinforced footings of the series of 1911 were 7 ft. long and three were 3 ft. long. The depth was, in most cases, 10 in. to the center of steel. In the test pieces made in 1908 and in 1909, the depth over all was 11 in., in 1911, 12 in. over all. In the series of 1908 one reinforced concrete footing was 6 in. to the steel instead of 10 in., and two of those reinforced with I-beams had other depths. In the series of 1908 two footings had their upper surfaces sloped from 11 in. to 5½ in. at the end of the projection and two were stepped as shown in Fig. 14, page 40. In the greater number of the footings the reinforcing bars were carried straight throughout their length. In some the bars were bent up with easy curves to points near the upper surface of the footing. In a few U-shaped stirrups were used, passing around and

TABLE 4.

Compression Tests of 6-in. Cubes
Series of 1908, 1909, 1910, 1911, and 1912.

No.	Kind of Concrete.	Age at Test days.	Maximum Load lb. per sq. in.	No.	Kind of Concrete.	Age at Test days.	Maximum Load lb. per sq. in.	No.	Kind of Concrete.	Age at Test days.	Maximum Load lb. per sq. in.
1301	1–3–6	64	1315	1535	1–2½–5	121	2510	1718	1–1–2	546	4640
1302	1–3–6	65	1137	1536	1–2½–5	94	2768				
1303	1–3–6	60	1073	1541	1–2½–5	109	2438	1721	1–2–4	640	2980
				1542	1–2½–5	79	2272	1722	1–2–4	606	2890
1306	1–1½–3	64	3970	1551	1–2½–5	121	2290	1723	1–2–4	368	2910
1307	1–1½–3	64	2107	1552	1–2½–5	101	2857	1725	1–2–4	606	2840
1308	1–1½–3	62	1807	1553	1–2½–5	120	2835	1726	1–2–4	368	2910
				1554	1–2½–5	103	2242	1727	1–2–4	641	2810
1311	1–3–6	63	2210	1561	1–2½–5	114	2550	1728	1–2–4	606	2840
1312	1–3–6	64	1610	1562	1–2½–5	87	2187	1729	1–2–4	373	3225
1313	1–3–6	64	2097	1563	1–2½–5	113	2462	1731	1–2–4	641	2810
1314	1–3–6	62	1420	1564	1–2½–5	83	2322	1732	1–2–4	436	2590
1315	1–3–6	62	1372	1631	1–2½–5	63	1173	1733	1–2–4	360	2570
1316	1–3–6	61	1433	1632	1–2½–5	60	1412	1741	1–2–4	629	3040
1317	1–3–6	62	1203	1633	1–2½–5	63	1173	1742	1–2–4	436	2590
1318	1–3–6	61	1433	1634	1–2½–5	66	1288	1743	1–2–4	360	2570
1319	1–3–6	60	1263	1635	1–2½–5	64	814	1744	1–2–4	629	3040
1322	1–3–6	60	1507	1636	1–2½–5	69	1666	1745	1–2–4	590	2705
1325	1–3–6	64	1910	1641	1–2½–5	69	1538	1746	1–2–4	359	2890
1326	1–3–6	60	1553	1642	1–2½–5	60	1412	1747	1–2–4	628	2870
1341	1–3–6	60	1383	1645	1–2½–5	61	1663	1748	1–2–4	590	2705
1342	1–3–6	64	1457	1646	1–2½–5	59	1235	1749	1–2–4	359	2890
1351	1–3–6	59	1283	1651	1–2½–5	68	1797	1751	1–2–4	634	2685
1352	1–3–6	60	1403	1652	1–2½–5	62	1350	1752	1–2–4	585	2725
1361	1–3–6	58	2103	1655	1–2½–5	61	1663	1753	1–2–4	354	3030
1362	1–3–6	59	1215	1661	1–2½–5	73	1757	1754	1–2–4	640	2980
1371	1–3–6	62	1211	1662	1–2½–5	75	1692	1755	1–2–4	585	2725
1372	1–3–6	59	1290	1665	1–2½–5	73	1842	1756	1–2–4	354	3030
1375	1–3–6	62	1228	1666	1–2½–5	55	1319	1757	1–2–4	640	2980
1376	1–3–6	59	960	1671	1–2½–5	59	1235	1758	1–2–4	591	3410
				1672	1–2½–5	75	1530	1759	1–2–4	354	3030
1411	1–2½–5	92	1395	1673	1–2½–5	63	1735	1761	1–2–4	634	2685
1412	1–2½–5	75	1523	1674	1–2½–5	60	1465	1762	1–2–4	591	3410
1414	1–2½–5	77	1533	1675	1–2½–5	69	1538	1763	1–2–4	354	3030
1415	1–2½–5	88	2028	1676	1–2½–5	55	1319	1806	1–2–4	70	2094
1416	1–2½–5	64	1307	1681	1–2½–5	63	1735	1807	1–2–4	73	1831
1417	1–2½–5	73	1343	1682	1–2½–5	62	1350	1808	1–2–4	64	1610
1418	1–2½–5	64	1292	1685	1–2½–5	57	1293	1809	1–2–4	66	2645
1421	1–2½–5	72	1877	1687	1–2½–5	67	1573	1810	1–2–4	63	1562
1422	1–2½–5	61	864	1688	1–2½–5	75	1692	1811	1–2–4	62	2203
1425	1–2½–5	94	1717	1692	1–2½–5	60	1465	1812	1–2–4	135	3123
1426	1–2½–5	60	1468	1693	1–2½–5	57	1293	1813	1–2–4	118	3197
1429	1–2½–5	82	1622	1694	1–2½–5	67	1573	1814	1–2–4	77	2450
1431	1–2½–5	100	1718					1815	1–2–4	62	2223
1432	1–2½–5	64	1385	1701	1–3–6	638	2640	1816	1–2–4	61	1877
1435	1–2½–5	104	1245	1702	1–3–6	298	1862	1817	1–2–4	66	2577
1436	1–2½–5	70	1151	1702a	1–3–6	422	2550	1818	1–2–4	61	2030
1437	1–2½–5	84	1543					1819	1–2–4	73	2772
1439	1–2½–5	100	1605	1703	1–2–4	520	3845	1820	1–2–4	69	1526
1447	1–2½–5	78	1482	1704	1–2–4	628	2870	1821	1–2–4	86	1678
1448	1–2½–5	58	1583	1704a	1–2–4	367	4170	1822	1–2–4	80	1692
1449	1–2½–5	92	1720					1823	1–2–4	61	2640
1451	1–2½–5	67	1418	1705	1–1–2	586	4816	1831	1–2–4	71	2620
1501	1–2½–5	112	2910	1706	1–1–2	431	4403	1832	1–2–4	108	3100
1502	1–2½–5	104	2939	1706a	1–1–2	546	4640	1833	1–2–4	67	2500
1503	1–2½–5	112	2307					1834	1–2–4	115	3240
1504	1–2½–5	109	1761	1707	1–2–4	640	2980	1835	1–2–4	67	2085
1505	1–2½–5	128	3618	1708	1–2–4	430	3090	1836	1–2–4	110	3410
1506	1–2½–5	116	2260	1708a	1–2–4	367	4170	1837	1–2–4	74	2625
1507	1–2½–5	121	3180	1710	1–2–4	430	3090	1838	1–2–4	109	3710
1508	1–2½–5	109	1988	1710a	1–2–4	367	4170	1839	1–2–4	73	2785
1515	1–2½–5	127	2980	1712	1–2–4	680	2175	1840	1–2–4	98	3235
1516	1–2½–5	118	2679	1713	1–2–4	606	2890	1841	1–2–4	116	2755
1522	1–2½–5	94	2935	1714	1–2–4	373	3225	1842	1–2–4	105	3050
1526	1–2½–5	88	1917					1843	1–2–4	116	3580
1531	1–2½–5	120	3193	1716	1–1–2	678	3870	1844	1–2–4	102	3630
1532	1–2½–5	114	2637	1717	1–1–2	431	4403				

TABLE 5.

Flexure Tests of 6 in. x 8 in. x 36 in. Control Beams. Series of 1908, 1909, 1910, 1911, and 1912.

No.	Kind of Con-crete.	Age at Test days.	Modulus of Rup-ture, lb. per sq. in.	No.	Kind of Con-crete.	Age at Test days.	Modulus of Rup-ture, lb. per sq. in.	No.	Kind of Con-crete.	Age at Test days.	Modulus of Rup-ture, lb. per sq. in.
1302	1–3–6	65	249	1516	1–2½–5	99	385	1708a	1–2–4	597	432
1303	1–3–6	71	167	1521	1–2½–5	107	357	1709	1–2–4	82	214
				1522	1–2½–5	91	296	1710a	1–2–4	597	432
1306	1–1½–3	63	437	1525	1–2½–5	107	294	1712	1–2–4	101	266
1307	1–1½–3	64	355	1526	1–2½–5	93	287	1713	1–2–4	36	226
1308	1–1½–3	71	385	1531	1–2½–5	93	342	1714	1–2–4	597	291
				1532	1–2½–5	96	440				
1312	1–3–6	64	140	1535	1–2½–5	105	377	1716	1–1–2	100	402
1313	1–3–6	64	286	1536	1–2½–5	94	230	1718	1–1–2	597	656
1314	1–3–6	64	258	1541	1–2½–5	104	387				
1315	1–3–6	63	151	1542	1–2½–5	85	397	1721	1–2–4	89	220
1316	1–3–6	60	194	1551	1–2½–5	88	363	1722	1–2–4	36	226
1317	1–3–6	71	175	1552	1–2½–5	96	368	1724	1–2–4	82	214
1318	1–3–6	60	194	1553	1–2½–5	87	365	1725	1–2–4	55	169
1319	1–3–6	59	284	1554	1–2½–5	99	362	1728	1–2–4	55	169
1321	1–3–6	63	379	1561	1–2½–5	81	375	1729	1–2–4	597	291
1322	1–3–6	59	309	1562	1–2½–5	84	408	1732	1–2–4	113	384
1325	1–3–6	67	289	1563	1–2½–5	80	306	1742	1–2–4	113	384
1326	1–3–6	77	210	1564	1–2½–5	89	313	1745	1–2–4	84	367
1341	1–3–6	69	237	1631	1–2½–5	56	231	1747	1–2–4	58	231
1342	1–3–6	60	219	1632	1–2½–5	65	283	1748	1–2–4	84	367
1351	1–3–6	60	182	1633	1–2½–5	56	231	1751	1–2–4	64	265
1352	1–3–6	56	228	1634	1–2½–5	65	267	1752	1–2–4	78	350
1361	1–3–6	60	296	1635	1–2½–5	63	301	1754	1–2–4	63	272
1362	1–3–6	60	191	1636	1–2½–5	67	323	1755	1–2–4	78	350
1371	1–3–6	63	258	1641	1–2½–5	81	274	1757	1–2–4	63	272
1372	1–3–6	60	179	1642	1–2½–5	65	283	1758	1–2–4	84	335
1375	1–3–6	63	250	1645	1–2½–5	72	341	1761	1–2–4	64	265
1376	1–3–6	60	174	1646	1–2½–5	63	252	1762	1–2–4	84	335
				1651	1–2½–5	61	260	1806	1–2–4	98	260
1411	1–2½–5	58	282	1652	1–2½–5	80	269	1807	1–2–4	88	346
1412	1–2½–5	74	151	1655	1–2½–5	72	341	1808	1–2–4	106	289
1414	1–2½–5	82	224	1661	1–2½–5	63	306	1809	1–2–4	67	362
1415	1–2½–5	78	295	1662	1–2½–5	86	325	1810	1–2–4	68	215
1416	1–2½–5	69	214	1665	1–2½–5	63	306	1811	1–2–4	74	358
1417	1–2½–5	71	322	1666	1–2½–5	65	255	1812	1–2–4	100	243
1418	1–2½–5	69	264	1671	1–2½–5	63	252	1813	1–2–4	93	224
1421	1–2½–5	70	287	1672	1–2½–5	86	315	1814	1–2–4	78	339
1422	1–2½–5	64	183	1673	1–2½–5	62	290	1815	1–2–4	74	432
1425	1–2½–5	87	301	1674	1–2½–5	60	227	1816	1–2–4	56	259
1426	1–2½–5	64	274	1675	1–2½–5	81	274	1817	1–2–4	67	415
1429	1–2½–5	75	287	1676	1–2½–5	65	255	1818	1–2–4	63	246
1431	1–2½–5	105	283	1681	1–2½–5	62	290	1819	1–2–4	74	405
1435	1–2½–5	109	287	1682	1–2½–5	80	269	1820	1–2–4	69	217
1436	1–2½–5	68	171	1685	1–2½–5	57	290	1822	1–2–4	95	277
1437	1–2½–5	83	265	1687	1–2½–5	66	288	1823	1–2–4	612	341
1439	1–2½–5	67	240	1688	1–2½–5	86	325	1831	1–2–4	301	401
1447	1–2½–5	72	294	1692	1–2½–5	60	227	1832	1–2–4	280	394
1448	1–2½–5	58	281	1693	1–2½–5	57	290	1833	1–2–4	298	331
1449	1–2½–5	98	294	1694	1–2½–5	66	288	1834	1–2–4	287	310
1451	1–2½–5	60	240					1835	1–2–4	293	303
1501	1–2½–5	79	309	1701	1–3–6	205	230	1836	1–2–4	281	405
1502	1–2½–5	87	423	1702a	1–3–6	597	281	1837	1–2–4	295	354
1503	1–2½–5	79	373					1839	1–2–4	303	358
1504	1–2½–5	109	425	1703	1–2–4	205	415	1840	1–2–4	267	453
1505	1–2½–5	95	376	1704	1–2–4	58	231	1841	1–2–4	287	356
1506	1–2½–5	99	272	1704a	1–2–4	597	432	1842	1–2–4	277	312
1507	1–2½–5	105	409	1706	1–2–4	668	383	1843	1–2–4	288	310
1508	1–2½–5	93	363	1706a	1–2–4	597	656	1844	1–2–4	274	387
1515	1–2½–5	95	371	1707	1–2–4	89	220				

under the longitudinal reinforcing bars. The reinforcing bars when straight were 4 ft. 11 in. and 4 ft. 11½ in. in length, except that in 1909 a length of 4 ft. 6 in. was used. The reinforcement of the footings is

described in Tables 9 to 11, and the position of the bars is shown in Fig. 14. Except where otherwise noted, the wall and footing were poured at the same time. In the footings made in 1909 and 1911 four ¼-in. bars were placed vertically in the corners of the wall, extending down into the footing, to prevent displacement of the wall in handling.

The brick footings were 5 ft. long and about 12 in. wide. The wall was 12 in. in thickness. The depth, offsets, and number of courses are shown in Fig. 13.

9. *Column Footings.*—The general form of a column footing is shown in Fig. 4, page 12. The column, or pier as it will be called, was 12x12x12 in. The footings were 5 ft. square. The depth over all varied from 6 in. to 18 in. One footing (No. 1451) had a sloping upper surface, the depth being 7 in. less at the edges than at the face of the pier. The dimension given in the table for the position of the reinforcement is the distance from the upper surface of the footing to the center of the two layers of bars, or to the center of the four layers in the case of four-way reinforcement. Unless otherwise noted the reinforcing bars were straight throughout their length. In the series of 1909 the reinforcing bars were generally 4 ft. 6 in. long; in 1910, 4 ft. 10 in. long; in 1911 and 1912, about 4 ft. 11½ in. long. In a few cases in 1909 the reinforcing rods were 9 in. shorter, and alternate bars were run within 12 in. of one face of the footing, the other bars going to within the same distance from the opposite face. For the footings made in 1909 the depth over all was in most cases 11 in., it being in a few 11½ in. and 12 in. For the footings made in 1910, 1911, and 1912, the depth over all was 12 in., except for the shallower footings. The general make-up of the footings and the disposition of the reinforcing bars is given in Tables 14 to 18, and in the diagrams. Eyes U-formed of steel rods were embedded in the footings at two points; hooks were inserted in these eyes when the footings were lifted and moved.

10. *Making and Storing Footings.*—The footings were built in wooden side forms directly on the concrete floor of the mixing room with a strip of building paper beneath the forms. The forms were generally removed after 7 days. In the work of the first two years the wall footings were left on the floor of the mixing room until the test, when they were removed to the Materials Testing Laboratory, but in the later years they were piled one above the other for storage. The column footings were piled one above the other for storage; they were tested in the mixing room. The specimens were wet down with water from

a hose at frequent intervals for some time after making. The temperature of the mixing room ranged from 50° to 70° F. and was somewhat irregular, so that the average temperature for the hardening for different specimens varied. As noted under Article 29, "Phenomena of Tests of Column Footings," part of the column footings were not removed from the place of making until just before the test, and the difference in moisture conditions probably affected the rate of hardening.

11. *Minor Test Pieces.*—In Tables 4 and 5 are given the results of compression tests of 6-in. cubes and of flexure tests of 6x8x36-in. plain concrete control beams. These minor test pieces were made from the same batch of concrete as the corresponding footings and serve to give an estimate of the strength and quality of the concrete used. The control beams were tested with a 3-ft. span and one-third-point loading upon a wooden base, so arranged as to insure a good distribution of the loads and pressures across the width of the beam.

12. *Testing Wall Footings.*—The wall footings were tested in the 200 000-lb. Olsen testing machine of the Laboratory of Applied Mechanics, except that in the tests made in 1912 the 600 000-lb. Riehle testing machine was used. A nest of springs was placed on the bed of the machine. These springs were "car springs," one set being 2¾x7x½-in. springs and the other set 3x12x9/16-in. springs. The first size closed 1.7 in. with a load of 1 700 lb., and the second size about 3 in. with a load of 2 000 lb. A calibration of a considerable number of these springs showed close uniformity among them, and their shortening was directly proportional to the load. The springs were held in place in setting the footing by ¼x3x12-in. plates with a dowel fastened on the under side which extended down into the opening of the spring. The springs were spaced to suit the load expected, and were most commonly 3 in. center to center, both lengthwise and crosswise of the footing. The 7-ft. footings of the series of 1907 had the sets of springs spaced 4 in. apart in the lengthwise direction. A view of a wall footing in the testing machine is given in Fig. 10. The footing rested directly on the spacing plates. On top of the stem or wall an iron plate was bedded in plaster of paris. On this plate a spherical bearing block was centered with respect to the wall and adjusted to the head of the machine and the load was applied directly to this bearing block, or the load was centered by using a rod across the plate to act as a pivot.

As the load was applied by the testing machine, the springs compressed and the ends of the footing deflected somewhat. Vertical measurements were taken from the bed of the machine to marks on

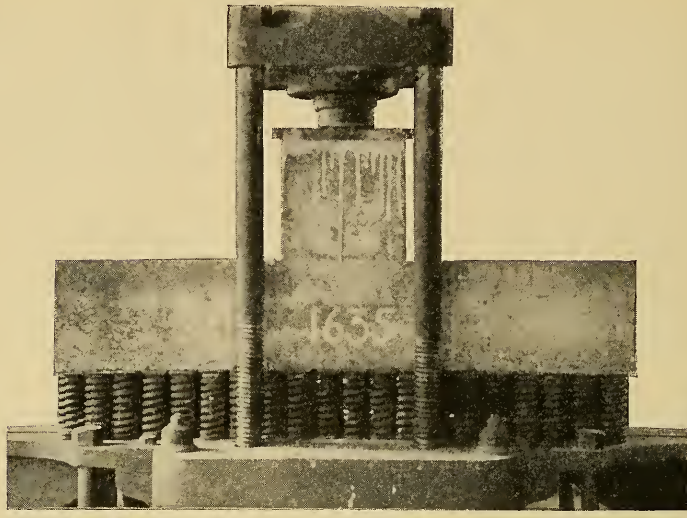

FIG. 10. VIEW OF WALL FOOTING IN TESTING MACHINE.

both sides of the footing at five points along its length. These measurements were taken at a load of 2 000 lb., and at the various loads applied thereafter. A careful watch was made for cracks and their appearance and growth was recorded. After failure the footing was broken up and examined. Measurement of slip of bar and of deformations in the bar were made in some cases, as is described elsewhere.

It was sometimes difficult to keep the test specimen in its place at high loads, as the bed of springs canted or otherwise got out of place. In those failures in which the footing separated into pieces, parts were thrown violently from the testing machine.

13. *Testing Column Footings.*—The column footings were tested in a machine built especially for the purpose. Fig. 11 (a) gives a view of the apparatus which was used in the tests in 1910, 1911, and 1912. A bed was formed by placing 10-in. I-beams side by side, the edges of the flanges touching. On this rested a bed of car springs on which the test footing was placed. Transversely under the bed of I-beams and near their ends were two 12-in. x 55-lb. I-beams 6 ft. long which took the load from above. Under these I-beams were two cast-iron blocks through which eight rods passed to similar cast-iron blocks on the upper part of the machine. The two hydraulic jacks by which the load was applied were placed between these blocks and a 24-in. I-beam. This I-beam transmitted the load through blocks to the top

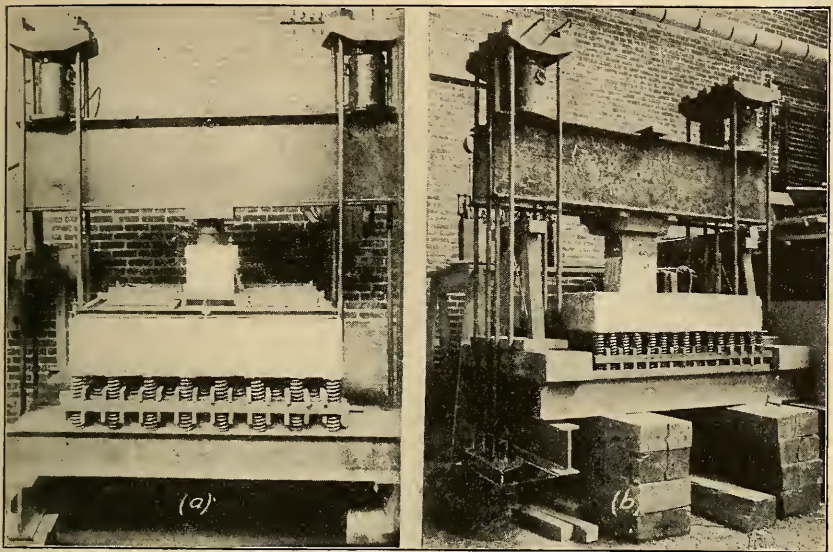

FIG. 11. TESTING MACHINES FOR COLUMN FOOTINGS.

of the pier. One of these blocks was an adjustable spherical bearing block. The lower or base block was bedded to the top of the pier with plaster of paris. The whole of the machine rested on a timber foundation. The pumps which operated the hydraulic jacks were placed on a platform near by, a gage being connected with each pump.

In the 1909 tests four jacks were used and two 24-in. I-beams were placed across the top with four sets of vertical rods running from their ends. A heavy steel block carried the load from the two I-beams. In some of the 1909 tests no adjustable bearing blocks were used, the bearing plates consisting of flat plates only and the adjustment being made by the jacks. This apparatus is shown in Fig. 11 (b). On account of the low loads on the jacks the results with this machine are less satisfactory than with the 1910 machine.

The springs used were the 3x12x9/16-in. helical car springs used in tests of wall footings, ground to a length of 12 in. In the 1910 tests there were generally 113 springs used, though for the heavy loads the number was increased to 225. In the 1909 tests seven footings were tested with 225 springs. The amount of shortening with this number of springs was so small at the lower loads as not to give a good distribution of the load over the bottom of the footing. A bed composed of 113 springs would compress about 1 in. under a load of 75 000 lb.

The springs completely closed with a shortening of about 3 in., so that with some deflection in the footing not more than 200 000 lb. could generally be carried by 113 springs.

In the operation of the tests the load was applied in varying increments. It was kept as nearly as possible equally divided between the jacks. In some cases, due to imperfect bearing, the springs tilted to one side and a release of load and a readjustment was necessary. The load was taken from the gage indication, and was corrected by means of a calibration graph which had been prepared from the calibration of the jacks in a testing machine. Many of the tests were continued beyond the critical load and in some cases the rupture of the specimen was followed by a violent throwing of large pieces of the footing from the machine.

Measurements of the compression of the springs were made at the corners of the footings. In the 1910 and 1911 tests a frame made up of 1¼x1¼-in. angles was supported at three points on the upper surface of the footing close to the pier. Measurements taken from this frame at numerous points on the upper surface of the footing by means of Ames test dials enabled the deflection of the footing at these points to be determined. In the 1909 tests threads were stretched along two opposite lateral faces of the footing and the deflection at the faces obtained by means of mirror-and-scale apparatus. A yoke clamped to the sides of the pier gave a basis of measurements for the transverse deflections.

Observations were taken of cracks after they became visible on the lateral faces. Due to the form of construction of the machine and the presence of the nest of springs, no observations could be made on the bottom surface of the footing during the progress of the test. The footings were examined after being taken from the machine, but it must be borne in mind that the cracks formed and fractures obtained indicated conditions that may have been brought into existence after the critical load was applied.

III. Experimental Data and Discussion.

A. Wall Footings.

14. *Tables.*—Table 6 gives values of j used in the calculations in connection with equations (13), (17) and (18). Tables 7, 9, 10 and 11 give descriptions of the wall footing test pieces, the results of the tests, and calculated quantities. These quantities are calculated by the formulas and methods given on pages 8 to 10. In all the calcula-

tions, a uniform distribution of load is assumed. This is done because in some cases the measurements of deflection and of compression of springs are not available, or are available only at loads below the critical load considered, and their use would put some tests on a different basis from the others. In the stiffer footings and for the lower loads the error in assuming that the distribution is uniform is slight. For a

TABLE 6.

VALUES OF j USED IN CALCULATIONS.

Reinforcement per cent	1–2½–5 and 1–3–6 concrete	1–1–2 and 1–2–4 concrete
0.20	.92	.93
0.30	.90	.915
0.40	.89	.90
0.50	.88	.895
0.60	.87	.89
0.70	.865	.88
0.80	.86	.875
1.00	.85	.86
1.20	.84	.85
1.50	.825	.84

very few cases the calculations will give perhaps 7% greater tensile stress than the real distribution, the same excess in bond stress, and perhaps as much in the vertical shearing stresses at the section used. Generally the error is well within this limit. Of course, after the yield point of the reinforcement is reached, or failure by bond occurs, the effect of uneven distribution of pressure is far greater than that outlined above.

It should be noted that there was some difficulty and an element of uncertainty in determining the amount of the critical load, especially when readings were not taken close together. Generally the critical load was taken at the marked increase in deflection, though dropping of load, marked increase in deformation in steel, and other features were considered when the information was available. The additional load beyond the critical load carried by most footings, will be useful as a safeguard, but is not available for design purposes.

In these tables the tensile stresses and bond stresses are calculated for a section at the face of the wall, the vertical shearing stresses for a section distant d from the face of the wall. The reason for using this particular section for the vertical shearing stresses is discussed under Article 25, "Vertical Shearing Stresses and Diagonal Tension Failures." Table 12 gives values of the vertical shearing stresses for wall footings

TABLE 7.

TESTS OF WALL FOOTINGS WITHOUT REINFORCEMENT.

Footing No.	Year made	Kind of Concrete	Cement		Length feet	Age at Test days	Load at Failure pounds	Modulus of Rupture lb. per sq. in.	Control Beams		6-in. Cubes.	
			Kind	Per cent					Modulus of Rupture lb. per sq. in.	Age days	Maximum Load lb. per sq. in.	Age days
1301	1908	1–3–6	U	8.5	5	64	14 600	289	...	..	1 315	64
1302	1908	1–3–6	AA	9.9	5	63	12 200	242	249	65	1 137	65
1303	1908	1–3–6	AA	10.1	5	61	12 000	238	167	71	1 073	60
1306	1908	1–1½–3	U	18.7	5	64	23 050	457	437	63	3 970	64
1307	1908	1–1½–3	AA	20.2	5	61	22 300	442	355	64	2 107	64
1308	1908	1–1½–3	AA	19.4	5	61	20 000	396	385	71	1 807	62
1701	1911	1–3–6	U	12.0	5	86	20 300	339	230	205	2 660	519
1702	1911	1–3–6	U	12.2	5	355	15 000	250	...	...	1 862	298
1702a	1911	1–3–6	L	11.8	5	295	23 300	389	...	...	2 630	360
1703	1911	1–2–4	U	15.5	5	86	28 100	469	415	205	3 845	520
1704	1911	1–2–4	U	17.2	5	89	15 400	257	231	58	.....	...
1704a	1911	1–2–4	L	17.1	5	296	22 700	379	...	...	4 170	367
1705	1911	1–1–2	U	36.5	5	100	27 900	464	...	...	4 920	363
1706	1911	1–1–2	U	34.3	5	355	26 500	442	...	...	4 403	431
1706a	1911	1–1–2	L	35.0	5	304	24 600	410	...	...	3 680	367
1707	1911	1–2–4	U	17.4	3	88	49 600	344	220	89	.....	...
1708	1911	1–2–4	U	17.1	3	374	35 100	244	...	...	3 090	430
1708a	1911	1–2–4	L	17.1	3	304	56 700	394	...	...	4 170	367
1709	1911	1–2–4	U	17.2	7	83	11 800	316	214	82	.....	...
1710	1911	1–2–4	U	17.1	7	375	11 900	319	...	...	3 090	430
1710a	1911	1–2–4	L	17.1	7	329	16 400	439	...	...	4 170	367

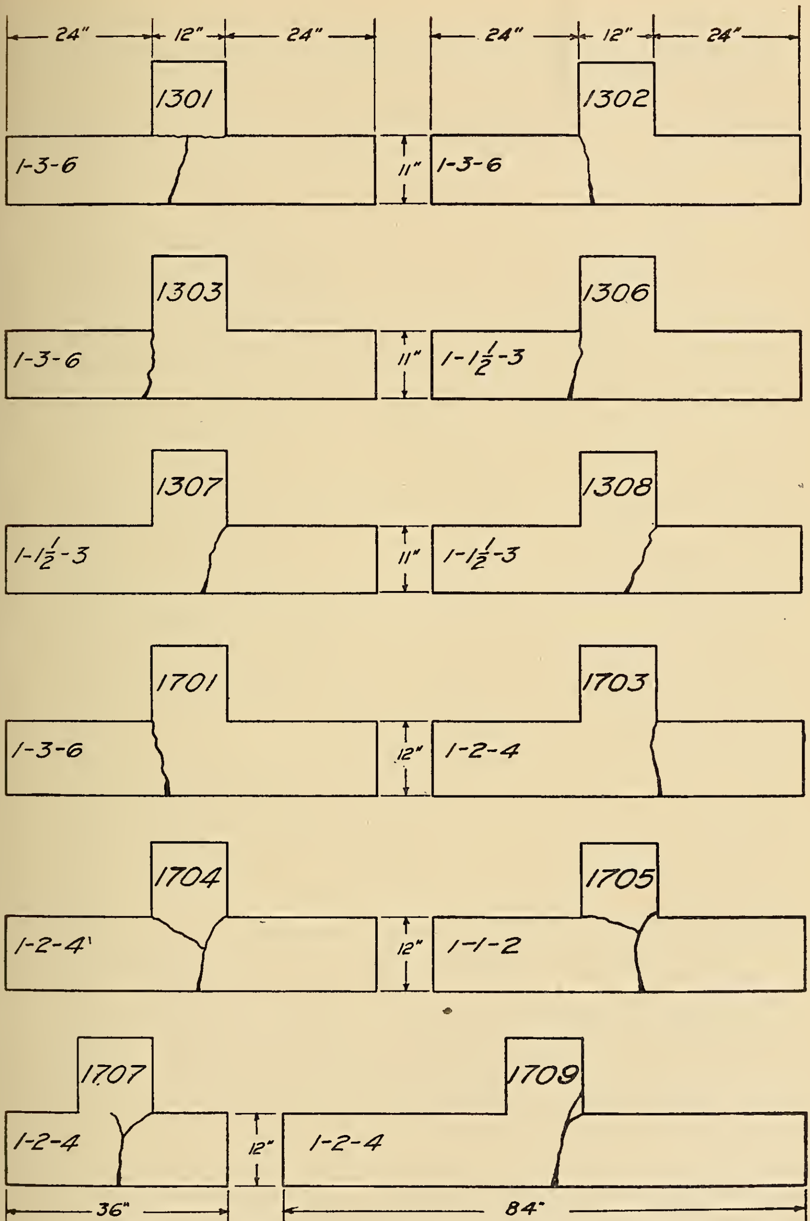

Fig. 12. Unreinforced Concrete Wall Footings.

failing by diagonal tension and for footings which developed high vertical shearing stresses; these are given both for a section at the face of the wall and for one distant *d* from the face. Table 13 gives the calculated values of the bond stresses developed, where failure was by bond, or high bond stresses were developed. In this case the term "nominal bond stress" is employed to cover cases where bars are bent up or where stirrups are used.

15. *Unreinforced Concrete Wall Footings.*—In the unreinforced concrete footings failure occurred very suddenly, no cracks being observed until the instant of failure. The failure crack (see Fig. 12) formed most frequently from a point almost directly under the edge of the wall and passed vertically upward to the face of the wall. In No. 1301, in which the wall had become separated from the footing before the test, the crack formed from a point under the middle of the footing and at failure the wall separated from the footing. Even with this form of failure, the footing developed a higher strength than its companion test pieces. In No. 1308 the crack formed from a point below the middle of the wall and extended to the edge of the wall; but its strength was less than that of its companion test pieces. The cubes from the mix from which this footing was made (see Table 4) gave a lower strength than did those of the companion cubes. It is considered that the critical section is at the edge of the pier and the calculations in Table 7 are made on this basis.

The modulus of rupture calculated for the section at the face of the wall is given in Table 7. The control test beams (6x8, 36-in. span) gave values of the modulus of rupture in general somewhat less than the modulus of rupture found in the corresponding unreinforced concrete footings. It may be expected that the modulus of rupture for the footings will not differ much from that obtained in tests of plain concrete beams. The footings of different lengths gave quite similar values of the modulus of rupture.

The effect of richness of concrete on the strength of the footings is apparent. Attention is called to the fact that the ratio of thickness of footing to projection of footing is much greater in these wall footings than is usual in practice, and that the load per square foot on the footing is low.

16. *Brick Masonry Footings.*—In connection with the tests of brick and terra cotta columns, described in Bulletin No. 27, four wall footings of brick masonry were built, and the results of the tests of these brick footings will be recorded here. The form and dimensions of these

footings are shown in Fig. 13. All the footings were well laid up in 1-3 portland cement mortar, joints being broken in workmanlike manner, and the workmanship was the same as that described in Bulletin No. 27 for the brick columns. Two grades of brick were used, shale build-

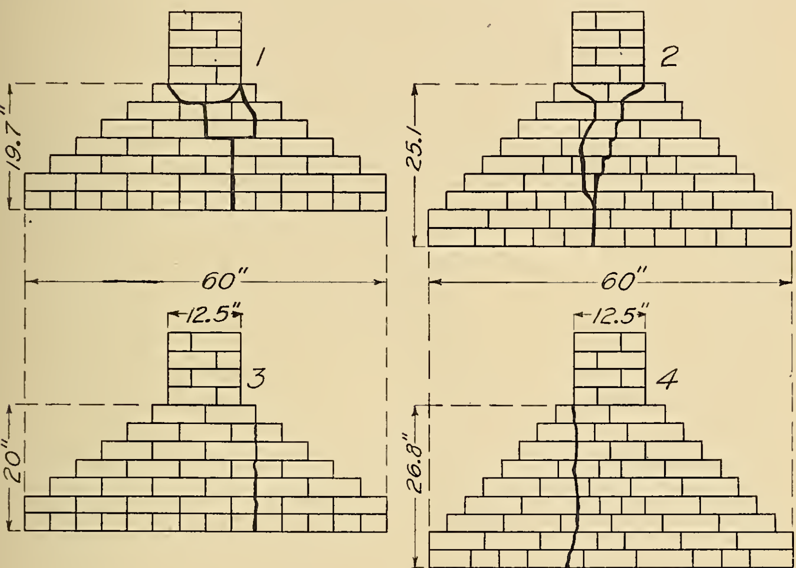

Fig. 13. Brick Wall Footings.

ing brick and underburned clay brick, as described in that bulletin. The tests were made in the same manner as the tests of the concrete footings.

The footings failed suddenly at the maximum applied load. The section of failure was usually at the edge of the wall or stem of the footing. The failure was along vertical joints and through the bricks. In the footings made with hard building brick the line of fracture was irregular, a second break running to the other face of the wall. In the footings made with soft brick there was but one line of failure.

The modulus of rupture calculated for the section at which failure occurred is given in Table 8. The average for the two footings made with the hard building brick is 281 lb. per sq. in., and for the two made with soft brick 76 lb. per sq. in. Table 8 also gives the results of tests made on brick beams built at the same time and in the same way. In No. 1 and No. 3 the courses were not well laid as to joints,

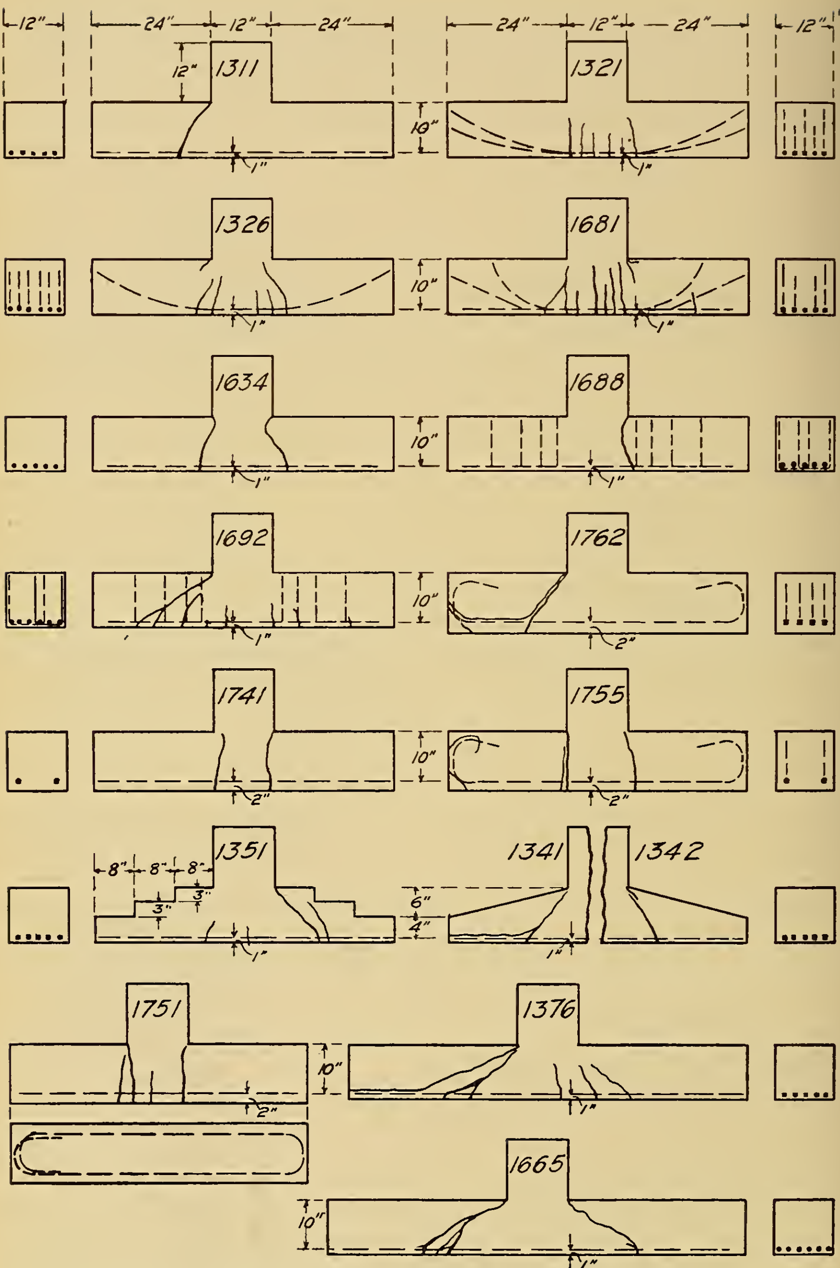

Fig. 14. Reinforced Concrete Wall Footings.

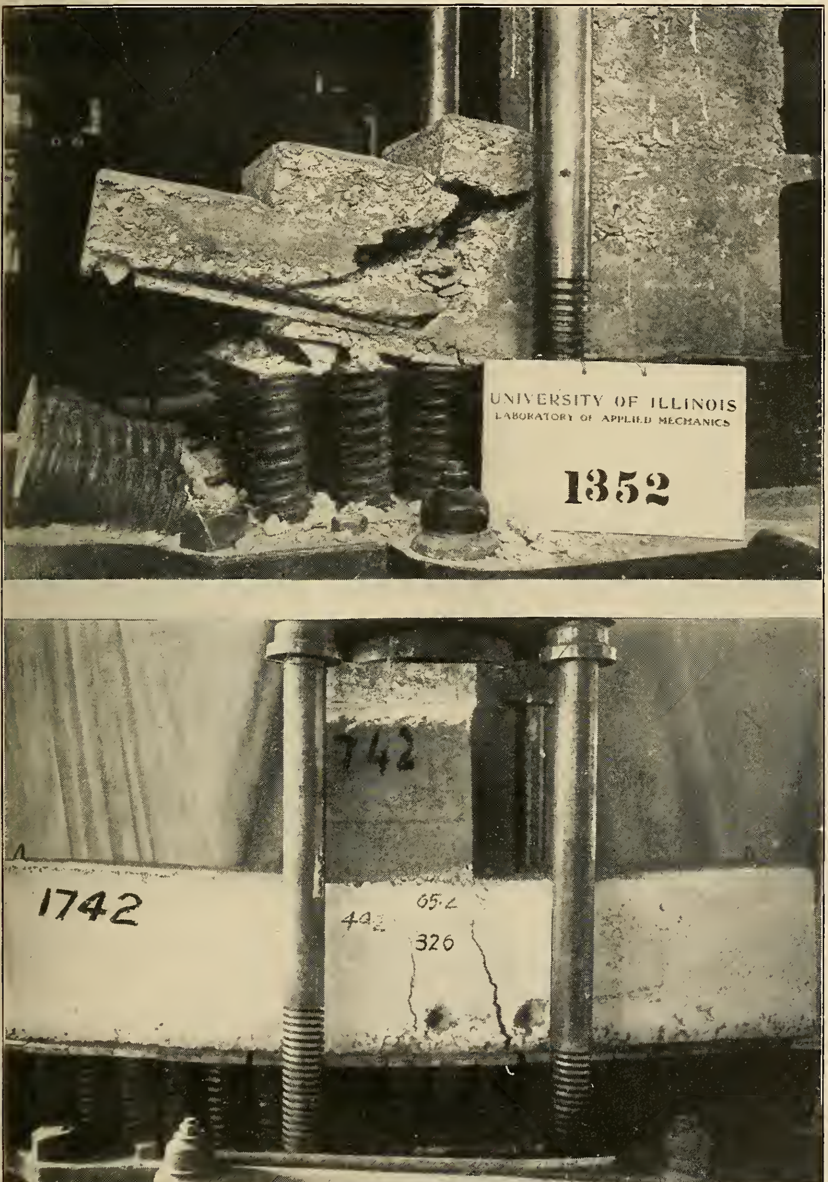

FIG. 15. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

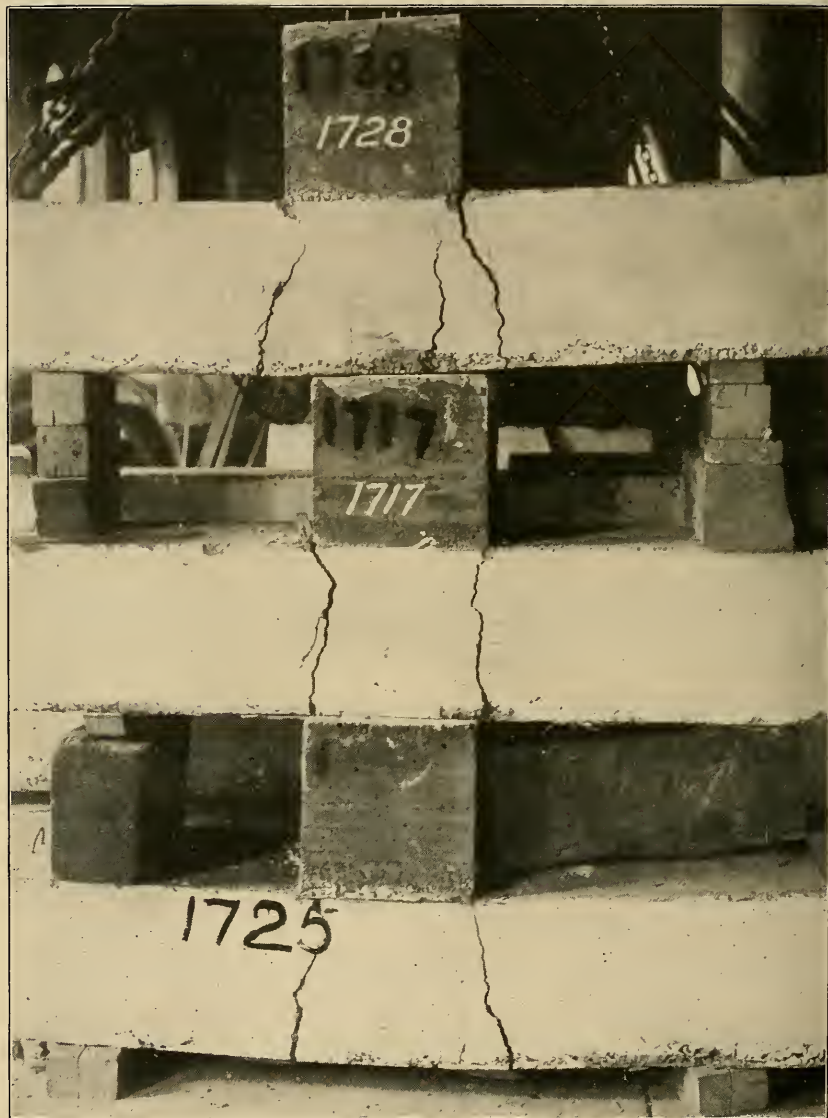

FIG. 16. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

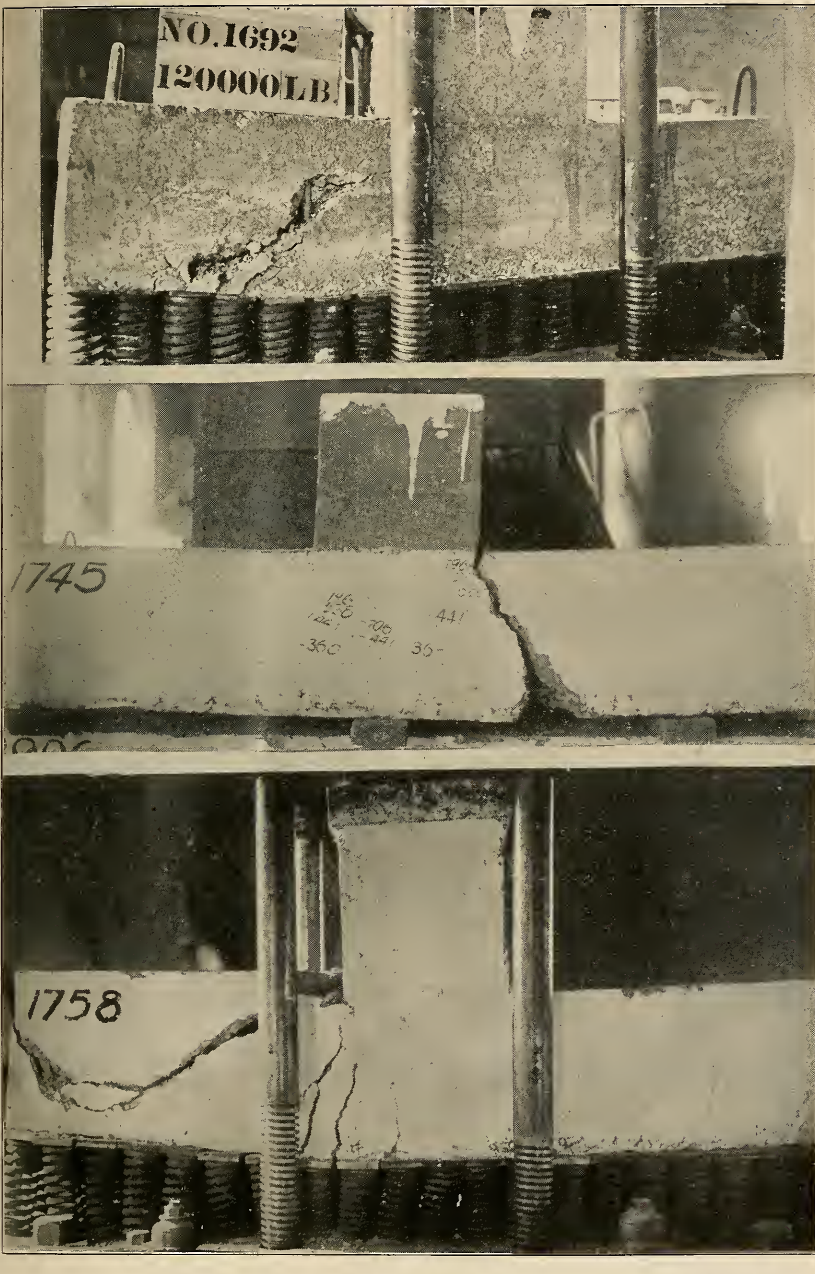

FIG. 17. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

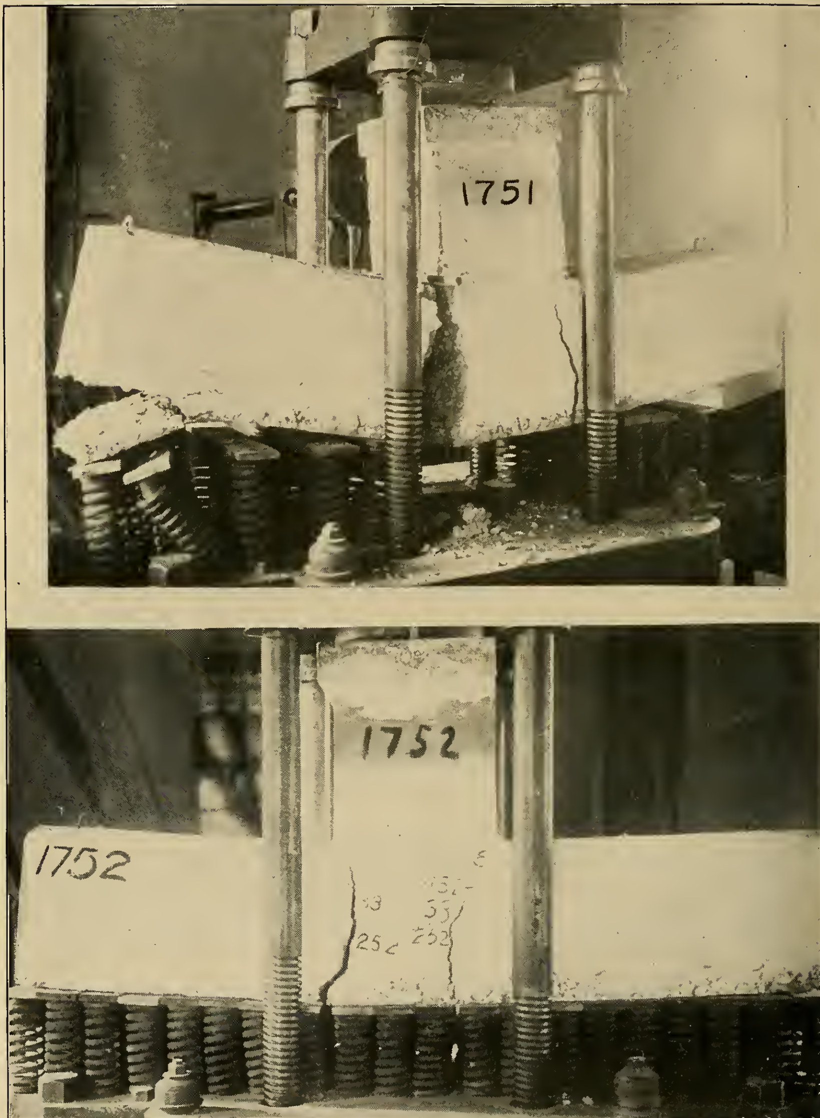

Fig. 18. Views of Reinforced Concrete Wall Footings after Test.

TABLE 8.

Tests of Brick Beams and Footings.

1–3 cement mortar used. Beams tested with a 54-in. span.

Speci- men	Age days	Length feet	Breadth inches	Depth inches	Material	Maxi- mum Applied Load pounds	Modulus of Rup- ture, lb. per sq. in.	Remarks
Beam No. 1	76	5	12.5	13.9	Shale building brick	10 450	235	Courses not carefully laid as regards joints.
Beam No. 2	76	5	12.75	13.6	Shale building brick	20 900	478	Courses carefully laid.
Beam No. 3	76	5	13.0	14.75	Underburned clay brick	2 960	57	Courses not carefully laid.
Beam No. 4	74	5	13.1	14.50	Underburned clay brick	4 310	85	Courses carefully laid.
Footing No. 1	76	5	12.5	19.7	Shale building brick	50 000	292	
Footing No. 2	77	5	12.5	25.1	Shale building brick	75 200	270	
Footing No. 3	74	5	12.5	20.0	Underburned clay brick	14 800	85*	Depth at line of fracture 18 in.
Footing No. 4	76	5	12.5	26.8	Underburned clay brick	21 000	66	

*Calculations made for section at line of fracture.

but in No. 2 and No. 4 more care was given to the position of the cross joints with respect to the adjacent bricks. It will be seen that the modulus of rupture for the brick beams is not far from that for the brick footings, but it is also apparent that the method of laying will greatly affect the strength of the structure. As the modulus of rupture of the shale brick was found to average 1 670 lb. per sq. in., and of the soft clay brick 481 lb. per sq. in. (see page 11, Bulletin No. 27), and the footings broke across the brick, it appears that with the excellent mortar used the strength of the footing is dependent upon the strength of the brick. However, it should be noted that the modulus of rupture of the footings is far below that of the brick, and the strength of the footing is greatly affected by the joints, and probably also by the thickness of the brick.

17. *Phenomena of Tests of Reinforced Concrete Wall Footings.*—In the tests, as the load was increased the springs forming the bed compressed, the amount of compression or shortening being proportional to the load applied. Although the deflection of the ends of the footing modified somewhat the distribution of the load over the bottom of the footing, this deflection was so slight, below what may be called the critical load on the beam, as compared with the total shortening of the springs, that the distribution up to the point of failure was not far from uni-

form. In those cases in which the deflection became considerable, as after the longitudinal reinforcement was stressed well beyond the yield point or the reinforcing bars had slipped considerably, an appreciably smaller load came at the ends than the middle. In some cases before the test was discontinued, the load was finally carried principally by the springs immediately under the wall or stem of the footing, due to the large deflection of the ends or to the closing up of the springs, and the applied load was not representative of the load taken in flexure. The point of failure or critical load was determined from the point of marked change in the deflection curve. By reason of the lack of definiteness of this point, the load at failure is somewhat uncertain in some of the tests, as has already been discussed. In a few of the tests the load seemed not to have been centrally applied on the wall or stem, and the springs were compressed much more at one end of the footing than at the other. As was to be expected, these footings failed at lower total loads than their companion test pieces. In the calculations, for simplicity the loads have been used as symmetrically applied, but it must be understood that in these cases the load on one projection of the footing was larger than the normal proportion of the total load.

As in ordinary beam tests, in the reinforced footings cracks formed in the concrete generally at loads well below the load which produced failure. In some cases these were tension cracks and in other cases diagonal tension cracks, while in some the cracks were evidently caused by slip of bar. The failures were usually slow, especially in the case of the tension failures and in some of the bond failures. With slow failures and in cases where deflection of end of footing became large, the load could finally be applied in an amount considerably above the load which may be considered to be the failure load. In diagonal tension failures, the failure was usually sudden and violent, often part of the footing being thrown off the weighing table of the machine. It will be appreciated that the amount of energy stored in the compressed springs was very large, and the sudden release of this energy resulted in a violent displacement of the specimen.

As high percentages of reinforcement were not used, no failures by compression were found. In two specimens the concrete in the stem or wall proved to be very poor or the wall was poorly made, and the wall failed before the full strength of the footing was developed. These tests have not been included in the tables.

The following are brief notes of the tests. The location of the cracks is shown in Figs. 14 and 19. Heavy lines indicate the crack

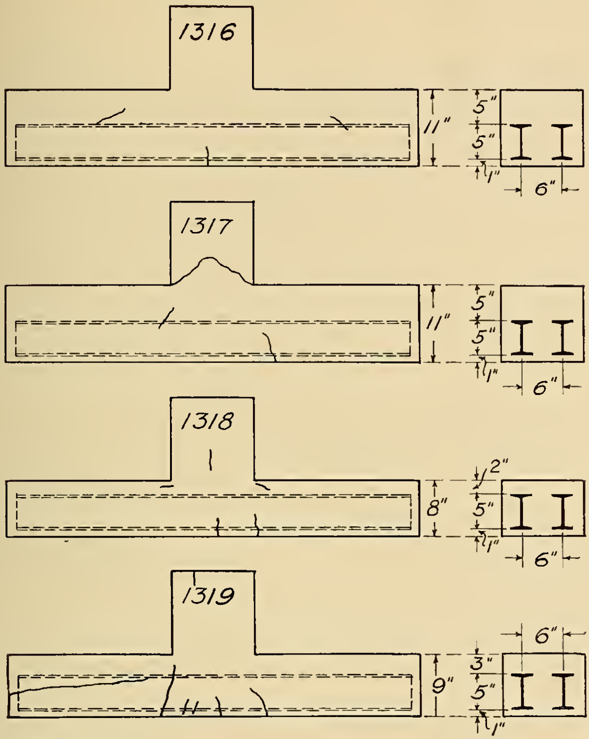

FIG. 19. WALL FOOTINGS REINFORCED WITH I-BEAMS.

along which failure took place. Reference may be made to Tables 8, 9, 10 and 11, page 41 and 55-58.

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No. 1311. At a load of 100 000 lb. springs became closed under the load and load was released. No sign of failure. Reloaded the following day, an additional number of springs being used. Slight cracks (diagonal) noted at load of 115 000 lb. Failed by diagonal tension at a load of 129 000 lb. See Fig. 14.

No. 1312. At load of 60 000 lb. small tension cracks were noted. At 80 000 lb. diagonal tension crack was noted. Failed by diagonal tension at load of 96 000 lb.

No. 1313. Failed at 95 000 lb. by diagonal tension.

No. 1314. At load of 78 500 lb. small tension crack was noted. At 84 000 lb. tension crack noted under left face of wall. Failure occurred by bond at 111 300 lb.

No. 1315. Fine diagonal crack noted at load of 59 000 lb. about 5 in. to the left of the wall. At 65 000 lb. another diagonal crack noted

a little to the right of the wall. Failed by diagonal tension at load of 89 500 lb.

No. 1316. This footing was reinforced with two 5-in. x 9¾-lb. I-beams, 4 ft. 9 in. long, 6 in. apart. Depth over all, 11 in. (See Fig. 19.) Small tension crack observed at load of 115 000 lb. At 130 000 lb. two small diagonal cracks were observed at either end of footing about 12 in. from wall. Footing was loaded to 140 000 lb., but as middle springs were entirely closed before this load was applied less than the normal proportion of load was taken at the ends, and the test was discontinued, although the footing had not failed.

No. 1317. Reinforced with two 5-in. I-beams; arrangement same as in No. 1316. (See Fig. 19.) First crack observed at load of 115 000 lb. Failure occurred by crushing of wall at 134 400 lb. Concrete split lengthwise.

No. 1318. Reinforced with two 5-in. I-beams 6 in. apart. Depth over all 8 in. (See Fig. 19.) Tension crack observed at 105 000 lb. Maximum load 120 000 lb. Failure occurred by tension in steel.

No. 1319. Reinforced with two 5-in. I-beams. (See Fig. 19.) Wall crooked. First crack observed at 117 000 lb. under left face of wall. At 120 000 lb. a longitudinal crack appeared about 6 in. from top of footing at left end and extended toward wall. Maximum load 140 600 lb. Probably failed by flexure of I-beam.

No. 1321. Bars bent up to different heights. Failed by tension in steel at 128 000 lb. (See Fig. 14.) Continued to take load, the ends finally deflecting 0.4 in.

No. 1322. Reinforcement the same as No. 1321. Tension crack observed at load of 74 000 lb. At 80 000 lb. a diagonal crack was noted a little to right of wall. Numerous tension cracks appeared up to the maximum load of 130 000 lb. Tension failure.

No. 1325. Bars bent up to different heights. At 81 000 lb. first crack appeared under right face of wall. Load at failure 100 000 lb. Failure occurred by tension in steel.

No. 1326. Bars bent up to different heights. (See Fig. 14.) At load of 67 000 lb. a tension crack was noted near center and a crack near right edge of wall somewhat inclined to the vertical. At 75 000 lb. another inclined crack was noted to left of wall. At 80 000 lb. the inclined cracks were growing rapidly. At 105 000 lb. tension cracks were noted under right and left faces of wall. Footing failed by tension in steel.

No. 1341. Sloped footing. (See Fig. 14.) At 69 000 lb. a diagonal crack appeared about 8 in. to left of wall. Failed by diagonal tension at 80 000 lb.

No. 1342. Sloped footing. Poor concrete. At 70 000 lb. diagonal crack noted 18 in. from right end. Failure occurred suddenly at 80 300 lb. by diagonal tension.

No. 1351. Stepped footing. (See Fig. 14.) Small tension cracks noted at load of 86 300 lb. At 90 000 lb. a diagonal crack was noted

about 12 in. from right end. Failure occurred suddenly by diagonal tension at 107 300 lb.

No. 1352 Stepped footing. (See Fig. 15). At 80 000 lb. a diagonal crack was noted. At 85 800 lb. a diagonal crack was noted a few inches left of wall. Failure occurred suddenly by diagonal tension at 86 800 lb.

No. 1362. At load of 70 000 lb. small crack was observed a little to right of wall. At 80 000 lb. tension crack noted under left face of wall. At 90 000 lb. a tension crack was noted near right face of wall and steel passed its yield point. Load continued to be taken and final rupture occurred suddenly by diagonal tension at load of 94 100 lb.

No. 1371. Footing 6 ft. 8 in. long. Small diagonal cracks noted near right and left faces of wall at 52 700 lb. Tension crack noted toward the middle of footing at load of 68 000 lb. Failure by tension at 69 300 lb., followed by diagonal tension.

No. 1372. Footing 6 ft. 8 in. long. Small inclined cracks noted near right and left faces of wall at load of 50 200 lb. At 67 000 lb. tension cracks appeared near center of footing. Failure by tension at 67 000 lb., followed by diagonal tension.

No. 1375. Footing 6 ft. 8 in. long, reinforcement, 5 ½-in. corrugated bars. At 60 000 lb. small inclined cracks noted a little to right of wall. At 69 000 lb. another small inclined crack noted near left face of wall. Failure occurred suddenly by diagonal tension at load of 75 500 lb.

No. 1376. Footing 6 ft. 8 in. long, similar to No. 1375. (See Fig. 14.) Diagonal cracks were noted at 40 000 lb. at both ends of footing near wall. At 60 000 lb. a diagonal crack was noted at right extending toward wall. At 67 000 lb. small tension crack near center of footing. Failure occurred by diagonal tension at load of 67 000 lb.

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No. 1631. Low percentage of reinforcement. At a load of 40 000 lb. a crack appeared 6 in. to the left of the wall and extended diagonally upward toward the wall. At this load a crack was also noted at 2½ in. inside the right edge of wall extending vertically 6 in. As the load was increased the cracks grew and at 55 000 lb. the yield point of the steel had been passed. With continued application of the load, the test piece deflected considerably at the ends, allowing much of the load to be taken directly under the wall. Failure by tension.

No. 1632. At 55 000 lb. vertical cracks were noted under the edges of the wall. Tension failure at 60 000 lb.

No. 1633. At 43 000 lb. a vertical crack was noted 2½ in. to left of the wall. Failure occurred slowly by tension and bond between steel and concrete at 78 000 lb.

No. 1634. (See Fig. 14.) At 40 000 lb. cracks were noted 2½ in. from each edge of wall extending almost vertically. Failure occurred slowly by slipping of bars (bond) at load of 73 000 lb.

No. 1635. In this test no cracks were noted before failure, which occurred suddenly at a load of 55 000 lb. by bond.

No. 1636. Diagonal cracks appeared at a load of 80 000 lb. 7 in. on each side of and extending toward wall. The footing failed slowly at load of 89 500 lb., probably by bond, although crack was inclined. Tension failure in steel probably imminent.

No. 1641. At 60 000 lb. a diagonal crack appeared 7 in. to the left of the wall. Failure occurred suddenly at 92 000 lb. by bond, the suddenness of failure possibly being occasioned by diagonal tension weakness. An examination of the end of the specimen showed that the bars had slipped.

No. 1642. At 60 000 lb. a diagonal crack was noted 5 in. to the left of the wall. Failure occurred slowly by bond at 80 000 lb.

No. 1645. At 60 000 lb. a crack was noted 2 in. to the left of the wall and inclined slightly toward it. The footing failed suddenly at 80 000 lb., the end of the footing being thrown off the machine. Failure probably by bond.

No. 1646. At 60 000 lb. a crack appeared on each side of the wall 8 in. from it. Failure occurred at 100 000 lb. by bond.

No. 1651. Failure occurred violently at 80 000 lb. by diagonal tension. The test piece became tipped so that there was more load on the left end than on the right end.

No. 1652. At 60 000 lb. a diagonal crack was noted 6 in. to right of wall. Failure occurred violently at 116 000 lb. by diagonal tension, the right end of the specimen being thrown off the machine.

No. 1655. At 60 000 lb. a diagonal crack was noted 8 in. to the left of the wall. Failure occurred suddenly by diagonal tension at 72 000 lb., the left end of the specimen being thrown off the machine. The specimen tipped so that there was more load on the left end.

No. 1656. At 60 000 lb. a diagonal crack was noted 6 in. to the left of the wall. The specimen failed violently at 108 000 lb. by diagonal tension.

No. 1661. At 80 000 lb. a diagonal crack appeared on each side of the wall and 8 in. from it. At 141 000 lb. failure occurred violently by diagonal tension and stripping of bars.

No. 1662. At 80 000 lb. a diagonal crack was noted 8½ in. to the left of the wall. At 114 000 lb. failure occurred suddenly by diagonal tension. The footing tipped slightly so there was more load on the left end.

No. 1665. Footing 7 ft. long. (See Fig. 14.) At 60 000 lb. cracks were noted on each side of the wall 15 in. from it. Failure occurred suddenly at 84 500 lb. by diagonal tension. There was more load on left end.

No. 1666. Footing 7 ft. long. Two cracks appeared at 55 000 lb., one 8 in. to the left of the wall and the other 2 in. to the right. The specimen failed violently at 94 000 lb. by diagonal tension along crack

which ran from point on bottom 10 in. from wall, the concrete below reinforcement stripping off from bars.

No. 1671. One bar straight, 4 bent up at two points. At 70 000 lb. a vertical crack was noted under left edge of wall. Beyond 80 000 lb. the crack had opened wide and the load was very unevenly distributed because of the large deflection at the ends. At 100 000 lb. the crack was 1/4 in. wide. 80 000 lb. considered as the critical load. Failure probably by tension followed by bond.

No. 1672. One bar straight, 4 bent up at two points. At 60 000 lb. a crack was noted 2 in. to the right of the wall and inclined slightly toward it. Vertical cracks also appeared under the left edge of the wall and 2 in. from it. Beyond 80 000 lb. these cracks opened up and much of the load was transmitted from wall direct to springs below. At 100-000 lb. the cracks extended to the top of the footing and were 1/4 in. wide at the bottom. Failure probably by tension and bond.

No. 1673. One bar straight, 4 bent up at two points. This footing was first loaded up to 80 000 lb. on 3-in. x 12-in. springs placed 3 in. center to center. At this load the springs bent and the specimen was thrown out of position. The first cracks had appeared at 60 000 lb. one on each side of the wall and about 1 in. from it. The springs were reset and a load of 98 000 lb. applied when the footing again swung out of position. The specimen was then placed on the 2 3/4 x 7-in. springs, as in the other tests, and loaded. The notes are indefinite, and the critical load is not known. After being taken from the machine the cracks were 1/4 in wide at the bottom. It seems that failure probably occurred at a load greater than 98 000 lb. by tension or bond.

No. 1674. One bar straight, four bent up at two points. At 60 000 lb. the first crack was noted just under the right edge of the wall and nearly vertical. The diagonal tension crack 8 in. to the left of the wall opened slowly. The footing failed at 99 500 lb., probably by a combination of bond and diagonal tension. Final failure was sudden. Upon examination it was found that the turned-up bars had slipped 1/2 in.

No. 1675. Two bars straight, four bent up at two points. At 80 000 lb. two cracks appeared on right 2 in. and 12 in. from wall. Failure was slow and occurred at 135 000 lb., slipping of the bars permitting diagonal tension crack to be formed. Bars found to have slipped at right end.

No. 1676. Two bars straight, four bent up at two points. At 75 000 lb. the first crack was noted 4 in. to the left of the wall. Failure occurred slowly at 99 000 lb., probably by bond, the crack beginning 7 in. to the left of the wall. The bars were found to have slipped slightly.

No. 1681. Six corrugated bars, two straight, four bent up. (See Fig. 14.) First cracks noted at 80 000 lb. Load was applied up to 125 000 lb., six vertical cracks opening under the wall and one a little to the left of it. As the springs were practically closed the load was removed. The specimen was then tested as a simple beam on supports 4 ft. 4 in. apart.

By this method of loading the footing failed at 60 000 lb., evidently by tension, although in this test one of the inner bars slipped.

No. 1682. Six corrugated bars, two straight, four bent up. At 80 000 lb. the first cracks appeared, one 3 in. inside left edge of wall and one 4 in. outside, extending diagonally toward edge of wall. Beyond 140 000 lb. the action of the springs was untrustworthy as the middle springs had entirely closed. No failure.

No. 1685. Four round stirrups near wall at each end, one stirrup exposed to view on face of footing. At 60 000 lb. a diagonal crack was noted 3½ in. to the right of the wall. Failure occurred slowly at 61 500 lb. by bond.

No. 1686. Four round stirrups near wall at each end. At 60 000 lb. a vertical crack appeared 1 in. inside right edge of wall. Footing failed suddenly at 82 000 lb. by bond. Examination of bars showed that they had slipped ⅜ in.

No. 1687. Four corrugated stirrups near wall at each end. At 40 000 lb. a vertical crack appeared 2 in. to the left of wall. Failure occurred slowly at 80 000 lb. by slip of bars. The inner stirrup also slipped.

No. 1688. Four corrugated stirrups near wall at each end. (See Fig. 14.) At 60 000 lb. a crack appeared 1½ in. to the right of the wall, extending 1 in. inside of wall near top at failure. Failure occurred slowly by bond at 108 000 lb.

No. 1692. Four round stirrups near wall at each end. (See Fig. 14 and 17.) At 61 000 lb. a vertical crack was noted 1 in. to left of wall. The footing failed slowly at 120 000 lb. by diagonal tension.

No. 1693. Four corrugated stirrups near wall at each end. At 80 000 lb. a crack was noted under the left edge of wall, extending inward slightly but almost vertically. Failure occurred suddenly at 106 600 lb. by diagonal tension between two stirrups.

No. 1694. Four corrugated stirrups near wall at each end. Two cracks were noted at 60 000 lb. 2 in. and 5 in. to left of wall and joining 5½ in. above base. Failure occurred suddenly at 113 000 lb. at a crack 9 in. to the left of the wall by diagonal tension. The bars and inner stirrups were found to have slipped.

Series of 1911.

No. 1712. At a load of 78 500 lb. small cracks appeared on both sides of footing near right face of wall extending toward edge of wall. The critical load was 85 000 lb. Failure occurred by tension or bond.

No. 1713. At a load of 40 000 lb. a vertical crack appeared under right face of wall and at a load of 47 000 lb. a similar crack appeared under left face of wall. A small tension crack was noted near center of footing at 65 000 lb. At a load of 71 700 lb. a diagonal crack appeared 16 in. from right end and at a load of 119 800 lb. another diagonal crack appeared 16 in. from left end. Critical load was 85 000

lb., the steel passing its yield point. The load was increased to 125 300 lb. when complete failure occurred suddenly along the diagonal crack which appeared at 119 800 lb. On account of the greater load taken by the springs near the middle after the yield point of the steel was passed, the increase in load does not represent the increase in bending moment.

No. 1714. Deformation in steel measured. At a load of 100 000 lb. cracks under the left and right faces of wall had extended vertically about 10 in. At the critical load of 100 000 lb. the measured stress in the steel was 42 000 lb. per sq. in. Failure was by tension, though under continued loading the bars finally slipped considerably.

No. 1716. 1-1-2 concrete. At a load of 22 800 lb. a crack appeared directly under the right face of wall. At a load of 86 400 lb. a number of fine cracks, which extended diagonally upward to the left, appeared just to the left. The critical load was 90 000 lb. At a load of 106 300 lb. a vertical crack was noted near the center of the footing. The deflection of the springs became very unequal, and at a load of 139 000 lb. the springs at the center of the footing had completely closed.

No. 1717. (See Fig. 16.) 1-1-2 concrete. At a load of 35 600 lb. cracks appeared under right and left faces of wall and 2 in. to right of center of the footing. At a load of 60 100 lb. a diagonal crack was noted 2½ in. to right of wall. Critical load 85 000 lb. At a load of 97 900 lb. the concrete appeared to be failing in compression at the upper surface of footing near wall and the cracks opened appreciably. At a load of 131 400 lb. the springs at the center had completely closed.

No. 1718. 1-1-2 concrete. Deformation in steel measured. At the critical load of 100 000 lb. the measured tension in the steel was 42 000 lb. per sq. in. At a load of 122 800 lb. the steel deformations had exceeded the range of the extensometer. Failure occurred by tension in steel.

No. 1721. Wall made one day later than footing. At a load of 48 100 lb. cracks appeared on both sides of footing about 2 in. inside right face of wall. Critical load was 60 000 lb. At a load of 63 800 lb. cracks appeared 18½ in. from left end and 20 in. from right end. At a load of 72 000 lb. the cracks first noted opened perceptibly.

No. 1722. Wall made one day later than footing. At a load of 34 700 lb. cracks appeared 2 in. and 3½ in. to right of left face of wall. The critical load was 60 000 lb. As load was increased to 70 200 lb. other cracks were noted under both faces of wall. The load then fell off and cracks opened. Failure occurred by tension in steel.

No. 1723. Deformation in steel measured. Wall made one day later than footing. At the critical load of 60 000 lb. the measured stress was above 40 000 lb. per sq. in. Failure occurred slowly by tension in steel.

No. 1724. Wall made one day later than footing. Building paper placed between wall and footing. At a load of 32 300 lb. a vertical crack was noted 4 in. to the left of center of footing. At a load of 40 000 lb. a vertical crack appeared under right face of wall. At a load of 48 000

lb. a crack appeared 21 in. from left end of footing and extended toward the wall. The critical load was 55 000 lb., failure occurring by tension in the steel. At a load of 62 000 lb. the crack under right face of wall opened appreciably.

No. 1725. (See Fig. 16.) Wall made one day later than footing. Building paper placed between wall and footing. At a load of 26 500 lb. the first crack appeared 3 in. inside left face of wall. At a load of 33 700 lb. a vertical crack appeared directly under the right face of wall. At a load of 40 300 lb. a crack was noted 1 in. to left of left face of the wall. Critical load 55 000 lb. At a load of 62 300 lb. the load was being taken slowly and cracks were opening. Failure occurred by tension in the steel.

No. 1726. Deformation in steel measured. At the critical load of 60 000 lb. the measured tension in the steel was 42 000 lb. per sq. in.

No. 1727. At a load of 32 000 lb. cracks appeared under right and left faces of pier. At a load of 40 500 lb. a small crack appeared near center of footing extending vertically 5 in. At a load of 49 500 lb. cracks appeared 20 in. from right end and 17½ in. from left end. The critical load was 55 000 lb. The cracks opened considerably at a load of 62 300 lb. Failure occurred by tension in the steel.

No. 1728. (See Fig. 16.) First crack appeared at a load of 24 000 lb. 4 in. to left of left face of wall and extended vertically. At a load of 30 600 lb. a crack appeared 2 in. to left of left face of wall. At a load of 36 900 lb. a crack appeared 2½ in. to right of wall. The critical load was 55 000 lb.

No. 1729. Deformation in steel measured. At a load of 40 000 lb. a small crack was noted near center of footing. Measurement of the deformation in steel at the critical load of 60 000 lb. indicated that the yield point had been passed.

No. 1731. Reinforced with six ½-in. corrugated square bars bent up in a manner similar to that shown in Fig. 14 for No. 1681. At a load of 35 800 lb. the first crack was noted 18½ in. from the left end of the footing extending upward toward the wall. At a load of 61 600 lb. a crack appeared under the left face of the wall. The critical load was 145 000 lb. Capacity of springs was reached at a load of 155 600 lb. Failure occurred by tension in the steel.

No. 1733. Reinforcement the same as No. 1731. Deformation in steel measured. At a critical load of 160 000 lb. the measured tension in steel was 58 000 lb. per sq. in. At 110 000 lb. load the wall crushed.

No. 1741. (See Fig. 14 and 22.) Deflection of the spings was not measured. Extensometer dials were attached to the reinforcing rods to measure slip. The curves in which slip is plotted against load (Fig. 23) show that at the face of the wall movement of steel relative to concrete began between loads of 6 000 lb. and 20 000 lb. At a load of 35 000 lb. the slip at this position was .001 in. The critical load was 50 000 lb. and at this load a slip of .0001 in. was observed at the end of one bar. The slip at this point progressed with the load, reaching .0007 in.

at a load of 65 000 lb. At a load of 68 900 lb. complete failure occurred by sudden slipping of the bars at the end where slip had previously been observed. If the curve above referred to, of slip at the end, is produced to the load of 68 900 lb. it indicates a slip of about .001 in., which pull-out tests indicate as the critical amount of slip. The last measurement taken (that at 65 000 lb. load) showed that the slip at the ends of all the other bars was very small, not over .0002 in. Failure was primarily by tension in the steel. At a load of 60 000 lb. crushing of concrete was observed at the intersection of the wall and the footing.

No. 1742. (See Fig. 15.) Deformation in steel was measured on one side of footing under face of wall. First crack appeared at a load of 32 600 lb. Critical load 53 000 lb. Measured stress at this load, over the gage length used, was 37 000 lb. per sq. in. Failure by tension.

No. 1743. The measured deformation of the bars shows tension failure. Critical load was 52 000 lb.

No. 1744. Extensometer dials attached to rods to measure the slip. Slip under face of wall began at a load of about 25 000 lb. At the critical load of 70 000 lb. bars at right end had slipped about .004 in. and about 3/8 in. at the maximum load of 84 000 lb. Bond failure.

No. 1745. (See Fig. 17.) At a load of 36 000 lb. first crack was noted 1 in. inside the right face of wall and extended vertically 5 in. At a load of 44 100 lb. cracks appeared at the center of the footing, 19½ in. from the left end and 20 in. from the right end. Critical load 70 000 lb. At a load of 79 600 lb. the cracks opened appreciably and at a load of 98 200 lb. the middle springs had closed. Bond failure.

No. 1746. Deformation in steel measured. At the critical load of 75 000 lb. the measured steel stress, over the gage length used, was 40 000 lb. per sq. in. Failure occurred by tension in steel.

No. 1747. Reinforcement two ½-in. square corrugated bars. At a load of 19 500 lb. first crack was noted 25 in. from right end. At a load of 25 000 lb. cracks appeared under left face of wall. At a load of 55 300 lb. the cracks were widening but not extending very much. Critical load 60 000 lb. At a load of 82 000 lb. the rods had slipped at the left end. Tension failure.

No. 1748. Reinforcement same as 1747. First crack was noted at a load of 21 800 lb. under right face of wall. As the load increased cracks appeared under left face of wall and about 8 in. to the right of the wall. Critical load 50 000 lb. Tension failure.

No. 1749. Reinforcement same as 1747. Critical load 60 000 lb. Tension failure.

No. 1751. (See Fig. 14 and 18.) At a load of 32 600 lb. a vertical crack was noted at the center of the footing. At a load of 40 400 lb. vertical cracks appeared under the right and left faces of the wall. Critical load 50 000 lb. At a load of 57 800 lb. a crack appeared 1½ in. to left of wall, and at this load the cracks opened considerably. Concrete split off bottom of footing at the hooked end of the bars. Tension failure.

No. 1752. (See Fig. 18.) Reinforcement same as in 1751. At a load of 25 200 lb. cracks appeared under the left face of the wall and 9 in. to right of the center of footing. Critical load 46 000 lb. At 50 800 lb. load the cracks opened perceptibly. Bars did not slip at the ends. Tension failure.

No. 1753. Reinforcement same as in 1751. Measured stress in steel at critical load of 58 000 lb. was 42 000 lb. per sq. in. Tension failure.

No. 1754. Reinforcement two ⅝-in. plain round bars curved up at ends to within 2 in. of top and back 10 in. First crack appeared at center of footing at a load of 17 500 lb. At a load of 24 300 lb. cracks appeared 3 in. to left of wall and under the right face of wall. Critical load 40 000 lb. At 40 200 lb. the latter crack opened appreciably and after 47 500 lb. the load was taken on much slower than before. Tension failure.

No. 1755. (See Fig. 14.) Reinforcement same as in No. 1754. Deformation in steel measured. At a load of 29 800 lb. a crack appeared under right face of wall. At a load of 39 800 lb. a crack appeared under left face of wall. At a load of 47 400 lb. the cracks had not opened very much. Critical load 47 500 lb. Measured steel stress 40 000 lb. At a load of 56 300 lb. the cracks opened rapidly. Tension failure.

No. 1756. Reinforcement same as in No. 1754. Critical load was 60 000 lb. Tension failure.

No. 1757. Reinforcement four ⅝-in. round rods curved up at ends to within 2 in. of top and back 10 in. First crack appeared at a load of 42 000 lb. at center of footing. At a load of 50 000 lb. a crack appeared 1½ in. to the left of the wall. At a load of 75 300 lb. a crack was noted 4 in. to right of wall. Critical load 92 000 lb. At 105 000 lb. the cracks opened appreciably and load was taken more slowly. Springs at the center closed at a load of 112 900 lb. Tension failure.

No. 1758. (See Fig. 17.) Reinforcement same as in 1757. First crack appeared at a load of 32 500 lb. under the right face of wall. As the load increased several cracks appeared under the wall and at a load of 82 200 lb. the cracks had not opened much. Critical load 90 000 lb. Load was increased up to 121 000 lb. when the footing failed suddenly by diagonal tension. Initial failure was by tension in steel.

No. 1759. Reinforcement same as in 1757. Deformation in steel measured. Critical load 90 000 lb. Failure occurred by tension followed by diagonal tension.

No. 1761. Reinforcement four ½-in. square corrugated bars curved up at ends to within 2 in. of top and back 10 in. First crack appeared under the right face of wall at a load of 27 700 lb. At a load of 35 000 lb. a crack appeared 2 in. to right of center and at a load of 44 100 lb. a crack appeared under the left face of the wall. At a load of 61 000 lb. a diagonal crack appeared 16 in. from the north end extending upward about 5 in. At a load of 69 500 lb. a fine diagonal crack appeared 7½ in. to left of wall. Critical load was 98 000 lb. At a load of 107 200 lb.

crack near center opened appreciably and at a load of 115 200 lb. failure occurred suddenly by diagonal tension. Initial failure due to tension.

No. 1762. Reinforcement same as in 1761. (See Fig. 14.) First crack appeared at a load of 40 200 lb. at center. At a load of 67 400 lb. a diagonal crack appeared 16 in. from left end of footing and extended toward wall. Critical load was 100 000 lb. At 112 000 lb. the load dropped off but cracks did not open perceptibly. At a load of 123-000 lb. crushing of concrete was observed. At a load of 132 700 lb. failure occurred suddenly by diagonal tension. Initial failure due to tension.

No. 1763. Reinforcement same as in 1761. Deformation of steel measured. Critical load 125 000 lb. Measured stress 65 000 lb. per sq. in. Failure occurred by tension in steel. At a load of 147 000 lb. the concrete split off above the bars.

18. *Reinforced Concrete Wall Footings: Bars Straight.*—In the series of 1908 (see Table 9) ten of the reinforced concrete footings having the longitudinal reinforcement straight throughout the length of the footing gave diagonal tension failures. One gave a bond failure and four failed by tension. In the series of 1909 (see Table 10), there were nine diagonal tension failures, seven bond failures, and two tension failures. In the series of 1911 (see Table 11), there were two bond failures and twenty-two tension failures. In many cases diagonal tension failures occurred at loads which gave high tensile stresses in the reinforcement. With a high percentage of reinforcement, diagonal tension failures were frequent.

19. *Reinforced Concrete Wall Footings: Bars Bent Up.*—The four footings with longitudinal reinforcement bent up (as shown in Fig. 14), of the series of 1908, No. 1325, No. 1326, No. 1321, and No. 1322, failed by tension in the reinforcement at calculated stresses generally somewhat above the yield point of the steel and at very high values of the bond and shearing stresses. There is some uncertainty in the manner of failure in some of the series of 1909, but it seems that all of the failures were by bond, although in No. 1671 and No. 1672 the calculated stress in the steel was above the yield point of the material and the cracks opened up somewhat and although in several cases the failures were complicated by diagonal cracks which ordinarily might be considered to be diagonal tension cracks, No. 1681 and No. 1682, reinforced with deformed bars, did not fail under the highest load applied, and No. 1681 when tested as a simple beam on supports 4 ft. 4 in. apart failed by tension and one of the bars bent up next to the wall was found to have slipped. In the series of 1911, footings No. 1731 and No. 1733 reinforced with deformed bars (two bars straight, two

curved upward, and two bent up, in a manner similar to that shown in Fig. 14 for No. 1681), gave high resistance to diagonal tension and developed the elastic strength of the steel. The other footings of this series having the bars bent up did not have the bending at such a point as to have any material effect upon resistance to diagonal tension.

The effect of bending up bars (anchorage of bars is not referred to here) was to increase the resistance to diagonal tension, higher values of vertical shearing stresses being obtained than in footings of similar reinforcement and proportions in which the bars were laid straight and failure was by diagonal tension. An increase in the tendency to failure by slip of bars was also apparent. The amount of bond stress developed will be discussed under Art. 26, "Bond."

20. *Reinforced Concrete Wall Footings with Stirrups.*—Of the series of 1909, all the footings having stirrups failed by bond or diagonal tension. Not considering the two footings in which the stirrups were exposed to view, the footings having deformed bars gave higher loads and developed higher bond stress, vertical shearing stress, and tensile stress than those having plain bars. The failures of the footings reinforced with plain rods were definitely bond failures, and the footings reinforced with deformed bars gave diagonal tension failures, though in No. 1694 both longitudinal bars and stirrups were found to have slipped. The footings having deformed bar stirrups gave somewhat higher loads than those having stirrups made of plain rods. As the stirrups were made without end anchorage it was expected that slip might occur, the purpose here being (as in former tests) to determine at what loads slipping occurred. It seems to be apparent from these tests that there was concentration of bond stress at the stirrup points and that bond failures are more likely to occur when this web reinforcement is used.

21. *Stepped and Sloped Wall Footings.*—(See Table 9, page 55). All the failures in the stepped and sloped footings were by diagonal tension. Fig. 15 is a view of a stepped footing after failure. The loads at failure were generally less than for the same amount and kind of reinforcement in the other forms of footings, but the calculated vertical shearing stresses in the section a distance *d* from the face of the wall were as large as in the other footings having similar reinforcement. It should be noted that the depth of the footing at the section considered was used in these calculations.

22. *Wall Footings Reinforced with I-beams.*—(See Table 9, page 55, and Fig. 19). The footings reinforced with two 5-in. x 9.75 lb.

TABLE 9.

TESTS OF REINFORCED CONCRETE WALL FOOTINGS. SERIES OF 1908.

All footings of 1–3–6 concrete, hand-mixed. Depth to center of steel 10 in. Length of footing 5 ft. Stresses are given in lb. per sq. in. Chicago AA Portland cement.

Footing No.	Age days	Longitudinal Reinforcement			Critical Load pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance *d*	Nominal Bond Stress	Manner of Failure
		Description	Per cent	Disposition					
1313	61	6 ½-in. round	0.98	Bars straight	95 000	45 500	217	475	Diagonal tension
1315	60	do.	0.98	do.	89 500	42 900	205	447	do.
1362	58	do.	0.98	do.	90 000	43 200	206	449	Tension followed by diag. tension
1371*	61	do.	0.98	do.	69 300	50 000	205	367	do.
1372*	58	do.	0.98	do.	67 000	48 400	197	355	do.
1375*	61	5 ½-in. cor. square	1.04	do.	75 500	60 500	222	378	Diagonal tension
1376*	58	do.	1.04	do.	67 000	53 600	197	335	do.
1311	61	do.	1.04	do.	129 000	58 200	295	607	do.
1312	61	do.	1.04	do.	96 000	43 300	219	451	do.
1314	61	4 ¾-in. round	1.47	do.	111 300	36 700	263	575	Bond
1321	62	5 ½-in. cor. square	1.04	{Bars bent up to different heights}	128 000	57 800	292	602	Tension
1322	57	do.	1.04		130 000	58 700	298	611	"
1325	63	6 ½-in. round	0.98	do.	100 000	48 000	229	500	"
1326	57	do.	0.98	Bars bent up to same height	105 000	50 400	240	525	"
1341†	59	5 ½-in. cor. square	1.04	Bars straight	80 000	36 100	250	376	Diagonal tension
1342†	61	do.	1.04	do.	80 300	36 200	251	378	do.
1351‡	61	do.	1.04	do.	107 300	48 500	362	505	do.
1352‡	61	do.	1.04	do.	86 800	39 200	292	408	do.
1316	60	2 5-in. I-Beams	....	See Fig. 19	130 000	....	...	...	No failure at this load.
1317	61	do.	....	do.	134 000	.....	...	...	Wall crushed
1318	60	do.	....	do.	120 000	.....	...	...	Tension
1319	58	do.	....	do.	140 600	.....	...	...	"

* Length 6 ft. 8 in. † Sloped footing. ‡ Stepped footing.

TABLE 10.

Tests of Reinforced Concrete Wall Footings. Series of 1909.

All footings 5 ft. long and of 1–2½–5 hand-mixed concrete unless otherwise noted. Chicago AA Portland cement. Depth to center of steel 10 in. Stresses are given in lb. per sq. in.

Footing No.	Age days	Longitudinal Reinforcement			Load at Failure pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance d	Nominal Bond Stress	Manner of Failure
		Description	Per cent	Disposition					
1631	63	6 ⅜-in. round	0.55	Bars straight	55 000	46 800	122	355	Tension in steel
1632	64	do.	0.55	do.	60 000	49 900	133	388	do.
1633	63	5 ½-in. round	0.82	do.	78 000	44 400	176	467	Tension and bond
1634	63	do.	0.82	do.	73 000	41 600	165	436	Bond
1635	57	6 ½-in. round	0.98	do.	55 000	26 400	126	274	"
1636	64	do.	0.98	do.	89 500	41 800	204	446	Probably bond
1641	70	5 ⅝-in. round	1.28	do.	92 000	34 200	213	445	Bond and possibly diagonal tension
1642	64	do.	1.28	do.	80 000	29 800	185	387	Bond
1645	59	6 ⅝-in. round	1.53	do.	80 000	25 300	189	318	Probably bond
1646	76	do.	1.53	do.	100 000	31 700	236	412	Bond
1651	65	4 ½-in. cor. square	0.84	do.	80 000	44 700	181	465	Diagonal tension
1652	73	do.	0.84	do.	116 000	64 800	262	675	do.
1655	59	5 ½-in. cor. square	1.04	do.	72 000	31 800	165	339	do.
1656	67	do.	1.04	do.	108 000	47 800	247	509	do.
1661	70	6 ½-in. cor. square	1.25	do.	141 000	53 600	327	560	do.
1662	69	do.	1.25	do.	114 000	43 400	264	453	do.
1665*	62	do.	1.25	do.	84 500	51 800	260	360	do.
1666*	58	6 $\frac{9}{16}$-in. cor. round	1.24	do.	94 000	57 900	289	453	do.
1671	76	5 ½-in. round	0.82	1 bar straight, 4 bars bent up at different points	80 000	45 500	181	474	Probably tension
1672	66	do.	0.82		80 000	45 500	181	474	do.

*Length 7 ft.

TABLE 10 (Continued).

Footing No.	Age days	Longitudinal Reinforcement			Load at Failure pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance d	Nominal Bond Stress	Manner of Failure
		Description	Per cent	Disposition					
1673	60	5 ⅝-in. round	1.28	1 bar straight, 4 bars bent up at different points	98 000‡	36 400	227	475	Uncertain
1674	60	do.	1.28		99 500	37 000	230	482	Bond and diag. tension
1675	69	6 ⅝-in. round	1.53	2 bars straight, 4 bars bent up at different points	135 000	42 600	318	555	Bond
1676	58	do.	1.53		99 000	31 200	234	400	"
1681	60	6 ½-in. cor. square	1.25	do.	125 000	47 600	289	496	Did not fail at this load; see notes of test.
1682	73	do.	1.25	do.	140 000	53 300	324	555	do.
1685§	61	5 ⅝-in. round	1.25	Bars straight	61 500	22 900	142	298	Bond
1686§	67	do.	1.25	do.	82 000	30 500	190	396	"
1687†	64	5 ⅝-in. round	1.28	Bars straight	80 000	29 800	185	387	Bond, inner stirrup also slipped
1688†	69	do.	1.28	do.	108 000	40 200	250	522	Bond
1692§	60	6 $\frac{9}{16}$-in. cor. round	1.24	do.	120 000	46 000	278	540	Diagonal tension
1693†	61	6 ½-in. cor. square	1.25	do.	106 600	40 800	247	423	do.
1694†	64	do.	1.25	do.	113 000	43 000	262	449	do.

§ ½-in. round stirrups. † ½-in. sq. cor. stirrups. ‡ Critical load may be beyond 98 000 lb.

TABLE 11.

Tests of Reinforced Concrete Wall Footings. Series of 1911.

All footings of 1–2–4, hand-mixed concrete unless otherwise noted. Depth to center of steel 10 in. Length of footing 5 ft. Stresses given in lb. per sq. in.

Footing No.	Age days	Cement	Longitudinal Reinforcement			Critical Load pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance *d*	Nominal Bond Stress	Manner of Failure
			Description	Per cent	Disposition					
1712	73	U	6 ½-in. round	0.98	Bars straight	85 000	40 200	192	418	Tension or bond
1713	98	U	do.	0.98	do.	85 000	40 200	192	418	Tension
1714	339	L	do.	0.98	do.	100 000	47 200	226	493	Tension
1716*	72	U	6 ½-in. round	0.98	Bars straight	90 000	42 500	203	443	Tension
1717*	94	U	do.	0.98	do.	85 000	40 200	192	418	"
1718*	338	L	do.	0.98	do.	100 000	47 200	226	493	"
1721†	64	U	6 ⅜-in. round	0.55	Bars straight	60 000	48 800	131	382	Tension
1722†	99	U	do.	0.55	do.	60 000	48 800	131	382	"
1723†	347	L	do.	0.55	do.	60 000	48 800	131	382	"
1724‡	67	U	6 ⅜-in. round	0.55	Bars straight	55 000	44 800	120	350	Tension
1725‡	88	U	do.	0.55	do.	55 000	44 800	120	350	"
1726‡	345	L	do.	0.55	do.	60 000	48 800	131	382	"
1727	66	U	6 ⅜-in. round	0.55	Bars straight	55 000	44 800	120	350	Tension
1728	88	U	do.	0.55	do.	55 000	44 800	120	350	"
1729	345	L	do.	0.55	do.	60 000	48 800	131	382	"
1731	83	U	6 ½-in. cor. sq.	1.25	Two bars straight, two curved, and two bent up.	145 000	54 500	331	568	Tension
1733	359	L	do.	1.25		160 000	60 300	365	628	"
1741	61	U	2 ⅝-in. round	0.52	Bars straight	50 000	43 800	109	571	Tension followed by bond
1742	101	U	do.	0.52	do.	53 000	46 400	116	605	Tension
1743	346	L	do.	0.52	do.	52 000	45 500	114	595	"
1744	73	U	3 ⅝-in. round	0.77	Bars straight	70 000	41 800	156	544	Bond
1745	73	U	do.	0.77	do.	70 000	41 800	156	544	"
1746	358	L	do.	0.77	do.	75 000	44 800	167	584	Tension

*1–1–2 concrete. †Wall made one day after footing. ‡Wall made one day after footing, building paper placed between wall and footing.

TABLE 11 (Continued).

Footing No.	Age days	Cement	Longitudinal Reinforcement: Description	Longitudinal Reinforcement: Per cent	Longitudinal Reinforcement: Disposition	Critical Load pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance d	Nominal Bond Stress	Manner of Failure
1747	65	U	2 ½-in. cor. sq.	0.42	Bars straight	60 000	64 000	129	666	Tension
1748	73	U	do.	0.42	do.	50 000	53 300	108	555	"
1749	359	L	do.	0.42	do.	60 000	64 000	129	666	"
1751	77	U	2 ⅝-in. round	0.52	Reinforcement of one horizontal bar looped at one end and bent in at other	50 000	43 800	109	571	Tension
1752	69	U	do.	0.52		46 000	40 400	100	525	"
1753	336	L	do.	0.52		58 000	50 600	127	663	"
1754	67	U	2 ⅝-in. round	0.52	Curved up to within 2 in. of top and back 10 in.	40 000	35 000	88	457	Tension
1755	70	U	do.	0.52		47 500	41 700	104	542	"
1756	340	L	do.	0.52		60 000	52 300	131	685	"
1757	69	U	4 ⅝-in. round	1.04	Curved up to within 2 in. of top and back 10 in.	92 000	41 800	208	544	Tension
1758	76	U	do.	1.04		90 000	40 750	204	535	"
1759	354	L	do.	1.04		90 000	40 750	204	535	"
1761	74	U	4 ½-in. cor. sq.	0.85	Curved up to within 2 in. of top and back 10 in.	98 000	54 000	219	562	Tension
1762	76	U	do.	0.85		100 000	55 100	224	574	"
1763	353	L	do.	0.85		125 000	68 800	279	720	"

I-beams carried high loads. One did not fail under a load of 130 000 lb., and in another the stem crushed under a load of 134 000 lb. without sign of failure in the footing. In No. 1318 the failure was a tension failure in the lower flange of the I-beams and in No. 1319 the failure was coincidently by tension of flange of I-beam and by bond, the steel slipping or splitting from the concrete. The calculated tensile stress in the steel at a section at the face of the wall using the lower flange of the I-beam as the tension area of the steel and considering the combination to act as a reinforced concrete beam, was somewhat higher than the yield point of mild steel. The total vertical shear was very high. The loads carried by these footings were among the highest in the experimental wall footing tests. Of course, the amount of steel in the I-beams was much larger than in the footings reinforced with longitudinal rods. The load carried was about double what would be carried by counting I-beams alone to take the full bending moment at a section at the face of the wall, using 35 000 lb. per sq. in. as the value of the modulus of rupture of the I-beams.

23. *Effect of Pouring Wall Separately from Footing.*—In construction it is generally necessary from the standpoint of convenience to pour the wall after the footing has taken its set. To determine whether this method of construction has an effect upon the choice of section which should be taken as the critical section in design, in a number of cases the wall or stem of the footing was built 24 hours after the footing had been finished. In three footings, No. 1724, 1725, and 1726, a layer of building paper was placed over the footing and the wall was constructed upon this. Two pieces of wire 0.1 in. in diameter passed from footing to wall to resist breakage in handling. The conditions were such as to make the bond very slight. Three footings, No. 1727, 1728, and 1729, were constructed monolithically under otherwise similar conditions. There was no marked difference in the loads carried, the method of failure, or the phenomena of tests for footings constructed under these different conditions, all giving tension failures at the face of the wall. By calculation the horizontal shearing stress at the face of the wall may be shown to be less than the probable coefficient of friction. These tests corroborate the view that the critical section for design and calculation may properly be taken at the face of the wall.

24. *Tension Failures and Tensile Stresses.*—In the footings which gave tension failures, the vertical cracks which had formed enlarged, similarly to the action in ordinary beam tests, and at the critical load the cracks opened and a marked increase in the end deflection occurred.

Beyond the critical load the footing usually took an increase of load, the ends bending up so that the distribution was no longer uniform, and generally the ultimate failure was slow. The failure crack was at or near the section at the face of the wall. The stress in the steel for this section, as calculated by the method given on page 8, was in general somewhat larger than the yield point of the steel determined by tests on coupons taken from the same bars.

The calculated value of the tensile stress in the reinforcement for the beams tested at an age of nearly a year is in some cases considerably higher than the yield point of the steel and higher than the calculated stress in the companion test pieces which were tested at an age of about 60 days and which are given as failing by tension. Part of the difference may be due to the use of the same values of jd in the older beams as in the others. As has already been stated, it was in many cases difficult to determine the manner of failure, as the phenomena of tension failure and bond failure have points in common, and it is possible that some of the footings reported as failing by tension in reality failed by bond.

In a number of footings of the series of 1911 measurements of the deformation of the steel were made by inserting an extensometer of the Berry type in gage holes drilled in the reinforcing bars at the side of the footing. Fig. 20 gives the results of some of the measurements, the deformations being translated into equivalent stresses. Generally, one gage line (usually 6 in. in length) was placed so that it was bisected by the plane of the face of the wall (marked B in the figure). Gage line A (when used) was bisected by the center line of the footing, gage line C (when used) was adjacent to B and nearer the end of the footing. As the stress varies from point to point, the instrument reading will give the average stress over the gage length and not the maximum stress. Especially may the average stress over gage line B be less than the stress at the section at the face of the pier. The measured deformation at the center gage line was found to be generally somewhat greater than that on gage line B. Evidently little bond stress is developed over the thickness of the wall. The amount of the measured stress was generally lower than the calculated values given in the tables, but the difference perhaps was not more than that due to the effect of the smaller deformation toward the outer gage point and the greater stiffness found in the older footings.

To determine whether the reinforcing bars had been stressed beyond their yield point, several test pieces were afterward broken up and the

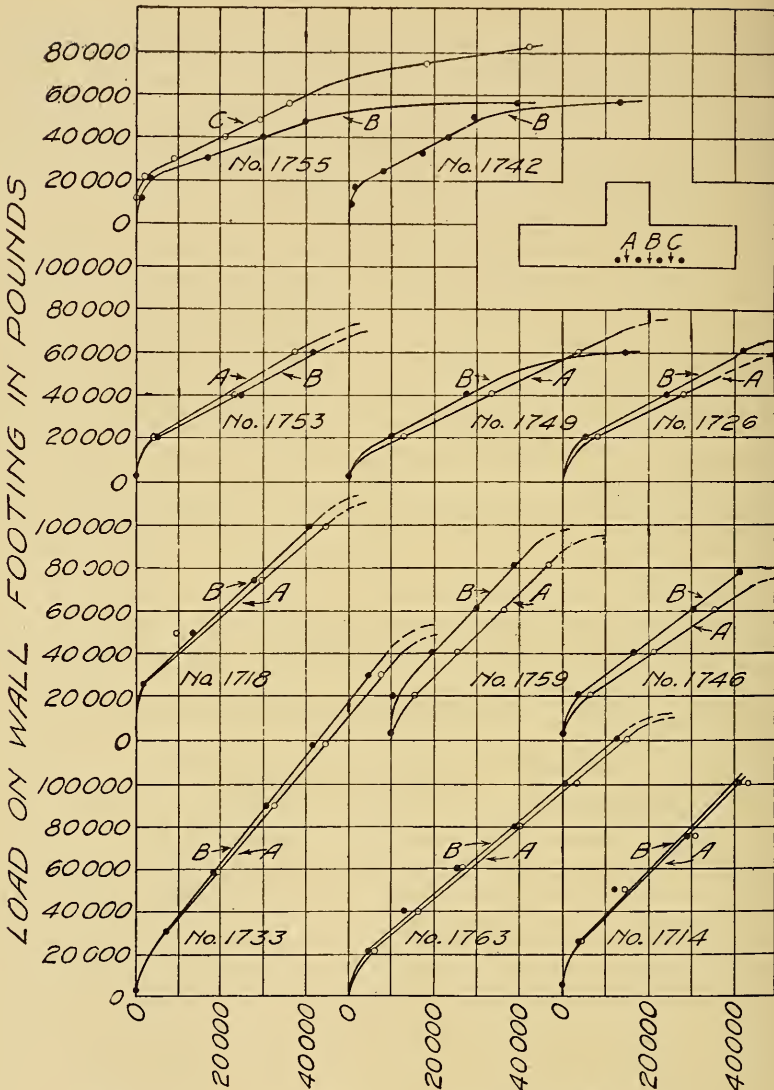


Fig. 20. Diagram of Observed Stresses in Reinforcing Bars.

bars examined and calipered. In Fig. 21 the diameter of bars at various sections along their length is plotted. Although there is always considerable variation in the original diameters of such bars, these measurements were useful in helping to determine the method of failure.

The fact that the tension failures occurred at the section at the face of the wall, together with the approximate agreement of the calculated stress with the observed deformation and with the yield point of the material, justifies the use of the section at the face of the wall as the critical section in calculations of bending moment and of tensile stress in the reinforcing bars. This is apparent with footings of different richness of concrete, different percentages of reinforcement, and different grades of steel.

25. *Vertical Shearing Stresses and Diagonal Tension Failures.*—As was noted under Article 4, "General Theory," the diagonal tension stresses developed in reinforced concrete beams may be expected to be roughly proportional to the vertical shearing stresses, though the diagonal tension may vary from one to two times the vertical shearing stress. As was stated on page 9, the value of the vertical shearing stress has come to be used as a convenient means of measuring the resistance to diagonal tension, although, of course, it is not the numerical equivalent of the stress. There is, however, some question as to what section should be taken as the critical section in short cantilever

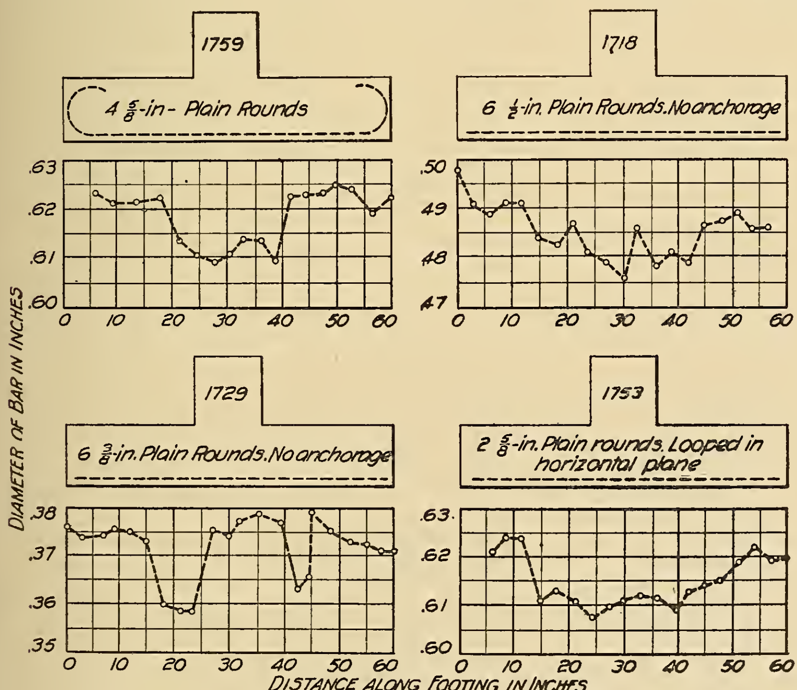


FIG. 21. DIAGRAM SHOWING DIAMETER ALONG BARS AFTER TEST OF FOOTING.

TABLE 12.

Values of Vertical Shearing Stress.

Footing No.	Reinforcement: Disposition	Reinforcement: Per cent	Nominal Vertical Shearing Stress lb. per sq. in.: At face of pier	Nominal Vertical Shearing Stress lb. per sq. in.: Distance d from face of pier	Manner of Failure
1311	Cor. bars straight	1.04	506	295	Diagonal tension
1312	do.	1.04	376	219	do.
1313	Round bars straight	0.98	372	217	do.
1315	do.	0.98	351	205	do.
1371*	do.	0.98	289	205	Tension followed by diagonal tension.
1372*	do.	0.98	279	197	do.
1375*	Cor. bars straight	1.04	314	222	Diagonal tension
1376*	do.	1.04	279	197	do.
1651	do.	0.84	310	181	do.
1652	do.	0.84	450	262	do.
1655	do.	1.04	282	165	do.
1656	do.	1.04	424	247	do.
1661	do.	1.25	560	327	do.
1662	do.	1.25	452	264	do.
1665§	do.	1.25	359	260	do.
1666§	do.	1.25	400	289	do.
1314	Round bars straight	1.47	450	263	Bond
1634	do.	0.82	283	165	"
1635	do.	0.98	216	126	"
1636	do.	0.98	351	204	Probably bond
1641	do.	1.28	365	213	Bond and possibly diagonal tension.
1642	do.	1.28	317	185	Bond
1645	do.	1.53	323	189	Probably bond
1646	do.	1.53	404	236	Probably diagonal tension and bond
1341	Cor. bars straight	1.04	314	244	(Sloped footing) Diag. tens.
1342	do.	1.04	315	245	do.
1351	do.	1.04	421	351	(Stepped footing) Diag. tens.
1352	do.	1.04	340	284	do.
1673	Round bars, 1 straight, 4 bent up at different points	1.28	388	227	Uncertain
1674	do.	1.28	394	230	Bond and diagonal tension
1675	Round bars, 2 straight, 4 bent up at different points	1.53	545	318	Bond
1676	do.	1.53	400	234	"
1685	Round bars straight with plain round stirrups	1.25	244	142	"
1686		1.25	325	190	"
1687	Round bars straight with cor. stirrups	1.28	317	185	Bond
1688		1.28	429	250	"
1692	Cor. bars straight with plain round stirrups	1.24	476	278	Diagonal tension
1693	Cor. bars straight with cor. stirrups	1.25	423	247	do.
1694		1.25	449	262	do.
1712	Round bars straight	0.98	329	192	Tension and bond
1713	do.	0.98	329	192	Tension
1714	do.	0.98	387	226	Tension
1716	do.	0.98	348	203	Tension
1717	do.	0.98	329	192	"
1718	do.	0.98	387	226	"

* Length 6 ft. 8 in. § Length 7 ft.

TABLE 12 (CONTINUED).

Footing No.	Reinforcement: Disposition	Reinforcement: Per cent	Nominal Vertical Shearing Stress lb. per sq. in.: At face of pier	Nominal Vertical Shearing Stress lb. per sq. in.: Distance d from face of pier	Manner of Failure
1731	Cor. bars, 4 bent up at different points	1.25	567	331	Tension
1733		1.25	628	365	"
1757	Round bars curved up at ends	1.04	356	208	Tension
1758		1.04	349	204	Tension
1759	do.	1.04	349	204	Tension
1761	Cor. bars curved up at ends	0.85	374	219	Tension
1762		0.85	382	224	"
1763	do.	0.85	480	279	Tension

beams supported and loaded as were these footings. It has been stated on page 10 that the application of the load on the wall (Fig. 1, page 8) and the uniform support of the footing along its bed may be expected to cause a different distribution of shear throughout the vertical section at the face of the wall than is usually assumed in normal beam action, and it seems probable that the vertical shear is more largely taken by the compression area and that relatively less of it is borne in this section below the neutral axis. As a result, the diagonal tension may be expected to be less at this section than would be the case with normal beam action. Tests of beams show that diagonal tension failures start at the reinforcing bars some distance from the support, even if the total vertical shear is much greater at the support. It may then be expected that the critical section for diagonal tension will be some distance from the face of the wall. In the tests of wall footings the diagonal crack is generally formed at the level of the reinforcing bars at a point distant from the face of the wall about equal to the vertical distance d from surface of footing to center of reinforcing bar. Without knowing exactly what point to select we may use a section through this point tentatively as the critical section (that is, a vertical section at a distance from the face of the wall equal to the depth of the footing down to the center of the steel) and compare the vertical shearing stresses obtained in footings having a variety of proportions of depth to length.

In Table 12 are given calculated values of the vertical shear for a section at the face of the wall and for a section distant d from it for those footings in which diagonal tension failures were found, and also for others in which high vertical shearing stresses were developed. It will be seen that the values of the vertical shear at the new section are much more consistent among themselves in the diagonal tension failures

than is the case with the section next to the face of the wall. The values of the shearing stress for this section are generally greater in the footings which failed by diagonal tension than the values found for other forms of failure. The fact that in the tests diagonal cracks generally formed first at a point somewhat near this section also favors its use. It would seem to be safe practice, where the vertical shearing stress is to be used as the measure of the resistance of the footing to diagonal tension failure, to consider the section distant d from the face of the wall as the critical section.

It should be noted that the values of the vertical shearing stress at this so-called critical section are larger than those which have been found in beam tests. This, probably, is due partly to the fact that short beams give higher resistance to diagonal tension (possibly on account of less deflection and on account of less frequent tension cracks in the concrete), as has been shown in Bulletin No. 29, and possibly partly to not taking the critical section far enough from the face of the wall.

26. *Bond Stresses.*—The analysis given in Article 5, "Analysis of Wall Footings," indicates that in wall footings with a uniformly distributed load the bond stress is greatest at the face of the wall and decreases uniformly to the end of the projection, if ordinary beam action is to govern. The distortion of the concrete at the wall, necessary to develop the tensile stress in the bar at this point, or a slip of the bar to produce the same effect, and the formation of tension cracks in the concrete (which take the place of much of the general deformation of the concrete), and other considerations which detract from true beam action, lead us to expect that equation (17) may not express the actual bond stress developed. It is even possible in the case of short bars that after slip of bar begins at the face of the wall the bond stress may for a time be fairly uniform along the bar and thus its intensity at the face of the wall be, say, only half of that given by this equation. However, for simplicity and because slip is very undesirable, the bond stress u given in the tables has been calculated on the basis of equation (17) for a section at the face of the wall. Although the ordinary analysis does not hold for stepped and sloped footings nor where the longitudinal reinforcing bars are bent up at the ends, for the sake of comparison equation (17) has been used for these also. It is realized that the bond stress so calculated will not be the true bond stress.

In failures by bond a crack, vertical or nearly vertical, formed at a section near the face of the wall and opened somewhat as the test progressed. Generally only one crack of this kind formed, though some-

times one formed at the second face. If the bar slipped at its end the crack opened widely. In some cases the bar slipped ¼ in. or more. In all the cases in which instruments were used for detecting the movements of the bar, motion was detected first at a section at the face of the wall and later at the end of the bar. The load for which at a section at the face of a wall a movement of the bar with respect to the concrete was first detected, corresponded with the load at which the concrete would be expected to fail in tension; and it seems that this early slip is intimately connected with the formation of tension cracks in the concrete and that it is more or less local. The amount of the movement was affected by the position of the crack with reference to the location of the instrument. The development of local slip will affect the distribution of the tensile stress in the bar and also will increase the bond stress at other points. In some cases the passing of the yield point of the steel and a considerable slip of the bar came at loads close together and it was difficult to tell which developed first. In other cases the cause of failure is uncertain, but the statements given in the tables were decided upon from a study of the notes of the tests, the position and growth of the cracks, the instrument readings when taken, and an examination of the test piece after failure including the calipering of the bars.

Calculated values of bond stresses are given in Tables 9, 10, and 11. Table 13 repeats these values for footings failing by bond or developing high bond stresses. It will be seen that in footings having ⅜-in. bars failure by slip did not occur. There were a few bond failures with ½-in. bars, and there were a number of bond failures with ⅝-in. bars. In the footings with plain straight rods it will be noted that the values of the bond stress, as calculated by the method used, range somewhat higher than the values of bond resistance which have been found in bond tests of plain rods. In footings with straight corrugated bars, like No. 1747, 1748, and 1749, high bond stresses were developed, and no failure of a footing reinforced in this manner is attributable to bond, though in No. 1694, at the end of the test, bars were found to have slipped. In some cases the concrete in front of the corrugations was found to have the appearance of being powdered, and slight movements were detected.

In three of the footings, measurement of slip of bars was made at different points along the length of the bar. A graduated dial carrying a pointer was attached to a reinforcing bar. A silk-covered wire, weighted at its free end to keep it taut, was wrapped around the shaft which carried the pointer and attached at its other end to the concrete

TABLE 13.

Values of Bond Stress.

Footing No.	Reinforcement: Disposition	Reinforcement: Per cent	Nominal Bond Stress lb. per sq. in.	Manner of Failure
1314	Round bars, straight	1.47	575	Bond
1633	do.	0.82	467	Tension and bond
1634	do.	0.82	436	Bond
1635	do.	0.98	274	"
1636	do.	0.98	446	Probably bond
1641	do.	1.28	445	Bond and possibly diagonal tension
1642	do.	1.28	387	Bond
1645	do.	1.53	318	Probably bond
1646	do.	1.53	412	Bond
1311	Cor. bars, straight	1.04	607	Diagonal tension
1312	do.	1.04	451	do.
1313	Round bars, straight	0.98	475	do.
1315	do.	0.98	447	do.
1371*	do.	0.98	367	Tension followed by diag. tension
1372*	do.	0.98	355	do.
1375*	Cor. bars, straight	1.04	378	Diagonal tension
1376*	do.	1.04	335	do.
1651	do.	0.84	465	do.
1652	do.	0.84	675	do.
1655	do.	1.04	339	do.
1656	do.	1.04	509	do.
1661	do.	1.25	560	do.
1662	do.	1.25	453	do.
1665§	do.	1.25	360	do.
1666§	do.	1.25	453	do.
1362	Round bars straight	0.98	449	Tension followed by diag. tension
1631	do.	0.55	355	Tension
1632	do.	0.55	388	Tension
1673	Round bars, 1 straight, 4 bent up two points	1.28	475	Uncertain
1674		1.28	482	Bond and diagonal tension
1675	do.	1.53	555	Bond
1676	do.	1.53	406	"
1321	Cor. bars, bent up at two points	1.04	602	Tension
1322		1.04	611	"
1325	Round bars bent up at two points.	0.98	500	"
1326	Round bars bent up at one point.	0.98	525	"
1671	Round bars, one straight 4 bent up at two points	0.82	474	Probably tension
1672		0.82	474	do.
1681	Cor. bars, two straight, 4 bent up at two points	1.25	496	No failure at maximum applied load. See notes of test.
1682		1.25	555	
1341	Cor. bars straight	1.04	376	(Sloped footing) Diagonal tension
1342	do.	1.04	378	do.
1351	do.	1.04	505	(Stepped footing) Diagonal tension
1352	do.	1.04	408	do.
1685	Round bars, straight with plain round stirrups	1.25	298	Bond
1686		1.25	396	"
1687	Round bars, straight with cor. stirrups	1.28	387	"
1688		1.28	522	"
1692	Cor. bars, straight with cor. stirrups	1.24	540	Diagonal tension
1693		1.25	423	do.
1694	do.	1.25	449	do.
1712	Round bars straight	0.98	418	Tension and bond

*Length 6 ft. 8 in. §Length 7 ft.

TABLE 13 (Continued).

Footing No.	Reinforcement: Disposition	Reinforcement: Per cent	Nominal Bond Stress lb. per sq. in.	Manner of Failure
1714	Round bars straight	0.98	493	Tension
1716	do.	0.98	443	Tension
1718	do.	0.98	493	"
1731	Cor. bars, 2 straight and 4 bent at two points	1.25	568	"
1733		1.25	628	"
1741	Round bars straight	0.52	571	Tension followed by bond
1742	do.	0.52	605	Tension
1743	do.	0.52	595	"
1744	do.	0.77	544	Bond
1745	do.	0.77	544	"
1746	do.	0.77	584	Tension
1747	Cor. bars straight	0.42	666	Tension
1748	do.	0.42	555	"
1749	do.	0.42	666	"
1751	Round bars looped at ends	0.52	571	"
1752	do.	0.52	525	"
1753	do.	0.52	663	"
1754	Round bars curved up and back at ends	0.52	457	Tension
1755		0.52	542	"
1756	do.	0.52	685	"
1757	do.	1.04	544	"
1758	do.	1.04	535	"
1759	do.	1.04	535	"
1761	Cor. bars, curved up and back at ends	0.85	562	Tension
1762		0.85	574	"
1763	do.	0.85	720	"

adjacent to the bar. The slip measuring apparatus is shown in Fig. 22. Fig. 23 gives the results for No. 1741 and No. 1744, the location of the points of measurement being shown in the plan at the right of the figure. Distance to the right of the zero line represents movement of the bar toward the center of the footing relatively to the concrete. In No. 1741 the measured movement at the face of the wall for a load of 35 000 lb. was 0.001 in. At a load of 65 000 lb. the end slip was nearly 0.001 in. At a load of 68 900 lb. complete failure occurred by sudden slipping of the bars at the end where slip had previously been observed, but there were indications of previous critical failure by tension in the steel. In No. 1744 at a load of 70 000 lb. the ends of the bars had slipped about 0.004 in., and this slip rapidly increased under a slightly larger load. The measurements go to show that there is a complicated relation between the slip of bars and the formation of tension cracks in the concrete. Tests have since been made to determine the relation between bond and slip movement, and the subject will be more fully treated in a forthcoming bulletin.

Of the footings with anchored bars, attention may be called to the

FIG. 22. INSTRUMENTS IN PLACE TO MEASURE SLIP OF BARS.

ones with horizontally looped bars, No. 1751, 1752, and 1753, which developed high strength and gave tension failures. The footings having bars bent or hooked back in a long curved bend, No. 1754, 1755, 1756, 1757, 1758, and 1759, did not show failure in bond, but for some reason No. 1754 did not carry a high load.

B. COLUMN FOOTINGS

27. *Tables.*—Table 14 gives data of the unreinforced concrete column footings, results of the tests, and calculated values of the modulus of rupture. Tables 15, 16, 17, and 18 give data of the reinforcement of the column footings for the series of 1909, 1910, 1911 and 1912, the results of the tests and the calculated stresses. The stresses were calculated by the methods outlined on pages 20 to 24. The values of j used are those given in Table 6. In the calculation of tensile stress and bond stress in footings of 10-in. depth, except where otherwise noted, the area of steel in an equivalent beam width of $\frac{46}{60}$ of the width of footing was used.

28. *Unreinforced Concrete Column Footings.*—The concrete footings without reinforcement generally failed suddenly and without warning at the maximum load applied. In some, as in No. 1506, the maxi-

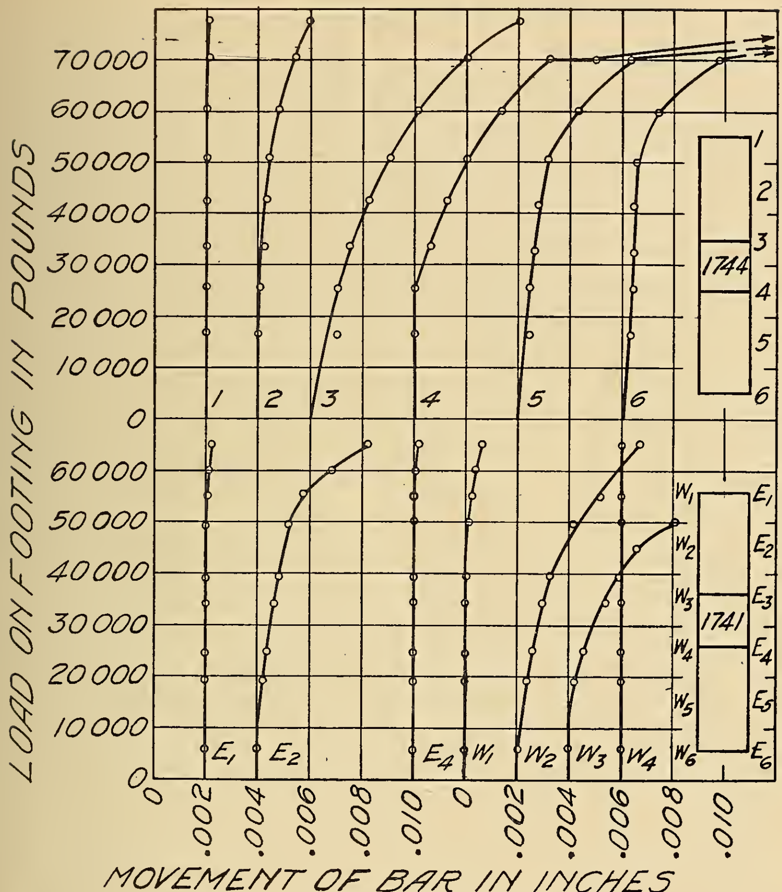


Fig. 23. Diagram Showing Slip of Bars.

mum load was held some little time before failure occurred. In the thicker footings, having at the higher loads much energy stored in the springs, the failure was violent, heavy pieces being thrown to one side and the testing machine giving the appearance of a wreck. In every case, failure was by tension and the footings broke into two or more pieces. The fracture generally occurred in vertical planes, except as it became inclined toward the edge of pier. Fig. 24 shows the position of the lines of fracture. The fractures on the side faces are not shown but the cracks on those faces were vertical in all cases. Along the top surface of the footings the fracture coincided with one face of the pier and in one of the failures cracks jogged back somewhat along

an adjacent face of the pier and ran from this point to the middle of the side of the footing.

The deflections of the footing at the top surface were measured in No. 1505, 1506, 1507, and 1508 in an effort to determine roughly the distribution of stresses. The results are not sufficiently definite for the purpose. The curvature at an edge was less than that through the face of the pier. It would seem to follow from this that the stresses developed in the concrete at the outside edge were less than those developed in a parallel direction at the face of the pier.

The moduli of rupture for the footings given in Table 14 were calculated by using a resisting moment based upon the full width of the footing, that is by considering the fiber stress in the concrete at the bottom of the footing to be uniform over the length of a section passing through the face of the wall, instead of taking into account the variation in stress across the section. The method of calculating the bending moment was the same as that used in the reinforced footings. Variations in concrete and in the actual distribution of load over the footing masked any effect due to the variation in proportion of depth to projection. It should be noted that the values of the modulus of rupture in the table are smaller than the modulus of rupture found in the control beams, averaging perhaps two-thirds as great if some allowance be made for the greater age of the control beams. The projection in an unreinforced footing usually is relatively short, and it may be more convenient in designing to use the full width of section, but it appears that the working stress used should be based on a modulus of rupture smaller than that found by beam tests. The probability of variation in tensile strength of concrete also must be taken into account in choosing the working stresses for unreinforced footings.

29. *Phenomena of Tests of Reinforced Concrete Column Footings.*—In the tests of the reinforced concrete column footings, as the load was applied the springs forming the bed compressed. The deflection in these footings was so slight and the consequent difference in the amount of shortening in the springs was so small that in the calculations the load was considered to be uniformly distributed over the footing up to the point of failure. In cases where the failure was by tension or by slip of bars, there followed bending up of the edges of the footing which modified this distribution of the load as soon as failure became evident. Three forms of failure may be distinguished: (a) tension in reinforcement; (b) bond between steel and concrete; and (c) diagonal tension or shear.

TABLE 14.

Data of Unreinforced Concrete Column Footings. Series of 1910.

All footings 5 ft. square. 1–2½–5 concrete, hand-mixed. Universal Portland cement. Weight of cement averaged 12.6 per cent of weight of aggregate.

Footing No.	Age days	Depth inches	Load at Failure pounds	Modulus of Rupture lb. per sq. in.	Control Beams: Modulus of Rupture lb. per sq. in.	Control Beams: Age days	6-in. Cubes: Maximum Load lb. per sq. in.	6-in. Cubes: Age days
1501	77	6	30 000	272	309	79	2910	112
1502	74	6	28 000	254	423	87	2939	104
1503	77	8	49 000	250	373	79	2307	112
1504	73	8	31 000	158	425	109	1761	109
1505	86	12	86 000	195	376	95	3618	128
1506	77	12	67 000	152	272	99	2260	116
1507	75	18	238 000	240	409	105	3180	121
1508	73	18	190 000	191	363	93	1988	109

(a) In the failures by tension in the steel the cracks which had appeared at the bottom or on the lateral faces of the footings near the middle portion of the length opened at the maximum load, and the maximum load was maintained for some time under steady pumping of the jacks, the edges of the footing meanwhile deflecting considerably.

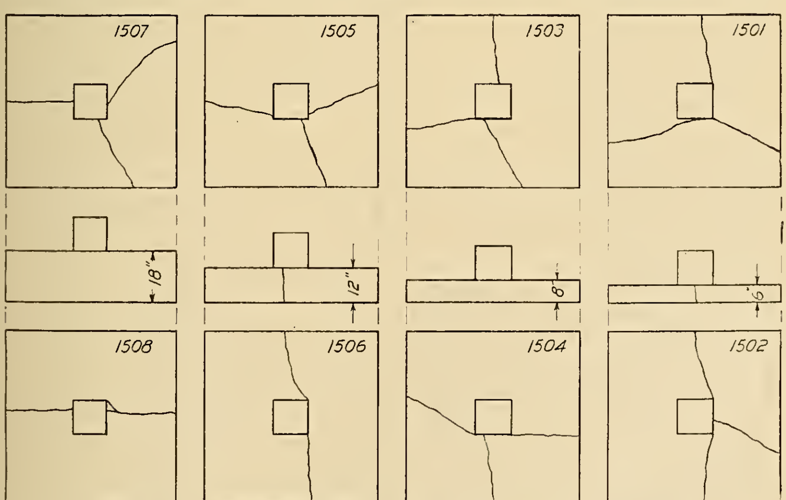


Fig. 24. Unreinforced Concrete Column Footings.

TABLE 15.

REINFORCED CONCRETE COLUMN FOOTINGS. SERIES OF 1909.

All footings 5 ft. square. 1–2½–5 concrete, hand-mixed. Chicago AA Portland cement. Total depth 11 in., depth to center of steel 10 in., unless otherwise noted.

Footing No.	Age at Test days	Reinforcement: Description	Reinforcement: Per cent	Reinforcement: Disposition	Load at Failure pounds	Calculated Stresses lb. per sq. in.: Tensile Stress in Steel	Bond Stress	Vertical Shearing Stress	Manner of Failure	Remarks
1411*	81	15 ⅜-in. round	0.28	Bars spaced 4 in. c. to c. in each of two directions.	112 000	31 900	220	69	Tension	
1412	76	do.	0.28		160 000	45 600	314	99	"	
1413*	82	12 ½-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	144 000	29 200	269	90	Bond	
1414	71	do.	0.39		192 000	38 900	358	120	"	
1415	80	do.	0.39	do.	160 000	32 400	299	100	"	Bars cut 3 ft. 9 in. long and ends staggered.
1416	68	do.	0.39	do.	128 000	25 900	239	81	"	
1417*	81	12 ¾-in. round	0.88	do.	160 000	14 900	206	104	"	Load eccentric.
1418	61	do.	0.88	do.	176 000	16 400	226	114	"	do. Total depth, 12 in.
1421*	84	1 ¾-in., 2 ⅝-in., 4 ½-in. and 4 ⅜-in. round.	0.38	do.	136 000	24 300	...	85	"	
1422	57	3 ¾, 4 ½, and 4 ⅜-in. round	0.42	do.	160 000	25 200	...	100	Not determined	Load eccentric. Total depth, 11½ in.
1425*	79	12 ½-in. round	0.39	Bars spaced from 3 to 7 in. in each of two directions	160 000	29 900	275	100	Probably tension	Load eccentric
1426	65	do.	0.39		160 000	29 900	275	100	do.	do.
1429	77	Wire mesh	0.13	Two double layers	128 000	53 700	...	77	....	
1431†	76	15 ⅓-in. cor. sq.	0.28	Bars spaced 4 in. c. to c. in each of two directions	156 000	44 200	271	97	Probably tension	
1432†	71	do.	0.28		136 000	38 500	236	84	do.	
1435	77	5 ¾-in., 2 ½-in., 2 ⅓-in. and 2 ¼-in. cor. sq.	0.61	Bars spaced 5 in. c. to c. in each of two directions	208 000	22 000	...	134	Diagonal tension	Total depth, 11½ in.
1436	64	5 ¾-in., 4 ½-in., 2 ¼-in. cor. sq. bars	0.67	do.	176 000	17 500	...	113	do.	Total depth. 12 in.
1437	82	3 ½-in., 4 ⅓-in., 4 ¼-in. cor. sq. bars	0.24	do.	128 000	34 800	...	79	Tension	
1439*	82	1 ¾-in., 4 ½-in., 4 ⅓-in. cor. sq. bars	0.33	Bars spaced 6 in. c. to c. in each of two directions	160 000	32 600	...	98	"	Load eccentric
1447	78	½-in. round	0.31	6 bars parallel to each side	208 000	42 100	...	130	Diagonal tension	Total depth, 12 in.
1448	63	do.	0.31	5 bars parallel to each diag.	176 000	35 600	...	110	do.	do.
1449	75	½ and ⅝-in. round	0.27	5 ½-in bars 5 in. c. to c. parallel to each side, 3 ⅝-in. bars 7 in. c. to. c. parallel to each diagonal	192 000	33 400	...	122	Tension	do.
1451	63	12 ½-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	88 000	17 800	131	59	Bond	Sloped. 3 in. to steel at edges

* Tested with a flat bearing plate. † Mild steel.

TABLE 16.

REINFORCED CONCRETE COLUMN FOOTINGS. SERIES OF 1910.

All footings 5 ft. square. 1–2½–5 concrete, hand-mixed. Universal Portland cement. Total depth 12 in., depth to center of steel 10 in. unless otherwise noted.

Footing No.	Age at Test days	Reinforcement: Description	Reinforcement: Per cent	Reinforcement: Disposition	Load at Failure pounds	Calculated Stresses lb. per sq. in.: Tensile Stress in Steel	Calculated Stresses lb. per sq. in.: Bond Stress	Calculated Stresses lb. per sq. in.: Vertical Shearing Stress	Manner of Failure	Remarks
1515	81	12 ½-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	185 000	37 600	347	117	Tension	
1516	80	do.	0.39		170 000	34 600	317	107	Tension followed by bond	
1521*	78	do.	0.58	do.	116 000	38 300	352	158	Tension	Total depth, 8 in.
1522†	71	do.	0.56	do.	122 000	38 700	356	156	Bond with possible tension on west side	do.
1525‡	79	do.	0.78	do.	63 000	29 700	273	144	Bond	Total depth, 6 in.
1526‡	71	do.	0.78	do.	85 000	40 000	368	194	Bond; tension probably imminent	do.
1531	81	18 ½-in. round	0.59	Bars spaced 3¼ in. c. to c. in each of two directions	280 000	38 700	357	180	Diagonal tension	
1532	79	do.	0.59		252 000	34 800	321	162	do.	
1535	85	8 ¾-in. round	0.59	Bars spaced 8 in. c. to c. in each of two directions	194 000	20 600	372	125	Bond	
1536	70	12 ⅝-in. round	0.61	Bars spaced 5 in. c. to c. in each of two directions	182 000	24 200	279	117	"	
1541‡	78	8 ¾-in. round	1.18	Bars spaced 8 in. c. to c. in each of two directions	52 000	16 700	231	122	Bond, lack of concrete below bars	Total depth, 6 in.
1542‡	61	12 ⅝-in. round	1.23	Bars spaced 5 in. c. to c. in each of two directions	95 000	29 400	340	222	Bond	Total depth, 6½ in.
1551	82	10 $\frac{9}{16}$-in. cor. round	0.42	Bars spaced 6 in. c. to c. in each of two directions	225 000	43 400	448	142	Diagonal tension	
1552	72	10 ½-in. cor. square	0.42		236 000	45 100	415	149	do.	
1553	81	15 $\frac{9}{16}$-in. cor. round	0.62	Bars spaced 4 in. c. to c. in each of two directions	327 000	43 000	445	211	do.	
1554	75	15 ½-in. cor. square	0.62		288 000	37 800	346	185	do.	
1561	77	½-in. round	0.59	Bars spaced 7 in. c. to c. uniformly in four directions	240 000	33 300		154	Probably combination of bond and tension	
1562	64	do.	0.59		210 000	29 100		135		
1563	77	do.	0.59	Bars spaced 7 in. c. to c. in four directions, 3 short diag. bars at corners omitted.	174 000	24 100		112	Probably bond	
1564	64	do.	0.59		210 000	29 100		135	do.	

*6¾ in. to center of steel. †7 in. to center of steel. ‡5 in. to center of steel.

TABLE 17.

Reinforced Concrete Column Footings. Series of 1911.

All footings 5 ft. square. 1–2–4 concrete, hand mixed. Universal Portland cement. Total depth 12 in., depth to center of steel 10 in. unless otherwise noted.

Footing No.	Age at Test days	Reinforcement			Load at Failure pounds	Calculated Stresses lb. per sq. in.			Manner of Failure
		Description	Per cent	Disposition		Tensile Stress in Steel	Bond Stress	Vertical Shearing Stress	
1806 1807	62 62	22 ⅜-in. round do.	0.41 0.41	Spaced 2¾ in. c. to c. in each of two directions	179 000 210 000	34 800 40 800	241 283	111 130	Probably bond Tension
1808 1809	62 62	27 ⅜-in. round do.	0.50 0.50	Bars spaced 2¼ in. c. to c. in each of two directions	198 000 236 000	31 600 37 700	218 260	124 148	Diagonal tension Tension followed by diagonal tension
1810 1811	59 60	33 ⅜-in. round do.	0.61 0.61	Bars spaced 1 13/16 in. c. to c. in each of two directions	219 000 261 000	28 700 34 200	198 236	138 164	Diagonal tension do.
1812 1813	78 65	12 ½-in. round do.	0.39 0.39	Bars spaced 5 in. c. to c. in each of two directions	171 000 121 000	34 300 24 300	315 223	106 75	Bond "
1814 1815	62 60	22 ½-in. round do.	0.72 0.72	Bars spaced 2¾ in. c. to c. in each of two directions.	301 000 294 000	33 700 33 000	310 303	192 187	" Diagonal tension and bond
1816 1817	65 59	8 ⅝-in. round do.	0.41 0.41	Bars spaced 7½ in. c. to c. in each of two directions.	132 000 159 000	25 400 30 600	293 353	82 99	Bond "
1818 1819	64 60	21 ⅜-in. cor. round do.	0.39 0.39	Bars spaced 2⅞-in. c. to c. in each of two directions.	198 000 261 000	40 400 53 300	279 367	123 162	Diagonal tension Tension followed by diagonal tension.
1820 1821	64 30	10 ½-in. cor. square do.	0.42 0.42	Bars spaced 6 in. c. to c. in each of two directions.	179 000 159 000	33 800 30 000	313 278	111 99	Diagonal tension do.
1822 1823	60 60	22 ⅜-in. round do.	0.41 0.41	Bars spaced 2 3/16 in. c. to c. in each of two directions No rods under pier	198 000 210 000	38 500* 40 800*	266* 282*	123 131	Tension "

* $\frac{48}{66}$ of steel considered as effective.

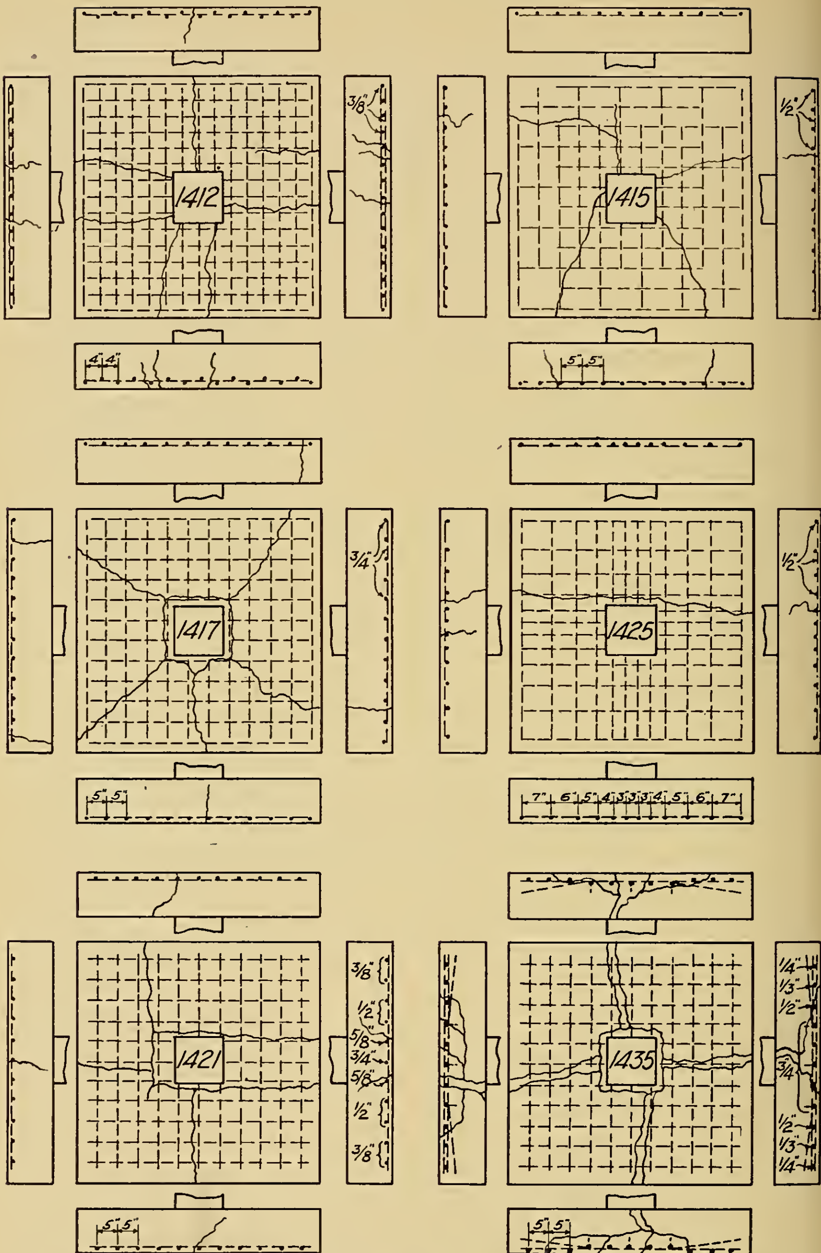
TABLE 18.

Reinforced Concrete Column Footings. Series of 1912.

All footings 5 ft. square. 1–2–4 concrete, machine-mixed. Universal Portland cement. Total depth 12 in., depth to center of steel 10 in.

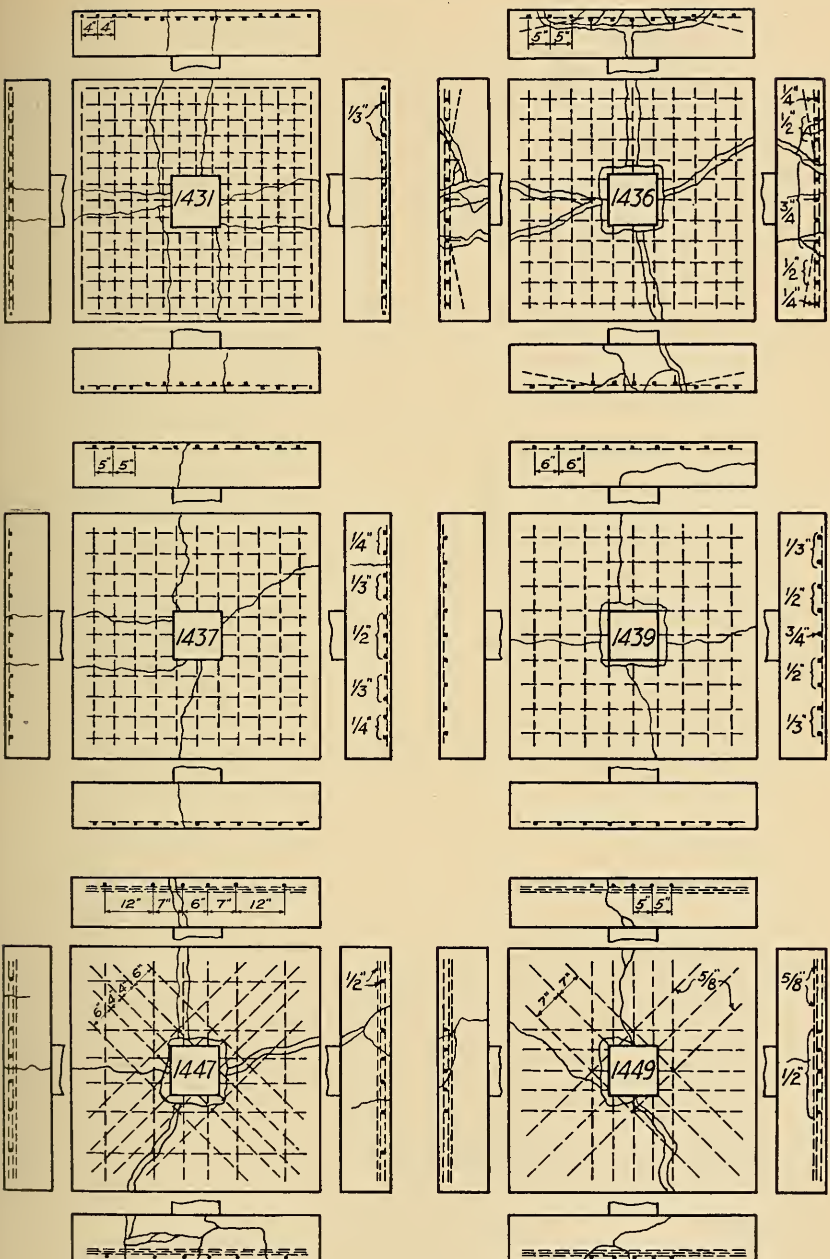
Footing No.	Age at Test days	Reinforcement: Description	Reinforcement: Per cent	Reinforcement: Disposition	Load at Failure pounds	Calculated Stresses lb. per sq. in.: Tensile Stress in Steel	Calculated Stresses lb. per sq. in.: Bond Stress	Calculated Stresses lb. per sq. in.: Vertical Shearing Stress	Manner of Failure
1831	69	8 ⅝-in. round	0.41	Bars spaced 7½ in. c. to c. in each of two directions	161 000	31 000	357	100	Bond
1832	73	do.	0.41		192 000	37 100	425	119	"
1833	66	22 ⅜-in. round	0.41	Bars spaced 1⅛ in. c. to c. in each of two directions. No bars nearer center of footing than 18 in.	113 000	22 000*	152*	71	"
1834	71	do.	0.41		152 000	29 800*	206*	95	"
1835	76	do.	0.41	Bars spaced 1⅝ in. c. to c. in each of two directions. No bars nearer center of footing than 12 in.	183 000	35 800*	248*	114	"
1836	66	do.	0.41		212 000	41 200*	285*	132	"
1837	77	do.	0.41	Bars spaced 1⅛ in. c. to c. in each of two directions. No bars farther from center of footing than 12 in.	192 000	28 700	198	119	"
1838	69	do.	0.41		247 000	36 900	254	154	Tension followed by sudden bond failure.
1839	69	8 ⅝-in. round	0.41	Bars spaced 7½ in. c. to c. in each of two directions. Bent from two long bars.	192 000	37 100	425	119	Tension followed by diagonal tension
1840	65	do.	0.41		201 000	38 800	445	125	
1841	71	do.	0.41	Bars spaced 7½ in. c. to c. in each of two directions. Curved up at ends to within 2 in. of top and back 8 in.	186 000	35 900	412	116	Tension followed by bond
1842	70	do.	0.41		203 000	39 200	450	126	Tension followed by diagonal tension
1843	71	8 ⅝-in. cor. round	0.41	Bars spaced 7½ in. c. to c. in each of two directions.	227 000	43 800	507	141	Diagonal tension
1844	67	do.	0.41		269 000	52 000	596	167	Bond followed by diagonal tension

* 43/60 of steel considered as effective.



The Drawings of No. 1412, 1415 and 1421 show the Cracks as they appeared on the Bottom Surface of the Footing.

FIG. 25. REINFORCED CONCRETE COLUMN FOOTINGS.



The Drawings of No. 1431 and 1437 show the Cracks as they appeared on the Bottom Surface of the Footing.

FIG. 26. REINFORCED CONCRETE COLUMN FOOTINGS.

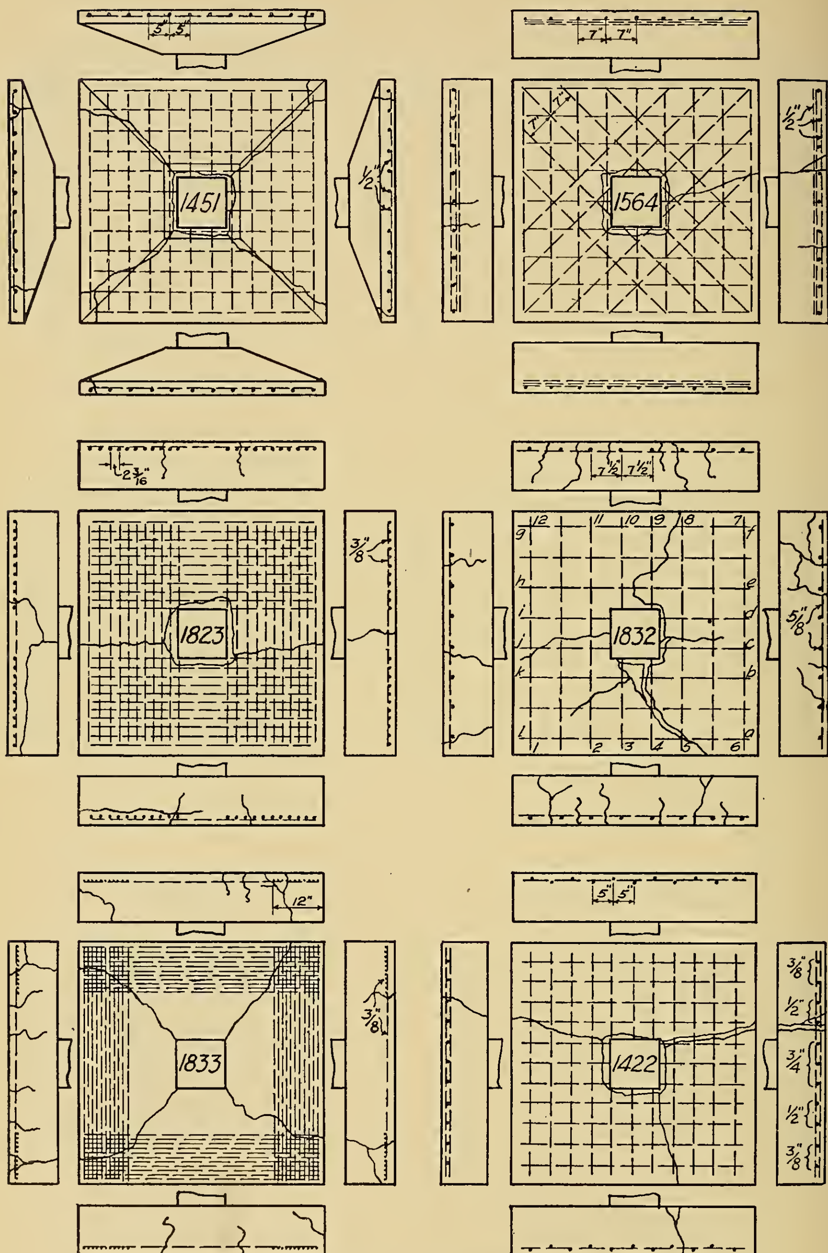


FIG. 27. REINFORCED CONCRETE COLUMN FOOTINGS.

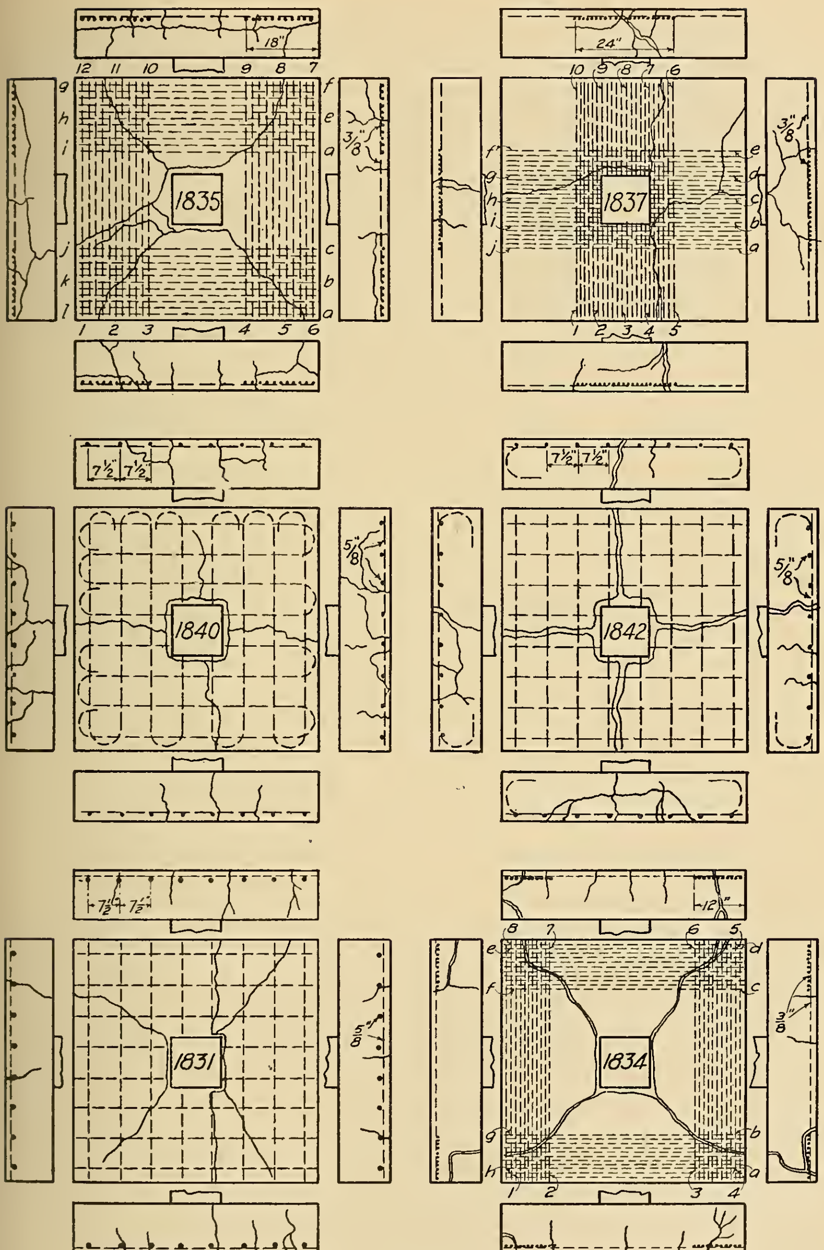


FIG. 28. REINFORCED CONCRETE COLUMN FOOTINGS.

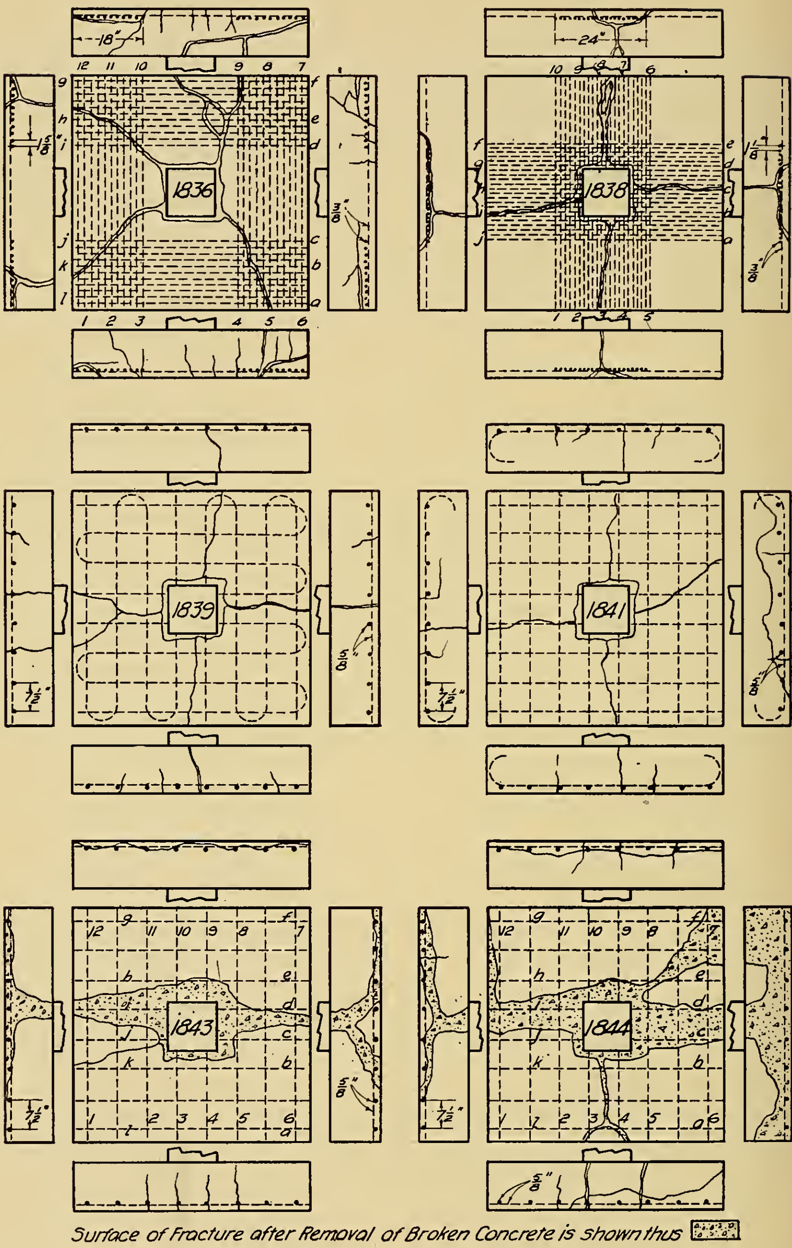


FIG. 29. REINFORCED CONCRETE COLUMN FOOTINGS.

When the operation of the jacks was continued after the maximum load was reached, the pier finally sheared or punched through the footing (see No. 1838, Fig. 31), the angle of the shearing face generally being steeper than 45° with the vertical. This shearing took place after considerable deflection and when the projection of the footing was taking only a fraction of its share of the load. After the phenomena just noted the cracks on the upper surface of the footing extended from the corner of the pier to a point on the lateral face of the footing opposite the face of the pier (see No. 1412, Fig. 25, page 78). In a few cases the line of fracture ran to a point near the middle of the pier. It should be borne in mind that the shearing and formation of cracks referred to occurred after the yield point of the reinforcement had been reached. In some footings examined the reinforcing rods were found to be necked.

(b) In the bond failures the failure was gradual, cracks forming near the ends of the lateral faces and finally opening up considerably, while the tension hair cracks which had opened in the middle of the lateral faces finally closed. (See No. 1415, Fig. 25, and No. 1417, Fig. 25). In general, the large cracks formed in the bond failures ran diagonally inward from near the corners of the footing. In many cases the bars were seen to have slipped at the ends. In cases where reinforcement was placed close to the lower surface of the footing, the tension cracks which formed by beam action in a direction parallel to the lower reinforcing bar, loosened the bond of these bars and caused failure at lower loads than might otherwise be expected. The manner of this loosening is apparent from Fig. 8 (c), page 18. Several of the footings of the series of 1909 suffered from this cause. In a number of cases, when the test was continued farther than the maximum load, an ultimate failure by punching through the footing in the manner noted under "Tension Failures" was found. The measurements of slip of end of bar taken in the tests of 1912 and discussed under "Bond Failures" give information on the first slip of bar.

(c) In the form of failure termed diagonal tension or shear failure the footing below the pier suddenly separated from the outer portion of the pier, leaving a mass in the form of a frustum of a pyramid below the pier, the reinforcing bars being stripped from a part or all of the remainder of the footing. The outer portion of the footing was generally intact, except as to tension cracks which had formed as usual. The face of the fracture was about 45° with the vertical. No. 1843, Fig. 31, failed in this way. These failures are similar to the failures

in ordinary reinforced concrete beams known as diagonal tension or shear failures in suddenness, in direction of the fractured face, and in the stripping of the reinforcing bars. Since these failures are in the interior the formation of diagonal tension cracks could not be observed. In all probability the failure is due to diagonal tension stresses, the cracks forming first at or near the reinforcing bars. For the highest loads obtained incipient compression failure was observed near the junction of pier and upper face of the footing.

The following are brief notes of tests. The location of cracks is shown in Fig. 25-29. The heavy lines on the lateral faces of the pier indicate cracks along which failure took place. Reference may be made to Table 15-18. It should be remembered that when first observed the cracks noted were hair cracks, which continued to be very fine cracks until they opened when the steel reached its yield point or bond resistance was overcome. As the faces of the piers varied in roughness and as they were not whitewashed, the load when a crack was first observed may be expected to be different in one footing from that in a companion test piece. The footings marked "Stored in place of making" were left on the floor of the mixing room in the place they were made until just before the test. In this position they were subject to considerable dampness during the time of seasoning, as the floor was frequently wet from the work. It seems evident that the concrete in this condition hardened more slowly and did not attain the same strength in the storage period as did those footings which were stored above the floor.

Series of 1909.

No. 1411. Tested with flat bearing plate. Shortening of springs at north end nearly double that at south end. Very uneven distribution of load. Tension crack formed at 96 000 lb. Failure at 112 000 lb. by gradual opening of tension cracks.

No. 1412. Load was applied through a spherical-seated bearing block. At 120 000 lb. tension crack appeared on east face directly in line with north face of pier. At 144 000 lb. a second tension crack appeared on the east face directly in line with the south face of the pier. Footing failed slowly at 160 000 lb., tension cracks opening. (See Fig. 25.)

No. 1413. This was the first footing tested and much of the work was experimental, and many of the springs were without bearing at the beginning of the test. A flat bearing block was used and the load was not uniformly distributed. Possibly some error in data. No cracks noted until directly before failure. The main cracks ran from points

on the lateral faces near the corners of the footing towards points under the pier. Slip of bars was observed. Bond failure.

No. 1414. First crack observed at 120 000 lb. in middle of west lateral face. A second crack formed at 136 000 lb. and closed up at the failure of the footing. Failed at 192 000 lb. by bond, new cracks forming on the bottom from near the corners.

No. 1415. (See Fig. 25.) Short rods extended alternately to within 3 in. and 12 in. of faces of footing. Southeast corner carried less load than remainder. Tension cracks which formed finally closed up. Failure at 160 000 lb. by bond, cracks forming and opening on lateral face near corners.

No. 1416. Tension crack at 120 000 lb. on two opposite faces. Gradual failure at 128 000 lb. Manner of failure not definitely known, probably bond. The use of shorter rods may have caused concentration of bond stresses.

No. 1417. (See Fig. 25 and 30.) No cracks were noted until the maximum load of 160 000 lb. was applied, when cracks formed gradually near the corners. Bond failure. Reinforcing bars were very close to surface, especially on east side.

No. 1418. No cracks were noted until maximum load of 176 000 lb. was applied, when cracks formed slowly on the lateral faces near the corners and failure was gradual. The slipping of reinforcing bars was very noticeable. Cracks formed in horizontal plane of rods and the concrete below split off.

No. 1421. (See Fig. 25 and 30.) Flat bearing plate used. Reinforced with rods of varying size. At 128 000 lb. tension cracks appeared on three faces. These cracks followed the lines of the reinforcing bars and reduced the effectiveness of the bond resistance. Method of failure not definitely known, probably bond.

No. 1422. (See Fig. 27.) Footing stored in place of making. Load not uniformly distributed. No cracks appeared until sudden failure occurred at 160 000 lb. Method of failure not determined.

No. 1425. (See Fig. 25.) Flat bearing plate. Reinforcing rods with varying spacing across the footing. Load not uniformly distributed, the north side taking a greater load. At 112 000 lb. cracks were seen on two faces. Maximum load 160 000 lb. Probably tension failure.

No. 1426. Tension crack at 112 000 lb. on east face and at 144 000 lb. on south face. Failure gradual at 160 000 lb. Seemingly tension failure, perhaps beginning at edge where rods were spaced far apart.

No. 1429. Reinforced with wire mesh. First crack at 96 000 lb. on east and west faces. Broke suddenly at 128 000 lb. Tension failure.

No. 1431. (See Fig. 26.) Reinforced with mild steel corrugated bars. Two cracks on east face in line with north-and-south face of pier at 112 000 lb. and one on west face and one on south face at 128 000 lb. Cracks grew as load was increased and at the maximum load of 156 000 lb. failure occurred. Tension failure.

No. 1432. Reinforced with mild steel corrugated bars. Crack at 128 000 lb. and others appeared later. Principal cracks opened. Gradual failure at 136 000 lb. Tension failure.

No. 1435. (See Fig. 25.) Reinforced with corrugated bars of varying sizes bent up somewhat at ends. At 136 000 lb. tension crack appeared in west face in line with the south face of pier, at 144 000 lb. on east face opposite north face of pier. Failed at 208 000 lb. by diagonal tension, the angle of the faces of fracture being about 45° on all four sides. The lower layer of concrete to the top of reinforcement fell away and outer portion of footing broke into four pieces. One ¼-in. bar near north face was found broken near the center of its length.

No. 1436. (See Fig. 26.) Reinforced as No. 1435. Stored in place of making. At 160 000 lb. tension crack appeared on east and west faces. At 176 000 lb. failed suddenly much as No. 1435.

No. 1437. (See Fig. 26.) Light reinforcement of corrugated bars. At 104 000 lb. tension cracks appeared on east and west faces on line with north face of pier; at 112 000 lb. on north and south faces. Failed by tension at 128 000 lb.

No. 1439. (See Fig. 26.) Flat bearing plate. At 136 000 lb. tension cracks appeared at center of length of east face and at 144 000 lb. near center of west and north faces. Failed at 160 000 lb. by tension in steel. Pier finally sheared through.

No. 1447. (See Fig. 26.) Reinforced with rods in four directions. At 144 000 lb. two cracks appeared on west face, one directly in center and one 12 in. from corner, the latter crack closing before final failure. At 160 000 lb. crack in north face in line with east face of pier and at 167 000 lb. on south face about center. Sudden failure at 208 000 lb. Diagonal tension failure.

No. 1448. Footing stored in place of making. At 144 000 lb. crack appeared on east face, one on the south face and two on the west face. Three were near center and one 8 in. from corner, the last nearly closing up before final rupture. Sudden failure at 176 000 lb. by diagonal tension.

No. 1449. (See Fig. 26.) At 96 000 lb. crack appeared at center of length of east face and on west face in line with north face of pier. At 160 000 lb. at center of south face and at 192 000 lb. at center of north face. Tension failure at 192 000 lb.

No. 1451. (See Fig. 27.) Sloped footing. Stored in place of making. At 80 000 lb. crack appeared on west face 9 in. from corner. Failed at 88 000 lb., evidently by bond. It would seem that in this form of footing bond stresses would be more concentrated towards the ends of bars than in footings of rectangular cross section.

Series of 1910.

No. 1515. Tension cracks appeared at the middle of the two lateral faces at a load of 120 000 lb. At 166 000 lb. tension cracks appeared

at the middle of the other two lateral faces. At 185 000 lb. gradual failure by tension occurred, the tension cracks opening up. With a continuation of the test there was a punching through the footing by the pier.

No. 1516. First crack (tension) at 102 000 lb. Instruments removed at 138 000 lb. Load released at 143 000 lb. to adjust the testing machine. Load again brought to 138 000 lb. and then released to adjust upper nuts of testing machine. Load again applied and released because of dangerous leaning of the springs. The test was continued the following day. The footing held the maximum load at 170 000 lb. for several minutes under steady pumping of the jacks, while the tension cracks on the sides slowly opened. Failure was by tension in the reinforcement. With continued operation of the test, rods in the lower layer slipped, cracks having formed along the lower surface beneath them. The pier finally sheared or punched through along diagonal planes.

No. 1521. Seven-inch depth to center of reinforcement. First cracks (tension) at 52 000 lb. When the load of 116 000 lb. was reached the tension cracks widened perceptibly and the load fell off at once to 112 000 lb. Tension failure along these cracks at this load. The pier finally punched through as shown in diagram.

No. 1522. Seven-inch depth. First crack (tension) at 85 000 lb. Instruments removed. At maximum load of 122 000 lb. cracks slowly opened. Failure was very gradual. Examination of footing after failure showed that rods had slipped at northeast corner and at south edge. Examination of three rods along west side showed no indication of slip. In general the slip of bars was accompanied by cracks in the concrete immediately underneath and in the direction of the bar. Rods which were calipered showed no indication of reduction of section.

No. 1525. Five-inch depth. First cracks (tension) at 45 000 lb. Failure at 65 000 lb. by gradual opening of cracks on face near corners and final appearance of diagonal cracks on top face. General indications of failure by bond. In the final punching through by the pier, the faces of the fractures were nearly vertical. Examination after failure showed that most of the rods had slipped, many of them at both ends. The bottom bars showed no reduction of section.

No. 1526. Five-inch depth. First crack (tension) at 38 000 lb. Instruments removed at 49 000 lb. At 85 000 lb. cracks on lateral faces near corners gradually opened. Bond failure, diagonal cracks finally reaching the top surface. After failure all the rods in this footing were examined and all but two found to have slipped at one or both ends. These two were in the bottom layer and next to the south edge. They were gaged with a micrometer caliper and found not to have necked.

No. 1531. Ten-inch depth. First crack (tension) at 85 000 lb. Springs leaned considerably and load was released, machine adjusted, and load reapplied. At 177 000 lb. the springs again needed adjustment

and the load was released. Four weeks later additional springs were placed on the bed and a second test made. At 280 000 lb. concrete above the base of the pier began to show signs of compression failure. The load fell off slowly and the footing finally failed suddenly by shearing through from the edge of the pier at the top of the footing to a line about as shown in Fig. 8(b) at the bottom, and the reinforcement together with the concrete layer below it was stripped off. The tension cracks which had formed closed up. Diagonal tension failure. Where concrete remained on rods examination was made after failure and no rods could be found to have slipped. The angle of fracture with the horizontal was about 45°.

No. 1532. First crack (tension) at 138 000 lb. Instruments removed at 198 000 lb. At 242 000 lb. the springs had closed up. The load was then released and one week later with additional springs placed on the bed the footing was loaded to failure with 252 000 lb. Failure was sudden by the pier shearing through. The fracture made an angle of about 45° with the horizontal. Of the few rods still encased in concrete when examined after failure none had slipped. As to the rest nothing could be determined.

No. 1535. First crack (tension) at 158 000 lb. Gradual failure at 194 000 lb. Failure was accompanied by opening of cracks near corners on lateral faces and by closing of tension hair cracks which had appeared near the center of the lateral faces earlier in the test. Bond failure. After failure all the rods were examined and all but two were found to have slipped at one or both ends and it is possible that even they had slipped at one end. These two were next to the north and east edges and in the top and bottom layers respectively.

No. 1536. Very gradual failure at 182 000 lb. Indications of failure by slipping of reinforcement. All bars which were found to have slipped were in the upper layer. Under the ends of bars found to have slipped cracks were found running in the direction of the bar. No such cracks were found at the ends of bars which had not slipped.

No. 1541. Five-inch depth. First crack (tension) at 45 000 lb. Instruments removed at 52 000 lb. Load fell off slowly while instruments were being removed and load could not be raised above 52 000 lb. by further pumping. This maximum load was held under steady pumping for about two minutes while footing deflected visibly. The failure was gradual, the bars slipping ½ in. to ¾ in. As the load was released some of the concrete below the bars dropped off. It seems probable that rods were placed too close to lower surface and that tension cracks reduced bond resistance.

No. 1542. Five-inch depth. First cracks (tension) appeared at 67 000 lb. on two opposite faces. Instruments removed at 85 000 lb. Gradual failure at 95 000 lb. Crack on lateral face near corners formed and opened on application of maximum load. Bond failure.

No. 1551. (See Fig. 30.) Ten-inch depth. Reinforced with round corrugated bars. Tension cracks formed at 102 000 lb. Instru-

Fig. 30. Views Showing Column Footings after Failure.

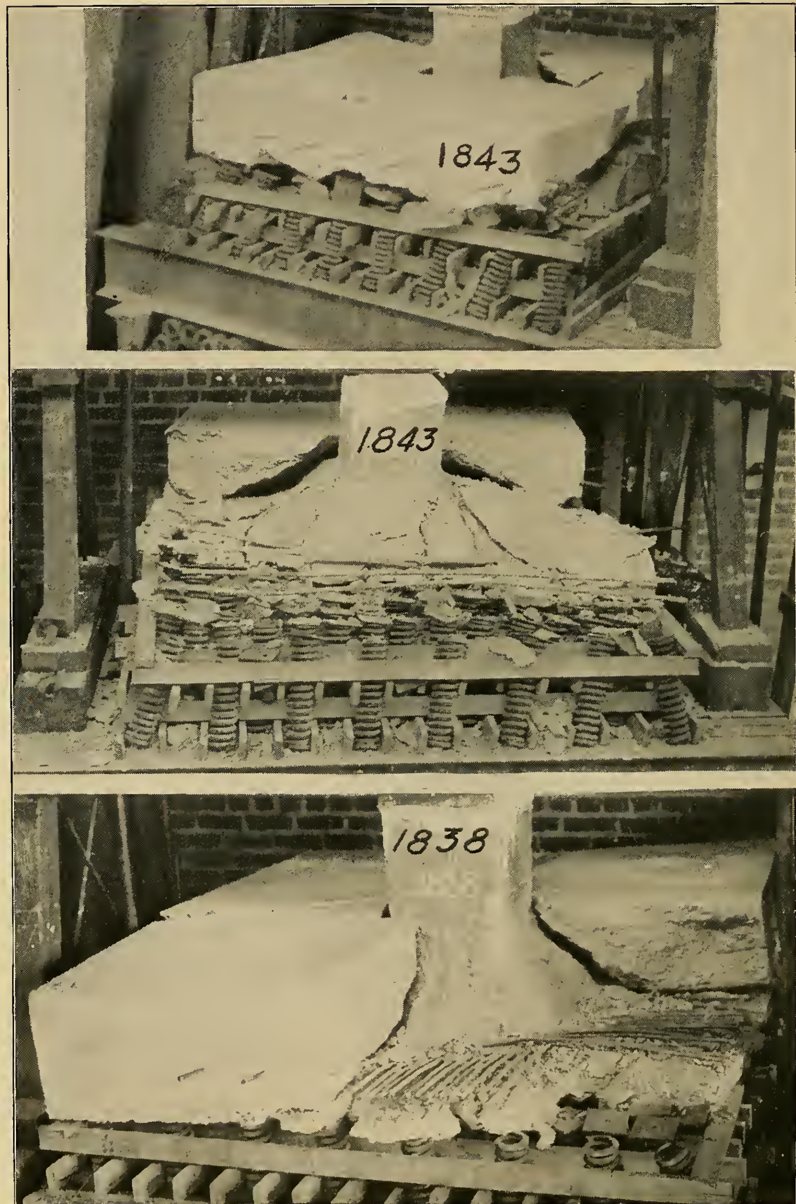


Fig. 31. Views Showing Column Footings after Failure.

ments removed at 102 000 lb. At 218 000 lb. tension cracks formed at middle of third lateral face. At 225 000 lb. failure by diagonal tension, as shown in Fig. 30, the reinforcing bars stripping off.

No. 1552. Reinforced with square corrugated bars. First crack (tension) at 102 000 lb. on three lateral faces. Instruments removed at 177 000 lb., and at 198 000 lb. load was released, footing removed, additional springs put in, and the test was continued. Failure at 236 000 lb., sudden and violent and similar to No. 1551. Diagonal tension. The reinforcing stripped off and the footing broke across as a plain concrete footing.

No. 1553. Reinforced with 0.6% of corrugated bars in each of two directions. At 218 000 lb. the springs had closed and the test was discontinued and the footing removed from the machine. A month later with additional springs on the bed of the machine the test was completed. At 327 000 lb. the footing suddenly failed by diagonal tension. Highest load of any test.

No. 1554. First crack (tension) at 120 000 lb. Final failure by diagonal tension at 288 000 lb. Angle of face of fracture about 45° with the vertical.

No. 1561. Reinforcement laid in four directions. First crack (tension) at 138 000 lb. Load released twice to adjust machine. Failure gradual at 240 000 lb., the tension cracks on the four faces opening. Examination after failure showed that of the rods in the upper layer those close to the edges of the pier had slipped at the ends while those passing under the center of the pier and those close to the edges of the footing had not slipped. Some of the diagonal rods had slipped.

No. 1562. Reinforcement in four directions. First crack (tension) at 120 000 lb. Instruments removed at 198 000 lb. Failed at 210 000 lb. by gradual opening of tension cracks on east and west faces, followed after a slight falling off of load by a sudden shearing around pier. Of the several rods on each side examined after failure many showed conclusively that they had not slipped and the remainder were in such condition that nothing about slip could be determined.

No. 1563. Reinforced in four directions. First crack (tension) at 120 000 lb. Failure at 174 000 lb. by gradual opening of cracks on east and west faces, followed finally by shearing.

No. 1564. (See Fig. 27.) First crack (tension) at 138 000 lb. Failure at 210 000 lb. by gradual opening up of tension cracks on two opposite faces followed by shearing around pier. After failure it was found that of the rods in the bottom layer those in the north half only had slipped at the west end. Of the north and south rods (third layer from bottom) those in about the middle third had slipped at the south end. Of the diagonal rods in the second layer from the bottom those passing under the center of the pier had slipped while those passing directly under the center of the pier had not slipped.

Series of 1911.

No. 1806. At a load of 138 000 lb. fine cracks were noted on all faces of footing over reinforcing bars, extending upward about 5 in. At a load of 159 000 lb. the cracks opened somewhat. As the load was increased, these cracks opened very little and at a load of 179 000 lb. failure occurred gradually by bond.

No. 1807. At a load of 159 000 lb. the cracks which had formed on the north face and the east face were opening and extending. Another crack was noted on west face of footig 6 in. north of pier. As the load was increased the cracks lengthened and became more prominent and at a load of 198 000 lb. they were gradually opening up. Failure occurred slowly at a load of 210 000 lb., probably by tension.

No. 1808. At a load of 120 000 lb. a vertical crack 4 in. high was noted on north face of footing in line with the west face of the pier. At a load of 138 000 lb. cracks were noted at the center of the north, east, and west faces 5, 7, and 6 in. high, respectively. A crack 6 in. high was also noted on west side in line with south face of pier. At a load of 198 000 lb. the pier began to fail and the load was released, a new pier set in place, and the footing again loaded. Failure occurred at a load of 198 000 lb. by diagonal tension. The cube tests show that the concrete in this footing was not quite up to standard.

No. 1809. At a load of 219 000 lb. the cracks on the north face were gradually opening up and failure was imminent. Maximum load was 236 000 lb. Failure occurred by tension followed by diagonal tension. Pier punched through footing.

No. 1810. At a load of 138 000 lb. fine vertical cracks were noted on north face of footing in line with east face of pier, on west face in line with north face of pier 6 in. high, and on east face of footing at center 6 in. high. At a load of 158 000 lb. the cracks became more prominent. At a load of 178 000 lb. cracks were noted on north face 2 in. west of center 6 in. high, on west face at center and in line with north face of pier, and on south face in line with east and west faces of pier 6 in. high. At a load of 198 000 lb. cracks were noted on north face 2 in. east of center 6 in. high and on west face 6 in. south of pier 6 in. high. Failure occurred at a load of 219 000 lb. by diagonal tension.

No. 1811. At a load of 219 000 lb. cracks previously observed were extending but no new ones were noted. At a load of 261 000 lb. the cracks began to widen, one of the rods in the upper layer slipped at its west end and failure occurred suddenly by diagonal tension.

No. 1812. At a load of 159 000 lb. all the cracks were prominent. Failure occurred at a load of 171 000 lb. by bond. Pier finally punched through the footing. Examination afterward showed that middle third of both layers of bars had slipped ¼ in. at their end.

No. 1813. At a load of 121 000 lb. the cracks were opening and extending. It was difficult to maintain the load. Load was removed

and the under side of footing examined and it was found that the bars had slipped. Bond failure.

No. 1814. At a load of 219 000 lb. the cracks were not opening very fast. As the load was increased to 260 000 lb. the cracks became more prominent but there were no signs of failure. At a load of 301 000 lb., the ends of bars were found to be slipping at holes cut into the concrete, and the cracks at reinforcing bars opened up gradually. Failure occurred suddenly at this load by bond and with great violence.

No. 1815. Cracks formed at reinforcing bars. At a load of 282 000 lb. the cracks had widened very little. Fine hair cracks were noted around some of the bars. Failure occurred at a load of 294 000 lb. by diagonal tension and bond.

No. 1816. As the load was increased to 138 000 lb. the cracks on the north face of footing opened up considerably and the rods slipped. Failure occurred by bond. Bars showed slip at three faces.

No. 1817. (See Fig. 30.) At a load of 159 000 lb. the bar at center running in a north and south direction had slipped and a crack at this bar had opened up considerably. Some of the bars at the west face of footing showed indications of slip. The load was released and cracks closed up very little. The footing was again loaded to 159 000 lb., the cracks opened up considerably, and failure occurred by bond. A number of the rods were observed to have slipped.

No. 1818. Reinforced with 3/8-in. round corrugated bars. Failure occurred at a load of 198 000 lb. by diagonal tension.

No. 1819. Reinforced with 3/8-in. round corrugated bars. At a load of 198 000 lb. a prominent crack was noted on north face 13 in. to west of pier 5 in. high. At this load the cracks on west side were opening up. As the load was increased the cracks opened gradually and failure occurred suddenly at a load of 261 000 lb. Failure due to tension followed by diagonal tension.

No. 1820. Reinforced with 1/2-in. square corrugated bars. As the load was increased small cracks were noted at the reinforcing bars on north, east, and west faces. Failure occurred at a load of 179 000 lb. by diagonal tension.

No. 1821. Reinforced with 1/2-in. square corrugated bars. Tested at an age of 30 days. At a load of 159 000 lb. failure occurred by diagonal tension. There was no indication that the bars had slipped. After failure the vertical cracks on sides of footing had practically closed. A prominent horizontal crack was visible at plane of bars.

No. 1822. Reinforced with 3/8-in. round rods. No rods under the pier. At a load of 102 000 lb. first crack was noted on north face of footing 6 in. east of center 8 in. high. At a load of 138 000 lb. a vertical crack was noted on west face 8 in. north of center. At a load of 158 000 lb. other cracks noted in the middle fourth of length on east, west, and north faces. Failure occurred gradually at a load of 198 000 lb. by tension in steel. Examination of bars showed that they had necked.

No. 1823. (See Fig. 27.) Reinforced in same manner as No. 1822. As the load was applied up to 159 000 lb. vertical cracks were noted in middle third of length on the north, east, and west faces of footing. At a load of 178 000 lb. the cracks opened considerably and a new crack was noted on east face 8 in. south of pier 6 in. high. At a load of 198 000 lb. the cracks opened up gradually and at a load of 210 000 lb. failure occurred. Tension failure. Examination of the bars showed necking, as is discussed on page 62.

Series of 1912.

No. 1831. Cracks on north and south faces were higher than those in east and west faces, the lower layer of bars running north and south. At 153 000 lb., during the process of reading, the load fell off slightly. The maximum load was 161 000 lb. At this load the cracks on the north face had opened about ¼ in. while those on the east and west faces had not opened appreciably. Failure was gradual and probably by slipping of north and south rods.

No. 1832. (See Fig. 27.) Cracks were as shown in sketch. Graphs showing slip of bars indicate the critical point for slipping to have occurred at about 140 000 lb. At 184 000 lb. measurements showed that the rods continued to slip after the increase of load had been discontinued. At 192 000 lb., the maximum load, the rods on all faces could be seen to be slipping. Failure was by bond.

No. 1833. (See Fig. 27 and 30.) Cracks were as shown in sketch. At a load of 92 000 lb. the pier failed, no cracks having been previously noted. A 12-inch cube about 4 years old was put in its place, embedded in plaster of paris, and the test continued. Failure took place violently at a load of 113 000 lb. about 20 seconds after pumping of the jacks had been stopped. The main cracks after failure were found where none had been observed during the test. Examination after failure showed that the bars lying near the north face of the footing (bottom layer) had slipped about ½ in. and ¾ in. at their ends. No other slipping was apparent.

No. 1834. Cracks were as shown in sketch. Careful search for cracks was made at lower loads but none were found. Failure was sudden at 153 000 lb., developing an entirely new set of cracks. Those previously observed did not open appreciably. At this load most of the bars slipped from ½ in. to ⅞ in.

No. 1835. (See Fig. 28.) The cracks were as shown in the sketch. The first crack occurred under rod number 3, the point at which the measurements detected slip first. Failure was gradual and took place at a load of 184 000 lb. The cracks which formed at failure had not been observed previously. Rods slipped from ¼ in. to ½ in.

No. 1836. At a load slightly less than 211 000 lb. an accident to the apparatus interrupted the test. The measurements for slip showed about the same characteristics on the second test as on the first. Before the second test, the cracks observed in the first test had nearly closed. Failure was probably by bond.

No. 1837. (See Fig. 28 and 32.) At a load of 162 000 lb. the pier of this footing failed. After a 12-in. cube had been put in its place the test was continued. Cracks formed in the first test closed before the second test. Failure was by slipping of bars.

No. 1838. (See Fig. 31.) No cracks appeared except the four cracks shown in the sketch. These could not be detected at a load of 74 000 lb. but were found at 92 000. At 240 000 lb. the cracks had opened about 1/4 in. Failure was by tension followed by the slipping of all the rods at the maximum load of 247 000 lb. The slab portion of the footing broke into four separate pieces, each breaking from the pier at an angle of about 45° with the vertical as shown in Fig. 31.

No. 1839. First cracks were seen on north and south faces at the ends of the lower layer of bars. These cracks opened rapidly near maximum load and final failure came suddenly. Failure by tension followed by diagonal tension.

No. 1840. (See Fig. 28.) Just before failure the cracks at the middle of the faces had opened to a width of about 1/8 in. and a very large deflection of the center of the footing was apparent. At the time that the pier punched through the footing, failure seemed to be occurring also by crushing of the pier. Failure was by tension followed by diagonal tension.

No. 1841. (See Fig. 29.) At failure the first observed cracks had become about 1/16 in. wide. An examination after failure showed that although the bars were bent backward in vertical planes the ends of the four center bars in the top layer had slipped from 1/16 in. to 3/8 in. Of the bottom layer only the two center bars had slipped. The proximity of large vertical cracks probably is largely responsible for the slipping. The main slip apparently occurred at the time of the collapse of the footing. Measurement with caliper showed some indication of necking.

No. 1842. (See Fig. 28.) First cracks were seen at the ends of the upper layer of bars. Failure occurred at 203 000 lb. after this load had been held under continuous pumping about 2 minutes. Examination of bars with caliper indicated some necking. Failure probably by tension followed by diagonal tension. The pier punched through, the angle of fracture being about 60° with the vertical.

No. 1843. (See Fig. 31.) This footing was reinforced with eight 5/8 in. round corrugated bars in each layer. The first slip as indicated by the measurements was in a rod of the bottom layer which ended in the crack first observed. This rod passed under the pier very near one edge.

No. 1844. The reinforcement was the same as in No. 1843. The first cracks appeared simultaneously on the north and south faces at a load of 93 000 lb. The measurements indicate that the first slip occurred in the two rods ending in these two cracks and at the same end as that at which the cracks were noted. Slipping of the bars was visible to the eye at the maximum load and before complete failure.

One crack opened 1/16 in. at a load of 252 000 lb. and ⅛ in. at the maximum load. Failure was by bond followed by diagonal tension, though there are indications that the slip of part of the bars threw greater stress on other bars and that these bars may have been stressed beyond the yield point.

30.. *Reinforced Concrete Column Footings: Tension Failures.*—As stated on page 73, the tension failures were marked by an appreciable opening of tension cracks at the lateral faces of the footing. The maximum load was generally maintained for some time under a steady pumping of the jacks, the edges of the footing meantime deflecting upward. In many cases these tension cracks appeared at a point on the lateral face of the footings in line with a face of the pier, and these cracks were found to extend entirely along the lower surface of the footing, passing through points immediately below the face of the pier referred to. In other cases the cracks were nearer the middle of the length of the lateral face, and either extended directly across the bottom surface or offsetted somewhat toward or directly over the face of the pier. As the crack extended upward, it sometimes became directed towards the junction of the face of the pier and upper surface of the footing, or it made a square turn at the corner. It was difficult to find the condition of these cracks in the interior, and it is evident that the cracks seen on the upper surface were the results of conditions which obtained after the maximum load was reached. No. 1412, Fig. 25, and No. 1431, Fig. 26, may be referred to as illustrations of the formation and direction of these cracks. Of course, it seems probable, as the bending up was greater along a middle or central section of the pier than along a section near a lateral face of the footing, that the tension cracks formed first in the interior and also that the bars in the interior reached their yield point before this stress was reached by bars at the lateral face. Possibly after the yield point was reached in the interior there was an adjustment of the stresses through the bars and more was taken by the rods near the lateral faces. The general phenomena of failure indicate that the resisting moment developed must have been greatest at a section passing through the face of the pier or else at a combination section through the part of the footing just below the face of the pier and across the remainder of the footing just a little back of this face, as shown in Fig. 8(a), page 18.

For failures by tension in reinforcement the loads carried were, in a general way, proportional to the amount of reinforcement, though in some cases the weaker footing of two companion test pieces failed at a

load below what might be expected, due, no doubt, to some part of the footing receiving a larger proportion of the load than the remainder and for which the assumption of uniform distribution of load gives incorrect results. For footings with the heavy reinforcement, sufficient load was not carried to develop the yield-point strength of the reinforcement but instead failure was by bond or diagonal tension. It is evident that the amount of reinforcement which may be made effective is limited by the resistance to diagonal tension and bond stress which may be developed, and that bond and diagonal tension strength must be considered in the design of footings.

In footings with depths of 5 in., 7 in., and 10 in., the complication of bond and tension failures prevents the drawing of final conclusions, but there is nothing to indicate a difference in action for footings of different thicknesses or different relative lengths of projection.

31. *Reinforced Concrete Column Footings: Bond Failures.*—As outlined on page 83, the failures by bond were generally gradual failures, cracks first becoming visible on the lateral faces near the corners of those footings in which the reinforcement was spaced over the entire footing and the tension cracks in the middle of the lateral faces finally closing or closing when the load was released. It seems probable that cracks had formed somewhat earlier in the interior nearer the pier, the central bars slipping, and that after this slip of the central bars greater stress would be given to the outer bars, probably at points nearer their ends, and the bond stress at the ends of these outer bars would increase and finally slip would occur there. The result was a failure crack in a diagonal direction. After these cracks opened the bars were found to have slipped.

Values of the bond stress developed in the footings which failed by bond and in others developing high bond stresses, calculated by the method described on page 23, are given in Table 19. They are fairly consistent and are somewhat lower than values of bond stress derived from simple pulling tests. It must be borne in mind that the method of calculation is empirical and that the analysis does not apply to the arrangement of bars in exterior bands. Low values are explained by the nearness of the bar to the surface in some of the footings and by the formation of tension cracks across one set of bars, which cracks extended longitudinally along the other set of reinforcing bars and acted to loosen the bond, see Fig. 8 (c). The footings reinforced with ¾-in. round rods are especially noticeable in this respect.

Although the bond stresses developed in footings reinforced with

TABLE 19.

Values of Bond Stress Developed in Column Footings.

In the calculations, the bars within a width of 46 inches were considered, except as otherwise noted.

Footing No.	Reinforcement		Calculated Bond Stress lb. per sq. in.	Manner of Failure
	Kind	Per cent		
1413	Plain round bars	0.39	269	Bond
1414	do.	0.39	358	"
1415	do.	0.39	299	Bond, (bars staggered)
1416	do.	0.39	239	do.
1417	do.	0.88	206	Bond
1418	do.	0.88	226	"
1451	do.	0.39	131	Bond, (sloped footing)
1522*	do.	0.56	356	Bond with possible tension
1525§	do.	0.78	273	Bond
1526§	do.	0.78	368	Bond; tension probably imminent
1535	do.	0.59	372	Bond
1536	do.	0.61	279	"
1541§	do.	1.18	231	Bond; lack of concrete below bars
1542§	do.	1.23	340	Bond
1812	do.	0.39	315	"
1813	do.	0.39	223	"
1814	do.	0.72	310	"
1816	do.	0.41	293	"
1817	do.	0.41	353	"
1831	do.	0.41	357	"
1832	do.	0.41	425	"
1833‖	do.	0.41	152	"
1834‖	do.	0.41	206	"
1835‖	do.	0.41	248	"
1836‖	do.	0.41	285	"
1837	do.	0.41	198	"
1412	Plain round bars	0.28	314	Tension
1515	do.	0.39	347	"
1516	do.	0.39	317	Tension followed by bond
1521*a*	do.	0.58	352	Tension
1531	do.	0.59	357	Diagonal tension
1532	do.	0.59	321	do.
1806	do.	0.41	241	Probably bond
1807	do.	0.41	283	Tension
1815	do.	0.72	303	Diagonal tension and bond
1822‖	do.	0.41	266	Tension
1823‖	do.	0.41	282	"
1838	do.	0.41	254	Tension followed by sudden bond failure
1839†	do.	0.41	425	Tension followed by diagonal tension
1840†	do.	0.41	445	do.
1841‡	do.	0.41	412	do.
1842‡	do.	0.41	450	Tension
1552	Cor. square bars	0.42	415	Diagonal tension
1554	do.	0.62	346	do.
1820	do.	0.42	313	do.
1551	Cor. round bars	0.42	448	Diagonal tension
1553	do.	0.62	445	do.
1818	do.	0.39	279	do.
1819	do.	0 39	367	Tension followed by diagonal tension
1843	do.	0.41	507	Diagonal tension
1844	do.	0.41	596	Bond followed by diagonal tension

*Depth 7 in. to center of steel. § Depth 5 in. to center of steel. † Continuous bar looped. ‡ Bars curved up at ends. ‖ Reinforcement placed in exterior bands; $\frac{46}{60}$ of steel used in calculations. *a* Depth 6¾ in. to center of steel.

deformed bars were in some cases above those developed with plain bars, only one bond failure was found (No. 1844), the calculated bond stress being 596 lb. per sq. in.

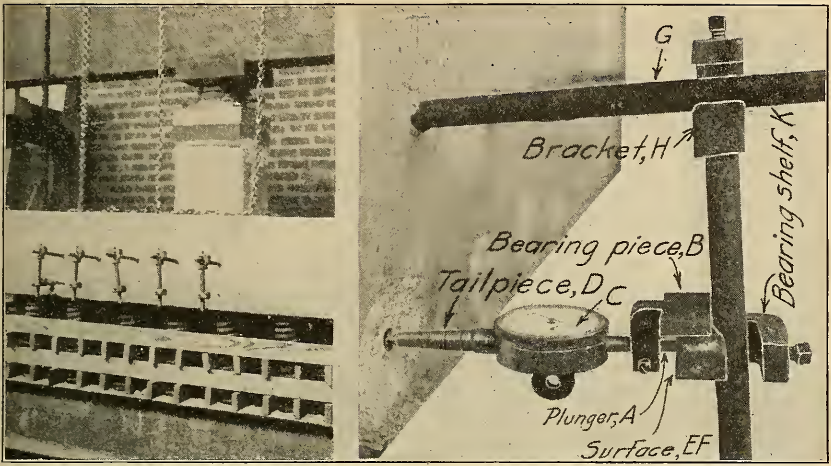


FIG. 32. APPARATUS FOR DETERMINING END SLIP OF BARS.

In order to determine definitely when first slip occurs at the end of the bars and whether bond is likely to be a primary cause of failure, a device was used in some of the 1912 tests by means of which an end movement of the bar as small as 0.0001 in. was measured with considerable certainty. This movement was determined by measuring the change in distance between the end of the reinforcing bar and a point in line with its axis and about 5½ in. from the face of the footing. The apparatus is shown in Fig. 32. The measuring instrument consists of an Ames gage micrometer equipped with a pointed tail piece D and a bearing piece B. Movement of the pointer C indicates change in distance between the point of the tail piece and the end of the micrometer plunger A. In operation the point of the tail piece is inserted in a small hole drilled in the end of the reinforcing bar, and the end of the plunger is brought to bear at a definite point upon the surface EF which has a fixed position relative to the face of the footing. The position of the surface EF is maintained by means of the auxiliary rod G, the cast-iron bracket H, and the bearing shelf K. The auxiliary rod G is embedded a short distance in the concrete at the time of pouring the concrete and the other parts are put in place at the time of the test. In order that the end of the plunger shall always have contact with the same point of the surface EF, two conical contact points attached to the under side of the bearing piece serve to insure that the bearing of the plunger of the micrometer shall always be in

the same horizontal element of the bearing surface while the engagement of one of these contact points in a groove of the bearing shelf insures that the bearing shall always be in the same vertical element. The bearing surface EF is sufficiently curved to insure pressure against the plunger spring while the instrument is being seated. Hence accuracy of results does not depend upon the plunger being forced into position by the stiffness of the spring. In the view at the left of Fig. 32, means of measuring slip on five bars are shown. The micrometer, a movable instrument, is shown in place against one of these bars.

By means of this instrument measurements of slip were taken on footings No. 1832, 1834, 1835, 1836, 1837, 1838, 1843, and 1844. It was found that with careful handling of the instrument results could be obtained which among themselves appear fairly consistent. Fig. 33, 34, and 35 give representative results showing the slip at the end of the bar. The position of the bars on which measurements were taken is shown by the letters and numbers on the diagrams of column footings given in Fig. 25-29. In a few instances there appears to be a progressive movement of the bars in the wrong direction for slip, and this indicates the possibility that some warping of the face of the footing may have been mistaken for movement of the bars. It seems unlikely that this would be of much importance in the results, since the slip was usually quite pronounced after it began, and since later observations usually strengthened the conclusion that slip actually began at the point where the slip curve makes a sharp bend to the right.

The tests of 1909, 1910, and 1911 had indicated that bond stress in column footings is an important consideration and that the formation of tension cracks in the footing must go along with a loosening of the bars which are parallel to these cracks, thus hastening bond failure. In the 1912 tests in which slip was measured, a careful record of cracks was kept, and these tests show an intimate relation between the formation of cracks on the lateral face of the footing and the slipping of the bars. Table 20 records the position of the cracks at the face of the footing and the position of the bars which give end slip, together with the corresponding loads. The position of the bar may be identified by reference to Figs. 27, 28, and 29, pages 80 to 82. In seven out of the eight footings in which observations were taken with the instrument the measurements showed slip. Of these seven, first slip at the end of bar occurred in four footings in bars which end at points where the first crack was detected. In one of the remaining three, first slip occurred where the second crack was detected. In the other two foot-

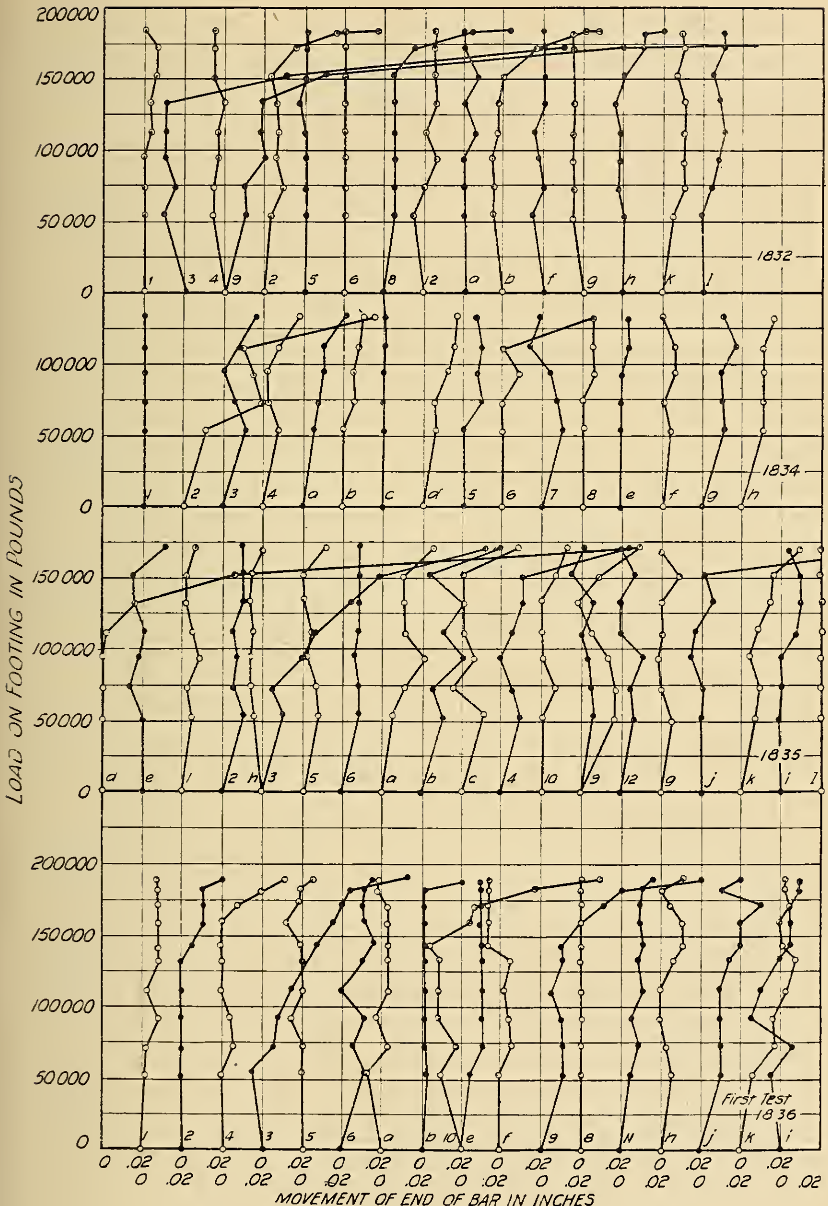


FIG. 33. DIAGRAM SHOWING END SLIP OF BARS.

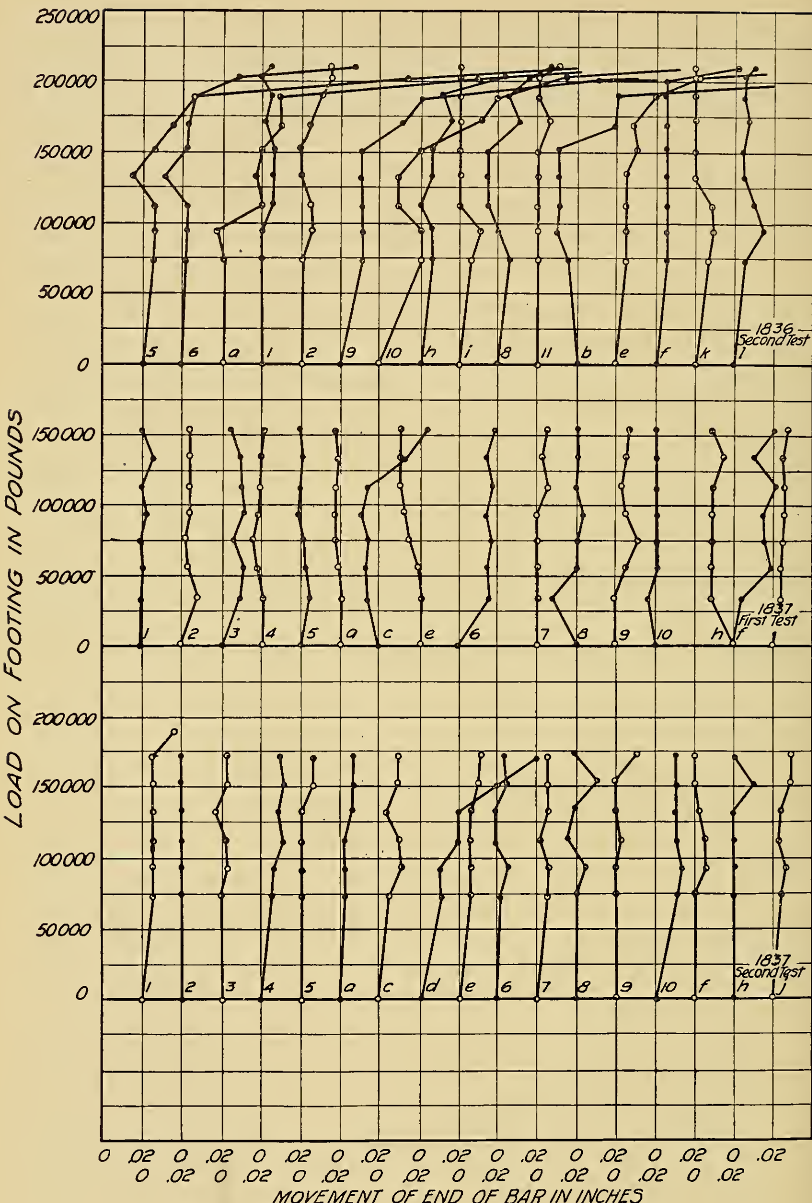


FIG. 34. DIAGRAM SHOWING END SLIP OF BARS.

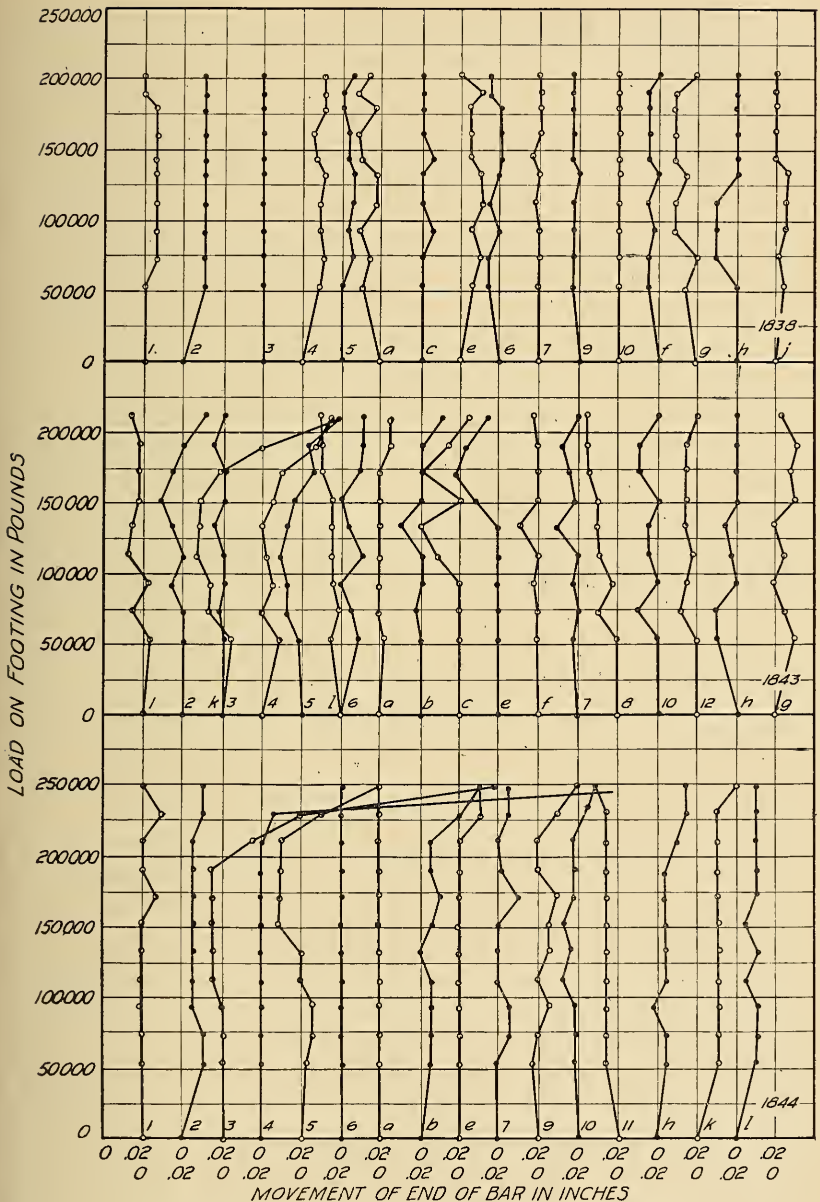


FIG. 35. DIAGRAM SHOWING END SLIP OF BARS.

TABLE 20.

Location of Cracks and Slips at Faces of Column Footings

No.	Diameter of Bars inches	Direction of: Top Layer	Direction of: Bottom Layer	First Cracks: Load pounds	First Cracks: No. of Bar	First Cracks: Face	First Cracks: Distance from Left Edge inches	End Slip: Load pounds	End Slip: Layer	End Slip: No. of Bar	End Slip: Face	End Slip: Distance from Left Edge inches	End Slip: Calculated Bond Stress lb. per sq. in.
1832	5/8	E & W	N & S	93 000	3	N	26	134 000	Bottom	3	N	26	295
				114 000	9	S	34	134 000	do.	9	S	34	295
				do.	10	S	34	152 000	do.	2	S	18	335
				do.	b	W		do.	Top	b	W	18	335
								do.	do.	8	N	18	335
1834	3/8	......	......	114 000		N	40	113 000	......	2	N	10	252*
						N	18	113 000	......	6	S	10	252*
1835	3/8	N & S	E & W	114 000	3	N	20	74 000	Top	3	N	20	100*
								95 000	Bottom	d	W	20	128*
1836	3/8	......	......	114 000		N	28	54 000	......	3	N	20	72*
				132 000		S	24	144 000	......	9	S	20	194*
				do.		S	36	144 000	......	10	S	40	194*
				do.	3	N	18						
1837†	3/8	N & S	E & W	73 000	d	W	24	112 000	Bottom	c	W	30	116
1837‡	3/8	......	......	......	..	..	....	120 000	Bottom	d§	W	24	124
1838	3/8	N & S	E & W	......	..	..	....	‖	......	....	...	....	....
1843	5/8 *a*	E & W	N & S	53 000	e	W	41¼	113 000	Bottom	5	N	41¼	252
				93 000	3	N	26¼	133 000	Bottom	4	N	33¼	295
				93 000	9	S	26¼	152 000	do.	3	N	26¼	337
								152 000	do.	2	N	18¾	337
1844	5/8 *a*	E & W	N & S	93 000	3	N	26¼	190 000	Bottom	3	N	26¼	420
				93 000	9	S	26¼	211 000	do.	9	S	26¼	467
				113 000	5	N	41¼	do.	do.	5	N	41¼	do.
				do.	4	N	33¼	do.	do.	4	N	33¼	do.
				do.	10	S	3¾	do.	do.	10	S	33¼	do.
								do.	do.	7	S	3¾	do.
								do.	Top	b	W	18¾	do.
								do.	do.	e	W	41¼	do.

* Bars placed in bands near edges of footing. † First test. ‡ Second test. § Readings at *d* not taken in first test. First slip occurred here in second test.
‖ Readings showed no slip until failure. *a* Corrugated round bars. All others plain round.

ings there seemed to be some relation between the formation of these cracks and the slipping of bars but the connection was not so close. In footing No. 1844 the first five points where slip occurred were coincident with the first five cracks detected. It may be expected that the loosening effect would begin as soon as any load is applied, and slip of bars was observed in two cases before any cracks had been detected. In all other cases a crack was found at a load lower than that at which slip occurred at the same point.

In all but one instance the bar in which first slip occurred was a bar which was located nearer the edge of the pier than any other on which measurement was taken. This was true whether the bars were spaced over the whole footing, grouped in the space between the edge of the pier and the edge of the footing, or confined to single bands somewhat wider than the pier. It may be noted then that the bar which showed the most marked tendency to slip lies in the vertical section for which the stress in the bars at right angles to those under consideration appears to be a maximum. Evidently stress in one system of bars tends to reduce the bond resistance of the bars at right angles, and the results are in keeping with the assumption that the critical section is at the face of the pier.

Attention should be called to the fact that the method of calculation of bond stress is not applicable to footings in which the reinforcement is placed in exterior bands, as is indicated by the very low values for No. 1834, 1835, and 1836. Footing No. 1837, in which the reinforcement was placed in central bands, was loaded twice, the first time to a load of 162 000 lb. and the second time to failure which occurred at 192 000 lb. On the first loading bar c gave indications of slip at 110 000 lb., and on second loading it showed no slip but bar d gave indications of slip at 120 000 lb. No other bar gave slip measurement. A crack had formed along bar d at a load of 73 000 lb. In the companion footing, No. 1838, no slip was observed until failure at a load of 247 000 lb.

32. *Reinforced Concrete Column Footings: Diagonal Tension Failures.*—The four faces of fracture found in the failures here named diagonal tension failures extended from the pier at the top of the footing at an angle of about 45° with the vertical to the bottom surface of the footing, forming a frustum of a square pyramid having the corners or edges rounded off somewhat. As the diagonal tension cracks would begin at or above the longitudinal reinforcement it seems a reasonable procedure to take as a measure of the diagonal tension stress the vertical shearing stress obtained by using the vertical sections located at a

TABLE 21.

VALUES OF VERTICAL SHEARING STRESS DEVELOPED IN COLUMN FOOTINGS

Footing No.	Reinforcement		Calculated Vertical Shearing Stress lb. per sq. in.	Manner of Failure
	Kind	Per cent		
1447	Plain round bars	0.31	130	Diagonal tension
1448	do.	-0.31	110	do.
1531	do.	0.59	180	do.
1532	do.	0.59	162	do.
1808	do.	0.50	124	do.
1810	do.	0.50	148	Tension followed by diagonal tension
1811	do.	0.61	164	Diagonal tension
1435	Cor. square bars	0.61	134	do.
1436	do.	0.67	113	do.
1552	do.	0.42	149	do.
1554	do.	0.62	185	do.
1820	do.	0.42	111	do.
1821	do.	0.42	99	do.
1551	Cor. round bars	0.42	142	do.
1553	do.	0.62	211	do.
1818	do.	0.39	123	do.
1843	do.	0.41	141	do.
1521*	Plain round bars	0.58	158	Tension
1522†	do.	0.56	156	Bond and possible tension
1525‡	do.	0.78	144	Bond
1526‡	do.	0.78	194	Bond; tension probably imminent
1542‡	do.	1.23	222	Bond
1807	do.	0.41	130	Tension
1809	do.	0.50	148	Tension followed by diagonal tension
1814	do.	0.72	192	Bond
1815	do.	0.72	187	Diagonal tension and bond
1823	do.	0.41	131	Tension
1836	do.	0.41	132	Bond
1838	do.	0.41	154	Tension followed by sudden bond failure
1839	do.	0.41	119	Tension followed by diagonal tension
1840	do.	0.41	125	do.
1841	do.	0.41	116	do.
1842	do.	0.41	126	Tension
1819	Cor. round bars	0.39	162	Tension followed by diagonal tension
1844	do.	0.41	167	Bond followed by diagonal tension

* 6¾ in. to center of steel. † 7 in. to center of steel. ‡ 5 in. to center of steel.

distance from the face of the pier equal to the depth of the steel reinforcement from the upper surface of the footing and to use as a length of section the four sides of the square base thus formed. The external vertical shear at this section would be the amount of load or upward pressure on the footing outside of this square base. The procedure in getting a measure of the diagonal tension is analogous to that used in ordinary beams, and the position of the section is analogous to that used in wall footings described on page 63. The formula for the vertical shearing stress already given on page 24 is

$$v = \frac{V}{4(a+2d)jd}.$$

Values of the vertical shearing stress thus calculated are given in Table 21. The values found seem to be fairly consistent with the results

obtained in beam tests. Higher values are noticeable with the larger percentage of reinforcement. This perhaps is explainable by the greater stiffness given by the larger reinforcement, as has been noted in the results for diagonal tension in ordinary beams. (See Bulletin No. 29.) The values obtained with the deformed bars were not greatly different from those with the plain rounds.

Attention should be called to the probability that the method here used of placing the critical section for diagonal tension may not be applicable in the case of stepped and built-up footings.

33. *Disposition of Reinforcing Bars.*—A variety of arrangement of reinforcing bars was used. In the two-way reinforcement the usual disposition of bars was to space uniformly across the full width of the footing, but in some cases a closer spacing was made across the middle portion and the remaining bars were spaced farther apart. Footings were also made with the bars spaced uniformly over a width somewhat greater than the width of the pier and no bars outside of this, thus making what may be considered as two central beams. In other footings the bars were placed in bands at the outer edge of the footing with no bars in the interior. In one set bars of a shorter length were used, and these were staggered in such a way that alternate bars ended near the face of the footing. Four-way reinforcement was also tried as shown in Fig. 26 and 27.

Uniform spacing of bars was used in the effort to determine the proportion of the reinforcement which may be considered to be effective in resisting the calculated bending moment or the amount to be used in the calculations to determine the stress in the most stressed bars. Judging from the calculated stresses in the footings of this kind which failed by tension in the steel, for footings of the proportions tested, about three-fourths of the steel is effective in resisting the calculated bending moment, or rather the stress in the highest stressed bars is the same as if three-fourths of the steel bars, equally stressed, made up the resisting steel.

The footings which had the reinforcement placed in the form of two central beams carried high loads. In No. 1837 the bond stress was the critical stress and in No. 1838 (which was made of unusually strong concrete, the test cubes giving an average strength of 3710 lb. per sq. in. at an age of 109 days) the yield point of the steel was exceeded and this was followed immediately by a bond failure. It seems probable that the bars were stressed nearly equally. It is obvious that with the central beam construction the corners outside the main reinforcement

must have the strength to carry the loads coming on this portion. In the case of No. 1838 it would seem that the tensile deformations in the concrete in the corner squares must have been very high. In fact, from another point of view, it does not seem probable that the steel in the bands could have reached its yield point at a section through the face of the pier without the concrete in the unreinforced space near it being stretched far beyond the ordinary limit of deformation of plain concrete. It seems probable that cracks would form in this space and that these cracks would preclude the development of the resistance needed in the corner square. The action in this portion of the footing warrants further study.

The footings in which the reinforcement was placed in outer bands and which had no reinforcement under the piers were made with a view of getting light on general footing action; it was not expected that this would be an effective way of placing reinforcement. It will be noted that No. 1822 and 1823 carried as high a load as the two footings with evenly spaced reinforcement made the same year (1911), No. 1806 and 1807. Similarly, in the series of 1912, No. 1835 and 1836 carried as high loads as No. 1831 and 1832, but No. 1833 and 1834 carried smaller loads. Of course the conditions for bond resistance were worse for the $\frac{5}{8}$-in. bars. The calculations for stresses given in Tables 17 and 18 for the footings with outer bands were made by using the same proportion of the bars as effective as was used for the footings with evenly spaced reinforcement ($\frac{46}{60}$), merely as a means of comparison and for want of any definite method of calculation which would be applicable to this kind of spacing, and the calculated bond stresses for the different dispositions of bars may not be comparable. It is seen that this method of calculation does not deal with the bending moment about a diagonal of the footing which for reinforcement in exterior bands may become an important consideration. The tests do not give information on the relation between the stresses in the different bars of any band. The tests bring out two points of interest: (1) the loads carried with this disposition of the reinforcing bars are large in comparison with what might be judged from the ordinary analyses and discussions which have appeared in engineering literature; and (2) there is seemingly a greater tendency to failure by bond when the bars are placed in bands near the edge of the footing, as is shown by the results in the bond failures in No. 1833, 1834, 1835 and 1836. The latter condition may result from a concentration of bond stress near the end of the bars. It goes to show the difficulties connected with the

calculation of bond stress in slabs. The location of the bars in which slip was first detected has already been discussed under the head of "Bond Failures."

A form of footing in which short bars are placed with their ends staggered, as shown in Fig. 25, is in line with designs which have been used in practice. This arrangement of bars is defended on the ground that there is the full amount of steel at the critical section and that there is no need of carrying all the bars to the face of the footing. In No. 1415 and 1416 one end of the bar extended to within 3 in. of the face of the footing while the other end was 12 in. from the face, the next bar alternating in position with this. As was to be expected this arrangement gave less bond resistance, and the footings failed by bond at lower loads than those in which the bars were made full length. It is hardly necessary to make the comment that this form of construction is not good practice, especially when the dimensions are such that resistance to bond stresses forms an essential part of the strength of the structure.

In the footings with the reinforcement placed in four directions (four-way reinforcement), the total weight of steel in the diagonal direction in the 1909 tests was made about the same as that in the other directions. In the calculations for No. 1447, 1448, and 1449, for want of a better method, the bending moment has been computed by the usual methods of the bulletin and one-half of this bending moment has been considered to be taken by one set of rectangular reinforcement. In the 1910 footings a larger amount of steel was used. Of these, No. 1561 carried a very high load. The significance of the results is obscured by the variety of manner of failure (bond, diagonal tension, and tension) and by variations in the quality of concrete, and a comparison with two-way reinforcement on the basis of load carried would not be of value. This type of distribution of reinforcement should receive further attention, and tests may well involve the measurement of deformation in the reinforcing bars.

The footings having the reinforcing bars looped in a horizontal plane (No. 1839 and No. 1840) developed high calculated bond stresses. Those having the reinforcing bars bent upward and backward in vertical planes (No. 1841 and No. 1842) also failed by tension, but it will be seen that the opening of vertical cracks will tend to reduce the effect of this kind of anchorage.

IV. Summary

34. *Wall Footings.*—The tests of wall footings cover a variety of reinforcement. The method used to secure a distributed upward pressure introduced difficulties in testing. It also made it difficult to determine the load which should be taken as the critical load, and the loads which have been so specified may not always be the true critical load. The use of the bed of springs on the whole proved very satisfactory and is probably the best available arrangement for tests of the number and range used. The tests bring out phenomena which might not be apparent from analytical considerations alone or which might not be accepted without physical verification. Variations in concrete add to the complications encountered in analyzing such a series of tests. The tables and diagrams and discussions present information and data of the tests in a detailed way. The following statements summarize in a general way some of the points which are brought out by the tests and which have a bearing upon the principles and methods of design:

1. Wall footings under load follow the general laws of flexure. The section for maximum moment, the critical section for calculation of vertical shearing stress for use in judging of resistance to diagonal tension, and the method of calculating bond stress received experimental consideration.

2. The values of the modulus of rupture found in the unreinforced concrete footings are not far from the values of modulus of rupture obtained in simple beam tests such as the control beams. Increasing the richness of the mixture gives the added strength which tests of simple beams would lead us to expect. Variations in the tensile strength of concrete are to be expected, and considerable variation was found in the moduli of rupture of the test pieces, the variation being augmented by differences incident to the method of testing. The tests on footings of different lengths, undertaken to determine whether the section at the face of the wall should be used for the critical section, do not disclose any marked differences in modulus of rupture.

3. The results of the tests and the measurements of deformation of the reinforcement indicate that the critical section may be considered to be at the face of the wall and that the calculated tensile stress in the bars at this section is probably somewhat above the maximum tensile stress developed. Whether the maximum compressive stress may properly be calculated in the same way was not determined. It may be expected that high compressive stresses exist at the intersection of

wall and projection. Indications of high compression and of incipient compression failure were found at the intersection of the wall and footing at loads above the critical loads.

Test pieces in which the wall was poured after the footing had taken its set, gave results which indicate that a section at the face of wall may properly be used in calculations of moments even when the wall is to be poured separately from the footing.

4. The calculations for bond stress, based upon the total external vertical shear at the section at the face of the wall and calculated by equation (17), evidently give stresses higher than the existing stresses. This is shown by the fact that the values calculated in this way are higher than those found in pull-out tests and beam tests. A study of the analytical conditions existing at this section tends to confirm the statement. However, as bond resistance is so important a strength element in a short cantilever beam, this method of calculation and the use of the working value of bond stress ordinarily assumed in design seems only reasonably conservative and may be recommended for general practice. Attention may properly be called to the importance of making calculation of bond stress in wall footings and other beams in which the length is short relatively to the depth. The advantage of using relatively small bars in such cases is also apparent.

Anchorage of bars by bending upward and back in a long curve or by looping the bar in a horizontal plane was found to add materially to bond resistance.

5. The tests indicate that the vertical shearing stresses developed at the face of the wall, calculated by the usual method, are higher than the vertical shearing stress which is found to exist in simple beams with concentrated loading when diagonal tension failures are developed. It was found that diagonal tension failures start at a point some distance away from the section at the face of the wall. This observation and certain analytical considerations such as the probable greater proportion of shear taken in the compression area at sections near the face of the wall show that, in calculating the vertical shearing stress which shall be used as a basis for judging the resistance to diagonal tension, a section some distance from the face of the wall should be used. The tests and the discussion indicate that a section d distant from the face of the wall (d being the distance from center of reinforcing bar to top of footing) may properly be used as the critical section for calculating the vertical shearing stress for this purpose, and that at this section the ordinarily accepted working stress may properly be used for calculating resistance to diagonal tension failure.

6. The bending up of bars at several points along the length of the projection gave added resistance against diagonal tension failure. Vertical stirrups also added to the resistance against diagonal tension failure but were not especially effective. Neither method of web reinforcement would be very convenient in construction. Generally speaking, it will be best to try to design the footing so that the vertical shearing stresses will be within the limit of the working stress permitted in beams without web reinforcement, and thus avoid the use of web reinforcement. In large important footings, when diagonal tension is a critical element, it would seem that some kind of unit frame with well-formed web reinforcement would be preferable to placing stirrups or to bending up bars at the necessary intervals. In stepped and sloping footings attention should be called to the larger diagonal tension and bond stresses developed. The increase in these stresses over those found in footings of uniform depth may be sufficient to decide against the use of stepped and sloping footings.

7. The footings having I-beams embedded in the concrete carried high loads, perhaps corresponding to the yield-point tensile strength of the lower flange of the I-beams and more than double what would be carried by naked I-beams. The weight of the I-beams, of course, was greater than that of the reinforcing bars used in the reinforced concrete wall footings.

35. *Column Footings.*—The requirement of uniform load and the presence of double-curved flexure complicate an investigation of column footings. In this investigation methods of testing were developed. As these are presumably the first tests on column footings, the phenomena of the tests and data of their action will be of interest to designers, especially in the directions in which tests have brought out weaknesses not always recognized and usually not guarded against. The results contribute data toward the settlement of methods of calculating of both the bending moment and the resisting moment for square footings, and the principles may with care be extended to other forms. The results may not easily be summarized, but the following statements are intended to cover the principal matters brought out in the tests:

1. A square column footing under load may be expected to take a bowl-shaped form. In slabs subject to bending in two directions, the stress in a fiber can not differ from that in an adjoining fiber at the same level without setting up longitudinal shear; and as there is considerable resistance to variation from equality of stress in adjoining fibers, it may be expected that in stiff thick pieces (as are footings

of ordinary design, where the thickness is large in comparison with the length of the projection) the deformations and consequent stresses will be distributed over the width of a cross section and that considerable stress will be developed even in the fibers at the edge of the footing.

2. For footings having projections of ordinary dimensions, the critical section for the bending moment for one direction (which in two-way reinforced concrete footings is to be resisted by one set of bars) may be taken to be at a vertical section passing through the face of the pier. In calculating this moment, all the upward load on the rectangle lying between a face of the pier and the edge of the footing is considered to act at a center of pressure located at a point half-way out from the pier, and half of the upward load on the two corner squares is considered to act at a center of pressure located at a point six-tenths of the width of the projection from the given section. By equating this bending moment and the resisting moment which is available at the given section, the maximum tensile stress in the concrete or in the reinforcing bars may be calculated.

3. As is usually the case when plain concrete is used in flexure, the unreinforced footings show considerable variation in results. The variations were such as not to permit a method of determining the effective width of resisting section to be established or to obtain a formula for resisting moment. Based upon the full section of the footing, the moduli of rupture obtained were considerably less than the moduli of rupture of control beams made with the same concrete.

4. In reinforced concrete column footings, resistance to non-uniformity of stress in adjoining bars will be given by bond and by longitudinal shear in the concrete, and the amount of variation from uniformity of stress in the various bars will depend upon the spacing of the bars as well as upon the relative dimensions of the footing. With two-way reinforcement evenly spaced over the footing, it seems that the tensile stress is approximately the same in bars lying within a space somewhat greater than the width of the pier and that there is also considerable stress in the bars which lie near the edges of the footing. For intermediate bars stresses intermediate in amount will be developed. For footings having two-way reinforcement spaced uniformly over the footing, the method proposed for determining the maximum tensile stress in the reinforcing bars, is to use in the calculation of resisting moment at a section at the face of the pier the area of all the bars which lie within a width of footing equal to the width of pier plus twice the thickness of footing, plus half the remaining distance on each

side to the edge of the footing. This method gives results in keeping with the results of tests. When the spacing through the middle of the width of the footing is closer, or even when the bars are concentrated in the middle portion, the same method may be applied without serious error. Enough reinforcement should be placed in the outer portion to prevent the concentration of tension cracks in the concrete and to provide for other distribution stress.

5. The method proposed for calculating maximum bond stress in column footings having two-way reinforcement evenly spaced, or spaced as noted in the preceding paragraph, is to use the ordinary bond stress formula, and to consider the circumference of all the bars which were used in the calculation of tensile stress, and to take for the external shear that amount of upward pressure or load which was used in the calculation of the bending moment at the given section.

An important conclusion of the tests is that bond resistance is one of the most important features of strength of column footings, and probably much more important than has been appreciated by the average designer. The calculations of bond stress in footings of ordinary dimensions where large reinforcing bars are used show that the bond stress may be the governing element of strength. The tests show that in multiple-way reinforcement a special phenomenon affects the problem and that lower bond resistance may be found in footings than in beams. Longitudinal cracks form under and along the reinforcing bar due to the stretch in the reinforcing bars which extend in another direction, and these cracks act to reduce the bond resistance. The development of these cracks along the reinforcing bars must be expected in service under high tensile stresses, and low working bond stresses should be selected. An advantage will be found in placing under the bars a thickness of concrete of two inches, or better three inches, for footings of the size ordinarily used in buildings.

Difficulty may be found in providing the necessary bond resistance, and this points to an advantage in the use of bars of small size, even if they must be closely spaced. Generally speaking, bars of ¾-in. size or smaller will be found to serve the purpose of footings of usual dimensions. The use of large bars, because of ease in placing, leads to the construction of footings which are insecure in bond resistance. In the tests the column footings which were reinforced with deformed bars developed high bond resistance. Curving the bar upward and backward at the end increased the bond resistance, but this form is awkward in construction. Reinforcement formed by bending long bars in a series

of horizontal loops covering the whole footing gave a footing with high bond resistance.

6. As a means of measuring resistance to diagonal tension failure, the vertical shearing stress calculated by using the vertical sections formed upon the square which lies at a distance from the face of the pier equal to the depth of the footing was used. This calculation gives values of the shearing stress, for the footings which failed by diagonal tension, which agree fairly closely with the values which have been obtained in tests of simple beams. The formula used in this calculation is $v=\frac{V}{bjd}$, where V is the total vertical shear at this section taken to be equal to the upward pressure on the area of the footing outside of the section considered, b is the total distance around the four sides of the section, and jd is the distance from the center of reinforcing bars to the center of the compressive stresses. This stress is somewhat larger than the average vertical shear over the section which is sometimes used. The working stress now frequently specified for this purpose in the design of beams, 40 lb. per sq. in., for 1-2-4 concrete, may be applied to the design of footings.

The punching shear may be calculated for the vertical sections which inclose the pier footing, although it may be expected that shear failure may not be produced exactly on this section. The value now generally accepted for punching shear, 120 lb. per sq. in. for 1-2-4 concrete, may be used for the working stress in this case.

7. No failures of concrete in compression were observed, and none would be expected with the low percentages of reinforcement used. The compressive stresses in the pier of the footing were in some cases very high and in a few instances the pier failed and was replaced by a cube of concrete. In frequent cases there were signs of distress near the intersection of pier and footing where there is an abrupt change in direction of surfaces and where the combined stresses are very high.

8. In stepped footings, the abrupt change in the value of the arm of the resisting moment at the point where the depth of footing changes may be expected to produce a correspondingly abrupt increase of stress in the reinforcing bars. Where the step is large in comparison with the projection, the bond stress must become abnormally large. It is evident that the distribution of bond stress is quite different from that in a footing of uniform thickness. The sloped footing also gives a distribution of stress which is different from that in a footing of uniform thickness. However, for footings of uniform thickness the bond stress

is a maximum at the section at the face of the pier; in a sloped footing the bond stress at the section at the face of the pier would be less accordingly than in a footing of uniform thickness, and a moderate slope may be found to distribute the bond stress more uniformly throughout the length of the bar. This is not of advantage if the full embedment of the bar is effective in resisting any pull due to bond.

9. The use of short bars placed with their ends staggered increases the tendency to fail by bond and cannot be considered as acceptable practice in footings of ordinary proportions. In footings in which the projection is short in comparison with the depth the objection is very great.

10. Footings having reinforcement placed in the direction of the diagonals as well as parallel to the sides (four-way reinforcement) gave good tests. The significance of the results is so obscured by the variety of manner of failure (bond, diagonal tension, and perhaps tension) and by variations in the quality of the concrete, that a comparison with two-way reinforcement on the basis of loads carried would not be of value. This type of distribution of reinforcement should be included in further tests. Measurements of deformation in the bars are needed to determine the division of stress among the four sets of bars.

36. *Concluding Remarks.*—The tests of wall footings and column footings leave uncertainty in some parts of the problem and there are gaps in other parts. The recent development of the portable extensometer or strain gage and the skill and experience which have been gained in its use in recent tests have opened opportunities for obtaining information on the stresses developed in such test pieces which were not available when the series of tests was undertaken. It is suggested that some of the remaining unsolved problems may most readily be attacked by measurement of deformations in the steel and concrete, and that further investigation may best be carried on by constructing a form of apparatus which will permit such measurements to be taken under the conditions of uniform loading.

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BY

HERBERT F. MOORE

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By Herbert F. Moore, Assistant Professor in Theoretical and Applied Mechanics

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LIST OF TABLES.

THE STRENGTH OF I-BEAMS IN FLEXURE.

1. *Introduction.*—The mathematical theory of the resistance of materials in flexure has been extensively developed, but much less has been done in the experimental study of the phenomena of flexural stress. A striking fact brought out in the tests which have been made is the tendency of metal beams to fail by reason of column action in fibers which are under compressive stress. This tendency is especially strong in I-beams, channel-beams, and other forms of beams having tension and compression flanges connected by a comparatively thin web. The tests of Marburg*, Christie†, and Burr and Elmore‡ show that this column action in I-beams may cause failure of test pieces by sidewise buckling or on account of excessive stresses in the web. These tests emphasize the importance of taking into account of stresses other than the direct flexure stresses in the flanges.

The wide-spread use of I-beams as flexural members makes the subject of their flexural strength a matter of general engineering interest. To obtain experimental data on the action of I-beams under load, several series of tests of I-beams were carried out in the Laboratory of Applied Mechanics of the University of Illinois. This bulletin records and discusses the results of these tests and of others of similar kind. A formula is deduced for the flexural strength of I-beams which are not restrained against sidewise buckling. There also is given a discussion of the stiffness of I-beams, a discussion of the action of I-beams restrained against sidewise buckling and restrained against twisting at the ends of the beams, and a discussion of web failure of I-beams.

2. *Acknowledgment.*—The experimental work was a part of the research work of the department of Theoretical and Applied Mechanics, and was done under the general direction of the head of that department, Professor A. N. Talbot. The tests were made under the direct supervision of the writer. Acknowledgment is hereby made to the following students in civil engineering who assisted in making tests: F. J. Weston and W. E. Deuchler of the class of 1910, and M. H. Froelich and F. C. Lohman of the class of 1911. Analytical methods devised by various investigators have been used in this bulletin, and experimental data from various sources have been quoted; in all cases an attempt has been made to give due credit for methods and data.

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*Proceedings of the American Society for Testing Materials, Vol. IX (1909), p. 378.

†Pencoyd Steel Handbook (1898 Edition), p. 23.

‡Selected Papers of the Rensselaer Society of Engineers, Vol. 1, No. 1. An abstract of the results of these tests is given in Burr's "Elasticity and Resistance of the Materials of Engineering," p. 694.

3. *Phenomena of Flexural Failure.*—The formulas commonly used for computing the stresses and deflections in beams are based on the assumptions (1) that a plane cross-section of a beam remains plane during flexure and (2) that the moduli of elasticity of the beam material for tension and for compression are equal and constant. For low stresses and for beams of medium or long spans these assumptions are very nearly exact. They become rough approximations when a beam is loaded to a point near failure.

The failure of beams of brittle material usually occurs by snapping of the extreme tension fibers at a computed fiber stress higher than the tensile strength of the material as determined by tension tests of specimens. Brittle material is nearly always much weaker in tension than in compression. As the fiber stress in beams of such material increases with increasing load, by reason of the change which takes place in the values of the moduli of elasticity, the tension side of the beam stretches more readily than the compression side shortens. The effect is to shift the neutral axis of the beam toward the compression side. This, together with the difference in strength, causes the actual tensile fiber stress to be less than the computed tensile fiber stress.

The failure of beams of ductile material may take place in one of a number of ways:

(1) The beam may fail by direct flexure. Under increasing load the usual flexure formulas are very nearly exact up to a load which stresses the extreme fibers of the beam to the yield-point strength of the material. When the yield point is reached in the extreme fibers, the deflection of the beam increases more rapidly with respect to an increase of load; and if the beam is of a thick, stocky section or is firmly held so that it can not twist or buckle, failure takes place by a gradual sagging which finally becomes so great that the usefulness of the beam as a supporting member is destroyed.

(2) In a beam of long span, the compression fibers act somewhat as do the compression fibers of a column, and failure may take place by buckling. Buckling failure, in general occurs in a sidewise direction. Sidewise buckling may be either the primary or the secondary cause of failure. In a beam in which excessive flexural stress is the primary cause of failure and in which the beam is not firmly held against sidewise buckling, the primary overstress may be quickly followed by the collapse of the beam due to sidewise buckling. The sidewise resisting strength of a beam is greatly lessened if its extreme fibers are stressed to the yield point. Sidewise buckling may in some cases be a primary cause of beam failure, in which cases the computed fiber

stress, in general, does not reach the yield-point strength of the material. Sidewise buckling not infrequently limits the strength of narrow, deep beams, especially beams of I-section or channel-section with tension and compression flanges connected by a thin web. Whether it is a primary cause of failure or a final manner of failure, sidewise buckling results in a clearly marked and generally quite sudden failure of a beam.

(3) Failure in an I-beam or a channel-beam may occur by excessive shearing stress in the web, or by buckling of the web under the compressive stresses which always accompany shearing stress. If the shearing fiber stress in the web reaches a value as great as the yield-point strength of the material in shear, beam failure may be expected and the manner of failure will probably be by some secondary buckling or twisting action. The inclined compressive stress always accompaning shear may reach so high a value that the buckling of web of the beam is a primary cause of failure. Danger of web failure as a primary cause of beam failure exists, in general, only for short beams with thin webs.

(4) In the parts of beams adjacent to bearing blocks which transmit concentrated loads or reactions to beams, high compressive stresses may be set up, and in I-beams or channel-beams the local stress in that part of the web nearest a bearing block may become excessive. If this local stress exceeds the yield-point strength of the material at the junction of web and flange, the beam may fail primarily on account of the yielding of the overstressed part and finally by a resulting twisting action of the beam.

4. *Earlier Tests of I-beams.*—Data of a considerable number of tests of I-beams have been published. A list of some important tests with references follows:

(1) Tests of twenty 6-in. wrought-iron I-beams loaded at the midpoint of the span and so held in the testing machine as to be free to buckle sidewise. The spans varied from 4 ft. to 20 ft. These tests were made by Burr and Elmore at the Rensselaer Polytechnic Institute and are reported in "Selected Papers of the Rensselaer Society of Engineers," Vol. 1, No. 1. An abstract of the results is given in Burr's "Elasticity and Resistance of the Materials of Engineering," 1905 edition, p. 694.

(2) A series of tests on wrought-iron I-beams made by Tetmajer at the Materialpruefungsanstalt at Zurich, Switzerland. The results of the tests are given in Heft IV. of the Mitteilungen of that laboratory. The results of nine of the tests are given in Lanza's "Applied Mechanics," 1905 edition, p. 443.

(3) A series of tests of twenty-one steel I-beams made by Christie at the Pencoyd Iron Works. In these tests the beams were somewhat restrained from sidewise buckling by the friction of the bearing blocks which were directly attached to heads of the testing machine. These tests were reported by Mr. Christie in the Transactions of the American Society of Civil Engineers for 1884. A summary of results is given in Burr's "Elasticity and Resistance of Materials of Engineering," 1905 edition, p. 689.

(4) Tests of wrought-iron and of steel I-beams made at the Massachusetts Institute of Technology. Results of twenty-nine such tests are reported in Lanza's "Applied Mechanics," 1905 edition, p. 444 and p. 497.

(5) Tests of thirty-one steel I-beams and rolled girders made by Marburg at the University of Pennsylvania. These tests included I-beams of standard cross-section, I-beams with specially wide flanges, rolled by the Bethlehem Steel Company, and broad-flanged girder beams rolled by the same company. In Marburg's tests great care was taken to secure the greatest freedom of motion possible for the beam in the testing machine. The tests are reported in the Proceedings of the American Society for Testing Materials for 1909, p. 378.

(6) A test of a built-up plate girder made by Turneaure at the University of Wisconsin. This girder was so designed and tested that failure occurred by buckling of the web.

The results of the first five series of tests are summarized in Table 2. The last named test is reported in the Journal of the Western Society of Engineers for 1907, p. 788, and a summary of results is given in Table 8.

5. *I-beam Tests at the University of Illinois.*—The tests made in the Laboratory of Applied Mechanics of the University of Illinois and described in this bulletin include forty steel I-beams. These tests are summarized in Tables 3, 5 and 8. The general features of the tests may be indicated as follows:

(1) Ten 8-in., 18-lb. I-beams (Tests 1, 2, 7, 8, 12, 13, 18, 19, 32 and 33) were tested under loads applied at the one-third points of the span with spans varying from 5 ft. to 20 ft. These beams were so held in the testing machine as to afford the maximum possible freedom of motion for the beam (see Fig. 4).

(2) Eight 8-in., 18-lb. I-beams (Tests 3, 4, 9, 10, 14, 15, 20, and 21) were tested under conditions similar to those described in (1) except that the ends of the beams were firmly held so that they could not twist. (See Fig. 6.)

(3) Seven pairs of 8-in., 18-lb. I-beams (Tests 5, 6, 11, 16, 17, 22, and 23) were tested under conditions similar to those described in (1) except that the beams were firmly restrained against sidewise buckling. (See Fig. 7.)

(4) Four 8-in., 18-lb. I-beams (Tests 24, 25, 26, and 27) were tested with 10-ft. span. Two beams were loaded at the mid-point of the span, and two were loaded at the one-sixth points of the span. The beams were not restrained against sidewise buckling or against end twisting action. (See Fig. 4.)

(5) Two 8-in., 25.25-lb. I-beams (Tests 28 and 29) were tested with loads at the one-third points of a 10-ft. span. The beams were without sidewise or end restraint. (See Fig. 4.)

(6) Two 8-in., 18-lb. I-beams (Test 31) were tested under a load continued on the beams for 107 days. The computed stresses under the applied load were about equal to the yield-point strength of the material. The beams were symmetrically loaded at two points in a span of 8 ft. 9¾ inches.

(7) One pair of 8-in., 18-lb. I-beams (Test 30) with their webs fastened together by separators were tested with loads at the one-third points of a 10-ft. span.

(8) Six 12-in., 31.5-lb. I-beams (Tests 34-39) were tested over a span of 3 feet, with two symmetrical loads near mid-span. Different web conditions were obtained by varying the thickness of web by planing down the webs of some beams. These beams all failed in the web.

Nearly all tests were made on a four-screw 200,000-lb. Olsen testing machine with long table for beam tests. The instrument used for measuring deflections consisted of a framework supported entirely on the test beam, fitted with an extensometer dial by means of which deflection could be measured to one one-thousandth inch. For measuring longitudinal fiber deformation in the beams various forms of extensometers were used. A strain gauge of the Berry type proved the most satisfactory extensometer for this purpose.

The I-beams tested were bought in the open market at various times, and the beams may be expected not to differ far from the range of material found in practice.

6. *Yield Point of Structural Steel in Tension and in Compression.*—For I-beams the yield-point strength of the steel is, perhaps, the most important physical characteristic. The tension test is the most usual test for determining the physical properties of structural steel, and the yield point reported (frequently but incorrectly called the "elastic limit") is the yield point in tension. As flexural members of

structural steel may fail through the yielding of the compression fibers, the yield point of structural steel in compression seemed worthy of investigation. A considerable amount of test data are available on this subject, and the conclusion seems fairly well established that for the softer grades of steel the yield-point determined for tension is about the same as the yield-point in compression.* This conclusion is corroborated by the results of tests made in connection with the investigation of I-beams made at the University of Illinois. Table 1 gives the values of the stress at yield-point in tension and in compression for specimens cut from the flanges of I-beams and from flat bars. Both in the compression tests and in the tension tests the specimens were held in wedge-shaped grips, a special head being used in the compression tests, as shown in Fig. 1. The specimens were of such length that column action was not noticeable below the yield-point in the compression specimens.

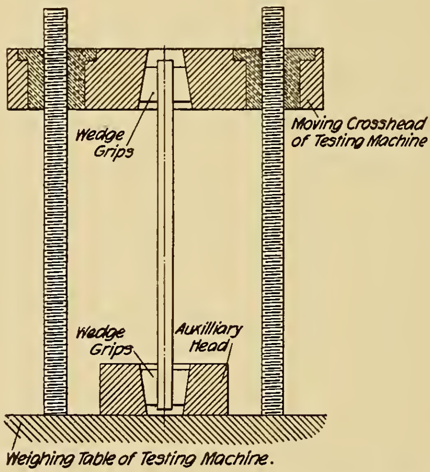

Fig. 1. Apparatus for Compression Tests of Steel.

The yield-point reported is the average of the values obtained by three methods: (1) the drop of the beam of the testing machine, which was well marked, as the speed used in testing below the yield-point was slow (0.1 in. per min.) and as the poise was easily kept in balance; (2) the "knee" of an autographic diagram of load and deformation, in which diagram the stretch or compression was magnified five times; and (3) the scaling of the specimens as shown by the flaking of the plaster of paris with which the specimen had been coated.

*J. B. Johnson, "The Materials of Construction," p. 502. This is a summary of tests by Chas. A. Marshall, and of tests at the Watertown Arsenal. Recent tests in British laboratories also show practically the same values for the yield-point strength of mild steel in tension and in compression.

7. *Failure of I-beams by Direct Flexure.*—In studying the failure of I-beams care must be exercised to distinguish between the primary failure and the final failure as judged by the shape of the beam after the test. Beams in which the primary cause of failure is excessive flexural stress not infrequently buckle sidewise *after* this excessive stress has weakened the flanges of the beam; in other cases the yielding of the flanges allows stress to be transferred to the web which then may twist or buckle. If a beam is held firmly against sidewise bending and has a thick web, the final failure under a load as applied in a testing machine will be by gradual sagging and the exact instant of failure will not be very clearly marked. Under excessive flexural fiber stress the time of application of load affects the deformation under load, and a beam may carry momentarily a load applied in a testing machine larger than that which if continued for several days would cause collapse of the beam.

In Table 2 are given test results obtained by various investigators in I-beam tests in which excessive flexural stress seems to have been the primary cause of failure. While it can not be certain that in every test the primary cause of failure was excessive flexural stress, yet, since every beam tabulated in Table 2 developed before failure computed stresses as high as might be expected for the yield-point strength of the material, and since in most cases friction of testing machine heads acted to prevent sidewise buckling, these tests appear to furnish a fairly satisfactory basis for the study of failure by direct flexure.

In Table 3 are given the test results for those I-beams tested at the University of Illinois for which excessive flexural fiber stress appeared to be the primary cause of failure. Figs. 10 to 15 inclusive (at the end of the text) give stress-deformation curves and stress-deflection curves for most of the beams tested at the University of Illinois.

A study of Table 2 and Table 3 shows that some beams which failed by direct flexure developed computed fiber stresses but little in excess of the usual values of yield-point strength of the material, while other beams developed momentarily stresses considerably higher. In Table 3 it will be seen that for nearly all the beams tested at the University of Illinois excessive deflection, large permanent set, or other sign of structural damage was observed at computed fiber stresses not much higher, if any, than the yield-point strength of the material. The load temporarily carried in a laboratory test depends in part on the speed of testing and the nature of the support of the beam in the testing machine. However, since a long-continued dead load or an oft-repeated live load would be more liable to injure a beam than an equal load ap-

plied by a testing machine, it is apparent that even under circumstances most favorable to the development of high fiber stresses, it is unsafe practice to regard as the ultimate fiber stress in flexure any value *higher* than the yield-point strength of the material of the beam.

Under long-continued static load the deformation of beams having their extreme fibers stressed to the yield-point of the material, increases for some time, frequently for several days, but the member does not necessarily fail. An illustration is furnished by a test made on two 8-in., 18-lb. steel I-beams loaded at two points each 12 inches to one side of the mid-point of an 8-ft. span. (Test No. 31, Fig. 15.) The load was applied to both beams until, as was shown by extensometers attached to the flanges, some fibers were stressed to the yield-point. Noticeable sidewise buckling of the beams had begun, and apparently failure was imminent. The load was kept constant for 107 days, and the extensometer on the most stressed flange was read from time to time. After a few days the fiber deformation reached a value practically constant, and the beam did not collapse during the test period. Thurston* reports tests showing similar results on transverse tests of steel of square section.

The excessive deflection of beams found when fibers are stressed beyond the yield-point, and the possibility of collapse emphasize the conclusion that for *I-beams the yield-point strength of the material in the flanges should be regarded as the ultimate fiber stress in flexure.* It is especially absurd to regard the ultimate tensile strength of steel as the ultimate fiber stress, as even under the most favorable conditions of service this fiber stress can not be developed before the I-beam collapses.

The results of those tests of I-beams in which failure occurred at stresses lower than the yield-point strength of the flange material due to sidewise buckling or to web failure, are tabulated and discussed in subsequent paragraphs.

8. *Inelastic Action of I-beams under Low Stress.*—Many recent writers on structural design have pointed out that in practically all steel structures the bending of beams and rods incidental to the erection of the structure and the use of drift pins in aligning rivet holes cause local stresses in excess of the yield-point strength of structural steel. These local stresses do not, however, cause the failure of the whole member. In Marburg's tests of I-beams and in those made at the University of Illinois frequent evidences of slight inelastic action at low stresses were observed. It is believed, however, that this inelastic action is the result of local stress and that it does not indicate the limit of load

*Thurston, "Text Book of the Materials of Construction," p. 516.

TABLE 1.

Yield Point of Structural Steel in Tension and in Compression.

Specimens cut from flanges of I-beams and from flat bars. The length of specimens was about 9 in., 4 in. between grips, the width was from 1 in. to 1.5 in., and the thickness 0.25 in. to 0.50 in. The specimens from flanges of I-beams were planed down till the cross-section was rectangular.

Specimen from	Number of Specimens		Fiber Stress at Yield Point, lb. per sq. in.		Ratio of Yield Point in Tension to Yield Point in Compression
	Tension	Compression	Tension	Compression	
I-beam 16a	1	2	34,700	34,500	1.00
I-beam 16b	2	2	34,100	34,800	0.98
I-beam 26	2	2	32,500	34,600	0.94
I-beam 27	2	2	34,200	34,400	0.99
I-beam 24	2	2	34,700	35,000	0.99
I-beam 17a	2	2	36,300	35,100	1.03
I-beam 17b	2	2	35,400	36,100	0.98
I-beam 25	2	1	31,100	35,400	0.88
I-beam 21	2	2	33,800	32,600	1.03
I-beam 23	2	2	37,900	40,300	0.94
I-beam 14	2	2	36,400	32,000	1.14
I-beam 15	2	2	33,800	33,500	1.01
Flat 1	1	1	41,700	46,000	0.91
Flat 2	1	1	42,500	42,900	0.99
Flat 3	1	1	41,400	40,000	1.03
Flat 4	1	1	39,700	39,500	1.00
Flat 5	1	1	40,700	39,000	1.04
Flat 6	1	1	38,500	37,500	1.02
Flat 7	1	1	39,600	44,800	0.88
Flat X	1	1	37,900	35,000	1.08
	31	31			Av. 0.993

carrying capacity for the beam as a whole. In support of this belief the following facts are presented:

(1) In any test of material if apparatus of sufficient delicacy is used inelastic action can be detected at comparatively low stresses.*

(2) The physical properties of the material in various places in an I-beam may vary considerably;† the material at the root of the flange is usually weaker than the material in the flange or than the material in the web, and inelastic action of the beam under low stresses may be due in part to yielding of this weaker material while the flange material can develop further strength.

(3) If the load which at first application causes inelastic action be removed and reapplied, this second cycle of loading and removal of load will usually show much less inelastic action than is shown during the first cycle of loading and unloading, and successive cycles will show

*Moore, "The Physical Significance of the Elastic Limit," Proceedings of the Sixth Congress (1912) of the International Association for Testing Materials.

†Marburg, Proceedings of the American Society for Testing Materials, Vol. IX (1909), p. 385.
Moore, Proceedings of the American Society for Testing Materials, Vol. X (1910), p. 235.
Hancock, Proceedings of the American Society for Testing Materials, Vol. X (1910), p. 248; Vol. XI (1911), p. 477.

gradual improvement of the elastic qualities of the beam. Fig. 2 shows the computed fiber stresses and the deflections observed with successive applications of load in a test of an 8-in. I-beam. (Test No. 13, Table 5.) The energy lost in inelastic action for a cycle of loading and unloading is shown by the shaded area. It will be seen that during the third cycle the action of the beam was almost perfectly elastic. Only

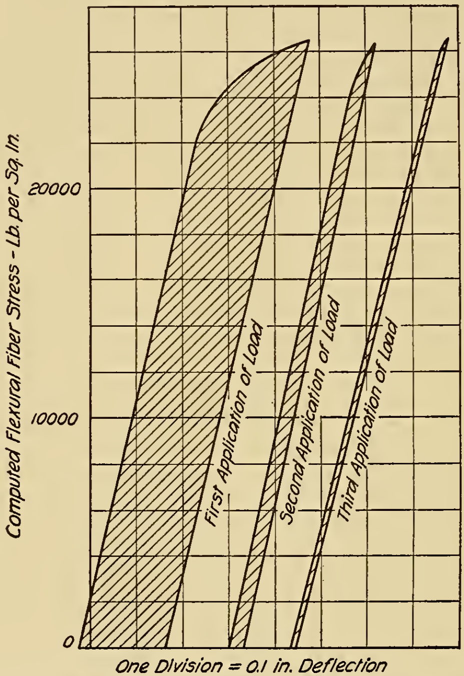

Fig. 2. Deflection of I-beam Under Repetitive Loading.

a few moments elapsed between successive loadings so that rest of material could not have played an important part in the result. Similar results were obtained in tests of seven other beams.

For static load or for loads repeated at infrequent intervals and always acting in the same direction, local inelastic action does not seem important. The writer has not been able to discover any record of the failure by fatigue of metal originally sound in a bridge, building, or other structure designed to carry static load, but in structures under loads applied successively in opposite directions or loads rapidly repeated many millions of times it might be expected that local inelastic action may cause minute cracks or miscroscopic flaws which, spreading, would eventually cause the failure of the structure by fatigue of metal.

TABLE 2.

TESTS OF I-BEAMS—PRIMARY FAILURE BY DIRECT FLEXURE.

Test Made by	Number of Beams Tested	Depth, inches	Section	Material	Span, feet	Loading	Computed Fiber Stress at Failure, lb. per sq. in.	Modulus of Elasticity of Beam, lb. per sq. in.†
Burr and Elmore.	2	6	Medium Wt.	Wrought iron	4.00	Mid-point*	45,100	22,800,000
Burr and Elmore.	2	6	Medium Wt.	Wrought iron	5.00	Mid-point*	43,600	23,600,000
Burr and Elmore.	2	6	Medium Wt.	Wrought iron	6.00	Mid-point*	41,500	22,000,000
Tetmajer	1	3.93	Light Wt.	Wrought iron	2.62	Mid-point	62,800	28,800,000
Tetmajer	1	5.91	Light Wt.	Wrought iron	3.96	Mid-point	56,500	28,200,000
Tetmajer	3	7.87	Light Wt.	Wrought iron	5.25	Mid-point	53,800	28,200,000
Tetmajer	1	9.45	Light Wt.	Wrought iron	6.30	Mid-point	51,600	27,400,000
Tetmajer	1	11.81	Light Wt.	Wrought iron	7.87	Mid-point	53,200	26,500,000
Tetmajer	1	13.39	Light Wt.	Wrought iron	8.93	Mid-point	53,900	27,700,000
Tetmajer	1	15.75	Light Wt.	Wrought iron	10.50	Mid-point	52,500	27,700,000
Lanza	1	5	Light Wt.	Wrought iron	12.00	Mid-point	46,600	29,500,000
Lanza	1	6	Medium Wt.	Wrought iron	12.00	Mid-point	42,400	26,700,000
Lanza	1	6	Heavy Wt.	Wrought iron	14.58	Mid-point	37,800	27,900,000
Lanza	2	7	Heavy Wt.	Wrought iron	12.89	Mid-point	42,400	26,800,000
Lanza	4	7	Heavy Wt.	Wrought iron	14.20	Mid-point	41,200	28,800,000
Lanza	2	8	Medium Wt.	Wrought iron	13.58	Mid-point	45,900	28,000,000
Lanza	3	8	Heavy Wt	Wrought iron	14.58	Mid-point	38,300	27,500,000
Lanza	1	9	Light Wt.	Wrought iron	13.67	Mid-point	45,800	27,400,000
Lanza	1	9	Medium Wt.	Wrought iron	14.50	Mid-point	41,500	27,000,000
Lanza	2	9	Heavy Wt.	Wrought iron	14.67	Mid-point	38,900	27,100,000
Lanza	1	6	Medium Wt.	Steel	14.58	Mid-point	44,900	28,200,000
Lanza	1	7	Medium Wt.	Steel	12.91	Mid-point	42,100	27,500,000
Lanza	1	7	Medium Wt.	Steel	14.50	Mid-point	42,900	29,000,000
Lanza	2	8	Light Wt.	Steel	14.54	Mid-point	45,300	29,200,000
Lanza	1	9	Light Wt.	Steel	14.50	Mid-point	40,200	29,900,000
Lanza	4	10	Medium Wt.	Steel	14.00	Mid-point	40,200	28,200,000
Christie	1	3	Medium Wt.	Steel	4.92	Mid-point	45,200	30,900,000
Christie	1	8	Medium Wt.	Steel	3.25	Mid-point	45,100	25,000,000
Christie	2	8	Heavy Wt.	Steel	20.00	Mid-point	44,200	29,000,000
Christie	1	8	Heavy Wt.	Steel	12.00	Mid-point	41,300	27,500,000
Marburg	3	15	Lt. Wt. Beth'm I-beam	Steel	15.00	Mid-point	46,100	26,900,000
Marburg	3	15	Lt. Wt. Stand. I-beam	Steel	15.00	Mid-point	42,200	26,200,000
Marburg	3	15	Lt. Wt. Beth'm Girder	Steel	15.00	Mid-point	53,900	26,900,000
Marburg	3	15	Lt. Wt. Beth'm I-beam	Steel	15.00	One-quarter pts.*	37,900	26,400,000
Marburg	3	15	Lt. Wt. Beth'm Girder	Steel	15.00	One-quarter pts.*	41,100	27,200,000
Marburg	3	24	Lt. Wt. Stand. I-beam	Steel	20.00	One-quarter pts.*	33,000‡	25,800,000
Marburg	3	24	Lt. Wt. Beth'm Girder	Steel	20.00	One-quarter pts.*	34,300‡	25,600,000
Marburg	1	30	Lt. Wt. Beth'm I-beam	Steel	20.00	One-quarter pts.*	32,300‡	29,400,000

*Beams mounted in testing machine so as to give freedom for sidewise motion.
†Computed from deflection readings.
‡Fiber stress in flexure at failure greater than the yield-point strength of test pieces cut from beam flanges.

TABLE 3.

TESTS OF I-BEAMS AT THE UNIVERSITY OF

In all tests included in this table the beams developed computed fiber stresses equal to

Test No.*	Beam	Span, feet	Loading	Yield-point Strength of Material in Flanges,° lb. per sq. in.
5	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	5	⅓ points	35,300
6	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	5	⅓ points	35,300
4	8-in., 18-lb. I-beam................. Restrained from end twisting.	5	⅓ points	33,800
1	8-in., 18-lb. I-beam................ No restraint.	5	⅓ points	37,000
2	8-in., 18-lb. I-beam................. No restraint.	5	⅓ points	37,000
11	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	7.50	⅓ points	35,200
10	8-in., 18-lb. I-beam................. Restrained from end twisting.	7.92	⅓ points	33,900
16	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	10	⅓ points	34,300
17	Pair of 8-in., 18-lb. I-beams......... Restrained from sidewise buckling.	10	⅓ points	35,900
15	8-in., 18-lb. I-beam................ Restrained from end twisting.	10	⅓ points	33,800
12	8-in., 18-lb. I-beam................. No restraint	10	⅓ points	33,800
30	Pair of 8-in., 18-lb. I-beams......... With separators.	10	⅓ points	32,400
28	8-in., 25.25-lb. I-beam.............. No restraint.	10	⅓ points	34,100
26	8-in., 18-lb. I-beam................. No restraint.	10	Mid-point	32,500
27	8-in., 18-lb. I-beam................. No restraint.	10	Mid-point	34,200
22	Pair of 8-in., 18-lb. I-beams......... Restrained from sidewise buckling.	15	⅓ points	34,200
23	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	20	⅓ points	35,300

*See Fig. 10-15 at the end of the bulletin.
°Determined from tests of specimens cut from flanges.

TABLE 3 (Continued)

ILLINOIS—PRIMARY FAILURE BY DIRECT FLEXURE.

or greater than the yield-point strength of the material.

Computed Fiber Stress at Maximum Applied Load, lb. per sq. in.	Deflection under Maximum Load, inches	Modulus of Elasticity, lb. per sq. in.°°	Remarks
41,500	0.30	26,500,000	0.28 in. set† after fiber stress of 33,400 lb. per. sq. in. Beams sagged gradually.
41,500	0.33	23,000,000	0.18 in. set after fiber stress of 33,400 lb. per sq. in. Beam sagged gradually.
35,300	....	....	Final failure by sidewise buckling *after* yield-point strength of material was developed.
38,400	0.20	....	Final failure by sidewise buckling *after* yield-point strength of material was developed.
37,800	0.13	30,300,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
37,300	....	....	Beam sagged gradually, excessive fiber deformation at 34,300 lb. per sq. in. fiber stress.
34,300	....	....	Final failure by very gradual sidewise buckling *after* yield-point strength of material was developed.
35,200	0.76	27,800,000	0.12 in. set after fiber stress of 33,700 lb. per sq. in. Beams sagged gradually.
36,600	0.70	26,300,000	0.08 in. set after fiber stress of 31,000 lb. per sq. in. Beams sagged gradually.
35,400	0.76	25,800,000	Ultimate strength apparently reached. Beam sagged gradually.
36,000	0.58	30,000,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
38,200	0.55	30,400,000	0.11 in. set after fiber stress of 34,800 lb. per sq. in. Final failure by sidewise buckling *after* yield-point strength of material was developed.
39,800	0.78	27,800,000	Flanges showed signs of crippling at fiber stress of 33,400 lb. per. sq. in.
36,200	0.63	28,400,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
37,600	0.45	28,000,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
35,300	....	....	Restraining batten plates clamped to beams. Hold of clamps loosened and final failure of beams was by sidewise buckling after yield-point strength of material was developed.
36,500	2.10	24,000,000	0.54 in. set after fiber stress of 31,600 lb. per sq. in. Beams sagged gradually.

°°Determined from deflection of beams.

†Set denotes deflection remaining after the removal of load from the beam.

9. *Buckling of Compression Flanges of I-beams: Equivalent Column Length.*—In an I-beam under a load which acts in the plane of the web there is a tendency for the compression flange to buckle sidewise due to column action. On account of this column action the ultimate available fiber stress for a beam with no restraint against sidewise buckling, as calculated by the flexure formula, may be less than the yield-point strength of the material in the flanges. Column action in a beam is more complex than the action of a strut under direct

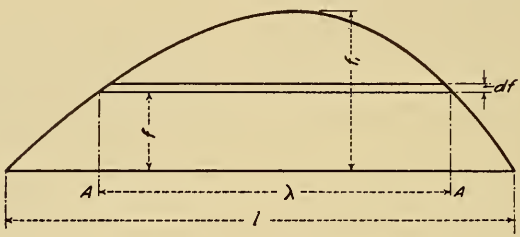

Fig. 3. Diagram of Stress in Compression Flange.

compression. In a strut a compressive load P is applied at one end and the average stress over the cross section due to the direct compression is the same all along the strut. For a strut the average stress over any cross section is given by the equation $f_c = \frac{P}{A}$, where P is the total compressive load, A the cross sectional area of the strut, and f_c the average stress due to direct compression. In a beam the compressive stress in the extreme fibers of the compression flange due to flexural action varies according to the location of the section, increasing from zero at an end support (or point of inflection for a fixed-ended beam) to a maximum at some point in the span. The compression flange of the I-beam then may be regarded as a strut loaded with a large number of elementary loads dP applied at successive points along the span. Each elementary load or increment of load may be considered to produce an increment in the compressive stress of the flange. The value of the compressive stress at any section will then be the summation of the increments of compressive stress from the point of zero stress, or $f = \int df$. This stress f for the remotest fiber of any section will be very nearly equal to the flexural stress produced by the flexure of the beam, which stress is computed by the usual flexure formula $f = \frac{Mc}{I}$, in which $M =$ the bending moment at the point of span considered, $I =$ the moment of inertia of cross-section of beam, and $c =$ distance from the neutral surface of the beam to the extreme compression fiber. To

find the effect of this kind of column loading it will be necessary to deduce an expression for the value of the compressive stress due to column action which will be developed with a column loaded with increments of load as here considered.

For a strut under direct compression the average compressive stress developed at failure f_c may be expressed fairly well by the formula

$$f_c = f_e - k\frac{l}{r}\text{*} \dots\dots\dots\dots\dots\dots (1)$$

in which

f_e = a value of stress about equal to the yield-point strength of the material,

l = length of strut,

r = minimum radius of gyration of cross-section of strut,

k = an experimentally determined constant.

We may regard the term $k\frac{l}{r}$ in either of two ways, (1) as representing a reduction from f_e of available ultimate fiber stress in the strut, due to the effect of column action, or (2) as a fiber stress which is due to a bending action in the column, produced by the same load as produces f_c, and which added to f_c brings the extreme fiber stress up to f_e. By the second conception the stress f_e is the sum of the direct compressive stress f_c and the stress due to column bending.

For the purposes of this discussion it will be convenient to use the second conception; i. e., to consider the last term as a stress produced by a bending action in the column.† In the case of the I-beam, the stress in the remotest fiber of the compression flange on the edge having the highest compression may be considered to be made up of the sum of the flexural fiber stress f_1 (computed by the usual flexure formula, $f_1 = \frac{Mc}{I}$) and the column bending stress f'. This stress will be a maximum at the section where the bending moment will be a maximum, and at failure by side buckling of the flange we may consider that $f_e = f_1 + f'$, where f_1 is the computed flexural fiber stress at the dangerous section, and f_e has a value not greatly different from the yield point of the material.

*While this "straight line" formula for columns is based directly on experiment rather than on mathematical reasoning, it is generally accepted as expressing with a good degree of accuracy the law of failure for columns whose $\frac{l}{r}$ is not greater than about 150 and which are of sufficiently stocky construction to avoid danger of failure by "wrinkling" of parts or local collapse.

†Other methods of analytical treatment of the sidewise buckling of I-beams have been proposed. Some of them are based on the Rankine-Gordon-Schwartz column formula, others on reasoning analagous to that used in developing the Euler column formula.

See Michell, in the Philosophical Magazine for 1899, p. 298; Reissner, in the American Machinist for March 10, 1906; H. D. Hess, in the Proceedings of the Engineers' Club of Philadelphia for April, 1909; Boyd, "Strength of Materials," p. 219; Cambria and Carnegie Steel Handbooks.

To determine the column bending stress f', it will be necessary to take into account the manner of application or of distribution of the assumed compressive loading along the length of the compression flange. At any point along the compression flange the column load may be taken to be the flexural fiber stress at that point, since by this conception the amount of the column load per unit of area of section is the flexural fiber stress. This flexural fiber stress increases from the point of zero bending moment in the beam to the dangerous section. To determine the effect of column action, the increment or differential of flexural fiber stress df applied along a differential of length of flange $d\lambda$ (which is the only column load which acts throughout the column length λ) will be considered as producing column bending stress at the dangerous section. This increment of load is applied at any two points A and A (Fig. 3) and acts upon a column of the length λ (the calculated flexural fiber stress being the same at A and A). The sum of all the loads on the whole length of column ($\int df$) will be the flexural fiber stress at the dangerous section (f_1). To determine the total column bending stress f' at the dangerous section A it will be necessary to make a summation of the effects of the increments of column load df over the length of flange to be considered. By analogy with the straight-line column formula, adopting q as the coefficient for the formula as applied to sidewise buckling of the flange, the term expressing the column bending stress will be of the form $q\frac{\lambda}{r'}$, where r' is the radius of gyration of a cross-section of the compression flange about a gravity axis parallel to the depth of the web. As the elements of column load df vary along the flange and as the proportional effect of each elementary load as compared with the sum of all the loads must be used in the summation, it is necessary to introduce the ratio $\frac{df}{f_1}$ into the term. Then

$$f' = \int q\frac{\lambda}{r'}\frac{df}{f_1} = \frac{q}{r'}\int \lambda\frac{df}{f_1} \quad\ldots\ldots\ldots\ldots(2)$$

This is the column bending stress at the dangerous section.

It will be found convenient to consider this stress as equal to the column bending stress in an ordinary strut loaded with a load f_1, and having a length of ml, where m is a coefficient depending upon the method of loading and conditions of continuity and l is the length of the beam. ml may be called the equivalent column length. Equation (2) may then be written

$$f' = \frac{q}{r'}\int\frac{\lambda\, df}{f_1} = q\frac{ml}{r'} \quad\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The equation for the computed flexural fiber stress at failure due primarily to sidewise buckling will be

$$f_1 = f_e - q\frac{ml}{r'} \qquad (4)$$

where f_e = a value not greatly different from the yield-point strength of the material in the flange.

q = a coefficient of column action.

ml = the equivalent column length of the flange of the beam, the coefficient m being found by equation (3) for different loadings and different conditions of continuity.

r' = the radius of gyration of the compression flange about a gravity axis parallel to the web. For practical purposes r' may be taken as the radius of gyration of the I-section about a gravity axis parallel to the web.

From equation (3) an expression for ml may be written $ml = \int \frac{\lambda df}{f_1}$. It may help in integration to consider that $\int \lambda df$ is the same as the area under the curve of flexural stress for the full length of the beam in simple supported beams, as is indicated in Fig. 3. Then, if f_a is the *mean* ordinate of the curve of flexural stress, $\int \lambda df = f_a l$.

$$ml = \int \frac{\lambda df}{f_1} = \frac{f_a l}{f_1 l}$$

That is, the equivalent column length of the compression flange of an I-beam is equal to the span multiplied by the ratio of the mean flexural fiber stress in the compression flange to the flexural fiber stress in the compression flange at the dangerous section. For a uniformly loaded beam of constant cross section m is found to be $\frac{2}{3}$. For a beam with a single load at any point between supports m is $\frac{1}{2}$. Table 4 gives values of m for various beam loadings. For beams fixed at the ends it will be

TABLE 4.

SIDEWISE BUCKLING OF I-BEAMS:

VALUES OF THE COEFFICIENT m FOR VARIOUS LOADINGS OF BEAMS.

Loading	Value of m
Simple beam, uniform load	0.667
Simple beam, mid-point load	0.500
Simple beam, single concentrated load at any point of span	0.500
Simple beam, one-third point loads	0.667
Simple beam, one-quarter point loads	0.750
Simple beam, one-sixth point loads	0.833
Cantilever beam, uniform load	0.667
Cantilever beam, end load	1.000
Fixed-ended beam, uniform load	0.281
Fixed-ended beam, mid-point load	0.250

seen that one limit for λ will be the distance between points of inflexion of the elastic curve.

10. *Buckling of Compression Flanges of I-beams; Tests.*—The value of the coefficient q of equation (4) is to be determined from the results of flexure tests of I-beams. All available data of tests of I-beams were studied and the test results given in Table 5 were chosen as furnishing the best basis for the determination of q. In selecting data suitable for the study of resistance to sidewise buckling, only those tests were considered in which the primary cause of failure was evidently sidewise buckling. Beams which were wholly or partially restrained laterally by the method used in supporting them in the testing machine were not considered. Fig. 4 shows the method used at the University of Illinois for supporting beams and preserving freedom in respect to sidewise buckling. Beams in which the yield-point strength of the material in the flanges was developed before failure were not considered, as the mere presence of such a fiber stress would explain the failure of a beam and might readily be the cause of sidewise buckling, which would then be a secondary and not a primary failure. The non-consideration of beams free to buckle sidewise which develop fiber stresses as great as the yield-point strength of the material affected only beams of short span or beams of medium span in which the flange material was unusually weak. As a matter of fact the results obtained for resistance to sidewise buckling would not be materially affected whether such beams were considered or not. In making up Table 5 tests were not considered in which web failure seemed to be the primary failure.

Fig. 5 shows graphically the results of the tests given in Table 5. The computed fiber stress at failure (generally called the modulus of rupture) was chosen as a criterion of the strength of an I-beam, rather than the "elastic limit" of the beam, for the following reasons: (1) The failure of beams which buckle sidewise is sharply marked, and the personal equation of the observer will affect the determination of the point of failure but slightly. On the other hand any determination of the elastic limit is dependent upon the sensitiveness of apparatus used in obtaining readings of deformation and upon the interpretation of a plotted curve, and it is much more subject to variations due to personal equation than is the computed fiber stress at failure. (2) The load at failure is more dependent upon the average physical properties of the beam material and less on local stresses and individual peculiarities than is the elastic limit. As the yield point of the material was not exceeded, the computed fiber stress at failure may be considered to vary but little from the actual fiber stress.

TABLE 5.

Tests of I-Beams—Primary Failure by Sidewise Buckling.

In all the tests of beams recorded in this table primary failure occurred by sidewise buckling of the compression flange at computed flexural stresses less than the yield-point strength of the material in the flanges.

Beam	Tested by	Test Numbers*	Number Tested	Span ft.	Loading	Slenderness Ratio for Whole Span l/r'	Equivalent Slenderness Ratio for Sidewise Buckling $\frac{ml}{r'}$	Computed Fiber Stress at Failure lb. per sq.in.	Modulus of Elasticity † lb. per sq.in.
15-in., 42-lb. I-beam	Marburg	......	3	15.00	One-quarter points	166	125	34,700	26,900,000
24-in., 72-lb. Beth'm I-beam	Marburg	......	3	20.00	One-quarter points	129	97	34,600	26,400,000
8-in., 18-lb. I-beam	Univ. of Ill.	7, 8	2	7.50	One-third points	107	71	36,600	25,100,000
8-in., 18-lb. I-beam	Univ. of Ill.	13, 32, 33	3	10.00	One-third points	143	95	32,000	28,600,000
8-in., 18-lb. I-beam	Univ. of Ill.	18	1	15.00	One-third points	214	143	28,900	....
8-in., 18-lb. I-beam	Univ. of Ill.	19	1	20.00	One-third points	286	191	28,100	32,300,000
8-in., 18-lb. I-beam	Univ. of Ill.	3	1	5.00	One-third points restrained against end twisting	71	47	31,400‡	27,600,000
8-in., 18-lb. I-beam	Univ. of Ill.	9	1	7.92	One-third points restrained against end twisting	113	75	34,300	....
8-in., 18-lb. I-beam	Univ. of Ill.	14	1	10.00	One-third points restrained against end twisting	143	95	33,800	30,200,000
8-in., 18-lb. I-beam	Univ. of Ill.	20	1	15.70	One-third points restrained against end twisting	224	150	32,800	....
8-in., 18-lb. I-beam	Univ. of Ill.	21	1	20.00	One-third points restrained against end twisting	286	191	29,200	25,000,000
8-in., 18-lb. I-beam	Univ. of Ill.	24, 25	2	10.00	One-sixth points	143	119	31,500	31,400,000
8-in., 25.25-lb. I-beam	Univ. of Ill.	29	1	10.00	One-third points	150	100	36,500	27,100,000

*See Figs. 10-15 at the end of bulletin.

†Computed from deflections of beams.

‡Very low stress at failure probably due to additional stress caused by imperfect bearing of restraining devices.

The advisability of adjusting the fiber stresses developed in tests of I-beams to compensate for variation in strength of material in the flanges of different test beams was considered, but it was decided to base conclusions on the stresses computed for the tests. Two reasons led to this decision: (1) Due to cold-straightening and other bending which a beam receives there is considerable variation in strength in different parts of the same beam, and the strength of test pieces from one part of the beam would not be wholly representative of the strength of other parts. (2) For beams of long span the resistance to sidewise buckling is dependent not so much on the strength of material as on its stiffness (of which the modulus of elasticity is an index); for beams of medium span the resistance to sidewise buckling is dependent partly on the strength of material and partly on its stiffness; hence the proper adjustment of stresses to compensate for variation of material would be a matter of no small difficulty.

From Fig. 5 it may be seen that the equation

$$f_1 = 40{,}000 - 60\,\frac{ml}{r'} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

represents the results with a fair degree of accuracy. The extreme values observed fall within 2,500 lb. per sq. in. of the values given by the above equation.

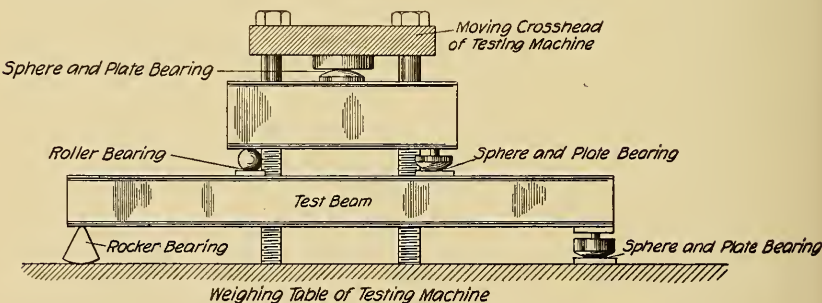

FIG. 4. APPARATUS FOR TESTING I-BEAM WITHOUT RESTRAINT OF ENDS OR OF COMPRESSION FLANGE.

A comparison of equation (4) with the results of tests of columns is of interest. Tests made by J. E. Howard at the Watertown Arsenal* on H-section steel columns with pin ends have been chosen as tests which furnish an excellent basis of comparison of column test results and I-beam test results. The results of Howard's tests of H-section columns with pin ends may be expressed by the equation

$$P/A = 36{,}000 - 100\,\frac{l}{r} \quad \ldots\ldots\ldots\ldots (6)$$

*Tests of Metals for 1909, p. 754; Proceedings of the American Society for Testing Materials, Vol. IX (1909), p. 413.

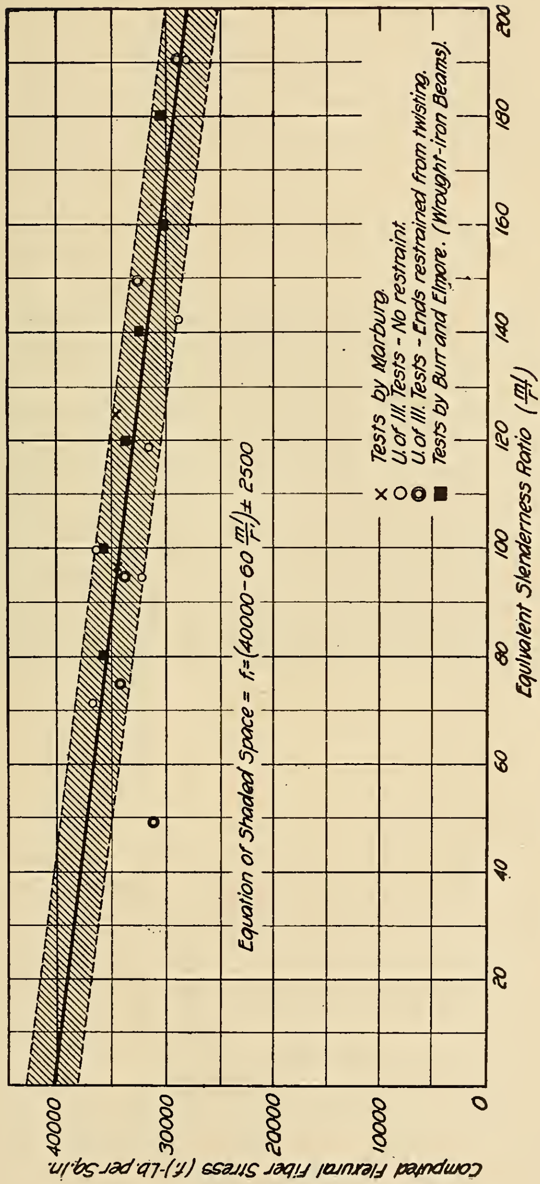

FIG. 5. RESULTS OF TESTS FOR SIDEWISE BUCKLING OF I-BEAMS.

in which P/A is the average intensity of compressive stress at failure, l is the length of the column and r is the least radius of gyration of the column section.

Comparing equation (5) with equation (6) it is seen that, as might be expected, the coefficient of equivalent slenderness ratio for the beam formula is somewhat less than the coefficient of slenderness ratio for the column formula. The smaller value of coefficient in the beam formula is doubtless due to the fact that in a beam there is end restraint against sidewise buckling and a restraining action of the web and the tension flange. The first term in the beam equation (5) is larger than the first term in the column equation (6). The flanges and webs of the I-beams were rolled thinner than the flanges and webs of the H-sections, and the additional work of rolling done on the I-sections may explain the increase in the yield-point strength of the material over that of the H-sections.

It should be noted that all but one of the beams given in Table 5 are "light" sections. The web and the tension flange of "heavy" I-beams would offer more restraint against sidewise buckling than do the web and the tension flange of "light" I-beams, and the fiber stresses developed at failure may reasonably be expected to be higher. Such a result is indicated by the tests made by Burr and Elmore at Rensselaer Polytechnic Institute, to which reference has already been made. The results of these tests are shown in Fig. 5 by small black squares. The tests were made on medium-weight wrought-iron I-beams, 6 in. deep, and it is seen that the greater strength and stiffness of steel I-beams was about offset by the greater stockiness of section of the Burr and Elmore wrought-iron test beams.

As test data are lacking for "heavy" steel I-beams, as the formula (equation 5) derived from tests of "light" I-beams gives results which err on the side of safety when applied to "heavy" I-beams, and as the reduction below yield-point strength of material of fiber stress at failure is not large for ordinary spans, no attempt will be made to derive a separate formula for I-Beams of medium-weight or heavy-weight sections.

Attention is called to the fact that in no case should the ultimate flexural stress be taken as higher than the yield-point strength of the material in the flanges. In the absence of special tests of material 35,000 lb. per sq. in. may be used as an average value for the yield point of structural steel. Especial attention is called to the fact that equation (5) gives ultimate values of fiber stress and not working values, which should, of course, be much lower.

11. *Tests to Failure of Beams Restrained from Twisting of Ends and Beams Restrained from Sidewise Buckling.*—Two series of tests were

carried out for the purpose of investigating the action of I-beams restrained against end twisting and of beams restrained against sidewise buckling. Fig. 6 shows the arrangement of apparatus used in testing beams restrained against end twisting. To each end of the web of the test beam heavy angles were bolted by one leg and the other leg of each angle was bolted to an end piece which rested on a roller. When the test beam was placed in the testing machine evenness of bearing under rollers was secured by the use of thin metal shims. This method of supporting the test beams proved effective in preventing end twisting and did not affect the tendency of the beam to buckle sidewise.

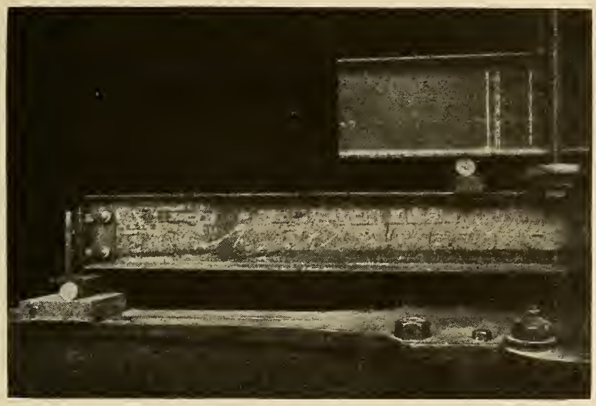

Fig. 6. Apparatus for Testing I-beam with Restraint against End Twisting.

The fiber stresses developed at failure in those beams which were restrained against end twisting are given in Tables 3 and 5. Short-span beams so restrained did not develop quite so high stresses at failure as did similar beams tested without restraint against end twisting. This was probably due to imperfect bearings at the ends. For all except the short-span I-beams the fiber stresses developed at failure by beams restrained from end twisting did not differ appreciably from the fiber stresses developed at failure by the beams not so restrained. Failure for both kinds of beams occurred by sidewise buckling, and it would seem that restraint of ends of I-beams against twisting does not appreciably increase their resistance to sidewise buckling.

The method of testing beams restrained against sidewise buckling is shown in Fig. 7. For each test two beams were fastened together along their compression flanges by means of batten plates spaced about ten inches apart. Each batten plate was fastened to the flanges of the beams by four studs of cold-rolled steel fitting snugly in drilled holes. This

device prevented appreciable sidewise buckling and all beams thus restrained failed very gradually by vertical sagging with the exception of the beams with 15-ft. span in which the batten plates were merely clamped to the flanges of the beams, and in which, though the full yield-point strength of the material was developed, the beams finally buckled sidewise. In all beams tested with restraint against sidewise buckling the maximum computed fiber stress developed in the test was equal to or slightly greater than the yield-point strength of the material in the flanges. It would seem that for beams effectively restrained against sidewise buckling the fiber stress developed before failure in flexure will be as great as the yield-point strength of the material, regardless of the length of the span. What constitutes effective restraint is discussed in the next paragraph.

TABLE 6.

EFFECT ON THE ELASTIC LIMIT OF I-BEAMS OF RESTRAINT AGAINST TWISTING OF ENDS AND AGAINST SIDEWISE BUCKLING.

All tests made on 8-in., 18-lb. I-beams loaded at the one-third points of the span.

Span ft.	Number of Beams Tested for Each Item	Computed Fiber Stress at the First Observed Elastic Limit lb. per sq. in.		
		No Restraint	Restrained against Twisting of Ends	Restrained against Sidewise Buckling
5	2	27,300	23,000	22,300
7.5	2	27,900	23,300	26,300
10	2	23,000	19,300	26,600
15	1	25,200	22,000	21,200
20	1	21,000	23,400	24,000

12. *Effectiveness of Sidewise Restraint of I-beams.*—In the tests of 8-in. I-beams, measurements of the extreme fiber deformation in the flange (stretch and shortening) at mid-span were made. By plotting the observed fiber deformations against the fiber stress computed by the usual flexure formulas, curves were obtained showing local action of the beams under load. These curves (Fig. 10-15) are given at the end of the bulletin. From these curves fiber stress at the elastic limit first observed at any part of the beam was determined,* and these stresses have been tabulated in Table 6. An examination of this table shows that inelastic action was detected in some restrained beams at computed fiber stresses lower than was the case for the corresponding unrestrained beams, and that, in general, the effect of restraint on elastic limit is not great. A reasonable explanation of this would seem to be that the re-

*The elastic limit was located by the method proposed by the late Prof. J. B. Johnson. His method consists in finding the point on a stress-deformation curve at which the deformation is increasing fifty per cent more rapidly than its initial rate of increase.
See Johnson, "The Materials of Construction," pp. 18-20.

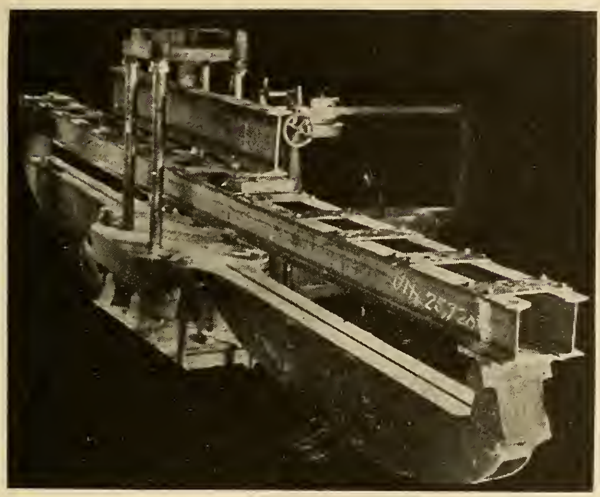

FIG. 7. APPARATUS FOR TESTING I-BEAM WITH RESTRAINT AGAINST SIDEWISE BUCKLING.

straining devices sometimes introduce additional stresses into the beam. As noted in "11. Tests to Failure of Beams Restrained from Sidewise Buckling and Beams Restrained Against Twisting of Ends," restraint against twisting of ends produced no marked effect on the *ultimate* strength of medium-span and long-span I-beams, while with restraint against sidewise buckling the fiber stresses developed before failure, even for the longest-span beams, were as high as the yield-point strength of the material.

The elastic limit observations in connection with the observations of ultimate fiber stress seem to indicate that the resisting effect exerted by restraint is not noticeable until failure is imminent. Observations on sidewise deflection tend to confirm this conclusion. In the tests of certain of the unrestrained I-beams the sidewise deflection was measured. Table 7 records the sidewise deflection observed at loads which give a computed flexural fiber stress of 16,000 lb. per sq. in. (an ordinary working stress). It will be seen that the sidewise deflection is small, and generally in the tests it continued to be small until failure of the beam became imminent. So small is this sidewise deflection for working stresses that it is questionable whether such restraining members as usually would be attached to beams in actual structures will be stiff enough to prevent it. In structures the usefulness of restraining I-beams against sidewise buckling lies mainly in the fact that such restraint renders available the full yield-point strength of the material in the flanges of the beams, and that should failure occur, with effective sidewise and

end restraint, the beam will fail by gradual sagging rather than by sudden collapse.

Any restraining devices to prevent sidewise buckling of I-beams should provide resistance to sidewise bending moments. Separators between the ribs of a pair of beams do not provide such resistance. A pair of beams held together merely by bolts and separators was tested (Test No. 30), and though the yield-point strength of the material was developed, final failure occurred rather suddenly by sidewise buckling. While comparatively slight sidewise restraint may enable a beam to develop the full yield-point strength of the material, a strong, stiff restraining system is needed to prevent sudden collapse when final failure does occur. Another illustration of this was furnished by testing a pair of 8-in., 18-lb. I-beams having a span of 15 ft. (Test No. 22) and restrained against sidewise buckling. In this test the batten plates holding the beams together were merely clamped to the flanges and not bolted. In the test the full yield-point strength of the flange material was developed, but soon afterward the beams failed quite suddenly by sidewise buckling. The clamps were not strong enough to hold the batten plates to the beam flanges under the large sidewise force developed when the yield point of the material in the beam was reached.

13. *Web Failure of I-beams.*—I-beams and built-up girders of short span are sometimes in danger of failure by crippling of the web. Web failure may be caused in several ways: (1) The fiber stress in shear at the middle of the web may exceed the yield-point strength in shear of the web material. (2) Accompanying the shearing fiber stress at any point of the web is a compressive stress of equal intensity acting in a direction inclined at 45 degrees with the direction of the shearing stress, and this compressive stress may become so great as to cause buckling. (3) There may be an excessive compressive stress near the junction of web and flange and adjacent to a concentrated load or reaction. The shapes assumed by a cross-section of an I-beam after web failure are shown in Fig. 8. The shape and position at (a) is that due to torsion of the beam as a whole; that at (b) to buckling of the web; and that at (c) to local compressive stress at root of flange. What has been referred to previously as failure by twisting of ends of I-beams is in most cases primarily caused by excessive local compression at the root of the flange.

An approximate method of computing the compressive stress at the root of the flange adjacent to a concentrated load or an end reaction, has been given by C. W. Hudson* as follows: Imagine a small piece cut

*Engineering News, December 9, 1909.

TABLE 7.

SIDEWISE DEFLECTION OF I-BEAMS AT A COMPUTED FIBER STRESS OF 16,000 LB. PER SQ. IN.—BEAMS FREE TO MOVE LATERALLY.

Beam	Material	Span ft.	Loading	Deflection in.
8-in., 18-lb. I-beam......	Steel.........	10	One-third points.....	0.046
8-in., 18-lb. I-beam......	Steel.........	20	One-third points.....	0.036
8-in., 25.25-lb. I-beam...	Steel.........	10	One-third points......	0.026
8-in., 25.25-lb. I-beam...	Steel.........	10	One-third points.....	0.019
17.5-in. built-up beam...	Wrought iron	12.9	One-third points.....	0.030
24-in. built-up beam.....	Wrought iron	14.2	Load 12 in. each side of center.........	0.067

from the flange and web of an I-beam immediately over a bearing block (as shown in Fig. 9), and imagine this piece to be held in equilibrium by the elastic forces which act on it while it is in its place in the beam. The forces are (1) the pressure of the reaction at the bearing block P; (2) the compression in the web which equals $f_w tb$, when $f_w =$ the average intensity of compressive stress, $t =$ the thickness of web, and $b =$ the length of bearing block; (3) a horizontal shearing force S_h; and (4) a vertical shearing force S_v. Very little of the total shear would be balanced by the small internal shearing stress in the flange of an I-beam, and if the section considered be taken at the root of the flange we may write without serious error

$$S_v = S_h = 0$$

Then the compressive stress on the web is balanced by the reaction on the bearing block. The compressive stress may be regarded as uniformly distributed, and we may write

$$f_w = \frac{P}{bt} \dots\dots\dots\dots (7)$$

In the above discussion the case considered is for the compressive stress adjacent to an end reaction. The reasoning for the compressive stress in the web adjacent to a concentrated load would be similar.

The compressive stress in the web of an I-beam necessary to cause buckling of the web is computed in most text books on strength of materials on the assumption that the web of the I-beam is in the same condition of stress as a fixed-ended column whose length is equal to the vertical distance between flanges multiplied by the secant of 45 degrees, and whose radius of gyration is equal to the thickness of the web divided by $\sqrt{12}$, and in which the average intensity of compressive stress is equal to the maximum intensity of shearing stress in the web of the I-beam. This shearing stress is very nearly equal to the total shear divided by the area of the web. The assumption of fixed-ended conditions and the

neglect of the restraint against the buckling of the web by tensile stress in the lower part of the beam render the accuracy of this method somewhat uncertain.

14. *Web Failure of I-beams; Tests.*—After gathering the data of tests it is realized that the whole amount of data on web failure of I-beams is small. The drawing of conclusions from these data is further complicated by the fact that several web failures of test beams seemed to be due partly to shearing stress in the web and partly to compressive stress in the web adjacent to bearing blocks.

In selecting test data for the study of web failure of I-beams, only those tests were taken in which at failure the fiber stress in the flanges was less than the yield-point strength of the material and in which the failure took place by crippling of the web.

Six of the tests selected were made at the University of Illinois. As noted on p. 33, variation in web dimension was obtained by planing down the webs of some of the beams.

The results of the tests selected for the study of web failure are given in Table 8. In the tenth line of the table is given the slenderness ratio of the web computed on the assumptions usually made in text books on mechanics of materials and named in the preceding paragraph. In the thirteenth line of the table is given the computed fiber stress at failure of the web by buckling as determined by Euler's formula for fixed-ended columns. Euler's formula was chosen on account of the high values of slenderness ratio. It will be seen that the calculated compressive stresses corresponding to the loads carried (tabulated in the twelfth line of the table) were in three cases very much in excess of the value given by Euler's formula. This excess is so marked that even these few tests may be taken to indicate that for computing the safety of I-beam webs against buckling the method common in texts on mechanics of materials gives results which are on the side of safety.

From the twelfth line of Table 8 it will be seen that in all beams but one the fiber stress in shear at mid-web was not much below the yield-point in shear for structural steel, which averages from 25,000 to 35,000 lb. per sq. in. Of course, even under the most favorable circumstances the web of an I-beam may not be counted on to develop without failure a stress in excess of the yield-point in shear of the web material.

In the fifteenth line of Table 8 is given the computed fiber stress in compression developed at the roots of the flange. This fiber stress is computed from equation (7). In the sixteenth line of the table is given the yield-point strength of the material of the beams at the root of the flange.

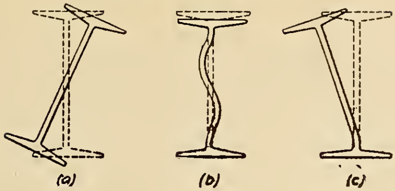

FIG. 8. SHAPES ASSUMED BY I-BEAMS AFTER WEB FAILURE.

This was determined by means of tests of specimens cut from the beams. The compressive fiber stress developed was in all cases not much greater than this yield-point strength of material. However, in all the tests for web failure made at the University of Illinois, before final failure occurred evident signs of structural injury, scaling, etc., had appeared. It is unwise to regard the ultimate compressive fiber stress in the web adjacent to a bearing block as higher than the yield-point strength of the material at the root of the flange. Moreover, the fact should be borne in mind that the material at the root of the flange of an I-beam has a yield-point strength somewhat lower than the material in the flange or in the web. In the absence of special tests the yield-point strength of the structural steel at the root of the flange of an I-beam may be taken as about 30,000

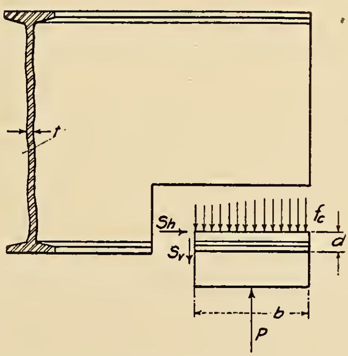

FIG. 9. DIAGRAM OF COMPRESSIVE STRESS IN WEB OF I-BEAM OVER A BEARING BLOCK.

lb. per sq. in., which is near the value obtained in various tests at Illinois and by Marburg and by Hancock. In view of the small amount of data of failure of I-beams by buckling of web, the conclusions given should be regarded as tentative.

15. *Stiffness of I-beams.*—In most of the tests of I-beams referred

to in this bulletin the value of the modulus of elasticity based on the beam deflections and the common theory of flexure is reported. Tables 2, 4 and 5 give values of the modulus of elasticity so computed. These values of the modulus of elasticity seem in general to be lower than the values usually obtained from tension tests of structural steel or of wrought iron. This difference in the values obtained in the two ways is confirmed by Marburg's tests. By means of extensometer tests of samples of material cut from I-beams, Marburg determined the modulus of elasticity of the beam material, and also computed the modulus of elasticity of the beam from load-deflection curves. Table 9 summarizes the average values obtained from the published data of Marburg's tests. It will be seen that the modulus of elasticity obtained from beam deflections is about 10 per cent less than the modulus of elasticity obtained from tension tests of samples of beam material; or in other words, the stiffness of the I-beams was about 10 per cent less than that indicated by tension tests of material.

16. *Summary.*—The following summary is given:

1. The yield-point strength, not the ultimate tensile strength, should be regarded as the ultimate fiber stress for structural steel in flexure.

2. The yield-point strength of structural steel in compression is about the same as the yield-point strength in tension.

3. The slight inelastic action which may be observed in steel I-beams under stresses as low as those used in practice is in general local in its effects and does not indicate the load-carrying capacity of the I-beam, if the load is not reversed in direction.

4. The computed *ultimate* fiber stress for steel I-beams not restrained against sidewise buckling of the compression flange is given by the formula

$$f_1 = 40{,}000 - 60 \frac{ml}{r'}$$

in which f_1 is the extreme fiber stress, in pounds per square inch, computed by the usual flexure formula, l is the length of span of beam in inches, r' is the radius of gyration of the I-section about a gravity axis parallel to the web, and m is a coefficient dependent on the method of loading, ml being a so-called equivalent column length. Values of m for various loadings are given in Table 4. In no case should f_1 be taken greater than the yield-point strength of the material in the flanges. It should be borne in mind that f_1 of this formula is an ultimate, not a working value.

5. A light system of sidewise bracing may so strengthen an I-beam that the full yield-point strength of the material will be developed before

TABLE 8.

Web Failure of I-Beams.

1. Beam	12-in., 31.5-lb. I-beam	12-in., 31.5-lb. I-beam. Web planed thin	12-in., 31.5-lb. I-beam. Web planed thin	12-in., 31.5-lb. I-beam. Web planed thin	Beth'm Girder Beam 30-in., 175-lb.	20-in. Special* Built-up Girder
2. Span, ft.	2.92	3.00	3.00	3.00	20	15
3. Loading	Two points 4⅞ in. each side of center	Two points 4⅞ in. each side of center	Two points 4⅞ in. each side of center	Two points 4⅞ in. each side of center	Quarter points	One load at quarter-point
4. Material	Steel	Steel	Steel	Steel	Steel	Steel
5. Number tested	2	1	1	1	3	1
6. Tested by	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Marburg (Univ. of Pa.)	Turneaure (Univ. of Wis.)
7. Fiber stress due to direct flexure, lb. per sq. in.	33,500	28,200	19,300	12,700	31,000	Sidewise buckling prevented by bracing
8. Vertical distance between flanges, inches (h)	10.52	10.52	10.52	10.52	26.8	14
9. Thickness of web, inches (t)	0.35	0.28	0.19	0.16	0.69	0.14
10. Slenderness ratio for web $\frac{l}{r}=\sqrt{12}\,\frac{h \sec. 45°}{t}$	148	184	272	324	190	490
11. Load at failure, pounds	190,100	160,500	109,600	72,100	538,400	106,000 (Approx.)
12. Computed fiber stress (shear and also compression) at middle of web, lb. per sq. in.	25,800	27,200	27,400	21,400	14,800	26,500
13. $\frac{4\pi^2 E}{(\frac{l}{r})^2}$	53,900	34,900	16,000	11,300	32,700	4,900
14. Length of block under support, inches	6	6	6	6	12	....
15. Computed fiber stress (compression) in web adjacent to support, lb. per sq. in.	45,300	47,800	48,200	37,600	32,500	Stiffeners used at ends and under load
16. Yield-point strength of material at root of flange, lb. per sq. in.	31,700	33,100	34,000	32,200	28,200	37,700

*Test reported in full in the Journal of the Western Society of Engineers for 1907, p. 788. Failure by sidewise buckling was prevented by bracing. The load at failure is given by Dean Turneaure as that at which very great distortion had taken place and noticeable buckling in the web occurred. Excessive compressive stress in the web adjacent to reactions and concentrated loads was prevented by using stiffeners, well fitted to the flanges.

TABLE 9.

MODULUS OF ELASTICITY OF I-BEAMS AND OF I-BEAM MATERIAL.

Values from results of tests by Marburg at the University of Pennsylvania.

Item	Standard I-beams	Bethlehem I-beams	Bethlehem Girder Beams
Average modulus of elasticity of tension test pieces cut from web, flange and root of flange of I-beam, lb. per sq. in. .... (A)	29,500,000	28,310,000	29,660,000
Average modulus of elasticity of beams determined from deflections, lb. per sq. in. .... (B)	26,300,000	26,570,000	26,120,000
(B) : (A)................................	0.892	0.937	0.882

failure occurs, but a stiff bracing capable of resisting sidewise bending moment is necessary to prevent sudden failure by sidewise buckling, once the yield-point of the beam flanges is reached. Separators between the webs of I-beams do not furnish a stiff bracing against sidewise buckling.

6. In investigating the safety of an I-beam as regards web failure three possible causes of failure should be considered:

(a) Failure by shearing stress in the web. The yield-point strength of structural steel in shear should be regarded as the ultimate fiber stress for the web.

(b) Failure by buckling of web. The buckling strength of a strip of web inclined 45 degrees to the flanges as computed by Euler's formula for fixed-ended columns was developed in several tests without collapse of the web.

(c) Failure by compressive stress in the part of the web adjacent to a bearing block. The value of this stress may be roughly estimated from the formula given by Hudson,

$$f_w = \frac{P}{bt}$$

in which f_w is the fiber stress in compression in pounds per square inch, b is the length of bearing block in inches, t is the thickness of web in inches, and P is the concentrated load or the reaction in pounds. The yield-point strength of the material at the root of the flange of the I-beam should be regarded as the ultimate value for f_w.

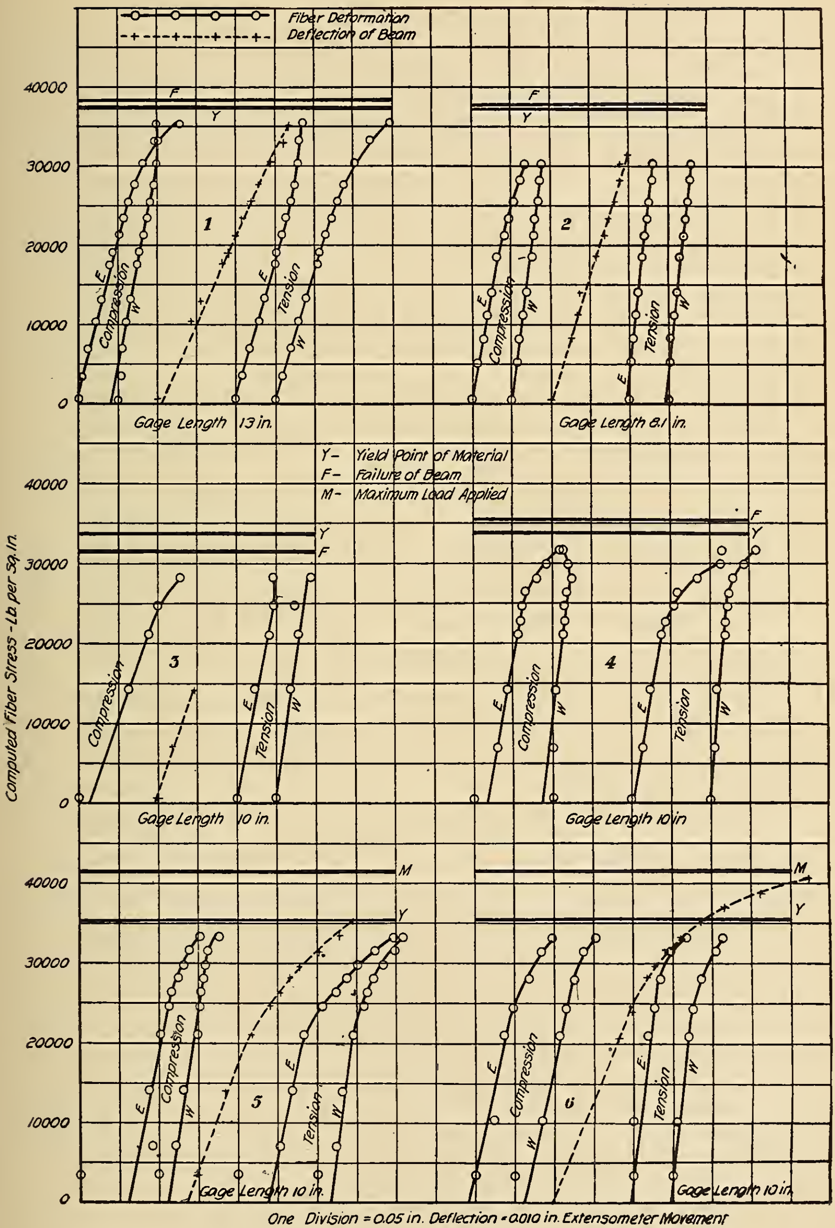

FIG. 10. RESULTS OF TESTS 1-6.

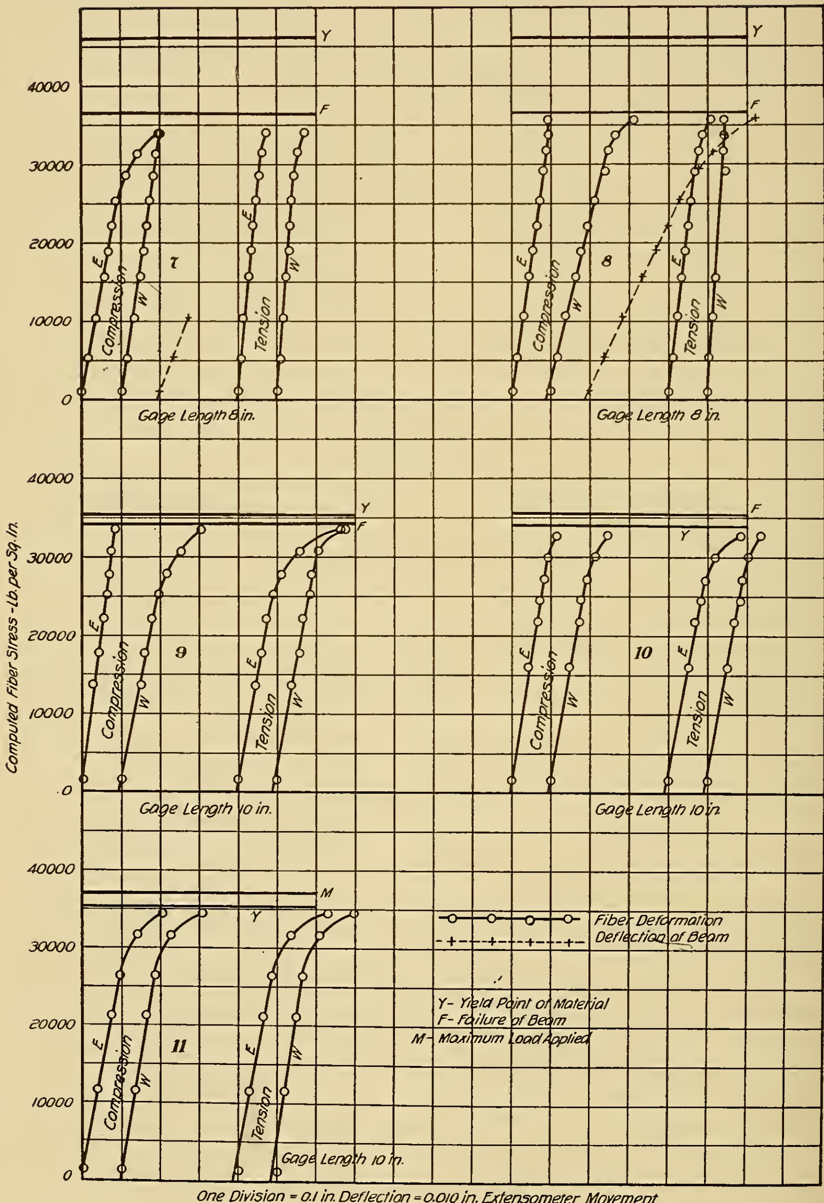

FIG 11. RESULTS OF TESTS 7-11.

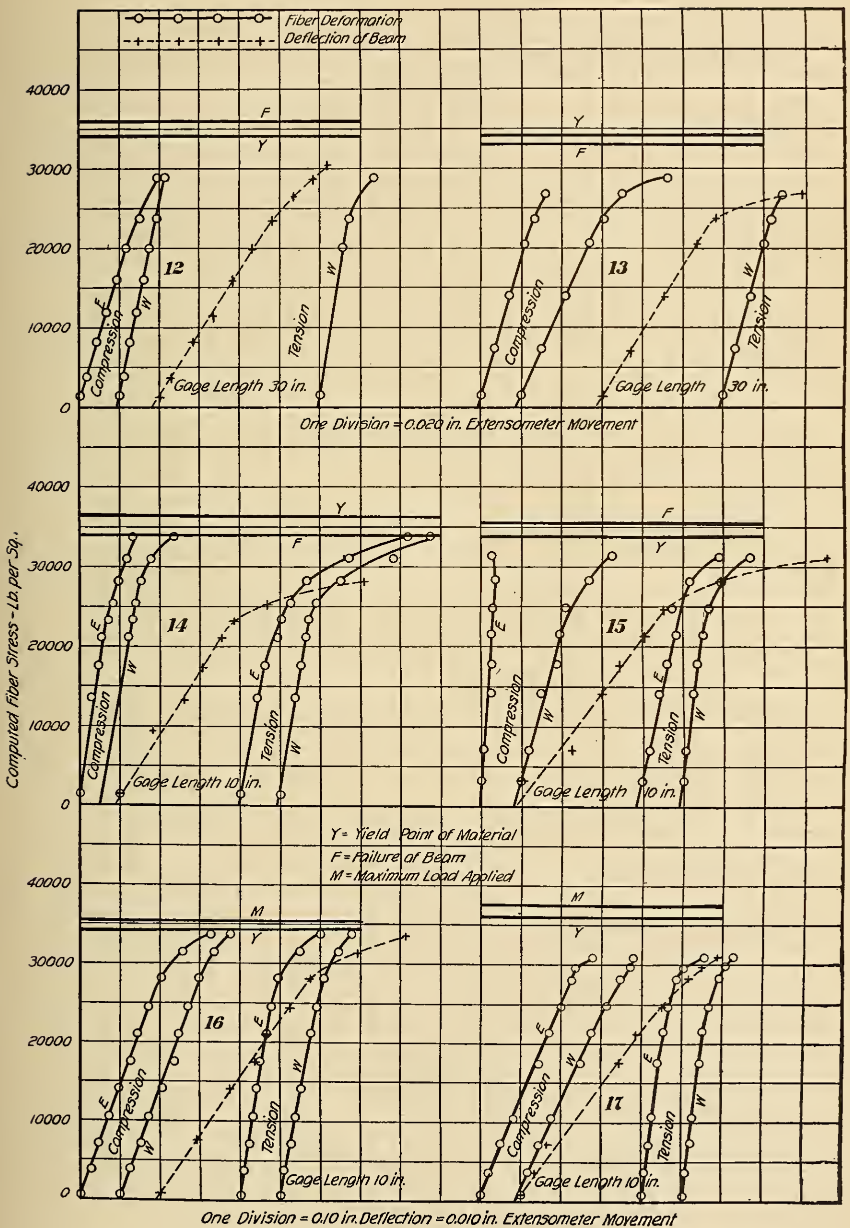

FIG. 12. RESULTS OF TESTS 12-17.

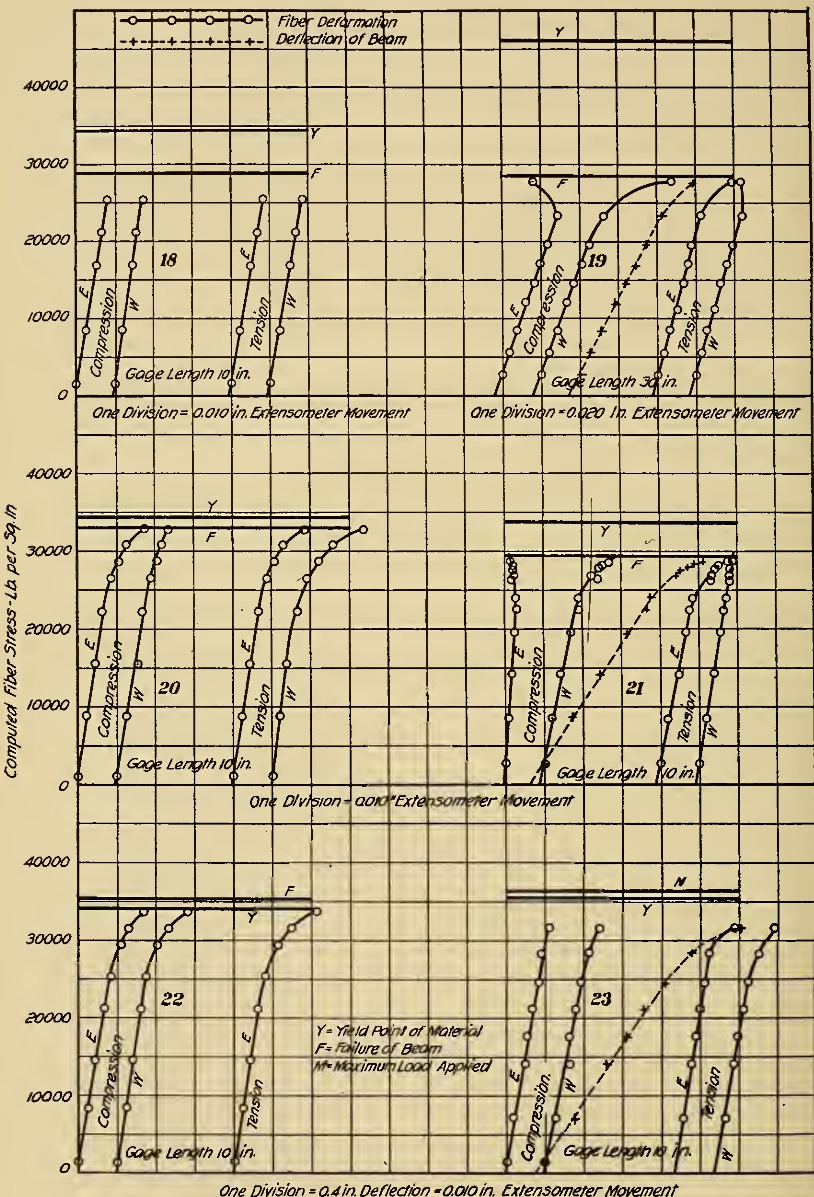

FIG. 13. RESULTS OF TESTS 18-23.

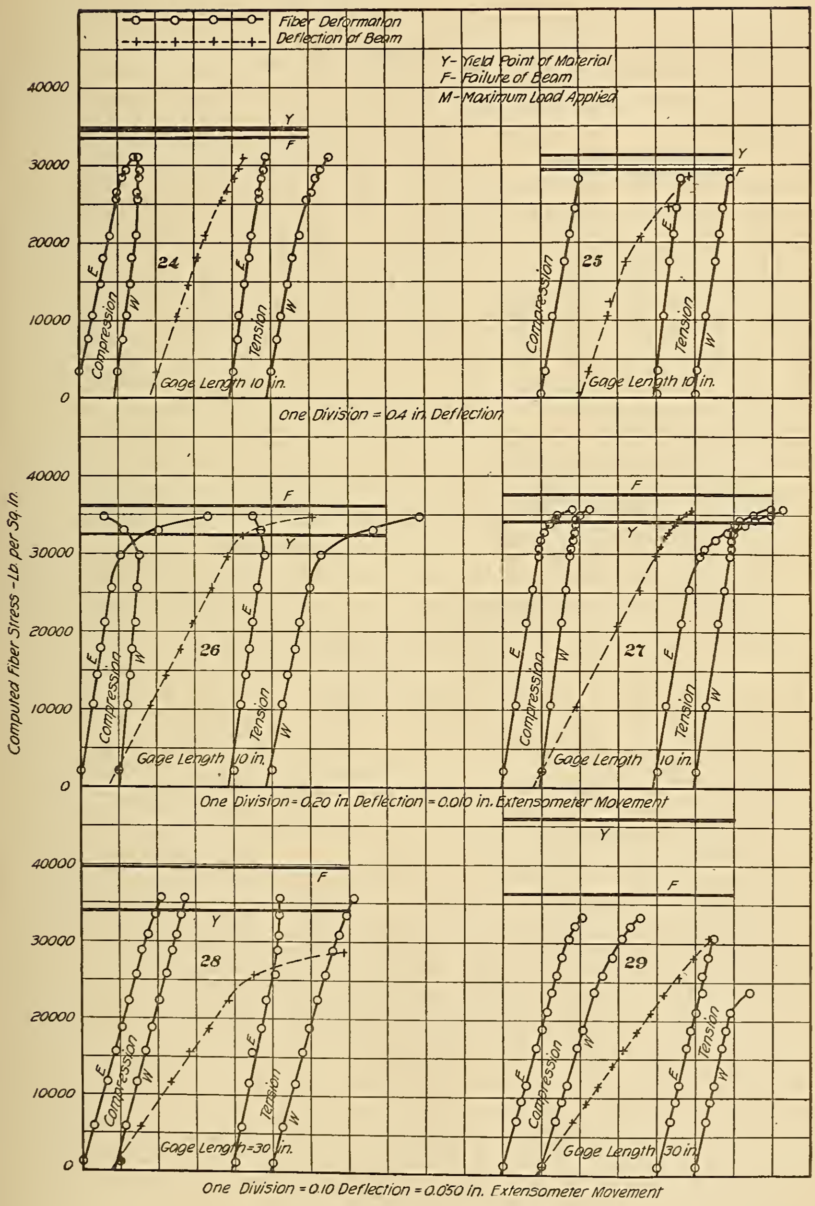

FIG 14. RESULTS OF TESTS 24-29.

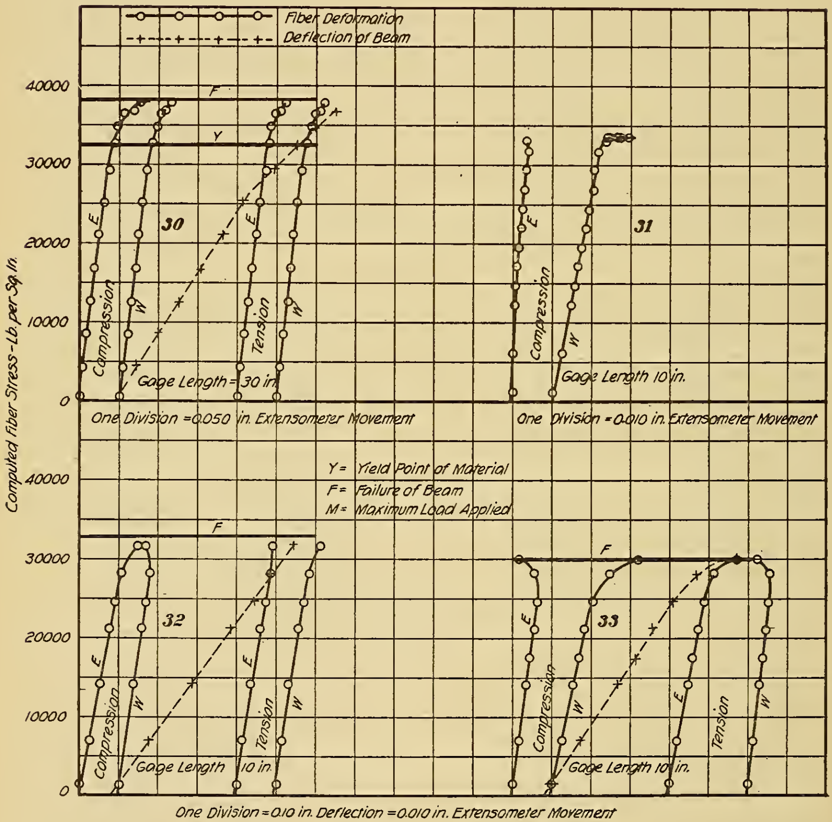

FIG. 15. RESULTS OF TESTS 30-33.

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Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None Available.*

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COAL WASHING IN ILLINOIS

BY

F. C. LINCOLN

BULLETIN NO. 69

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ENGINEERING EXPERIMENT STATION

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COAL WASHING IN ILLINOIS

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COAL WASHING IN ILLINOIS.

I. Introduction.

Coal washing is the partial purification of coal by mechanical treatment with water. It is one of a series of purifying processes to which the general term coal dressing is applied, among which are also included hand picking, mechanical picking, pneumatic separation and flotation. Coal dressing is the preparation of coal for use as a fuel or in the manufacture of coke, and comprises sizing, crushing, briqueting and purification, and combinations of two or more of these processes. Coal washing may therefore be more fully defined as that branch of coal dressing which is concerned with the partial removal of impurities from coal by mechanical treatment with water for the purpose of preparing it for use as a fuel, or in the manufacture of briquets or coke.

1. *Scope of Bulletin.*—This bulletin contains a general account of the history, practice, results and costs of coal washing in Illinois. It is a compilation of data obtained by reading, correspondence, personal interviews and field work. A period of two and one-half months was spent in the field. This time was sufficient for the inspection of all the operating coal washeries in the State and the collection of over one hundred samples.

It was too short, however, to permit of detailed studies being made of any of the washeries visited, although the value of such studies is fully realized and it is hoped they may be carried out in the future. The time was also too short for a study of the methods by which the raw screenings for the washeries were produced. Fortunately, however, mining methods in Illinois—including those parts of the State in which coal washeries are in operation—have been made the subject of an investigation undertaken jointly by the Department of Mining Engineering of the University of Illinois, the Illinois State Geological Survey and the United States Bureau of Mines. The co-operative publications on districts in which washeries are situated may profitably be consulted in connection with this bulletin.

2. *Acknowledgements.*—The investigation of Illinois washeries was suggested by Prof. H. H. Stoek of the University of Illinois, and his advice and criticisms have proved of great value. The owners and operators of washeries throughout the State have lent their assistance in arranging for the inspection of their plants and the sampling of their products. Many of them have also supplied costs, analyses and historical data which have added greatly to the value of this work. The companies engaged in the building of washeries have been most kind in supplying information. In several instances men not at present financially interested in washing operations have contributed historical details, notably Messrs. L. D. Howe, E. D. Meier and A. E. Tyler.

Pictures, drawings or cuts for some of the illustrations of this bulletin have kindly been furnished by the Allen and Garcia Co., Chicago; Allis-Chalmers Co., Chicago; American Coal Washing Co., Alton; American Concentrator Co., Joplin, Mo.; Mr. C. A. Herbert, Gen. Supt. Chicago, Wilmington and Vermilion Coal Co., Streator; Jeffrey Mfg. Co., Columbus, O.; Link-Belt Co., Chicago; and Roberts and Schaefer Co., Chicago.

Permission to employ many of the analyses in this bulletin was granted only with the understanding that they should be used in such a manner that it would be impossible to tell to what mine and washery they belonged. In order to carry out the spirit of this agreement, it has been necessary to omit specific acknowledgements of other analyses whose sources are not confidential. Some of the analyses published come from the co-operative work mentioned above, more were kindly furnished from unpublished records of the Illinois State Geological Survey and of the Commercial Testing and Engineering Company, others were copied from publications of the United States Bureau of Mines and from other works noted in the bibliography, Appendix A, while still others were obtained from private communications.

3. *Production of Coal.*—In the year 1912, 19 844 517 tons of bituminous coal were washed in the United States. This was 4.4 per cent of the total production. The washeries produced from the raw coal 17 538-572 tons, or 88.4 per cent of washed coal, and 2 305 945 tons, or 11.6 per cent of refuse. Table I shows the production of washed bituminous coal in the United States from 1906 to 1912, as published in "Mineral Resources." It will be observed that there were marked decreases in the productions of both washed and raw coals for 1911 compared with those for 1910. This was due to two causes. The principal one was the depressed state of the iron industry in 1911, leading to a smaller demand for coke for the furnaces and resulting in a decrease of about ten million tons in the quantity of coal used in the manufacture of coke. The production of washed coal was more seriously affected proportionally than that of the raw coal, since by far the larger portion of the washed coal produced is used in the manufacture of coke. The other factor in decreasing the production was the competition with California fuel oil in the Northwest. Table 2 shows the production of washed coal by States.

The washing of anthracite is an important industry in Pennsylvania, 3 533 768 short tons of washed anthracite having been produced in 1912. This was 4.8 per cent of the total production for that year. When this amount is added to the production of washed bituminous coal, the total production of washed coal for 1911 is found to be 23 378 285 tons, of which 15.1 per cent is anthracite and 84.9 per cent bituminous coal.

Out of a total production of 59 885 226 tons of coal in 1912, Illinois washed 3 522 760 tons, or 5.88 per cent This amount is greatly in

TABLE 1.

PRODUCTION OF WASHED BITUMINOUS COAL IN THE UNITED STATES IN SHORT TONS.

Year	Raw Coal Produced	Raw Coal Washed	Washed Coal Produced	Refuse	Number of States Producing
1906	342 874 867	10 425 455	9 251 946	1 173 509	17
1907	394 759 112	12 981 514	11 269 518	1 711 996	19
1908	332 573 944	13 660 478	11 870 438	1 790 040	18
1909	379 744 257	16 541 874	14 443 147	2 098 727	16
1910	417 111 142	18 395 382	16 035 387	2 359 995	21
1911	405 907 059	12 355 716	10 830 823	1 524 893	17
1912	450 104 982	19 844 517	17 538 572	2 305 945	20

TABLE 2.

BITUMINOUS COAL WASHED AT THE MINES IN 1912, WITH QUANTITY OF WASHED COAL AND OF REFUSE OBTAINED FROM IT, BY STATES, IN SHORT TONS, AND RANK OF STATES.

State	Quantity of Coal Washed	Quantity of Cleaned Coal	Quantity of Refuse	Rank of States
Alabama	7 187 211	6 325 946	861 265	1
Arkansas	72 753	50 563	22 190	15
Colorado	116 950	107 174	9 776	13
Georgia	111 923	87 300	24 623	14
Illinois	3 522 760	3 070 523	452 237	3
Indiana	18 784	17 077	1 707	19
Kentucky	164 496	150 626	13 870	9
Maryland	53 842	53 191	651	17
Michigan	128 738	113 623	15 115	12
Missouri	140 582	101 953	38 629	11
Montana	666 713	599 104	67 609	6
Ohio	336 639	305 629	31 010	8
Oklahoma	143 537	117 018	26 519	10
Oregon	12 501	10 501	2 000	20
Pennsylvania	4 819 330	4 326 162	493 168	2
Tennessee	449 847	390 994	58 853	7
Texas	25 599	20 639	4 960	18
Virginia	60 640	56 925	3 715	16
Washington	863 643	731 521	132 122	5
West Virginia	948 029	902 103	45 926	4
Total	19 844 517	17 538 572	2 305 945	

excess of the average proportion of bituminous coal washed in the United States and is about the same as the proportion of anthracite washed in Pennsylvania. There were produced 3 070 523 tons of washed coal and 452 237 tons, or 12.8 per cent, of refuse. This proportion of refuse is greater than the average for the United States, which is 11.6 per cent. Table 3 shows the production of washed coal in Illinois from 1906 to 1912. It will be noted that in the face of a greatly reduced production of washed coal for the country as a whole in 1911, Illinois made

a slight increase. This was due to the fact that Illinois washes coal for direct use as fuel and not for the manufacture of coke, and that there is no competition with fuel oil, so the causes of the decreased production elsewhere were not effective in this State.

TABLE 3.

PRODUCTION OF WASHED COAL IN ILLINOIS IN SHORT TONS.

Year	Raw Coal Produced	Raw Coal Washed	Washed Coal Produced	Refuse	Rank among Coal Washing States
1906	41 480 104	2 600 817	2 216 593	384 224	2nd (Pa. 1st.)
1907	51 317 146	2 988 386	2 465 767	522 619	2nd (Ala. 1st.)
1908	47 659 690	3 768 112	3 202 264	565 848	1st
1909	50 904 990	4 064 085	3 466 097	597 988	2nd (Ala. 1st.)
1910	45 900 246	2 453 208	2 019 396	433 812	3rd (Ala and Pa. 1st and 2nd.)
1911	53 679 118	2 553 381	2 154 697	398 684	2nd (Ala. 1st.)
1912	59 885 226	3 522 760	3 070 523	452 237	3rd (Ala. and Pa. 1st and 2nd.)

II. HISTORY OF COAL WASHING.

4. *Early History.*—In early times, the only mode of cleaning coal was to screen out the slack and throw it away and to pick the impurities out of the lump coal by hand. As competition grew, this wasteful method was in part superseded by washing, an adaptation to the purification of coal of processes long in use for the wet concentration of ores.

Coal washing was first practised in Europe, and as early as 1826 slack coal was step-washed in the valley of the Tharandt near Dresden, Saxony. The step washer was an early form of the trough washer and owed its name to the presence of two inclined planes, or "steps," in the bottom of its trough. The jigging process was applied to coal in 1837 at the St. Etienne collieries, France, hand piston jigs with capacities of 3/8 to 1/2 ton per day of 10 hours being employed. A mechanical piston jig capable of washing 80 tons in 12 hours was introduced by Berard in 1848 and rapidly came into use in France, Belgium, Germany and England. A number of other types of washers were invented shortly thereafter, and by 1860 the process of coal washing was firmly established in Europe.*

Pennsylvania anthracite was the first coal to be washed in the United States. Experiments with crude jigs were made in the vicinity of Pottsville in 1860 and 1865, and one or two small washeries were in operation prior to 1870, although the first extensive washing of anthracite was probably that undertaken by the Lehigh Coal and Navigation Co. about the year 1875.**

The washing of bituminous coal in the United States began almost

*Beckwith, A., Van Nostrand's Mag. 2, (1870), 337-350.
**H. H. Stoek, Editorial in Mines and Minerals 25, (1905), 245.

as early as that of anthracite. The first washery was built at Alpsville on the Baltimore and Ohio R. R. about 24 miles from Pittsburgh, Pa., in 1869. It was a piston jig plant with a capacity of 10 tons per hour and washed slack from the neighboring mines for conversion into coke on the spot. Mr. John J. Endres, a German mining engineer who had been employed at Prussian government mines, had charge of the construction.*

5. *Washeries in Illinois.*—The first washery in Illinois, and second bituminous washery in America, was that of the Illinois Patent Coke Co. at East St. Louis, begun in 1870 and completed in 1871. Mr. Adolphus Meier was president of the company, Mr. Joseph Lindenschmidt vice-president, and Mr. Louis Schantl had charge of the erection of the plant. The washery had a capacity of 10 tons per hour and employed the Osterspey jig,—a piston jig with differential motion and clap valves on its piston, adapted from a jig used to concentrate lead ore at Mechernich, Prussia. The washed coal produced supplied a battery of 24 Belgian coke ovens. As first erected this plant was unsatisfactory and in 1873 it was remodeled by Messrs. E. D. and J. W. Meier.

Late in 1871, Mr. Schantl erected another washery and 30 coke ovens for St. Louis capitalists at East St. Louis, Ill. This second plant never turned out any coke. In 1871 and 1872, Mr. Endres erected 6 more piston jig washing plants. Among these were two constructed in Illinois in 1872,—one at Joliet and the other at Equality. The panic of 1873-9 brought to a close this first period of washery building.

The sudden revival of business which took place in 1879 induced a second period of washery building. The Illinois Coal and Salt Co. constructed a piston jig washery with a capacity of 20 tons per hour in connection with a coke plant at St. Johns in 1880. By 1884, more washeries and coke works had been erected in Illinois, but none succeeded in reducing the sulphur sufficiently to make a high grade metallurgical coke, and the coal washing industry would have died out in Illinois had not another market for washed coal developed.

Among the washeries erected during the second period was one built by the Luther and Tyler Coal and Coke Co. on Otter Creek north of Streator, in 1883. This was a piston jig washery with a capacity of 35 tons per hour and was erected under the supervision of Mr. Endres. It was the first washery in Illinois to wash coal for fuel. The first screenings were washed on November 22, and regular operation began November 26, 1883. For over a year experiments were conducted in the hope of producing metallurgical coke from the washed coal, but without success. On account of lack of water in the dry period, this washery was moved in 1888 to the Vermilion River, south of Streator. When the Wilmington Washed Coal Co., successors to the Luther & Tyler Coal

*Diescher, S., Proc. Eng. Soc. West. Pa. 23, (1907), 202-3.

and Coke Co., put up a 27-ton jig washery on the Kankakee River opposite Wilmington in 1890, the washery on the Vermilion River was abandoned. Mr. L. D. Howe, who had been superintendent for the Luther and Tyler Coal and Coke Co., and was in charge of the new washery, invented a tub washer and installed it in place of the jigs in 1891. This was the first of a series of 6 Howe washeries erected in Illinois.

The third period of washery building in Illinois, beginning with the year 1894 when 3 fuel washeries were constructed, extends down to the present, and during this period an average of more than 3 washeries per year have been built. There were two years—1897 and 1903—when no washeries were erected; and the greatest number put up in any one year was 11 in 1907. A general view of the progress of washery building in Illinois is presented in Table 4.

6. *Luhrig Washeries.*—The first washery in the Southern coalfield was built under the supervision of Mr. Alexander Cunninghame who came from England in 1891 to introduce the coal washing system invented by Mr. C. Luhrig of Germany. The first Luhrig washery in America was erected at the City Furnace of the Sloss Iron Co. at Birmingham, Alabama, and although this washery was not at first a mechanical success, Mr. Cunninghame succeeded in interesting Mr. T. G. Warden of Chicago in the Luhrig System, with the result that the second Luhrig washer in America was constructed at the Brush Shaft, Carterville, in 1894. Since then, 11 more Luhrig washeries have been built in Illinois.

7. *Forrester Washeries.*—In 1894 two Forrester washeries were erected in the Northern coalfield by the Illinois Coal Washing Co. and employed the Forrester jig invented by Mr. W. W. Forrester of Chicago. Three similar washeries have since been erected by the same company.

8. *Pan Jig Washeries.*—Up to 1898, the only type of jig used for washing coal in Illinois was the piston jig. In that year, Mr. Ellwood A. Stewart invented a pan jig washer and erected the first Stewart washery at the Harrison mine near Murphysboro. Twenty-one other Stewart washeries have been built in the State since then, and eight American pan jig washeries have been built in Illinois beginning with that at Porterfield, near Toluca in 1904. Mr. Nicholas Shannon invented a type of pan jig which was tried out in the washery of the Bessemer Washed Coal Co. at East St. Louis in 1907, and since that time this jig has been installed in 6 washeries.

9. *Tub Washeries.*—From 1888 till 1902 the Howe was the only tub washer in Illinois. In the latter year the Wilmington Star Mining Co. erected a Robinson tub washery at Coal City, and since then 5 other Robinson plants have been constructed.

10. *Miscellaneous.*—During the third period of washery building in Illinois the following washeries have been built, in addition to those al-

TABLE 4.

PROGRESS OF WASHERY BUILDING IN ILLINOIS.

Date	Piston Jig Washeries: Forrester No.	Forrester Cap.†	Luhrig No.	Luhrig Cap.	Others No.	Others Cap.	Pan Jig Washeries: Stewart No.	Stewart Cap.	American No.	American Cap.	Shannon No.	Shannon Cap.	Combined Pan and Piston Jig Washer's No.	Cap.	Tub Washeries: Howe No.	Howe Cap.	Robinson No.	Robinson Cap.	Bumping Table Washer's Campbell No.	Cap.	Trough Washer's Scaife No.	Cap	Summary: Total No.	Ave. Cap.	Total Cap.
		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)		(tons)	(tons)
1870 2					4	40																	4	10	40
1880-4					5	138																	5	28	138
1888					1	40																	1	40	40
1890					1	27																	1	27	27
1891															1	30							1	30	30
1894	2	100	1	60																			3	53	160
1895	1	75																	1	20			2	48	95
1896			1	30											1	75			1	40			3	37	145
1897																							0	0	0
1898			2	110			1	75													1	10	4	49	195
1899							1	75													1	10	2	43	85
1900															1	50							1	50	50
1901	1	100			1	125	2	285											1	20			5	106	530
1902							4	380									1	50					5	86	430
1903																							0	0	0
1904			3	305			1	60	1	100					1	50							6	86	515
1905	1	75	2	325					1	140							1	75					5	123	615
1906			1	125	1	60	3	190	1	140							1	100					7	88	615
1907			2	160			4	280	3	490	1	125					1	150					11	110	1205
1908							1	50	1	280							1	50					3	127	380
Prior to 1909															2	90							2	45	90
1909							1	100					2	215									3	105	315
1910					2	40*			1	80													3	40	120
1911							3	310			1	125											4	109	435
1912							1	60					2	155**			1	120					4	84	335
Total	5	350	12	1115	15	470	22	1865	8	1230	2	250	4	370	6	295	6	545	3	80	2	20	85	78	6590

*Experimental washeries.
**Including one 5-ton experimental washery.
†Capacities are all given in tons per hour.

ready noted: 3 Campbell bumping table washeries, 2 Scaife trough washeries, 1 commercial New Century piston jig washery and 2 experimental New Century jig washeries, 1 Pittsburgh pan and piston jig washery and 1 experimental washery containing various types of coal washers. Foust piston jigs were installed in two of the Shannon washeries noted above, and are being installed in one of the Stewart washeries to wash the finer sizes of coal.

11. *Summary.*—From Table 3 it appears that at least 82 commercial washeries and 3 experimental washeries have been built in Illinois. Of these, 10 have burned, 29 have been dismantled or allowed to go to ruin and 8 are shut down, leaving 35 commercial and 3 experimental washeries in operation in the winter of 1912-1913. Further details concerning these washeries will be found in the chronology of coal washing in Illinois, Appendix B.

III. Impurities in Coal.

12. *Nature of Impurities.*—If by pure coal is meant coal free from moisture, sulphur and ash, then all coal is to some extent impure. The impurities are of two kinds: (1) Those impurities which are so intimately mixed with the coal as to form an integral part of it, and (2) those impurities which are less thoroughly mixed with the coal and do not enter into its constitution. Impurities of the first class can not be separated by mechanical means, those of the second class can. The principal separable impurities in Illinois coals are shale, fireclay, bone, pyrite, gypsum, calcite and moisture.

Fireclay is a rock formed from a rather pure mixture of the mineral kaolinite, ($Al_2 O_3 . 2SiO_2 . 2H_2 O$), with a little quartz, (SiO_2). A similar mixture with a lower degree of purity constitutes shale. Fireclay and shale occur in the coal beds as the filling of shrinkage cracks and faults crossing the beds; as rounded forms called "clay balls"; as partings in the beds; and as an intimate mixture with coal, forming a material intermediate between impure coal and carbonaceous shale, known as bone. Fireclay and shale from roof and floor also become mixed with the coal during the process of mining.

Pyrite, (FeS_2), also called iron pyrites and "fool's gold", is found in the coal beds in an even wider variety of forms. It occurs in individual isometric crystals in the clay and shale of faults and shrinkage cracks; in rounded shapes called "sulphur balls"; in partings; in cleavage cracks; as flakes; as minute disseminated particles known as "flour pyrites"; and finally as an intimate mixture with coal which is appropriately termed "sulphur bone", since it corresponds to the mixture of shale and coal which constitutes true bone.

Gypsum, ($CaSO_4 . 2H_2O$), and calcite, ($CaCO_3$), occur as thin laminae in cleavage cracks in the coal. The rock limestone is composed

of more or less pure calcite and pieces of this may become mixed with the coal during mining.

Moisture is the term applied to the water mechanically held by the coal and does not include that which is chemically combined in kaolinite and gypsum. The coals of Illinois as mined have a high and variable percentage of moisture. Those in the southern part of the State contain from 8 to 10 per cent and those in the northern part from 12 to 16 per cent. By air drying this moisture content is reduced to from 2 to 5 per cent.

13. *Density of Impurities.*—The separation of impurities from coal by washing depends upon differences in density between coal and its impurities. The specific gravities of all the impurities are higher than that of the coal, but in varying degree. Lord* made specific gravity determinations on inch lumps of coal with a Nicholson hydrometer and secured values for Illinois coals as follows:

Town	*Clean Coal*	*Average Coal*
Lincoln	1.22	1.31
Auburn	1.24	1.28
Donkville	1.22	1.26
Germantown	1.26	1.30
Zeigler	1.31	1.33

Smith** gave the following specific gravities for substances from Mine No. 17 of the Consolidated Coal Co. near Collinsville, Ill.:

Coal	1.25
Bone	1.45—1.80
Shale ("Slate")	2.25—2.50
Coal with pyrite	3.20—3.60
Pyrite	5.00—5.20

Table 5 gives the specific gravities of a number of Illinois coals and of their impurities, obtained by the writer by using a Jolly balance on half-inch lumps. The results are in accord with the earlier determinations of Smith and Lord. It will be observed that the specific gravity of Illinois coal varies from 1.14 to 1.31, being lowest in the Northern field and highest in the Southern, but showing considerable local variation. The bone varies from 1.42 to 1.95; the shale, including fireclay, from 2.03 to 2.64; the pyrite bone has about the same specific gravity as the shale; the purest pyrite that can be obtained from partings is low in specific gravity owing to the inclusion of thin flakes of coal, the only pure pyrite found being in octahedral crystals in a fault plane; and the calcite is also low in specific gravity owing to inclusions of coal.

It is quite probable that further investigations would fill up the gaps shown in Table 5 between coal and bone and between bone and shale,

*Lord, N. W. "Experimental work conducted in the Chemical Laboratory of the United States Fuel Testing Plant at St. Louis, Mo., Jan. 1, 1905 to July 31, 1906." U. S. Bur. of Mines Bull. 28.

**Smith, C. H. "Mine No. 17, Collinsville, Ill." Mines and Minerals 28, 16-17.

TABLE 5.

SPECIFIC GRAVITIES OF ILLINOIS COALS AND IMPURITIES.*

Coal Bed	Coal Field	Town	Bright Coal	Dull Coal	Bone	Shale	Pyrite	Calcite
2	Northern	Granville	1.14—1.16	1.20—1.21		2.03—2.64		
2	"	Streator	1.15—1.20	1.20	1.42	2.50	(Bone) 2.43	
6	Danville	Danville	1.20—1.22			2.54—2.62	(Parting) 4.17	2.10
6	Central	Pana	1.21—1.22	1.24			(Crystal) 4.95	
6	"	Collinsville	1.2)	1.25—1.31			(Parting) 3.77	
6	Southern	Carterville	1.26—1.30		1.44—1.73	2.21—2.50		
6	"	Herrin	1.24—1.31		1.47—1.91	2.41—2.59		
6	"	Marion	1.24—1.28		1.47—1.95	2.26—2.47		

*The values for specific gravity here given are lower than those given by other authors and than those determined subsequently in the mining laboratory of the University of Illinois. This difference is probably due to the fact that the values here given were determined without first heating the coal in hot water to remove the air.—H. H. STOEK.

giving every gradation in composition and density from pure coal to pure shale. A similar gradation from pure coal to pure pyrite would probably also be found, and possibly gradations from coal to the other impurities also. But even if there were no such gradations in composition, there would still be particles having specific gravities from that of pure coal to that of the densest impurity, because there would be mixed particles composed of part coal and part impurity in all proportions.

The ash varies in a general way with the content of solid impurities, but the variation is not exactly proportional to the impurity content because the impurities are not composed entirely of ash but contain varying percentages of it. Approximate analyses of the two most common impurities, shale and pyrite, will serve to illustrate this:

Impurity	*Volatile Matter*	*Fixed Carbon*	*Ash*
Gray Shale from Herrin	7.19	0.93	91.88
Octahedral Pyrite Crystal from Pana	25.97	7.24	66.79

It is evident from these analyses that of the two impure pieces of coal of the same specific gravity, one with shale as the impurity and the other with pyrite, the ash would be much higher in the case of the coal containing shale. In the case of any given coal, the ratios of the various impurities present are likely to remain fairly constant so that the variation in ash will be very nearly in unison with the variation in specific gravity. Similarly, the content of British thermal units in a given coal will decrease with a fair degree of regularity with an increase in ash and specific gravity.

14. *Advantages of Removing Impurities.*—Washed coal is better than the raw coal from which it was produced because part of the impurities which were present in the raw coal have been removed. All the impurities mentioned in Section 12, with the exception of moisture, may be removed to a greater or less degree by washing. The general advantages of removing these impurities from coal which is to be used for fuel are:

(1) Freight and haulage do not have to be paid on waste material which has been removed by washing.
(2) The fuel value of the coal is increased so that the required charge is less and firing is therefore easier.
(3) The coal burns more freely and with less production of smoke and soot, thus increasing its efficiency.
(4) The percentage of ash is decreased and the removal of the ashes and their ultimate disposal are therefore easier.
(5) The clinkering properties of the ash are decreased, thus improving the efficiency of the fuel and the ease of removing the ashes.

In addition to the advantages just mentioned, the removal of shale, fireclay, and bone has the further advantage that with them is removed their water of constitution which would otherwise require sufficient heat to free it from chemical combination and change it to steam. If the coal is to be used in coke manufacture, it may be necessary to remove the shale and fireclay, owing to a common requirement of not over 10 per cent ash in a metallurgical coke. The removal of these impurities before coking is also desirable, because it decreases the amount of fine coke, or "breeze," formed. Unlike the other impurities in coal, pyrite is combustible and generates considerable heat. Its disadvantages, however, more than offset this advantage. In burning it forms sulphur dioxide, a gas which corrodes fire grates and boilers, and ferric oxide, a solid which is an active agent in bringing about the clinkering of ash. In coking, only a portion of the sulphur is driven from the pyrite, a very serious matter when one considers that one per cent. of sulphur is commonly the maximum permitted in a coke to be used for smelting iron. Calcite and gypsum are not very actively harmful when present in a fuel, although the former requires heat to liberate its carbon dioxide, and the latter heat to free and gasify its water; but gypsum is undesirable when the coal is to be coked, since all its sulphur will enter the coke.

IV. Crushing.

15. *Objects of Crushing.*—Crushing is the reduction in size of material by the application of mechanical force. The crushing of bituminous coal may conveniently be divided into coarse crushing and fine crushing. Coarse crushing may be defined as the breaking of coal to a maximum size of more than ¾ in., and fine crushing to a maximum size of ¾ in. or less. Coarse crushing is commonly employed when coal is to be used for fuel, and fine crushing when it is to be used in the manufacture of coke.

Crushing in connection with washing may have for its object the freeing of attached particles of coal and impurities, or the production of small sizes to meet a demand for them. Crushing to produce small sizes of washed coal is not at present practised in Illinois, but Bement,* commenting upon the increasing demand for washed sizes Nos. 4 and 5, states that if this increase continues they will soon command a higher price than the larger sizes. In that event, the practice will undoubtedly be introduced.

Crushing to free impurities may be applied to the raw coal prior to washing, or to the refuse as a preliminary to re-washing. Flakes, balls, bands and strata of impurities adhere tightly to the coal as it lies in the ground. During the process of mining, the coal and impurities

*Bement, A. "The Screening Problem in Illinois." Coal Age 1. (1912) 1105.

become broken apart to a greater or less extent, and this is all the crushing that the screenings sent to Illinois washeries usually receive. Lump or run of mine is crushed and washed at some mines when the coal becomes temporarily more impure than usual.

Coarse crushers for reducing the size of the lump or run of mine to the maximum size washed have been installed in five of the commercial washeries at present operating in Illinois. When washing coal for coke, a greater reduction of ash and impurities is desirable than when washing it for fuel, so in two experimental coal washeries fine crushing is employed in order to free more of the impurities than would coarse crushing.

When large sizes of coal are washed, fair-sized pieces of good coal attached to pieces of impurities are carried into the refuse. By re-crushing and re-washing the refuse, a considerable amount of this coal may be recovered. This process was successfully introduced into one Illinois washery in the fall of 1912.

16. *Toothed Rolls.*—The crushers used for reducing refuse, and four of the five crushers employed in Illinois for breaking mine run and lump, are toothed rolls, consisting of two cylinders which rotate toward one another, as looked at from above, and are provided with projecting teeth which nip and crush the material fed to them. When this material is coal, about the same proportion of different sizes are produced as in the process of mining, if the following results obtained at a washery in southern Illinois on coal from Bed No. 6 are typical:

	No. 1 Extra	*No. 1*	*No. 2*	*No. 3*	*No. 4*	*No. 5*
Produced by mining	9.0 %	15.8 %	12.4 %	15.6 %	27.2 %	20.0 %
Produced by crushing	10.0%	17.2 %	15.0 %	9.7 %	29.7 %	18.4 %

No figures were obtainable as to the capacities and power consumptions of the rolls used for crushing coal in Illinois. The pair of toothed rolls crushing refuse worked on material between 3 in. and ¾ in. in size reducing it to less than ¾ in. They were 18 inches long and 18 inches in diameter and treated about 7 tons per hour.

One single-roll crusher is installed in an Illinois washery to do coarse crushing. The single roll crusher instead of having two toothed rolls rotating toward each other, as in the ordinary coal rolls, consists of a single toothed roll rotating toward a fixed plate.

The hammer pulverizer is well adapted to the fine crushing of coal. It consists of a number of hammers attached loosely to a shaft which is rotated at high speed. The hammers strike the coal in the air and crush it largely by impact. A Williams No. 00 hammer pulverizer was found to reduce Illinois coal to ⅜ in. at the rate of 1¼ tons per hour when running at 2200 r. p. m. A Pennsylvania crusher required 75 k. w. to reduce 75 tons of dry Pocahontas coal per hour from mine run to ¾ in. Fifteen per cent. less dry Illinois coal could be put through

per hour under the same conditions, and when this coal contained from 9 to 12 per cent. moisture only 50 tons per hour could be crushed, making the power consumption 1.5 k. w. per ton moist Illinois coal per hour.

V. Sizing Raw Coal.

17. *Sizing* is the separation of material into groups of particles lying within distinct ranges in size. When sizing is perfect, these groups do not overlap, but consist wholly of particles whose sizes lie within some definite range and are all smaller than those in the group above and larger than those in the group below.

Sizing is usually effected by means of screens made of bars, woven wire or perforated metal. The screens in use in Illinois washeries may be classified as follows:

CLASSIFICATION OF SCREENS.

Fixed	*Moving*	
Gravity Screens	Revolving Screens	(Cylindrical) (Conical)
Drag Screens	Shaking Screens	

18. *Fixed Screens* are constructed of bars or perforated metal forming a plane surface over which coal may slide by gravity ("gravity screens"), or be drawn by a scraper ("drag screens"). Gravity screens are inclined so that coal runs down hill over them, the larger pieces passing over the openings and the smaller pieces through them. They are placed in the bottom of chutes to relieve or supplement overloaded screens of other types, to remove chippings or to drain off water. Drag screens are either level or inclined, the coal being dragged up hill in the latter case. They are used for separating the two finest sizes of washed coal, for rinsing, and in one instance for preliminary sizing. No headroom is lost by their use—head being actually gained when they are inclined—and in this respect they are superior to moving screens, but they have the disadvantage in common with gravity screens that they do not stir up the coal as thoroughly as the moving screens and so are not as efficient.

19. *Revolving Screens.*—Moving screens are used in washeries for all sorts of sizing. Revolving screens are made of woven wire or perforated metal, bent to the shape of a hollow cylinder ("cylindrical revolving screens"), or the frustum of a cone ("conical revolving screens"). Cylindrical revolving screens are supported upon spiders extending from a central shaft or upon exterior rings resting on rollers, and are revolved by gears. They are mounted with a slight inclination which, combined with their rotation, causes the coal fed in at the high end, which does not pass through the openings in the screen, to be discharged at the lower end. Conical revolving screens are usually mounted with their

axes level and thus have simpler bearings than cylindrical screens. The movement of the coal over them is due to the downward slope of the bottom of the screen caused by the increasing diameter of the frustum of the cone, assisted, as in the cylindrical screen, by the revolution of the screen.

Two or three revolving screens of different diameters are sometimes arranged concentrically upon the same shaft, when the screen is said to be "double-jacketed" or "triple jacketed." Such an arrangement is not only economical of space and power, but it enables the coarser sizes to be removed first, so that they need not pass over the fine screen, which is the one most readily worn out. The disadvantages are that since the outer screen rotates more rapidly than the inner, when it is going at the proper speed the inner one is not going fast enough; again, since the outer screen is larger than the inner one but has less material to screen, if the inner screen is big enough the outer one will be too large; and, finally, the construction renders it difficult to clean and repair the inner screen or screens. These particular disadvantages are overcome by arranging a series of cylindrical revolving screens of the same diameter close together upon the same shaft, but unfortunately other disadvantages arise. More space and heavier shafting are required and since all the coal passes over the fine screen it wears out rapidly. Moreover, this screen must be longer than in the preceding case since it has more work to do.

20. *Shaking Screens* are of perforated metal forming a plane surface which is given a rapid reciprocal motion by means of eccentrics. They may be supported on rollers below, hung from rods above, or supported above or below by means of planks rigidly attached to the screen frame. The form of shaker with the planks placed below is called the Parrish screen and not only are its supports of wood fastened tightly to the frame, but its eccentric rods are also of wood and also firmly secured to the body of the screen. This wooden construction makes the Parrish screen light, while the rigid attachment of planks and rods results in a sharp upward jerk on each stroke of the screen which is very effective. Shaking screens are more efficient per square foot of screening surface than revolving screens, occupy less space, and are more readily cleaned and repaired. Their great disadvantage lies in the fact that they can not be perfectly balanced under all conditions of load, and so they are apt to cause harmful vibration of the framework of the washery. This effect may be reduced by mounting the shaking screens in pairs moving in opposite directions.

21. *Object of Sizing.*—Sizing in connection with coal washing is employed before washing, after washing, or both before and after. Sizing after washing is discussed in Chapter VII, the present chapter being concerned with sizing raw coal only. The object of sizing before

washing is to make possible a better separation of coal from impurities on the washing machines. When the impurities have specific gravities much higher than that of the coal and are not attached to the coal, a good separation may be made without preliminary sizing, but if the impurities are but slightly higher in specific gravity than the coal, or are firmly attached to it, preliminary sizing is essential to effective washing. The reason for this will become clear when the theory of washing given in the next chapter is considered.

In Illinois, preliminary sizing is employed in 10 out of 32 operating commercial washeries. This includes six piston jig washeries—which is all but one of the washeries of this type; two pan jig washeries—which is but one-tenth of the washeries of this type; one combined pan and piston jig washery, and one bumping table washery, as shown in Table 6. Two sizes are made in three washeries, three sizes in one, four sizes in five washeries and five sizes in the remaining washery. In six cases the smallest size washed is through a ¾ in. round hole; in four cases a size between ¾ in. and 1 in. is washed; in three a size between 1 in. and 1¾ in.; and in two a size between 1¾ in. and 3 in. It is evident that preliminary sizing is very irregular in Illinois, especially of the larger sizes. Theoretically, in order that similar results shall be achieved when washing different sizes, the screen scale (or sizes of openings of successive screens) must have a constant geometrical ratio. For example, with a ratio of two the sizes of openings might be ¼ in., ½ in., 1 in., 2 in., and 4 in. An examination of Table 6 will show that no screen scale

TABLE 6.

PRELIMINARY SIZING IN ILLINOIS.

Washery Type	Reference No.	Sizes Made		Screens Employed	
		No.	Dimensions in Inches	Type	Ref. No.
Pan Jig	C.2	2	1⅛*—¾, sq., ¾ sq.—0	Cylindrical	E III a 1
"	20	2	3—$\frac{9}{16}$ sq., $\frac{9}{16}$ sq.—0	"	3
Pan and Piston Jig	35	2	3—¾, ¾—0	Conical	12
Piston Jig.......	19	4	3—1¾, 1¾—1, 1—¾, ¾—0	Triple Jacketed Cylindrical	2
"	24	4	"	Triple Jacketed Conical	4
"	28	4	3—2, 2—1, 1—¾, ¾—0	Triple Jacketed Cylindrical	5
"	29	4	3—1¾, 1¾—1, 1—¾, ¾—0	"	6
			"	"	7
"	32	4	¾—0	Shaking	E II a 1
			"	"	2
			3½—2¾, 2¾—1¾, 1¾—¾	Cylindrical	E III a 8
			"	"	9
"	34	5	1¾—1, 1—¾, ¾—0	Conical	10
			3½—3, 3—1¾	Double Jacketed Cylindrical	11
Bumping Table	30	3	3¾—1¾ slot, 1¾—⅜ slot, ⅜—0	Drag	E I a 1

*All sizes made thru round holes unless otherwise specified.

with constant geometrical ratio is at present in use for preliminary sizing in Illinois. Revolving screens are in use for sizing raw coal in all but one of the washeries. Cylindrical revolving screens only are in use in six washeries, shaker screens supplement the cylindrical screen in one washery, one has both a conical and cylindrical screen, and one a conical screen only. The washery which sizes before washing entirely without the aid of revolving screens employs a drag screen, which is traversed by scrapers running at the rate of 180 ft. per minute. The first section of this screen receives 3¾ in. screenings and is perforated with ⅜ in. round holes. When the washery is running at its rated capacity this screen treats one ton of coal per hour for each 0.2 sq. ft. of screening surface. Examination of the screen in action shows that it is too small for the work.

The two shaking screens treating raw coal make 132 shakes per minute and screen 52 per cent of ¾ in. coal out of 3½ in. screenings at the rate of one ton per hour for each 0.7 sq. ft. of screening surface. The Coal and Metal Miners' Pocket Book gives the capacity of ¾ in. shaking screens working on bituminous coal as one ton per hour per 0.63 to 0.83 sq. ft. of screening surface,* of which 0.7 is about the average, but in anthracite practice a much larger screening surface is employed.

There are twelve revolving screens sizing raw coal in Illinois washeries, nine of which are cylindrical and three conical. Of these screens, one is a simple cylndrical and one a simple conical, two are single-jacketed cylindrical screens making two sizes, one is a double-jacketed cylindrical, and four are triple-jacketed cylindrical, while the remaining two are triple-jacketed conical revolving screens. The cylndrical screens have slopes ranging from three to seven degrees, with an average of five degrees, and the conical screens all have their axes level. The number of revolutions per minute varies from seven to thirty, giving peripheral speeds ranging from 126 to 471, with an average of 219 ft. per minute. The Coal and Metal Miners' Pocket Book‡ states that the peripheral speed of anthracite screens should be about 200, and Sterling** says that it should not exceed 250 ft. per minute. Thus while the average speed of these screens appears to be about right, seven out of the twenty-seven screening surfaces under consideration have speeds above what is considered good practice in anthracite preparation. The square feet of screen surface per ton treated per hour varies from 1.0 to 11.3 with an average of 4.2. F. E. Brackett† holds that this ratio should be 8 when the mesh is ¾ in. and 16 when it is ¼ in., indicating that the raw coal screens in Illinois are not as large as elsewhere in the United States.

Further details concerning the various screens enumerated above may be found in Appendix E, Parts Ia, IIa and IIIa.

*8th Edition, (1904) 433.
‡Page 434.
**Bul. Am. Inst. Min. Eng. 58, (1911) 762.
†Coal Age 3, (1913) 131.

VI. Washing.

A. Theory.

22. *Free Settling Ratio.*—If a piece of coal and a piece of shale of the same size and shape are dropped into water at the same time, the shale will reach the bottom first, because its weight is greater than that of the coal and the surface it presents to the water to be acted on by friction is similar. If the same piece of coal is dropped into water with successively smaller particles of shale of the same shape, the difference in time of fall will be reduced because the proportion of surface to weight, and therefore the friction per unit of weight, increases with decrease in size. Finally a particle of shale will be found of such a size that it will settle at the same rate as the piece of coal. The ratio of the diameters of this coal and shale will be inversely as the ratio of their weights in water. Expressed as a formula, this becomes

$$\frac{D}{D'} = \frac{S' - 1}{S - 1} \qquad (1)$$

where D is the diameter of the coal, S its specific gravity, D′ the diameter of the shale and S′ its specific gravity. For example, if coal has a specific gravity of 1.2 and shale a specific gravity of 2.4, the ratio is 7, which means that a particle of this coal with a diameter of 7 will settle at the same rate as a similarly shaped piece of shale with a diameter of 1. The average particle of shale is flatter than the average particle of coal, so when pieces of coal and shale which have been sized on screens are allowed to fall through water the shale will present a larger surface in proportion to volume than the coal and the ratio will therefore be less than if the grains were of the same shape. Experimentally, the equal-falling ratio, under free settling conditions, has been found by the writer to be about 6. In a similar manner, free settling ratios may be determined for all the impurities in coal.

23. *Trough Washer.*—When particles of coal and impurities are dropped into a stream of water moving horizontally, they will describe parabolas. The more rapidly falling particles will reach the bottom first after the shortest exposure to the horizontal force of the stream and will therefore have been carried the shortest distance, while the more slowly falling particles will have been carried a greater distance. Thus a horizontal stream divides a group of particles of different sizes and specific gravities into groups of equal-falling particles of various sizes, the size divisions being determined by distance from the point of introduction of the particles. This principle is used in the trough washer. A trough is constructed of such a length and operates with such a stream of water that the largest particle of coal treated will be

carried just the full length of the trough. With it will be carried equal-settling particles of all impurities together with all smaller sizes of impurities and all the coal, while all pieces of impurities larger than those which are equal-settling with the largest coal will remain in the trough, to be removed and disposed of as refuse. No trough washers are at present employed for washing coal in Illinois, but a modified form of trough washer is in use at one washery to remove pyrite from the circulating water.

24. *Grading Box.*—The grading box employed in the Luhrig system of coal washing makes use of the same principle, but in a different manner. The grading box is a trough with openings in its bottom at intervals, through which groups of particles which are equal-falling under free settling conditions are drawn off. The continuous removal of a portion of the stream with each group of grains decreases the strength of the current and results in a more rapid dropping of the particles than if the velocity of the stream remained constant, as in the trough washer. The object of the grading box is not to produce a finished product but to prepare the raw coal for further treatment on a series of jigs.

25. *Hindered Settling.*—If a crowded mass of particles of coal and impurities of different shapes and sizes is subjected to a rising current of water which is not so swift as their free settling velocities and so will not carry them away, but yet is strong enough to keep them in motion, they will arrange themselves in layers having definite ratios dependent upon their specific gravities and shapes. The ratios are larger than for the same particles under free settling conditions and are called hindered settling ratios. This increase in ratios may be explained on the ground that under hindered settling conditions the classification is brought about by means of a quicksand which is of higher specific gravity than water, but otherwise works in a similar way. The formula for ratios then becomes

$$\frac{D}{D'} = \frac{S' - s}{S - s} \tag{2}$$

with the letters D, D′, S and S′ having the same meaning as in the formula for free settling ratios and s being the specific gravity of the quicksand. Again, considering the coal of 1.2 specific gravity and the shale of 2.4, and supposing the mixture of coal, shale and water forming the quicksand to have a specific gravity of 1.1, the hindered settling ratio is 13 where the free settling ratio was only 7; or, correcting for the difference in shape between coal and shale, 11, where the free settling ratio was only 6. In all the washers in use in Illinois—tubs, jigs and bumping tables—the washing takes place under hindered settling conditions.

26. *Tub Washer.*—The tub washer is the simplest form of hindered settling washer. It consists of a hollow, inverted cone at the bottom of which a rising current is introduced, which, with the help of mechanical stirrers, keeps the mixture of coal and impurities agitated. The larger impurities settle to the bottom, where they are drawn off intermittently, while the coal and smaller impurities which are equal-settling with the coal under hindered settling conditions are discharged continuously at the top.

27. *Jig.*—In the jig, hindered settling conditions are attained by an intermittent rising current, a succession of pulsions, or by alternating rising and falling currents, pulsions and suctions. The currents pass through a screen upon which the material to be treated rests, and are either produced by the movement of a piston, in piston jigs, or by the movement of the screen in pan jigs. The coal is discharged continuously at the top and the refuse either continuously or intermittently at the bottom. In order that the water shall work evenly over the whole area of the jig, it is necessary that a thin, loose layer of the impurities mixed with the coal be maintained over the screen, a natural bed, or if this is undesirable, a thin layer of fragments of some foreign substance like feldspar, an artificial bed. The art of jigging depends largely upon keeping the bed regular and open so that the currents of water shall rise freely through the material with equal velocities in all parts of the jig.

28. *Bumping Table.*—In the bumping table, hindered settling conditions are induced by subjecting the wet material resting on an inclined table to a succession of bumps tending to move the material up the table. The material becomes stratified in equal-settling layers. The upper layers, containing big pieces of coal and small particles of impurities, are washed off the lower end of the table by the stream of water, while the lower layers, containing the large particles of impurities, are bumped up the table and off at the top.

B. TUB WASHERS.

Tub washers were in operation in five Illinois washeries in the fall of 1912. Two commercial washeries employed Robinson tubs, two used Howe tubs and one experimental washery was testing the Richards-Janney classifier. The construction and operation of these machines is described in this section, but the results obtained at commercial tub washer plants are not given, nor are the results at other types of commercial washeries given in the succeeding sections, as such results show the action of the washery as a whole and not of the washing machines alone. For this reason results of washing are not given until Chapter X, which comes after the arrangement of washeries has been discussed.

29. *Robinson Washer.*—The Robinson tub washer, shown in section in Figure I, has a washing chamber shaped like the inverted frustum of a cone, attached at the bottom to a cylindrical refuse chamber which has valves at top and bottom.

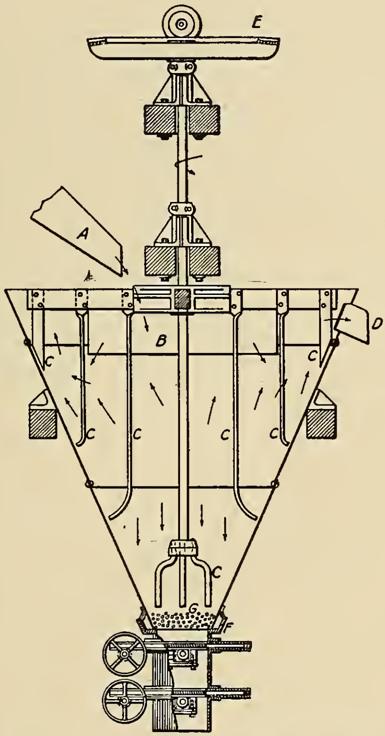


FIG. 1. ROBINSON TUB WASHER.

Raw coal is fed through the chute, A, into the center of the ring, B, while water under pressure enters the housing, F, and passes through the perforations, G, into the washing chamber. There coal and water are thoroughly mixed by the vertical stirring arms, C, driven by a gear from above. Hindered settling conditions are thus set up, causing the washed coal to rise and pass off with the water at D and the refuse to sink into the refuse chamber whose upper valve is left open. When it is desired to remove the refuse, this may be done without interfering with the operation of the tub by temporarily closing the upper valve and opening the lower one, when the refuse which has accumulated in the refuse chamber will be discharged.

The Robinson tubs in Illinois are 10½ ft. in diameter and 10½ ft. in height with 2 ft. discharge. Their rated capacity is 50 tons unsized

raw coal per hour. At one washery they are run at 20 r. p. m. and at the other at 24. In order to increase the refuse storage capacity of the tub, a second refuse chamber and third valve have been added at one washery.

30. *Howe Washer.*—A sectional view of the Howe tub washer is shown in Fig. 2. It differs from the Robinson washer mainly in that it has horizontal instead of vertical stirrers.

The Howe tub at one washery is 9½ ft. in diameter and 8 ft. in height. It is run at 20 r. p. m. and has a rated capacity of 50 tons unsized raw screenings per hour. At the other washery the tub is 8½ ft. in diameter and the stirring arms are given 18 r. p. m.

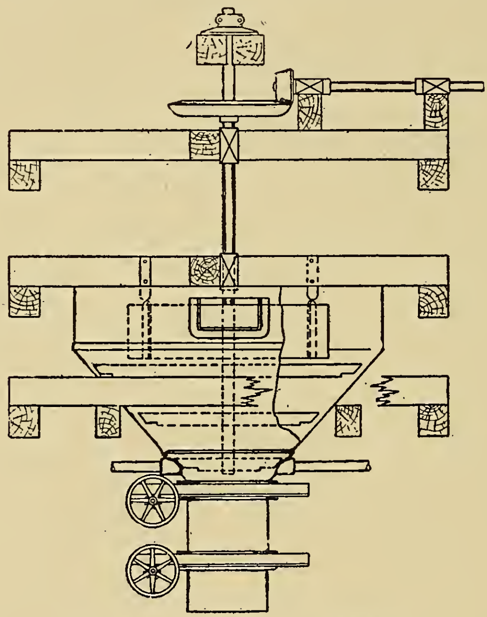


Fig. 2. Howe Tub Washer.

31. *Richards-Janney Classifier.*—The Richards-Janney classifier is shown in Figs. 3 and 4. Essentially it is a series of rectangular compartments (A) of increasing size each with a tub washer (B) having horizontal stirring arms like the Howe at its bottom. These washers differ from the Howe mainly in the valve employed. There is a single valve (C) above the refuse chamber (D), which is opened automatically at definite intervals, and in place of the lower valve a small opening or spigot (E) exists at the bottom of the refuse chamber, which permits the refuse discharged from the washing chamber to run away slowly and continuously. The action of the rectangular compartments

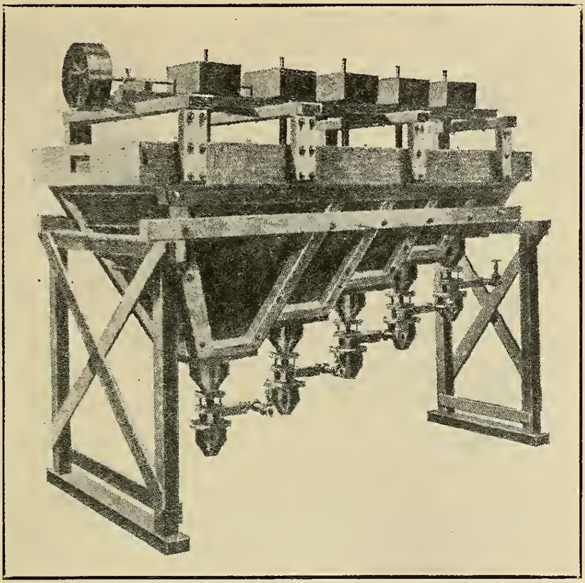


FIG. 3. FIVE COMPARTMENT RICHARDS JANNEY CLASSIFIER.

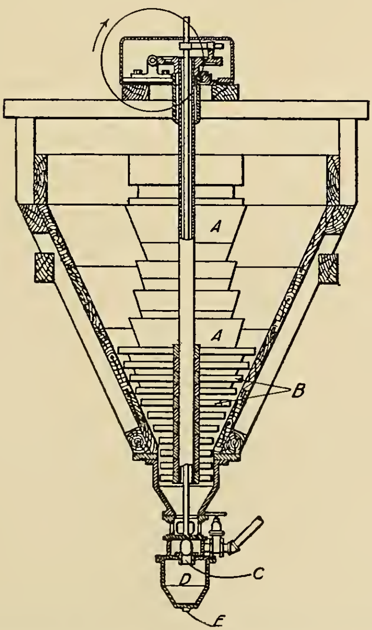


FIG. 4. RICHARDS-JANNEY CLASSIFIER, CROSS SECTION.

above the tub washers is that of a free settling classifier, or in other words these compartments deliver products which have been classified under free settling conditions to the tub washers, where they are reclassified under the more favorable hindered settling conditions. The Richards-Janney classifier is made in a small size capable of handling coal up to 3/8 in. in diameter only, and its stirring arms are run at a speed of 80 to 90 r. p. m.

Experimental work with a five-compartment Richards-Janney classifier demonstrated that a good separation could be made with the first three compartments. From the first spigot pure refuse was drawn off, from the second and third "second grade coal"—that is, coal of low grade but sufficiently good for use about the plant, a product similar to the "middlings" product of ore-dressing mills; while washed coal came from the fourth and fifth spigots and from an extra rotating valve. No water was supplied to the last two compartments, so their only function was to dewater the washed coal. The machine treated from five to six tons of raw coal per hour with a water consumption of 14000 to 20-000 gallons supplied under a pressure of 25 pounds, with a power consumption of a fraction of a horsepower. It is said that in treating an Illinois coal containing 14 per cent ash and 2.70 per cent sulphur, washed coal is obtained with about 8 per cent ash and 1.70 per cent sulphur, a small amount of secondary coal with about 16 per cent ash and 3 per cent sulphur, and refuse with 70 per cent ash and 13 per cent sulphur.

C. PISTON JIGS.

In the fall of 1912, piston jig washers were in operation in eight commercial and two experimental washeries in Illinois. They were the second in number among the classes of washers in use, being only exceeded by the pan jigs. The types represented were the Luhrig jig, the Forrester jig, the New Century jig and the Foust jig.

32. *Luhrig Jig.*—Luhrig jigs are single-compartment jigs whose pistons are given simple reciprocating motion by means of eccentrics located above, as shown in Figs. 5 and 6. They are used with sized or classified coal and the jig used for the large sizes and shown in Fig. 5 is radically different from that used for small sizes and shown in Fig. 6.

The Luhrig nut coal jig (Fig. 5) consists of a rectangular box with hopper bottom having a partition in the middle, extending about half way down from the top, or to a point slightly above where the hoppering begins. Upon one side of this partition, is a relatively close-fitting rectangular piston actuated by an eccentric.

Upon the other side of the partition there is a fixed screen slightly inclined away from the partition. The jig is filled with water, to which the piston imparts a pulsating motion, forcing it up and down through

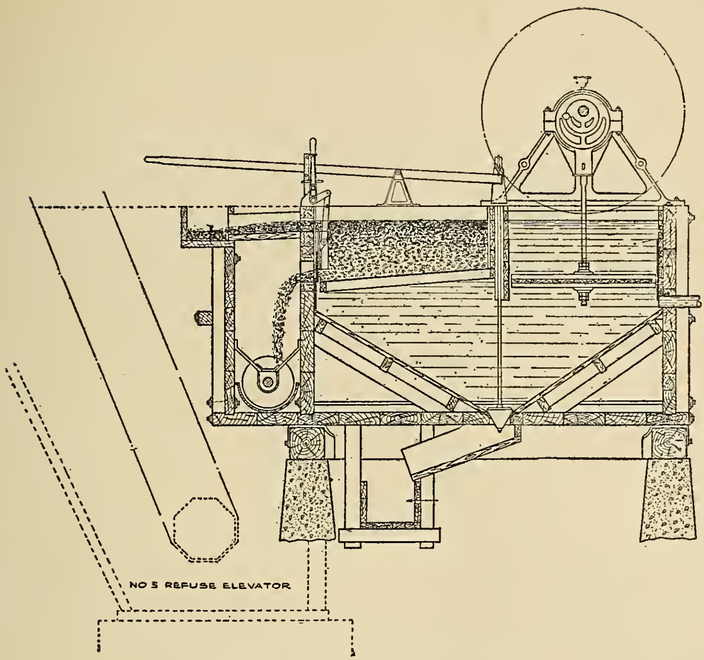


FIG. 5. LUHRIG NUT COAL JIG.

the screen. Sized raw nut coal is fed upon the screen near the partition and purified by the hindered settling action induced by the pulsation of the water through the screen. The washed coal flows from the top of the screen compartment at the opposite end from the feed, while the refuse works its way across, assisted by the slope of the screen, and the excess over that required to maintain a suitable bed is discharged through a valve just above the screen and below the washed coal overflow. The bed is kept thin enough to permit regular and even pulsations of water through the screen, and thick enough to prevent fine coal from working through the screen by the aid of suction and entering the hoppered bottom, or hutch, of the jig. The refuse which collects in the hutch is discharged at intervals, as required, through a valve at the bottom.

The Luhrig fine coal jig (Fig. 6) differs from the nut coal jig in three important particulars. The screen slopes toward the partition, an artificial bed of feldspar is provided, and all the refuse passes through the bed and screen into the hutch from which it discharges continuously. It is fed with fine coal which has been classified in a grading box. The reversal in slope of the screen is for the purpose of bringing the thickest portion of the bed near the piston where the rising current of water is strongest, thus equalizing the pulsations throughout the

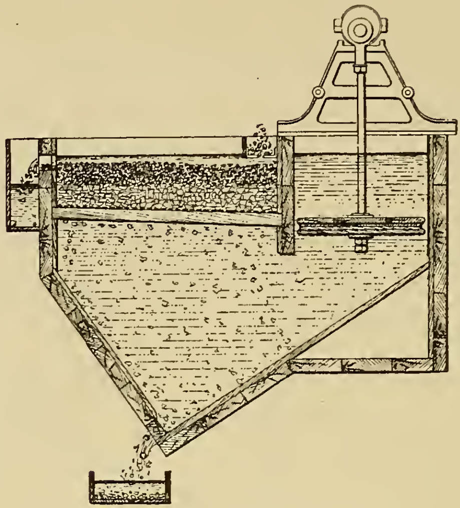


FIG. 6. LUHRIG FINE COAL JIG.

bed. The object of using a false bed is to overcome the tendency which exists in a bed of fine refuse to pack tightly and offer too much resistance to the passage of the water.

Luhrig jigs are the commonest type of piston jigs in use in Illinois, being employed in six washeries. They have screens ranging in size from 33 in. x 35 in. to 42 in. x 54 in. and treat on the average 7.2 tons raw coal per jig per hour. The proportion of nut coal to fine coal jigs is as 5 is to 4. The number of piston strokes per minute in the case of the nut coal jigs averages 79 and the length of stroke varies from 7 in. to 3 in., while the fine coal jigs make an average of 157 strokes per minute with lengths from 2 in. to ¾ in. The size of perforations in the screens of the nut coal jigs varies from ¼ in. to ¾ in., while in the fine coal jigs they are usually ¾ in. or larger, to permit of easy passage of the refuse through the artificial bed of feldspar, though in one washer there is a range of from ⅜ in. to 1⅛ in. in size of opening. Further details may be obtained through an examination of Table 7.

33. *Forrester Jig.*—The Forrester jig (Fig. 7) differs from all other piston jigs in that it is intended for use upon unsized and unclassified screenings. It differs from the Luhrig nut coal jig in three important particulars: It has a level screen, a different method of actuating its piston, and a different manner of disposing of its refuse. Reciprocating motion is imparted to the piston by means of pitmans attached to disc wheels set on a shaft behind and below the jig. This arrangement produces a very compact machine which takes up but little height. The refuse elevator raises the refuse into full view of the operator and thus keeps him in constant touch with the work the jig is doing.

TABLE 7.

LUHRIG JIG PRACTICE.

Washery		All Jigs		Nut Coal Jigs														Fine Coal Jigs		
						No. 1 Extra Jigs			No. 1 Jigs			No. 2 Jigs			No. 3 Jigs					
No.	Rated Capacity per h.	No.	Av. Capacity Tons per h.	No.	Strokes per Minute	No.	Size Treated (inches)	Length of Stroke (in.)	No.	Size Treated (inches)	Length of Stroke (in.)	No.	Size Treated (inches)	Length of Stroke (inches)	No.	Size Treated (inches)	Length of Stroke (inches)	No.	No. of Strokes	Length of Stroke (in.)
19	120	20	6.0	10	72				4	3 —1¾	5	3	1¾—1	5	3	1—¾	4½	10*	138	2
24	100	13	7.7	7	80				2	3 —1¾	6	3	1¾—1	4½	2	1—¾	3	6	174	2
28	60	11	5.5	7	80				2	3 —2	5½	3	2 —1	4½	2	1—¾	3½	4	150	2½—2
29	125	25	5.0	13	76				4	3 —1¾	5½	5	1¾—1	4½	4	1—¾	3½	12	128	1½— ¾
32	125	15	8.3	9	88	3	3½—2¾	7	3	2¾—1¾	6	3	1¾—¾	4				6	192	2 —1½
34	200	20	10.0	12	80	2	3½—3	6	4	3 —1¾	6	4	1¾—1	5	2	1—¾	4	8	160	1½— ¾
Sum	730	104																		
Av.	122	17	7.2	10	79			6½			5½			4½			3½	8	157	1

* Classified ¾ inches

There is but one Forrester washery now operating in Illinois. At this washery the original Forrester jig design has been modified to the extent of replacing the scraper elevator formerly used by a screw elevator which is said to give much better satisfaction. The jig screen is 30 in. by 36 in. and is perforated with 7/16 in. round holes and the jig has a rated capacity of 25 tons of 2½ in. screenings per hour. The piston makes 6 in. strokes at the rate of 40 per minute.

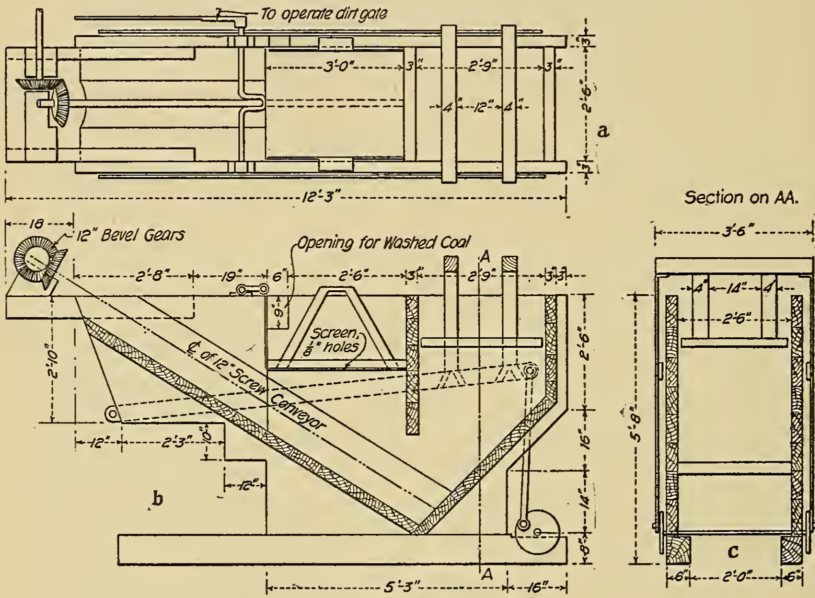


FIG. 7. FORRESTER JIG.

34. *New Century Jig.*—The term "New Century" is applied to all the jigs manufactured by the American Concentrator Company, including some eight or ten adapted to washing coal. In general, these jigs bear a resemblance to the Luhrig nut coal jig. Their principal differences from the Luhrig jig consist in the employment of some form of mechanism to cause the piston to move downwards with greater velocity than it ascends, thus increasing pulsion and decreasing suction; and in the use of a second compartment, or second jig, by which there is obtained not only washed coal and refuse but also an intermediate product called "secondary coal" that may be used about the washery or mixed with the first grade washed coal, as determined by circumstances. Specially designed refuse valves are also used on these jigs. A double New Century jig No. 900, shown in Fig. 8, has a cam and spring to give the desired differential movement to the piston.

TABLE 8.

Tests with New Century Jigs on Illinois Coals.

Locality		Size Tested	Raw Coal		Washed Coal			Second Grade Coal			Refuse		
Town	County	(Inches)	Ash	S	Ash	S	Proportion	Ash	S	Proportion	Ash	S	Proportion
Westville	Vermilion	¾—½ & ½—0*	14.45	3.23	6.50	1.76	72.2	12.06	2.47	19.5	47.81	5.76	8.3
Benton	Franklin	"	13.50	2.48	8.20	2.31	80.6	12.50	2.45	12.2	32.48	9.62	7.2
"	"	"	13.20	2.02	7.00	1.37	82.5	16.00	1.84	12.2	37.00	6.80	6.3
Rend	"	"	11.48	2.65	7.25	1.97	75.0	12.73	2.67	18.7	44.82	9.32	6.3
"	"	"	16.45	1.39	6.45	1.01	84.7	15.60	1.57	7.5	34.00	4.12	7.8
Christopher	"	"	10.40	1.23	6.85	0.95	87.4	17.52	1.89	5.0	46.80	2.89	7.6
"	"	"	12.36	5.11	7.40	2.56	79.9	13.26	3.05	9.0	57.00	16.67	11.1
Royalton	"	¾—⅜ & ⅜—0*	11.57	1.61	6.53	1.01	83.4	18.04	2.32	10.3	25.50	3.12	6.3
West Frankfort	"	⅜—0	11.00	3.00	7.50	1.30	84.0	34.00	8.00	9.7	57.00	18.20	6.3
Herrin	Williamson	¾—½ & ½—0*	9.10	2.04	8.00	1.89	78.4	11.40	4.71	12.3	35.50	11.39	9.3
"	"	"	12.70	2.22	8.50	1.80		16.90	4.31		48.18	12.02	
Harrisburg	Saline	⅜—0	11.22	2.27	6.80	1.55	85.5	37.41		9.6			4.9

*Sizes washed separately and then combined for weighing and analysis.

Tests with New Century jigs on Illinois coals were being conducted at two experimental washeries during the summer of 1912. One of these washeries used a No. 600 jig. It was found that this had a capacity of three tons per hour on ⅜ in. coal and that it worked best with 120 strokes of ⅝ in. per minute. One hundred seventy-five gallons of water per minute were consumed. The jig had a ⅛ in. screen and was operated with natural bed, a pebble bed having been tried first and found unsatisfactory because of the tendency of the rounded pebbles to choke the screen. The other washery employed New Century jigs Nos. 800 and 900, and washed the coal in two sizes. Some of the results obtained at these washeries have been kindly supplied by the operators and are given in Table 8.

35. *Foust Jig.*—The Foust jig (Figs. 9 and 10) is a compartment jig having several screens, each slightly lower than the preceding one, and each supplied with pulsating currents of water by a pair of pistons, one on either side. Like the Luhrig fine coal jig, it is intended for use on the smaller sizes of coal only, but in the case of the Foust jig the

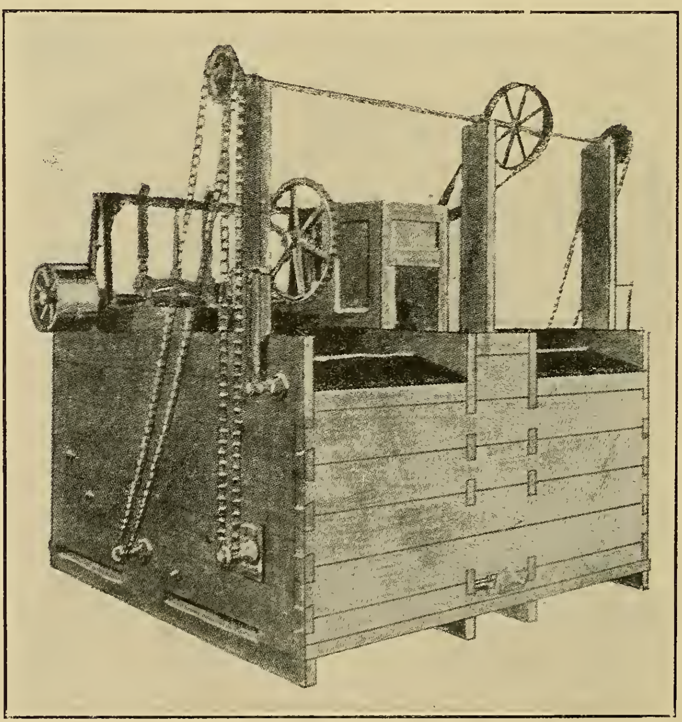


FIG. 8. DOUBLE NEW CENTURY JIG NO. 900.

coal is not classified prior to washing and the jig is not supplied with an artificial bed but makes its own from the refuse in the raw coal. The finer refuse passes through the screen into the hutch while the coarse refuse in excess of that required for a suitable bed is removed through valves in the screens. Only one Illinois washery had a Foust jig in operation in 1912 although two more have put them into commission since then. At the operating washery a two-compartment Foust jig was treating raw coal ¾ in.—0 in size. The pistons were making 130 strokes per minute, the stroke on the first compartment being 1 in. and on the second ⅞ in. The refuse was very clean, containing 74.13 per cent ash (Sample 89).

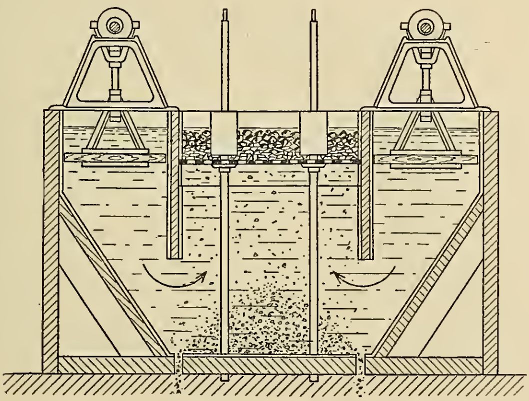


FIG. 9. FOUST JIG, CROSS SECTION.

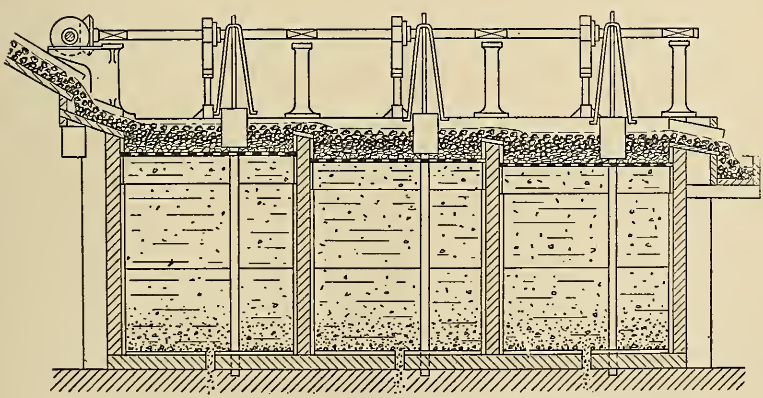


FIG. 10. FOUST JIG, LONGITUDINAL SECTION.

D. PAN JIGS.

The pan jig is the commonest class of washer in use in Illinois. Three types are in operation: the Stewart, the American and the Shannon.

36. *Stewart Jig.*—The characteristic feature of all pan jigs is the box, or pan, with perforated bottom upon which the coal is washed. A view of the pan of the Stewart jig looked at from above is shown in Fig. 11, and a sectional view of the entire jig is given in Fig. 12. Raw

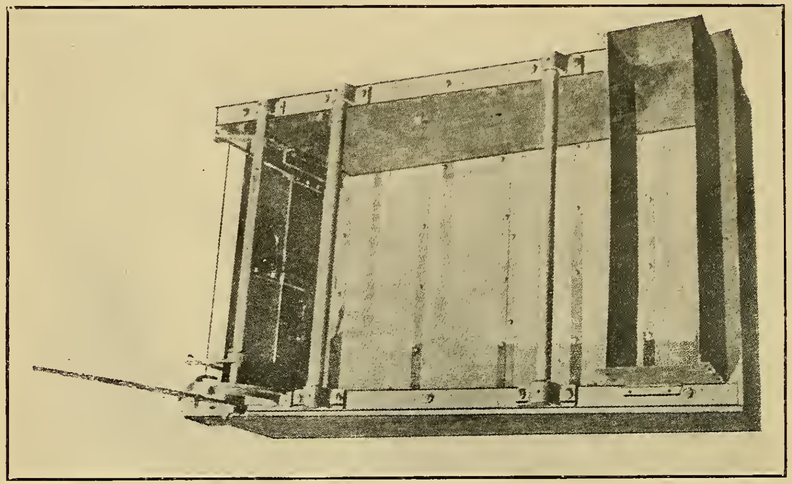


FIG. 11. PAN OF STEWART JIG.

screenings, commonly unsized, are fed into the hopper at one end of the pan, which is held in a jig box filled with water by supports attached to eccentrics located above the pan. These eccentrics give the pan a reciprocating motion and cause the water to rise and fall through the screen in the bottom of the pan, thus producing hindered settling conditions. The coal rises and the refuse sinks and both work their way across the pan, the movement of the refuse being assisted by the inclination of the screen. At the forward end of the pan the washed coal overflows into a launder and the refuse is discharged through an adjustable gate into the hutch where it mixes with the fine refuse which has passed through the pan screen. The mixed refuse is removed continuously by means of a bucket elevator. The water supply comes from a tank back of the pan compartment and connected with it by means of an opening in the hutch beneath the pan. This opening is fitted with a clap valve so arranged as to open by suction when the pan is raised, thus

TABLE 9.

STEWART JIG PRACTICE.

Washery No.	Rated Tons per Hour	No. of Jigs	Tons per Jig per Hour	Width of Jig Pan (feet)	Tons per Foot of Width per Hour	Size of Feed (inches)	Length of Stroke (inches)	Strokes per Minute	Centrifugal Pumps Returning Water (number)
1	50	2	25	4	6	1⅛—0	6	40	8
2	125	1 / 3	31	4	8	1⅛—¾ sq. / ¾ sq.—0	4	40	10
8	60	2	30	5	6	2 x 3½ ell.—0	4	34	(No pump. Water returns by gravity.)
9	100	2	50	6	8	3—0	4	38	8
12	150	4	37½	4	9	3—0	4	36	12
13	100	2	50	6	8	3—0	4	35	9
14	120	4	30	4½	7	3½—0	4	34	10
16	90	4	22½	4½	5	3—0	4	42	8
21	80	2	40	4	10	2—0	4	36	8
26	75	2	37½	5	7½	2½—0	4	40	8
27	60	2	30	3½	9	3—0	3½	32	8
31	160	4	40	4	10	3—0	4	40	10
Sum	1170	34							
Average	97½	3	34	4½	7½		4	37	

allowing water to flow into the hutch and reducing the amount of suction in the jig bed, and to close when the pan is lowered, thus forcing water through the bed with strong pulsion.

The Stewart jig is the commonest type of pan jig in use in Illinois, and is employed in more washeries than is any other type of washing machine. Details of Stewart jig practice at twelve washeries are shown in Table 9. The pan of the Stewart jig is always made 7 ft. in length but its width varies from 3½ ft. to 6 ft. The tons of raw coal per hour based on the rated capacity treated per foot of width of Stewart pan varies from 5 to 10 with an average of 7½. The number of strokes

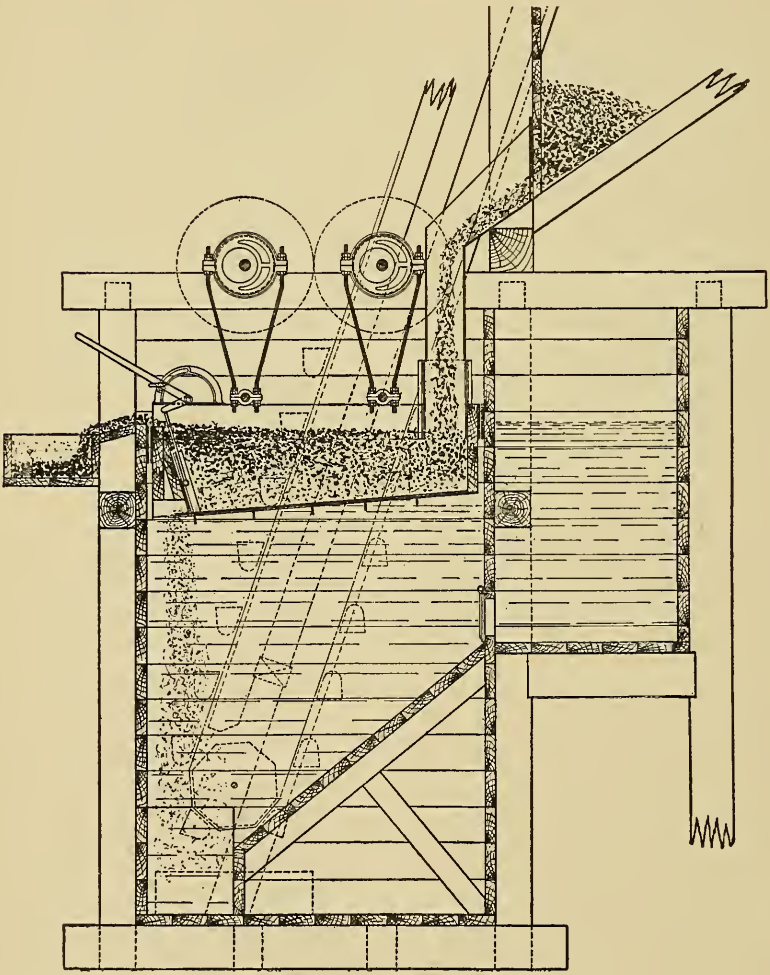


FIG. 12. SECTIONAL VIEW OF STEWART JIG.

made by the pan per minute varies from 32 to 48 with an average of 37, and the length of the stroke is usually 4 in., although in one case it is 3½ in., and in another 6 in. Thus the average Stewart jig treats 34 tons of raw coal per hour in a pan 4½ ft. x 7 ft. in size, which makes 37 strokes 4 in. in length per minute. In some instances, the feed hopper at the end of the jig is omitted, and in others a spray is added at this point to assist in wetting the raw coal as it falls into the pan. The perforations in the pan bottom are round holes of from ¼ in. to ⅜ in. in diameter. Various devices are employed to hold the slate valve open to the desired extent, such as a screw turned by a hand wheel and a pin on the lever fitting into any one of a series of holes in a swinging arm hung in the proper position.

37. *American Jig.*—The American jig, made by the American Coal Washer Company and shown in section in Fig. 13, differs from the Stewart jig mainly in the method of supplying water and in the operation of the refuse valve. While the returning water enters the hutch through a clap valve as in the Stewart jig, its movement through the valve is not dependent upon the suction produced by the rising pan aided by a slight difference in level of water in supply tank and jig box, as in the Stewart jig, but is the result of delivering the water directly to the clap valve by means of a large distributing pipe from a pump having a capacity slightly larger than actually required to fill the jig tank on the up stroke of the pan, thus reducing suction to a minimum. The refuse valve is designed to operate automatically, being opened at intervals as desired, but always to its fullest extent, thus preventing the accumulation and jamming of large pieces of shale in front of the gate. A safety device is provided which keeps the refuse valve in operation as long as raw coal is supplied to the jig, but disconnects it automatically when the feed ceases.

American jigs are used in five Illinois washeries. In the majority of these, the refuse valve just described has been discarded in favor of a simpler discharge like that employed on the Stewart jig. Details of American jig practice are shown in Table 10. From this it is apparent that the average American jig has a pan 4 ft. in width and treats 37 tons per hour, using 4 in. strokes at the rate of 41 per minute. Comparing this average with that for the Stewart jig as shown in Table 9, it is found that the American jig is rated as having 20 per cent greater capacity than the Stewart. At one American washery the coal is sized on a woven wire screen with 9/16 in. openings before washing.

38. *Shannon Jig.*—The Shannon jig (Fig. 14) does not attempt to force the water through the coal in the pan by making it fit the pan compartment snugly, but obtains a similar effect by prolonging the sides of the pan downwards below the screen and using comparatively rapid strokes. The pan is held free from the walls of the pan compartment by

TABLE 10.

AMERICAN JIG PRACTICE.

Washery No.	Rated Tons per Hour	No. of Jigs	Tons per Jig per Hour	Width of Jig Pan (feet)	Tons per Foot of Width per Hour	Size of Feed (inches)	Length of Stroke (inches)	Strokes per Minute (number)	Centrifugal Pump Returning Water (number)
4	100	2	50	4	12½	1½—0	4	48	8
15	80	2	40	4	10	3½—0	4	34	8
17	280	8	35	4	9	1¼—0	4	36	2—No. 10s
18	140	4	35	4	9	3 —0	4	36	10
20	140	2 2	35	4	9	3—9-16 sq 9-16 sq—0	4	48	10
22	140	1 3	35	3½ 4	9	3 —0	4	45	10
Sum	880	24							
Average	176	5	37	4	9		4	41	

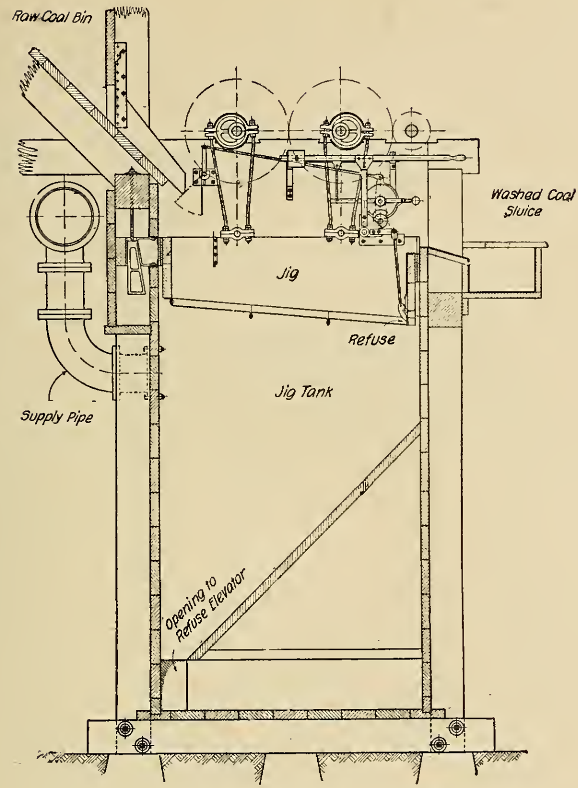


FIG. 13. SECTIONAL VIEW OF AMERICAN JIG.

means of rollers. Another peculiarity of the Shannon jig is that the washed coal compartment and pan compartment are connected at the top so that water is free to move back and forth between them, which greatly reduces the quantity of water which must be pumped to keep the jig going.

Three washeries employing Shannon jigs were in operation in Illinois in 1912. The jigs were all 4 ft. x 6 ft. in size and were rated to treat from 40 to 62½ tons raw coal per hour with an average of 47 tons. This is 25 per cent in excess of the average rating for a 4 ft. x 7 ft. American jig and 50 per cent in excess of that for a 4 ft. x 7 ft. Stewart. The number of strokes of the pan per minute varies from 72 to 90, which is about twice the number used on Stewart and American jigs, while the length of stroke is from 3 in. to 3½ in., which is less than for other pan jigs. At one washery the Shannon jigs are treating coal sized between 3 in. and ¾ in., the smaller size going to a Foust jig, while the other two are working on unsized screenings. The new water required by each jig is

supplied through a small pipe near the raw coal feed. At one washery the pipe at each jig was a 2½ in. pipe hammered flat so as to leave a ⅜ in. slot through which water was delivered under 30 pounds pressure.

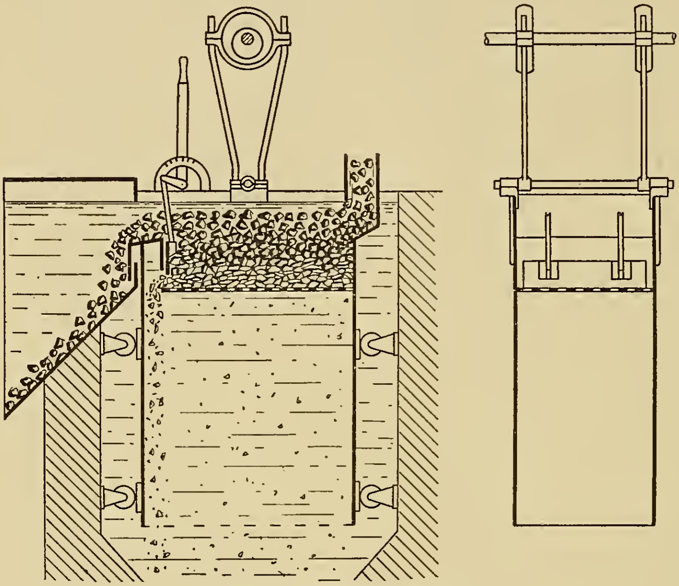


FIG. 14. SHANNON JIG.

E. BUMPING TABLES.

39. *Campbell Table.*—Campbell bumping tables are employed at one operating Illinois washery. Each table is 2½ ft. in width by 7½ ft. in length, with a working deck of sheet iron corrugated transversely and having a variable slope which is 5 degrees steeper at the back than at the front. It has sides of board and a wooden keel running lengthwise with a steel-shod bumper at the back end, the whole suspended by rods from above and provided with means of regulating slope and length of swing. The table is shown in Fig. 15. Swinging motion is produced by means of a cam of peculiar oval form working in conjunction with a rocker and rocker arm, arranged to give a slow forward motion and quick return, with velocity gradually increasing up to the moment of impact with the bumping block on the framework back of the table. The feed is delivered upon the center of the table, the refuse being jerked and bumped backward and upward until it falls over the back of the table and the coal washed downward and forward until it flows off the front.

The washery is rated at 40 tons per hour and contains seven tables,

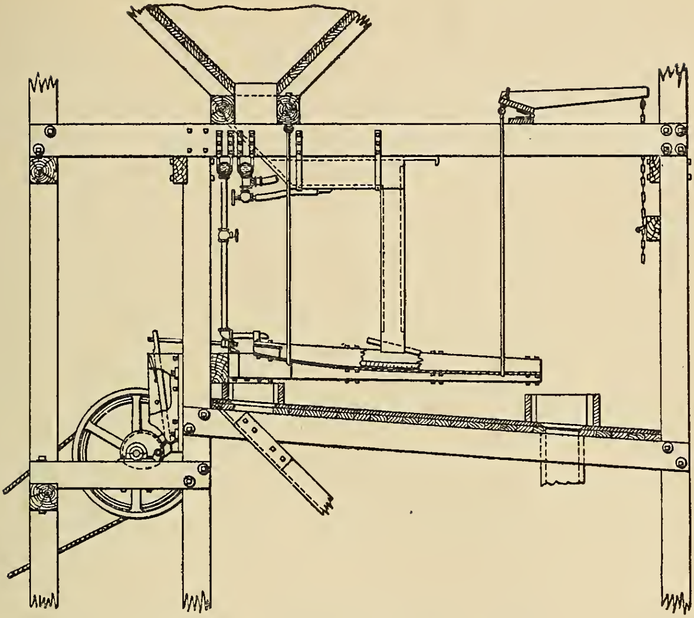


FIG. 15. CAMPBELL BUMPING TABLE.

three working on 3¾ in. —1¾ in. raw coal, three on 1¾ in. —⅜ in., and one on ⅜ in. —0. The slope is varied to suit the size, character and quantity of feed. The tables treating medium sized coal had a slope of 7 degrees in front and 12 degrees in the rear. The tables receive 76 bumps per minute, the stroke being 5 in. for the coarse, 2½ in. for the medium, and 1¼ in. for the fine coal.

VII. SIZING WASHED COAL.

The sizing of raw coal and general considerations connected with sizing have already been discussed in Chapter V. There are two reasons for sizing washed coal: to improve its appearance and to improve its burning qualities. Considerable breakage occurs during the process of washing, so in order to produce a well-sized product it is necessary to size after washing, even if the coal has been given a preliminary sizing prior to washing.

40. *Sizes and Trade Names.*—At the time the writer visited the Illinois washeries, all save one were making more than one size of washed coal, and that one had already installed a screen for making a second size. The number of sizes made varied from 2 to 7, the number being smaller, as a rule, in the Northern field than in the Central or Southern, as will be seen by reference to Table 11. Where two sizes are made

they are designated "washed nut" and "washed slack," and where more than two, they are designated by number, as shown in the table. But while a number of washeries are shipping washed coals under the same designation, curiously enough there are no two washeries making washed coals of exactly the same sizes. The range is as follows:

	No. 1 Extra	No. 1	No. 2 Extra	No. 2	No. 3	No. 4	No. 5
			INCHES				
Always under	3¾	3½	2¼	2¼	1½	⅞	$\frac{7}{16}$
Always over	2½	1¾	1⅜	⅞	⅝	$\frac{3}{16}$	0

The ranges are thus seen to overlap widely, and there is wide variation within the ranges, so that the No. 1 washed coal of one producer may correspond with the No. 2 of another, and the No. 2 produced at one washery with the No. 3 at another. The consumer buying washed coal by name or number is therefore unable to tell what size he will actually obtain except from previous experience or from a knowledge of the sizes of screens employed in the washery whence the coal comes. It is evidently desirable from the point of view of the purchaser, at least, that a standard set of sizes be adopted. In 1903, the operators of Williamson County agreed to the following standard sizes:*

Designation	Through	Over
No. 1	3 in. round holes	1¾ in. round holes
No. 2	1¾ in. round holes	1 in. round holes
No. 3	1 in. round holes	¾ in. round holes
No. 4	¾ in. round holes	¼ in. round holes
No. 5............	¼ in. round holes	

This agreement has not been adhered to, however, and the sizing of washed coal throughout Illinois is badly in need of standardization. It has been pointed out in Chapter V that in order to produce sizes the particles in each of which shall have the same variation in diameter, the openings in the washery screens must have a constant geometrical ratio. The advantages of adopting such a screen scale would be that each size, with the exception of the smallest, would be sized to exactly the same degree, and the series of sizes would therefore present the best possible appearance. It has already been shown that such a screen scale would be of advantage in the preliminary sizing prior to washing. For these reasons, the writer suggests the adoption of the following standard sizes for washed coals:

Designation	Through	Over
No. 1	4 in. round holes	2 in. round holes
No. 2	2 in. round holes	1 in. round holes
No. 3	1 in. round holes	½ in. round holes
No. 4	½ in. round holes	¼ in. round holes
No. 5............	¼ in. round holes	

*McGovney, C. S. "Tests of Washed Grades of Illinois Coal." U. of I. Eng. Exp. Sta Bull. 39.

TABLE 11.

Sizes and Trade Names of Illinois Washed Coals.

Washery No.	No. of Sizes	Diameter in Inches (Round holes unless otherwise noted)								
		"Nut"	"Slack"	"No 1 Extra"	"No. 1"	"No. 2 Extra"	"No. 2"	"No. 3"	"No. 4"	"No. 5"
Northern Field										
1	2	1⅛—¾ sq.	¾ sq ⅛							
2	2	1¼—¾ sq.	¾ sq - 3/16							
3	1		⅞ bar—3/32 & 1/16							
4	3				1½—1⅜ sq		1⅜ sq—⅞ sq	⅞ sq—3/32		
5	2	2½—2	2-0							
6	2	2¼ & 2½—1¼	1¼—1/16							
7	2	1½ bar—1¼	1¼—1/16							
8	5				2x3½ ell—1¾		1¾—1¼	1¼—¾	¾—⅜	⅜—0
9	5				3—2¼		2¼—1½	1½—¾	¾—5/16	5/16—1/16
*10	2	¾—⅜	⅜—0							
*11	1		⅜—0							
Central Field										
12	5				3—1¾		1¾—1¼	1¼—¾	¾—5/16	5/16—1/16
13	7			3½—2⅞	2⅞—2¼	2¼—1¾	1¾—1¼	1¼—¾	¾—5/16	5/16—0
14	5				3½—1¾ sq		1¾ sq—1 sq & 1⅜	1 sq & 1⅜—⅞	⅞—¼	¼—0
15	5				3½—2¼		2¼—1¼	1¼—⅞	⅞—⅜	⅜—0
16	5				3—1⅞		1⅞ 1¼	1¼—⅞	⅞—5/16 & ¼	5/16 & ¼—0
17	3							1¼—11/16	11/16—⅜	⅜—0
18	5				3—1¾		1¾—1⅛	1⅛—¾	¾—¼	¼—0
19	5				3—1¾		1¾—1	1—¾ & ½	¾ & ½—¼	¼—0
20	5				3—1¾		1¾—1⅛	1⅛—¾	¾—¼	¼—1/16
21	5				2 & 1¼—1½		1½—1	1—⅜	⅜—¼	¼—3/16
22	5				3—1¾		1¾—1⅛	1⅛—¾	¾—⅜	⅜—1/16
23	5				3—1¾		1¾—1¼	1¼—¾	¾—⅜	⅜—0
24	5				3—1¾		1¾—1	1—¾	¾—¼	¼—0
25	5				3—2		2—1⅛	1⅛—11/16	11/16—¼	¼—1/16
Southern Field										
26	4				2½—2		2—1¼	1¼—1		¼—0
27	4		¾—⅛		3—2		2—1	1 ¾		
28	5				3—2		2—1	1—¾	¾—¼	¼—0
29	6			3—2¾	2¾—1¾		1¾—1	1—⅝	⅝—⅜ & 3/16	⅜ & 3/16—0
30	6			3¾—3	3—1¾		1¾—1	1—¾	¾—¼	¼—0
31	5				3 1¾		1¾—1¼	1¼—¾	¾—7/16 & ⅜	7/16 & ⅜—0
32	6			3½—2¾ & 2½	2¾ & 2½—1¾	1¾—1⅜	1⅜—⅞		⅞—⅜	⅜—3/16
33	5				3½—1¾		1¾—1	1—¾	¾—5/16 & ¼ & ½ sq	5/16 & ¼ & ½ sq—0
34	7			3½—3	3—2¼	2¼—1¾	1¾—1 3/16	1 3/16—¾	¾—5/16	5/16—3/16
35	5				3—2		2—1	1—¾	¾ ¼	¼—0

* Experimental

In the proposed screen scale—¼ in., ½ in., 1 in., 2 in., 4 in.—the ratio employed is 2. The scale begins with ¼ in., the size most commonly used for making No. 5 coal, and ends with 4 in., which is ¼ in. larger than the largest size at present washed. If it were desired to make more than five sizes, the suggested sizes might be subdivided on the ratio $\sqrt{2}$, the larger particles being designated by the number and the affix "extra," and the smaller particles by the number and the affix "small." Carried to eighths of an inch, these sizes would be as follows:

Designation	Through	Over
No. 1 Extra	4 in. round holes	2⅞ in. round holes
No. 1 Small	2⅞ in. round holes	2 in. round holes
No. 2 Extra	2 in. round holes	1⅜ in. round holes
No. 2 Small	1⅜ in. round holes	1 in. round holes
No. 3 Extra	1 in. round holes	¾ in. round holes
No. 3 Small	¾ in. round holes	½ in. round holes
No. 4 Extra	½ in. round holes	⅜ in. round holes
No. 4 Small	⅜ in. round holes	¼ in. round holes
No. 5 Extra	¼ in. round holes	⅛ in. round holes
No. 5 Small......	⅛ in. round holes	

Gravity screens, drag screens, revolving screens and shaking screens are all employed to size washed coal in Illinois washeries, as will be seen by referring to Appendix E. Gravity screens are installed in chute bottoms and used only to relieve or supplement other types of screens. Drag screens are occasionally employed for general sizing but most frequently for rinsing fine coal, drag rinsing screens being in use in seven Illinois washeries.

Shaking screens are in use for general sizing of washed coal, especially in washeries where the coal has not been sized as a preliminary to washing. They are employed in three washeries for resizing washed coal, the average screen having a slope of 10½ degrees, making 155 strokes 4½ in. in length per minute, and having 1.35 sq. ft. screen surface per ton of coal per hour. For the fifteen washeries employing shaking screens for sizing washed coal not previously sized raw, the average, in so far as figures are available, is as follows: slope, 9 degrees, strokes, 138 of 5 in.; square feet of surface per ton per hour, 1.7. The figures 1.35 and 1.7 are unfortunately not comparable, 1.35 being an average for two screens only and 1.7 being reduced to 1.4 if a single exceptional screen is omitted. We have seen that 0.7 sq. ft. per ton dry coal per hour is the average bituminous practice. Wet coal requires more screening area, and the Illinois shaking screens conform to this requirement by having twice that area.

Revolving screens are used in Illinois for the general sizing of washed coal, and particularly for the resizing of coal which has been previously

sized raw. As in the case of raw coal screens, their peripheral speed averages above 200 feet per minute and below 250 feet, which is desirable, but in many cases it greatly exceeds the 250 feet per minute which is set by some writers as the maximum permissible speed. The average square feet screen area per ton per hour for raw coal revolving screens was found to be 4.2. Wet revolving screens should have larger proportional areas, and the averages 5.66 and 6.5 sq. ft. for resizing and sizing screens, respectively, appear on their face to show that this requirement is carried out. A closer inspection, however, demonstrates that this is not the case with the sizing screens, the figure 6.5 being lowered to 3.4 if a single exceptional screen be omitted from the average. Illinois washed coal revolving screens, like the raw coal revolving screens, appear to be smaller on the average than required by good practice in other parts of the United States.

The proportions of different sizes produced by most of the operating washeries are shown in Table 12. The sizes to which these proportions refer are indicated in Table 11. About one-sixth of the washed coal produced at the two-size washeries of northern Illinois makes nut coal and the other five-sixths makes slack. Five numbered sizes are commonly made in central and southern Illinois. On the average, more No. 4 is produced than any other size, No. 5 being next in the Central field, and No. 2, with No. 1 a close second, in the Southern field. In other words, the Central coal field produces larger proportions of the smaller sizes of coal than does the Southern.

VIII. Hand Picking.

Hand picking is used in conjunction with washing to remove, first, lumps of refuse which have remained with the coal as a result of imperfect washing, and, second, lumps which could not be removed by washing, but which contain numerous thin flakes of pyrite that while not materially increasing its specific gravity, ash or sulphur contents, detract from its appearance.

Only one Illinois washery employs a man regularly and solely to pick bad pieces from its washed coal. In this washery the largest two sizes are passed over an apron conveyor and from this the picker removes about 1½ tons of refuse per day. The hand-picked refuse contains 69.16 per cent dry ash and 3.54 per cent dry sulphur, as shown by Analysis 55, Appendix D. At two washeries there are men who employ part of their time picking refuse from the washed coal as it leaves the resizing revolving screens, and at one washery bad lumps are thrown out of the larger sizes as they are being loaded into the cars.

TABLE 12.

PERCENTAGES OF WASHED COALS PRODUCED AT ILLINOIS WASHERIES.

Field	Washery No.	Nut	Slack	No. 1 Extra	No. 1	No. 2 Extra	No. 2	No. 3	No. 4	No. 5
Northern	1	10.00	90.00							
	2	10.67	89.33							
	3		100 00		2.44		21.98	75.58		
	4									
	5	21.30	78.70							
	6	20 00	80.00							
	7	20.00	80.00							
	8				8.00		12.00	23.00	57.00	
	9				3.10		5.90	33.50	25.70	31.80
(a) Average for 5 washeries making 2 Sizes.		16.40	83.60							
Central	15									12.00
	17							60.70		39.30
	18				14.26		18.31	11.82	28.84	26 77
	19				10.12		22.98	23.27	21.72	21.91
	20				17.11		17.74	13 56	28.77	22.82
	21				15.00		18.00	23 00	26 00	18.00
	22				1.24		18.75	19.47	22.53	38.01
	23				17.20		12.30	20 30	32.00	18.20
	24				17.09		17.90	15.10	23.57	26.34
	25				2.80		17.40	21.20	30.30	28.30
(b) Average for 8 washeries making 5 Sizes.					11 85		17.92	18.47	26.72	25.04
Southern.............	27		36.10		16.90		21.60	25.40		
	28				9.80		28.00	8.70	34.20	19.30
	31				23.37		18.09	10.78	22.40	25.36
	32			9.00	15.80	12.40	15 60		27.20	20.00
	33				24 00		21.00	15.00	22 00	18.00
	35				26.80		18.60	18 50	27.90	8.20
(c) Average for 4 washeries making 5 Sizes.					20.99		21.42	13.25	26.62	17.72

IX. Arrangement of Washeries.

Washeries must be arranged to receive raw coal, separate it into washed coal and refuse, and dispose of both these products. They must be supplied with power to run the machinery and water to wash the coal. In the simplest Illinois washery (No. 6), the screenings from the tipple are delivered directly to the washing machine making washed coal and refuse. The washed coal is sized, making "nut" and "slack," which fall into bins whence they are drawn into cars for shipment. The refuse is drawn into a car and trammed to a refuse dump. A single engine supplies the power and a single pump provides fresh water. In a more complicated but not unusual washery (No. 29), the raw screenings from the tipple are delivered to a scraper conveyor which carries them to the top of the washery, where they pass into a raw coal bin. They are next sized and each size delivered to a different set of washing machines which produce washed coal and refuse. Keeping the sizes separate, they are next resized and all but the smallest size fall into bins whence they are drawn into railroad cars. The smallest size is collected in a settling tank and raised by a bucket elevator to its bin. All the refuse is delivered to a single refuse tank whence it is drawn into a car which hauls it up the refuse pile and dumps it there. Not only are there pumps for fresh water but a pump is provided for returning dirty water.

The arrangement of a washery is still further complicated when provision is made for crushing raw coal or for crushing and rewashing refuse. Poor original design and change in method of treatment after erection also have introduced serious complications in several instances.

41. *Receiving Coal.*—All but two of the operating Illinois washeries are situated near a mine from which they are receiving screenings, and might therefore have their raw coal delivered to them by means of some form of conveyor. In several instances, however, this would mean a conveyor of undesirable length, while in others the washery is receiving coal not only from the neighboring mine, but also from a mine or mines at a distance, raw coal being shipped on railroad cars to thirteen washeries. Mechanical car unloaders are employed at twelve of these washeries, but are far from giving satisfaction, because of the large amount of manual labor they entail. Not only must there be a man to operate the unloader and a crew to move cars, but there are required two or more men to shovel back of the conveyor in the car. At the thirteenth washery the raw coal is dumped into a hopper beneath the track, whence it is removed by a scraper conveyor delivering into a bucket elevator, which raises it to the top of the washery. This is a much more satisfactory arrangement when bottom-dumping cars are available.

In one case, the tipple is near enough to the washery so that by omitting the raw coal bin the screenings will run by a chute directly to the washing machine, while in another the coal is trammed by hand

from the tipple, but in the majority of cases some form of conveyor or elevator is necessary. There are seven scraper elevators in use conveying raw coal from beneath the tipple screen to the top of the washery, six conveying belts, five bucket elevators and one "Pacific Coast" conveyor, which is a cross between an apron conveyor and a bucket elevator. The belts employed are all of rubber. In one washery where a canvas belt was tried it lasted only one year where a rubber belt lasted five years.

When the coal is to be sized before washing, it passes from the receiving appliance into the raw coal screen from which the different sizes fall into separate bins. When the coal is to be sized after washing only, it usually falls directly into the raw coal bin. In two washeries, however, the coal is weighed on Richardson automatic scales before passing into the bin. From the raw coal bins the coal flows into the washing machines. Revolving pocket feeders are in use at two washeries and a number of the pan jig washeries employ a feeding device, consisting of a sheet iron gate with a rope attached to it and to the jig in such a way that the gate opens and closes with each stroke of the eccentrics. Just as the coal is leaving the bin and before it falls into the washer, it is frequently sprayed to insure its prompt and thorough wetting.

Washing machines and washed coal screens require but a passing mention at this point, having been described in the two chapters preceding. It is well to recall that the fine coal to be washed on Luhrig jigs is mixed with water after passing through the raw coal screen and classified in a grading box, the different products passing to separate Luhrig fine coal jigs arranged in a battery.

For purposes of flexibility, it is highly desirable that good-sized storage bins be provided for both raw and washed coal. A large raw coal bin permits the washery to run during periods when the hoisting of coal at the shaft is interrupted, while large washed coal bins permit it to operate during interruptions in the car service. The majority of Illinois washeries do not appear to be adequately supplied with bins. In the case of 24 washeries, with an average rated tonnage of about 100 tons raw coal per hour, the average raw coal bin is of about 200 tons capacity and the average washed coal storage about 250 tons. It is evident that in the majority of these washeries even a slight interruption in the car supply or raw coal delivery would necessitate a shut-down.

Where the washed coal bins have considerable capacity, coal dropping from the screens will be badly broken unless the bins are nearly filled. In order to prevent this, "telegraphs" are installed in many of the bins, designed to hold the larger sizes of washed coal. The telegraph is simply a chute, or series of chutes, so arranged that the screened coal will slide, instead of falling, into the bin. Telegraphs are in use for the larger sizes of washed coal in eight washeries. A single spiral chute is the most popular form, although in some cases a series of chutes is employed, having each successive member inclined in the

opposite direction and arranged in such a manner that the coal drops a short distance from the bottom of one to the top of the next.

The commonest method of disposing of refuse is to haul it up an inclined refuse pile and dump it. This is done at 25 washeries. At four washeries the refuse is trammed by hand to the dump, at three it is sluiced with water, and in one instance all the refuse is loaded into railroad cars for ballast.

The washery water is usually kept in continuous circulation by means of a centrifugal pump, a piston pump or pumps being employed to supply the fresh water needed to replace the water taken up by the washed coal and refuse and lost in other ways. Five washeries, however, use all fresh water, which is desirable if not too expensive, as better results can be secured when washing with clean water.

42. *Settling Tanks.*—There are settling tanks in 27 washeries. In all but three of these the settler is used to collect the No. 5 washed coal, but in three it is used to purify the circulating water and the settlings are added to the refuse from the washing machines. In the majority of the settling tanks there is a slow-moving drag, which forces the settled material toward one end of the tank, whence it is raised by a bucket elevator. If the raw coal contains fireclay the circulating water soon becomes charged with fine particles of clay which remain in suspension only so long as the fluid is in rapid motion. When there is a large quantity of this fine clay in the water, it deposits when the water slows up in the settling tank. For this reason, the settling tanks have recently been removed in two Illinois pan jig washers. The result is that the slowest part of the circulation is in the hutches below the pans and the clay deposits there and is removed with the other hutch refuse. At other washeries, the deposition of fireclay in settling tanks collecting fine coal only becomes troublesome when the machinery slows down or is stopped for the night. In these cases it is customary to run all the settlings into the refuse for a short time after resuming operations. Even under the most favorable conditions the water in time collects an undesirable quantity of clay and pyrite and it is necessary to provide fresh water for the entire system at intervals varying from once a day to once a week.

43. *Construction.*—The construction of a washery has a decided influence upon its arrangement. All but one of the operating Illinois washeries are of wooden construction and present appearances similar to that of washery No. 26, shown in Fig. 16. Contrast this with Fig. 17 showing washery No. 23, which is of steel and concrete construction, noting particularly the large conical storage bin for raw coal and the smaller conical bins for washed coal.

44. *Flow Diagrams.*—Flow diagrams for Howe, Robinson, Luhrig, Forrester, Shannon and Foust, Shannon, Stewart, American, and Campbell washeries will be found in Appendix F.

FIG. 16. WASHERY OF THE BIG MUDDY COAL AND IRON COMPANY AT HARRISON, ILL.

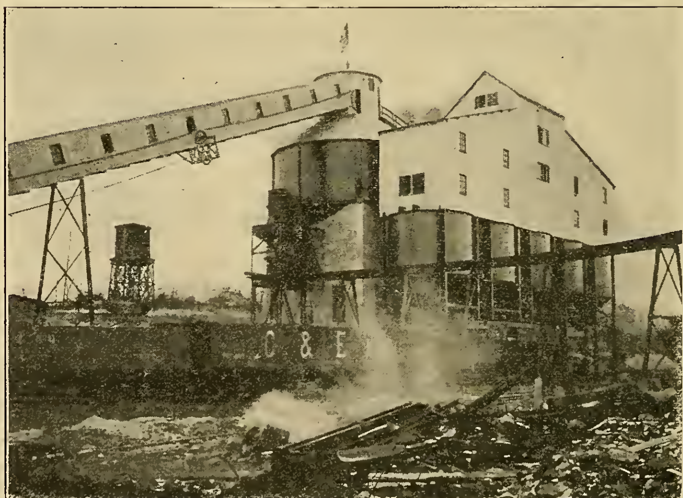


FIG. 17. SHANNON WASHERY OF THE SHOAL CREEK COAL COMPANY AT PANAMA, ILL.

X. Results of Washing.

A. GENERAL.

The effect produced by washing is usually measured by comparing the ash, sulphur and British thermal unit determinations on the washed coal with those on the raw coal. In the washed coal the percentages of ash and sulphur are lower and the number of British thermal units higher. A fair idea of the general effect of washing as practised in Illinois may be obtained by a consideration of the averages for the ten washeries in Tables 13, 14 and 15.

45. *Ash.*—In Table 13, "Face" designates the samples taken across the working face of the coal bed excluding shale and sulphur partings, which the miner is supposed to remove. Ash determinations on such samples indicate the lowest possible results which could be secured by mining. It will be observed that the coals as actually mined contain about 50 per cent more ash than the face samples, which is due to the inclusion of material from the roof and the floor, and to failure to exclude partings. Thus the average face sample in the table contains 9.60 per cent ash and the mine run 14.58 per cent. The additional 5 per cent ash is mainly in the shape of small impurities, for upon screening there is produced a lump coal which has about the same ash content as the face, and screenings with a little more than twice that amount. When these screenings with their high ash content of 20.81 per cent are treated in the washeries, washed coal with only 9.81 per cent ash are obtained. The average percentage for excess of ash for run of mine coal over face samples, as given by Table 13, has been found to vary quite widely in other analyses made at the University of Illinois. This is to be expected, however, owing to the difficulty of properly sampling run of mine coal, and until a better method of sampling such coal is found no general figure can be determined for the relation between the ash in run of mine and in face samples. The general effect of the process of washing upon the ash contents of the screenings is thus seen to be their reduction to about the same amount as in the face and in the lump coal. It should be noted, however, that while the washed coal as a whole is no better than the lump and face, Nos. 1, 2 and 3 are about 10 per cent better. No. 4 has about the same ash content as the face and lump, while No. 5 is about 30 per cent higher.

46. *Sulphur.*—From Table 14 the sulphur content of the face is seen to be about equal to that of the mine run. This may be explained on the ground that sulphur partings which should be excluded find their way into the coal as mined, but that the extra sulphur thus introduced is diluted with sulphur-free shale from floor, roof and partings and the sulphur content of the mixture is brought down to that of the face. The lump is about 20 per cent better than the face, and the screenings

TABLE 13.

Dry Ash Contents of Illinois Raw and Washed Coals.

Coal Bed	Coal Field	Raw Coal				Washed Coal					
		Face %	Mine Run %	Lump %	Screenings %	Screenings %	No. 1 %	No 2 %	No. 3 %	No. 4 %	No. 5 %
2	Northern	8.89	16 95	11.40	28.58	11.06	9.74	11 43			
2	"	7.33	18 91	6.20	29.30	9 05	6.77	4.93	5.13	11.83	
2	"	5.64	10.45	6.80	27.10	6 69	5.86	6.60	6.29	6 85	7.07
6	Central	11 01	13.62	11.72	16.48	10.29	9.61	9.99	10 79	10 29	12.92
6	"	10.51	18.00	10.00	18.43	12.52	10.30	9.70	10.40	10.90	18.31
6	"	12.75	15.15	12.93	25.76	10.83	11.57	6.04	10 60	8 80	14.25
6	Southern	12.43	17.03	10.58	20.51	9.07	5.63	8.88	8.78	7.44	14.14
6	"	7.94	11.73	10.68	12.43	10.08	9.12	9.00	9.28	10 76	11.46
6	"	9.73	11 42	8.35	13.47	9.15	9.09	8.16	8.78	9.46	11.60
6	"	9.80	12.50	7.90	16 00	9.39	7 65	8.41	8.10	8.25	13.45
	Average	9.60	14.58	9.66	20.81	9.81	8.53	8.31	8.96	9.60	12.65

TABLE 14.

Dry Sulphur Contents of Illinois Raw and Washed Coals.

Coal Bed	Coal Field	Raw Coal				Washed Coal					
		Face %	Mine Run %	Lump %	Screenings %	Screenings %	No. 1 %	No. 2 %	No. 3 %	No. 4 %	No. 5 %
2	Northern	3.23	3.38	3.12	3.93	3.76	3.72	3.77			
2	"	3 28	2 84	1 80	3.69	2.98	2 72	2.55	2.78	3.18	
2	"	2.59	2.41	2.30	2 90	2.77	2 64	2.80	2.81	2.78	2.74
6	Central	4.75	3.63	3.18	4.30	3.60	4.05	3.98	3 49	3 33	3.68
6	"	5 23	4 90	4.00	4.70	4.60	4.50	4 40	4.40	4.40	4.77
6	"	5.51	4.73	5.34	5 80	4.16	3.75	3 55	4 14	4.45	4.60
6	Southern	3.51	4.31	2.09	5.51	2 62	2.67	2.45	2.83	2.67	2.68
6	"	1.51	1.69	1.41	1.87	1 47	1.40	1 55	1 52	1.57	1.38
6	"	1.06	2.16	1.49	2.61	2.41	2 60	1.93	2.64	2 31	2.72
6	"	2.35	2.89	2.00	3.11	1.95				1.53	1.77
	Average	3.30	3.29	2.67	3.84	3.03	3.11	2 98	3.12	3.00	3.13

20 per cent worse as far as sulphur is concerned. The washed coal, with an average of 3.03 per cent sulphur, shows an improvement of about 10 per cent over the face and lump with 3.30 per cent and 3.29 per cent; and an improvement of about 20 per cent over the raw screenings with their average of 3.84 per cent sulphur. The slight variations between the sulphur contents of the different sizes of washed coals do not seem to be sufficiently regular to permit of any generalizations.

47. *British Thermal Units.*—In Table 15 another factor has been introduced: coal free from ash and sulphur, the "unit coal" of Parr.* The British thermal units in this substance are obtained by calculation, and indicate the highest heating values that coals could have if all their ash-forming impurities and organic sulphur were removed. The average for unit coal is 14 598 British thermal units, while the face is 13 007, or about 1600 units less. The mine run is about 800 units lower, but just as was the case with the ash, the lump contains approximately the same number of British thermal units as the face. The average for raw screenings is 11 166, or about 1900 units less than the face and lump, while for washed screenings it is 12 959, or nearly the same as the face and lump and thus again similar to the ash content. This similarity extends also to the individual sizes of washed coals, Nos. 1, 2 and 3 being better than the average, No. 4 about equal to it, and No. 5 worse.

48. *Summary.*—The general effect of coal washing as practised in Illinois is therefore to bring the ash and British thermal units in the screenings back to those in the face and in the lump, and to make the sulphur content slightly less than in the face and lump. Disregarding the slight improvement in sulphur content, it might be said that all the Illinois washeries are doing at present is to bring the screenings back to the condition in which they would be in the beginning if mined with the greatest possible care. This is hardly a fair statement of the case, however, since the additional cost of mining so as to exclude all particles of roof and floor and all thick partings would in most cases greatly exceed the cost of washing. Moreover, the larger washed sizes are now better than the face and the smaller sizes would be made so if the demand were sufficiently strong.

B. EFFICIENCY OF WASHING.

The process of coal washing is dependent upon differences in specific gravity and its efficiency may best be measured by methods based upon specific gravities. While the object of coal washing is to reduce the proportion of impurities in the coal, the extent to which it is desired to carry this reduction is commonly stated in terms of ash and British

*Parr, S. W. and Wheeler, W. F. "Unit Coal and the Composition of Coal Ash." U. of I Engineering Experiment Station Bull. 37.

TABLE 15.

BRITISH THERMAL UNITS ON DRY COAL IN ILLINOIS RAW AND WASHED COALS.

Coal Bed	Coal Field	Unit Coal (Free from Ash and Sulphur)	Raw Coal				Washed Coal					
			Face	Mine Run	Lump	Screenings	Screenings	No. 1	No. 2	No 3	No. 4	No. 5
2	Northern	14 814	13 329	11 895	12 750	10 103	12 903	13 116	12 845			
2	"	14 463	13 221	11 538	13 571	9 874	12 996	13 372	13 655	13 617	12 554	
2	"	14 579	13 613	12 909	13 495	10 239	13 269	13 402	13 281	13 330	13 243	13 209
6	Central	14 390	12 572	12 069	12 417	11 546	12 668	12 704	12 663	12 500	12 655	12 159
6	"	14 365	12 583	11 452	12 669	11 369	12 289	12 636	12 512	12 627	12 327	11 268
6	"	14 365	12 227	11 820	12 649	10 202	13 089	12 870	13 649	13 338	13 182	12 558
6	Southern	14 912	12 907	11 752	12 829	11 172	13 074	13 486	13 465	13 070	13 163	12 140
6	"	14 671	13 366	12 918	13 095	12 651	13 029	13 189	13 196	13 161	12 925	12 799
6	"	14 763	13 229	12 939	13 753	12 397	13 170	13 086	13 253	13 134	13 037	12 687
6	"	14 660	13 030	12 700	13 450	12 103	13 101	13 391	13 280	13 325	13 417	12 233
	Average	14 598	13 007	12 199	13 068	11 166	12 959	13 125	13 180	13 095	12 935	12 445

thermal units when coal is washed for fuel; so when it is desired to ascertain the efficiency of a washery it is necessary to translate ash and British thermal units into specific gravity, since it is upon a specific gravity basis that the coal and refuse are separated in the washery. It has been pointed out that there is a fairly regular increase in ash and decrease in British thermal units with increase in specific gravity in the case of any given coal. Evidently, by effecting a separation at the proper specific gravity any desired proportion of ash within the range of possibility for the coal in question may be obtained.

Separation by means of a solution of high specific gravity is 100 per cent efficient, and while this method is too expensive to be used on a large scale commercially it is admirably adapted to testing. Thus, if a raw coal is tested in heavy solutions of different specific gravities, and the float from each test weighed and analyzed, the specific gravity of a solution that will give float with the desired ash and British thermal units content may be determined. Suppose that 1.35 specific gravity is found to be the right one. The efficiency of the commercial washing process may be determined by testing the products with a solution of this specific gravity. The washed coal will be found to contain a certain amount of material with specific gravity higher than 1.35, and the refuse a certain amount with a specific gravity less than 1.35. Obviously, if the separation at 1.35 had been 100 per cent efficient there would have been no overlap whatever, and the proportions of the amount of overlap are direct measures of the efficiency of the washed coal separation and of the refuse separation respectively. The average of these efficiencies, expressing in a single figure as it does the efficiency of the washery, will be found very useful for purposes of comparison.

The method of determining washery efficiency may be illustrated by the following tests made in the laboratory of the Department of Mining Engineering of the University of Illinois. Samples of 1¼ in. washed coal screenings and refuse from a tub washery treating Bed No. 2 coal in the Northern Field, were tested for sink and float in a zinc chloride solution of 1.35 specific gravity. The estimation of efficiency was made on the basis that a perfect washed coal should contain no heavy or sink particles, and that a perfect refuse should contain no light or float particles. The following is a summary of the results:

	Washed Screenings		Refuse		General Efficiency of Process. Washed screenings plus refuse divided by 2
	Efficiency	Ash	Efficiency	Ash	
Float	91.29%	7.15%	1.21%	9.21%	95.04%
Sink	8.71%	21.44%	98.79%	69.86%	

The lowest washery efficiency comes in the separation of fine coal from fine refuse. Samples of the fine, or No. 5, washed coal and refuse, as produced from Coal Bed No. 6 in the Southern Field, were subjected to the same tests as above, with the results given in Table 16.

TABLE 16.

EFFICIENCY TESTS ON COAL NO. 6 FROM THE SOUTHERN FIELD.

Type of Washery	Product	Washed Coal %	Washed Coal Efficiency %	Washed Coal Ash %	Refuse Efficiency %	Refuse %	Refuse Ash %	General Efficiency of Process. Washed Coal plus Refuse divided by 2. %
Pan Jig	Float Sink	12.75	87.25	4.73 57.48	56 50	43.50	7.00 70.49	71.88
Pan Jig	Float Sink	19.23	80.77	4 56 61 10	90.38	9.62	8.50 87.58	85.58
Piston Jig	Float Sink	22.00	78.00	4.11 35.14	74.20	25.80	6.40 56.30	76.10
Bumping Table	Float Sink	14.73	85.27	4.71 52.66	80 47	19.53	5.71 69.49	82.87
Average	Float Sink	17.18	82.82	4 53 51.60	75.39	24.61	6.90 70.97	79.11

TABLE 17.

PERCENTAGES OF REFUSE PRODUCED AT ILLINOIS WASHERIES.

Field	Washery	Percentage
Northern	1	32
"	2	36
"	3	32
"	4	33
"	5	24
"	6	25
"	7	25
"	8	26
"	9	19
Average Northern		28
Central	14	10
"	15	10
"	17	11
"	18	22
"	19	11
"	20	21
"	22	19
"	23	10
"	24	20
"	25	13
Average Central		15
Southern	28	19
"	31	5
"	32	9
"	33	15
"	35	5
Average Southern		11
General Average		19

C. COMPARISONS.

Owing to the presence of numerous variables, comparisons of the performances of different washeries must be made with great care or they will prove misleading. The results obtained are dependent upon character

of coal bed, method of mining, method of raw coal supply to washery, size washed, type of washery, arrangement of washery and operation of washery. A variation in any one of these factors may materially affect the results.

This is evident when considering the proportions of refuse produced at different Illinois washeries, as shown in Table 17. So irregular are these results and so uncertain the causes, that only the most general comparisons can be made with safety. It can be seen that there is, on the whole, a general decrease in the proportion of refuse produced in going from north to south, the average for the Northern field being 28 per cent, for the Central 15 per cent and for the Southern 11 per cent. The percentages given cover all the material thrown away and lost at the washeries, and so include not only the coarse refuse which goes to the dump but also the suspended matter carried away by the water. The amount of this floating refuse is rather difficult of determination, but from several estimates it appears probable that the loss from this source is not infrequently as high as 2½ per cent of the weight of the raw coal washed. Reference to Tables 18 and 19 shows this suspended matter to be always lower in ash than the coarse refuse, while in some instances the ash is seen to be so low as to suggest that the material might profitably be collected and used locally as fuel. The sulphur content of some refuse, as shown in Appendix D, is so high that possibly the pyrite content might be concentrated and sold at a profit.

49. *Different Coalfields.*—The washed coals from the Northern field are on the whole lower in ash than those from the other fields, although in individual cases they may be higher. Southern Illinois coal is next best from point of view of ash content, for though but little refuse is removed from it, it is low in ash at the outset. The average washed coal of the Central field has a slightly higher ash content, while those of the Springfield and Danville fields have still higher. These conclusions are based on Tables, 13, 17 and 18. Table 18, which shows the results achieved at pan jig washeries, is especially valuable for purposes of comparison between the three principal coalfields, since there are more pan jig washeries than all other classes combined. This Table shows the average ash content of washed coals from four Northern washeries to be 8.34 per cent, from two Southern washeries 9.81 per cent and from four Central washeries 12.03 per cent.

50. *Pan Jig and Luhrig Washeries.*—Pan jig washeries of different types give similar results, but the results obtained by different types of piston jig washeries vary considerably. The commonest type of piston jig washery, the Luhrig, gives slightly better results than pan jig washeries, probably because Luhrig washeries always size before washing. The average ash content of the washed coals from two southern Luhrig washeries was 8.99 per cent and of two Central washeries 11.41 per cent as

TABLE 18.

Dry Ash in Products of Illinois Pan Jig Washeries.

Coal Field	Raw Screenings	Washed Coal						Refuse	Suspended Matter	Decrease in Ash Washed Coal under Raw Screenings		Increase in Ash Refuse over Raw Screenings	
		No. 1	No. 2	No. 3	No. 4	No. 5	All Sizes			Amt.	Prop'rt'n	Amt.	Prop'rt'n
	%	%	%	%	%	%	%	%	%	%	%	%	%
Northern	19 17	6.77	4.93	5.13	11.83		9.05	58.68	28.92	10.12	52 79	39.51	206.1
"					7.10			63.59					
"	(27.10)*	5.86	6.60	6.29	6.85	7.07	6 69	63.39		20.41	75 31	39.06	160.5
"	24.47	7.74	7.37	7.91			7.79	58.79		16.68	68.12	34.32	140.3
"	28 87	not made		8.75	9.96		9.83	50.05		19.04	65.95	21.18	73.4
"		not made		9 67	8.61								
Average	24 21	6.79	6.30	7.55	8.71	8.75	8 34	58.90		16.56	65 54	33.49	145.1
Springfield	(16.50)	(11.60)	(12.00)	(12.30)	(12.2)	(12 20)		(5.50)				43.40	374.1
Central						14 83							
"	(16.48)	(9.61)	(9.99)	(10 79)	(10.29)	(12.92)	(10 29)	(57.49)		6.19	37.56	41.01	249.5
"	(19 40)	(9.60)	(10.90)	(10.30)	(11.40)	(14 30)		(57.70)				38.30	197.4
"	(18.30)	(10.30)	(9.70)	(10.40)	(10.90)	18.31	12.52	(72.00)	(22.3)	5.78	31.58	53 70	293.4
"	(25.76)	(11.57)	(6.04)	(10.60)	(8.80)	(14.25)	10.83			14 93	57.96		
"	(26.00)	(10 80)	(10.40)	(10.10)	(13.80)	19.29	14 47	64 23		11 53	44.35	38 23	147.0
"				(8.05)	(9 22)								
"	(26 30)	(8.00)	(11.50)	(12.20)	(12.50)			65.70				39.40	139.8
Average	22.04	9.98	9.76	10.33	10 99	15.65	12.03	63.42		9.61	42.86	42.15	205.4
Southern	13.47	9.09	8.16	8.78				56.02				42.55	315.9
"	13.26	8 96	8.52	7.89	7.42	15.43	9.53	53 48		3.73	28.13	40.22	303.3
"	12.43	9.12	9.00	9.28	10.76	11.46	10.08	42.87	19.02	2.35	18.92	30.44	244.9
"	(16.00)	(11.00)	(10.80)	(10.00)	(8 25)	(13.45)							
"	(10.60)	(8.30)	(7.30)	(7.00)	(8.30)	(10.50)		(60.50)				49.90	470 8
Average	13.15	9 29	8 76	8 59	8.38	12.59	9.81	53.22		3.04	23.53	40.78	333.7
Average of Northern, Southern, and Central	19.80	8.69	8.27	8.82	9.36	12.33	10.06	58.51	23.41	9.74	43.98	38 81	228.1

* Results in parentheses were not obtained by the writer but are from sources believed to be reliable.

against 9.81 per cent and 12.43 per cent respectively, for pan jig washeries in the same fields.

It is characteristic of the pan jig washeries in the Northern and Southern fields that the ash content of the sizes rises in the order 2, 1, 3, 4 and 5, with the No. 5 washed coal markedly higher than the No. 4. In the Southern field, the order is 4, 3, 2, 1, 5, indicating a more careful washing which is probably due to the small amount of refuse to be removed as compared to the amounts in the other fields. It is characteristic of Luhrig washeries that the No. 3 size of washed coal has a higher ash content than the No. 4. This is owing to the fact that Nos. 4 and 5 are washed together and the bone and shale left are of small sizes which, on screening, go mainly into the No. 5. The Nos. 1, 2 and 3, being washed separately, should have decreasing ash contents, if washed under the best conditions, but as a matter of fact their ash contents increase, as may be seen by consulting Table 19.

Robinson tub washeries in the Northern and Central fields are doing as well as the pan jig washeries. A Forrester washery in the Northern field does not do so well. A Foust jig in the Southern field does not appear to be any improvement on a pan jig. A Campbell bumping table washery in the Southern field holds its own with other types of washeries, the results secured making it appear to be about the equal of a Luhrig washery. Table 20 gives further details of the results obtained in Illinois washeries of different types.

51. *Rescreener and Dry Cleaner.*—So free from bone and shale is some of the coal from the Southern field that the rescreened product may equal some of the washed coals in quality, as will be seen by the figures in Table 20. Dry cleaning is also seen to be very effective in reducing the ash in the No. 1 and No. 2 sizes. The No. 5 coal given beside "Dry Cleaner" in the table was not cleaned. The ash content of the No. 3 and No. 4 sizes is only slightly reduced by the process, but this is probably due to overcrowding rather than to a defect in the method.

XI. Costs of Washing.

The costs of washing coal are hard to determine for three reasons. The washeries do not work regularly throughout the year and a considerable and variable charge for idle time must be made against them. The washeries are frequently run in direct connection with the mine which supplies them with screenings, the same power plant and management serving for both, with the result that the charges which should be made to the washery are not always segregated from those properly belonging to the mine. Finally, the operators of washeries are naturally reticent about letting their business opponents know their costs. Upon the promise that costs supplied would be printed only in such a way that

TABLE 19.

DRY ASH IN PRODUCTS OF ILLINOIS LUHRIG JIG WASHERIES.

Coal Field	Raw Screenings	Washed Coal						Refuse	Suspended Matter	Decrease in Ash W'shed Coal under Raw Screenings		Increase in Ash Refuse over Raw Screenings	
		No. 1	No. 2	No. 3	No. 4	No. 5	All Sizes			Amt.	Proportion	Amt.	Proportion
	%	%	%	%	%	%	%	%	%	%	%	%	%
Central	(25.60)*	(10 10)	(9.40)	(10.20)	(9.50)	19.49	12.32	68.89	53.20	13.28	51.38	43.29	169.1
"	(20.40)	(7.30)	(8.20)	(13.00)	(9.90)	12.32	10.50	Coarse 38.10 Fine 65.70		9.90	48.53		
Average	23.00	8 70	8.80	11.60	9.70	15 91	11.41			11.59	50.21		
Southern			(8 47)	(8.41)	(8.07)	(8 76)							
"		(5.63)	(8.88)	(8.78)	(7.44)	(14.14)	9.07						
"	13.97	8.39	9 21	9.07	7.66	10.94	8.90	47.30 43.42					
"	(18.95)	(8.51)	(8.70)	(9.95)	(8.26)				47.12	5.07	36.30		
"	(15.37)	(8.13)	(5 58)	(7 97)	(9 66)	(8.91)							
Average	16 10	7.67	8.16	8.84	8 22	10 69	8.99						

*Results in parentheses were not obtained by the writer but are from sources believed to be reliable.

TABLE 20.

DRY ASH IN PRODUCTS OF ILLINOIS WASHERIES, DRY CLEANER AND RESCREENER.

Type of Plant	Raw Screenings	Treated Coal						Refuse	Suspended Matter	Decrease in Ash Tr'ted Coal under Raw Screenings		Increase in Ash Refuse over Raw Screenings	
		No. 1	No. 2	No. 3	No. 4	No. 5	All Sizes			Amt.	Proportion	Amt.	Proportion
		%	%	%	%	%	%	%	%	%	%	%	%
Northern Field													
Robinson Washery	29.96	not made			8.36		8 36	69 13	51.98	21 60	72.10	39.17	130.7
Forrester "	28 58	9.74			11.43		11.06	73.18	38.77	17.52	61.30	44.60	164.1
" "					(14 85)*								
Pan Jig Washeries	24.21	6.79	6 30	7.55	8.71	8.75	8.34	58.90	28.92	16.56	65.54	33 49	145.1
Danville Field													
Forrester Washery	(19.30)*			(10.80)	(11.30)	(17.50)		(66.80)				47.50	246.1
New Century "					(9.66)	(16.39)							
Central Field													
Robinson Washery			9.63			13.95		60.14	46.26				
Luhrig Washeries	23.00	8.70	8.80	11.60	9.70	15 91	11.41	68.89	53 20	11 59	50.21	43 29	169 1
Pan Jig "	22 04	9.98	9.76	10.33	10.99	15.65	12.03	63.42	22.30	9.61	42.86	42.15	205 4
Springfield Field													
Pan Jig Washery	(16.50)	(11.60)	(12.00)	(12.30)	(12 20)	(12.20)		55.00				43.40	374.1
Southern Field													
Luhrig Washeries	16.10	7 67	8.16	8.84	8.22	10.69	8.99	Coarse 47.30 Fine 43.42	47.12	5.07	36 30		
Foust Jig	18.90				9 46	11 60	11.31	74 13	16 07	7.59	40 16	55.23	292.2
Pan Jig Washeries	13 15	9 29	8.76	8.59	8.38	12.59	9 81	53.22	19 02	3.04	23 53	40.78	333.7
Campbell Washery		9 33	8 71	8.10	7 53	11.77		57.44	13.98				
Dry Cleaner		9 25	8.43	13.19	13.37	13 55	11.64	46 81	20.35†				
Rescreener		11.83	11 39	11 28	10 10		11.00	56 79‡					

*Results in parentheses were not obtained by the writer but are from sources believed to be reliable.
‡Hand picked. †"Bone."

no one save the operator supplying them could tell to what washery they applied, the writer has kindly been furnished with the figures presented in Table 21.

The costs of power, labor, supplies, repairs and renewals per ton of raw coal washed, as reported by fifteen washeries, varied from 3 to 18 cents, with an average of about 10½ cents. In obtaining this average, depreciation was included in one instance but this is probably more than offset by omission of costs of power from reported general washing costs which are likely to be made when mine and washery are operated with the same power plant, so it is believed that this average is low rather than high. The cost of building fifteen washeries with a combined capacity of 1740 tons raw coal per hour was $572 000. At the same rate the 33 operating commercial washeries of Illinois with their combined hourly tonnage of 3555 would have cost $1 169 000. These figures do not represent the total investment in washeries, as they do not include cost of land for washery, reservoirs and refuse dump sites. Costs of individual washeries could not be included in Table 21 without violating the confidences of some of the operators, but costs per ton rated capacity are given. These costs are for washeries with hourly tonnages ranging from 25 to 280 and averaging 116, and show costs of from $130 to $583 per ton capacity per hour, with an average of $351. The costs of individual washeries, while showing considerable irregularity, still vary in a general way with size and type. The average cost per ton capacity per hour of seven washeries with capacities of 100 tons and under was $448, while eight washeries with capacities in excess of 100 tons averaged $266. The average cost of three Luhrig jig washeries was $393 per ton capacity per hour, of six Stewart jig washeries $341 and of two Robinson tub washeries $322.

XII. Moisture in Washed Coal.

By Professor S. W. Parr.

The question not infrequently arises as to the amount of excess moisture carried in commercial shipments of washed coal. The particular samples upon which data have been obtained in this Bulletin covering the percentages of ash, sulphur, etc., were not received at the laboratory in a form which made it possible to derive trustworthy conclusions on this point.

In an earlier series of investigations carried on by the Engineering Experiment Station,* however, a large number of car lot shipments were received directly from the washeries. In the majority of cases these car lots were unloaded upon their arrival at the University and sampled in

*University of Illinois Engineering Experiment Station Bulletin No. 39, entitled "Tests of Washed Grades of Illinois Coal."

TABLE 21.

Costs of Washeries and of Washing.

Cost of Washery per Ton Rated Capacity per Hour (Dollars)	Power (Cents)	Labor (Cents)	Cost of Washing per Ton Raw Coal Washed — Supplies Repairs & Renewals (Cents)	Depreciation (Cents)	Insurance (Cents)
280	1.69	7 02	4.60		
375	2 50	7.50	2.19		1.20
583	2.016	13.290	2.671	2.754	0.599
	3.3		0.8	3.4	
294	0.5		10.6	(5—7½%)	
130			6.00		
			15.24		
268			8.60		
563			5.00—10.00		
277			9.50		
400			10.80		
217			3.00	(5%)	
583			16.60		
380					
280			3.00		
280					
350			14.00		
Average 351			10.42		

the process of unloading. It occasionally happened that some of the cars were unloaded into bins before the steam tests were begun. The moisture determination was then made on the coal as it was fired. Where the time of such storage was long and the size of the coal such as to seem to permit of a drying out of the moisture, the storage time is indicated in Table 22. We have thus a very fair indication of the average moisture carried by coals that have passed through the washery and been subjected to such drying out processes as would ordinarily occur in shipment and handling. In every case, also, it has been possible to give a reference factor for the normal moisture in the face samples taken in the usual manner from the same mines, and these factors are included in the table.

One or two points are to be borne in mind in this connection. First, the distance involved in shipment has not a little to do with the water content at the point of delivery. The shipments from Vermilion County to Urbana, for example, cover a distance of approximately 35 miles. As a rule, cars loaded at the Westville mine will be delivered in Urbana the next morning. The moisture increase over the normal or face moisture in this shipment is noticeably higher than the increase indicated for the shipments from Williamson County, which is at a distance of approximately 175 miles.

Another point to be taken into consideration is the fact that the normal or face moisture under ordinary conditions of shipment for unwashed coal will show a decrease in the amount of water present in the car sample. This fact is so well known that it does not need verification here, but it so happens that a number of the samples were duplicated by the unwashed product and received at the laboratory in this form in car lots. Samples 33 to 35 have the advantage, therefore, of giving this data directly. That is to say, with a normal moisture factor in the face of 8.79 per cent, the dry or unwashed coal was received at the University with an average moisture factor of 7.34 per cent. The moisture factors as given, therefore, for the washed coal might well be compared with this factor as the normal one which would accompany the shipments from that distance.

Another point of interest is the general uniformity of the moisture factors for any given series of shipments where conditions of distance and time are approximately constant.

In Table 22 as given, the sizes as obtained by screening at the washery are indicated and this serves to show that the finer sizes retained a higher percentage of moisture and the coarser sizes lost a relatively larger amount of moisture where opportunity presented itself for the drying out of the mass.

52. *Summary.*—Distance, size of the coal and weather conditions are the controlling factors in the amount of moisture retained by washed

TABLE 22.

MOISTURE IN COMMERCIAL SHIPMENTS OF ILLINOIS WASHED COAL.

Table No.	Lab. No.	Source of Sample	Sized by Screening (Inches)	Date of taking car samples	Normal Moisture in Vein Samples	Moisture in Washed Commerc'l Car Shipments as delivered	Remarks
1	220	Himrod Mine	Thru ⅞	Dec. '06	12.96	18.38	New
2	226	Westville,	Over ⅜	"	"	18.63	Century
3	228	Vermilion		"	"	17 35	Washer
4	230	County		"	"	19.11	
5	233			"	"	18 71	
6	‡253			"	"	18.03	
7	243	Shaft A,	Thru ¾	"	9.13	12 24	Stewart
8	246	Herrin	Over ¼	"	"	13.15	Washer
9	249	C & C. C. Co.,		"	"	12.49	
10	255	Williamson		Jan. '07	"	13 13	
11	258	County		"	"	11.30	
12	‡273			"	"	12.14	
13	261	Shaft A,	Thru ¼	"	"	12.41	Stewart
14	264	Herrin	Over 0	"	"	15.80	Washer
15	269	C. & C. C. Co.,		"	"	14.59	
16	271	Williamson		"	"	16.20	
17	284	County		"	"	14.40	
18	286			"	"	17.84	
19	‡303			"	"	15.69	
20	288	Himrod Mine	Thru ⅞	"	12.96	18.68	New Century
21	290	Westville,	Over 0	"	"	19.32	Washer
22	292	Vermilion		"	"	18.33	
23	294	County		"	"	18.79	
24	‡302			"	"	18.78	
25	296	Shaft 2,	Thru ¾	"	10.47	10.72	Stewart
26	274	Peabody C. C.	Over ¼	"	"	10.72	Washer
27	275	Marion		"	"	11.32	
28	276	Williamson Co		"	"	11.23	
29	278	Sh't A, Herrin,	Thru 1¾	Feb. '07	9.13	11.43	Stewart
30	280	C. & C. C. Co.,	Over 1	"	"	10.94	Washer
31	282	Williamson Co	Thru ¾	"	"	10.21	
32	‡304		Over ¼	"	"	10.69	
33	498	Sh't A, Herrin,	Thru 1¾	Apr. '07	"	8.79*	Stewart
34	500	C. & C. C Co.,	Over 1	"	"	8 06*	Washer
35	511	Williamson Co		"	"	8.44*	
36	‡502			"	"	9.60*	
37	520	Sh't A, Herrin,	Thru 1	May '07	"	10.12	In Storage Six
38	541	C. & C. C. Co.,	Over ¾	"	"	9.40	Weeks Before
39	556	Williamson Co		"	"	8.99	Sampling
40	548			"	"	9.38	
41	552			"	"	8.90	Stewart Washer
42	559			"	"	9.74	
43	560			"	"	9.25	
44	‡571			"	"	9.27	
45	561	Shaft No. 2,	Thru ¼	"	10.47	13.94	Stewart
46	563	Marion,	Over 0	"	"	13.97	Washer
47	565	Peabody C. Co.		"	"	21 49	
48	567	Williamson Co		"	"	18.52	
49	569			"	"	17.34	
50	577			"	"	19.27	
51	578			"	"	19.63	
52	579			"	"	19.49	
53	‡587			"	"	18.17	
54	581	Himrod Mine	Thru ⅜	"	12.96	21 61	New Century
55	588	Westville,	Over 0	"	"	21.72	Washer
56	589	Vermilion Co.		"	"	21.35	
57	590			"	"	21.25	
58	‡600			"	"	22.00	
59	591	Himrod Mine,	Thru ⅞	"	"	17.24	New Century
60	592	Westville,	Over ⅜	June '07	"	22 63	Washer
61	594	Vermilion Co.		"	"	17.95	
62	596			"	"	19.83	
63	598			"	"	17 68	
64	601	–		"	"	16.48	
65	603			"	"	19.29	
66	605			"	"	19.49	
67	‡608			"	"	18.97	

* Five samples, unwashed, from same mine and of same size gave on delivery in car-lots at Urbana a composite moisture factor of 7.34%

‡ Composite sample. Proportional weights taken.

coal at the point of delivery. With the finer sizes and the minimum distance, as in the Westville samples, Table 22, table Nos. 54 to 58 being all sizes through a 3/8 in. screen, the moisture content in the washed coal is 9 per cent higher than in the normal or face samples. This represents an increase of very nearly 10 per cent over the original weight of the coal in the mine. The next larger size from the same mine, as shown in table Nos. 20 to 24, namely, all sizes through a 7/8 in. screen, shows an average moisture factor of 18.78 per cent. This is an increase of 5¾ per cent over the normal or face moisture. The next size from the same mine, table Nos. 1 to 5, shows an average moisture content of 18.03 per cent, which is 5 per cent above the normal moisture.

Summarizing the values for the coals subjected to the longer haul, the finest sizes represented are found in table Nos. 13 to 19, the size of coal being all which would pass through a ¼ in. screen. The average moisture for the washed coal is shown to be 15.69 per cent, which is 6.5 per cent higher than the normal or face moisture. The next larger size, namely, through a ¾ in. screen and over a ¼ in. screen, is found in table Nos. 31 and 32, with an average moisture value of 10.69 per cent. This is approximately 1.5 per cent higher than the moisture in the face sample. Table Nos. 25 to 28 are of the same size and the same hauling distance with a moisture factor for the washed coal a little less than 1 per cent above the normal. Another set of values for this same size is shown in table Nos. 7 to 12 in which the increase of moisture in the washed coal is 3 per cent above normal. The largest screen size, as for example, through a 1¾ in. and over a 1 in. screen, as shown in table Nos. 29, 30 and 33 to 36, are not materially different from the screened size between ¼ in. and ¾ in.

The greatest uniformity in moisture increase is shown in the coals having the short haul and the widest variations are shown in the coals with the long haul. Taking into consideration the loss of moisture which would occur in unwashed coals from Williamson County, it may be said that the finer sizes will show an increase of moisture of from 2 to 6 per cent, while the coarser sizes will show an increase of from 2 to 4 per cent above what would normally be the case with unwashed coal at the point of delivery. The increase on the corresponding sizes of washed coals subjected to a short haul would be approximately from 6 to 9 per cent on the finer sizes and approximately 5 per cent on the coarser sizes.

APPENDIX A.

Bibliography of Coal Washing in Illinois.

1874

(1) *Luebbers, H. L.*

"The manufacture of Coke from Illinois Coal." Trans. Am. Soc. C. E. *2* (1874), 163-6, no figs. Contains a very general description of washing Illinois coals by jigging.

1876

(2) *Meier, E. D.*

"Coal Washing in Illinois," Engineering and Mining Journal *22*, 88-90. 3 figs. Also in Engineering News *3*, 228-229, 3 figs. Describes remodeling the Osterspey washery of the Illinois Patent Coke Co. at East St. Louis, Illinois.

1885

(3) *Weeks, J. D.*

"Manufacture of Coke. Illinois." Mineral Resources of the United States, Calendar Years 1883 and 1884, 160-163. Contains brief mention of washeries at Harrison, Saint Johns, Brussels, Equality, Brookside, Streator, and East St. Louis.

1893

(4) *Weeks, J. D.*

"The Manufacture of Coke. Illinois." Mineral Resources, Calendar Year 1891, 378.

Reports failure of attempts to wash Illinois coal sufficiently clean to make good coke.

1896

(5) (*Rutledge, J. J.*)

"The Braceville Coal Washer, Illinois," Eng. and Min. Jour. *62*, 511, 1 fig. Describes 100-ton Howe washery of the Wilmington Coal Washing Co. at Central City.

(6) *Rutledge, J. J.*

"Coal Mining in Illinois," Mineral Industry *4*, 194. Brief reference to coal washing in Illinois.

1897

(7) *Schaefer, J. V.*

"The Washing of Bituminous Coal by the Luhrig Process." Trans. Am. Soc. Mech. Eng. *18*, (1897), 84-90, 4 tables, 3 figs. Reprinted in Mines and Minerals *17*, (1897), 249-253. Reprinted with additional

matter in catalog of Cunninghame & Co. Includes analyses of raw and washed products of 3 Illinois washeries and detailed description of washery of St. Louis & Big Muddy Coal Co. at DeSoto.

1900

(8) *Dinsmore, A.*

"Coal Mines at Streator." Mines and Minerals *21,* 145. Describes Forrester washery of the Illinois Coal Washing Co.

1901

(9) *Breckenridge, L. P.*

"Boiler Tests with Illinois Coals." Jour. West. Soc. Eng. *6,* 230. Briefly refers to benefits derived from washing.

(10) *Kent, W.*

"Steam Boiler Economy," New York, 73. Gives Analysis of washed screenings from Wilmington.

(11) *Schaefer, J. V.*

"Washing of Bituminous Coals by the Luhrig Process." Jour. West. Soc. Eng. (1901), 6 figs. Reprinted in Mines and Minerals *22,* (1902), 366-370. Gives analyses of raw and washed coals at 2 Illinois washeries and list of 4 washeries in operation in Illinois.

1902

(12) *Parr, S. W.*

"The Chemical Analyses and Heating Values of Illinois Coals." Illinois Bur. Labor Stat. *20,* 99-113. Also published separately as a bulletin. Includes 15 analyses of washed coals.

1903

(13) *Bement, A.*

"Shipping Mines and Coal Railroads of Illinois and Indiana." Peabody Coal Co., Chicago, Jan. 1903, 51 and 53. Lists of 21 Illinois Coal Washeries.

(14) *Ross, D.*

"The Southern Illinois Coal Mining & Washing Co., Mine No. 3, Located at Marion, Illinois." Ill. Bur. Lab. Stat., Ann. Rep. *21,* 92, 1 fig. Briefly mentions washery.

(15) *Ross, D.*

"The Acme Coal Co. Mine and Coal Washer, Streator, Illinois." Ill. Bur. Lab. Stat. Ann. Coal Rep. *21,* 98, 1 fig. Briefly mentions washery.

1904

(16) *Bement, A.*
"Washed Coal and Its Preparation," Chapter IV of "The Economical Burning of Coal Without Smoke." Peabody Coal Co., Chicago, 28-37, 4 figs.

Brief description of Stewart washery of Southern Coal Mining & Washing Co. at Marion, Illinois.

(17) *Blakey, D. T.*
"Coal Washing by the Stewart System." Mines and Minerals *24*, 213. Makes brief reference to washing by Stewart system in Illinois.

(18) *Parr, S. W.*
"The Coals of Illinois: Their Composition and Analysis." The University Studies *1*, No. 7, (U. of I. Bull. *1*, No. 20), 24, 30, 31, 35 and 37. Analyses of Illinois washed coals.

(19) *Roberts & Schaefer Co.*
Business announcement. Briefly describes and gives views of 5 Illinois washeries.

1905

(20) *Link-Belt Machinery Co.*
"Washing Bituminous Coal for Coke or Fuel." Special Booklet *42*. Contains views and diagrams of Illinois washeries, lists of those built and those equipped by the company and 3 analyses of Illinois washed coals.

(21) *Parker, E. W., Holmes, J. A., & Campbell, M. R.*
"Preliminary Report on the Operations of the Coal Testing Plant of the U. S. Geological Survey at the Louisiana Purchase Exposition, St. Louis, Mo., 1904." U. S. Geol. Sur. Bull. *261*. Includes washing test of Illinois slack coal and coking tests of Illinois washed coal.

1906

(22) *Breckenridge, L. P.*
"Tests with Illinois Coals Under Steam Boilers." Ill. Geol. Sur. Bull. *3*, 79-83. Gives brief review of 3 boiler tests of washed coals.

(23) *Breckenridge, L. P.; Parr, S. W.; Dirks, H. B.*
"Fuel Tests with Illinois Coals." U. of I. Eng. Exp. Sta. Bull. *7*, Gives details of 3 boiler tests with washed coals and 23 analyses of washed coals.

(24) *Holmes, J. A.*
"Preliminary Report on the Operations of the Fuel-Testing Plant of the U. S. Geological Survey at St. Louis, Mo., 1905." U. S. Geol. Sur. Bull. *290*. Includes washing tests on Illinois coals and coking and steaming tests on Illinois washed coals.

(25) *Moss, R. S.*
"The Peabody Mines in Southern Illinois." Mining World, *25,* 66-7. Includes brief description of the Stewart washery of the Southern Illinois Coal Mining and Washing Co. at Marion and analyses of washed coals produced.

(26) *Parker, E. W.; Holmes, J. A.; & Campbell, M. R.*
"Report on the Operations of the Coal Testing Plant of the U. S. Geological Survey at the Louisiana Purchase Exposition, St. Louis, Mo., 1904." U. S. Geol. Sur. Prof. Paper *48.* Includes washing test of Illinois slack and coking tests of Illinois washed coal.

(27) *Parr, S. W.*
"Composition and Character of Illinois Coals, Illinois Geol. Sur. Bull. *3,* 27-78. Gives analyses of 8 washed coals.

(28) *Roberts & Schaefer Co.*
"Coal Washeries, Coal Tipples and Mining Equipment." Bull. *5.* Gives views of 3 Illinois washeries.

1907

(29) *Bement, A.*
"Ash in Coal and Its Influence on the Value of Fuel." Ill. State Geol. Sur. Bull. *8,* 209. Gives average analyses of Illinois washed coals.

(30) *Breckenridge, L. P.*
"How to Burn Illinois Coal Without Smoke." U. of I. Eng. Exp. Sta. Bull. *15.* Includes boiler tests with Illinois washed coals and brief mention of advantages of Illinois washed coal.

(31) *Diescher, S.*
"Process of Coal Washing," Proc. Eng. Soc. Western Pa. *23,* 202. Refers to first washeries in Illinois. Abstract in Iron Age, July 11, 96-100.

(32) *Lord, N. W.*
"Experimental Work Conducted in the Chemical Laboratory of the United States Fuel Testing Plant at St. Louis, Mo., Jan. 1, 1905, to July 31, 1906." U. S. Geol. Sur. Bull. *323.* Reprinted in 1911 as U. S. Bur. of Mines Bull. *28.* Includes specific gravity determinations and 3 washing tests of Illinois coals.

(33) *Parker, E. N.*
"Quantity of Coal Washed at the Mines in 1906." Mineral Resources, Calendar Year 1906, II, 610-511.

(34) *Parr, S. W. & Wheeler, W. E.*
"An Initial Coal Substance Having a Constant Thermal Value." Ill. State Geol. Sur. Bull. *8,* 157. Analyses of Illinois washed coals.

(35) *Roberts and Schaefer Co.*
"Modern Coal Washeries and Mining Plants." Bulletin No. *8*. Contains list with 10 Illinois washeries constructed by the company and views of 5.

(36) *Smith, C. H.*
"Mine No. 17, Collinsville, Ill." Mines and Minerals *28*, 16-17. Includes brief description of Luhrig washery of Consolidated Coal Co. of St. Louis near Collinsville.

1908

(37) *Holmes, J. A.*
"Report of the United States Fuel-Testing Plant at St. Louis, Mo., January 1, 1906, to June 30, 1907." U. S. Geol. Sur. *332*. Includes briquetting test on Herrin No. 5 washed coal and washing tests on 12 Illinois coals.

(38) *Roberts & Schaefer Co.*
"Modern Coal Mining Plants and Coal Washeries." Bull. No. *18*. Contains views and brief descriptions of washeries at Pana and DeCamp, Illinois.

(39) *Parker, C. W.*
"Quantity of Coal Washed at the Mines in 1907." Min. Res., 1907, II, 59-60.

1909

(40) *McGovney, C. S.*
"Tests of Washed Grades of Illinois Coal." U. of I. Eng. Exp. Sta. Bull. *39*, 146 pp. Fully describes boiler tests on 10 grades of Illinois washed coal.

(41) *Moss, R. S.*
"The Carterville Coal Field, Southern Illinois." Mining World, *30*, 676-8. Briefly describes history of coal washing at Brush shaft, Carterville.

(42) *Parker, E. W.*
"Quantity of Coal Washed at the Mines in 1908." Min. Res., 1908, II, 57-58.

(43) *Parr, S. W. and Wheeler, W. F.*
"Unit Coal and the Composition of Coal Ash." (U. of I. Eng. Exp. Sta. Bull. 37). Includes analyses of 2 Illinois washed coals.

(44) *Snodgrass, J. M.*
"The Use of Illinois Coal for Domestic Purposes." Ill. State Geo. Sur. Bull. *14*, 223-228. Includes boiler test on Illinois washed coal.

1910

(45) *Belden, A. W.; Delamater, G. R.; Groves, G. W.; & Way, K. M.* "Washing and Coking Tests of Coal at the Fuel-Testing Plant, Denver, Colo., July 1, 1908, to June 30, 1909." U. S. Bureau Mines Bull. *5*. Includes tests on coal from Sesser, Illinois.

(46) *Bement, A.* "The Illinois Coal Field." Ill. State Geol. Sur. Bull. *16*, 195-196. Gives average analyses of 10 washed coals.

(47) *Parker, E. W.* "Quantity of Coal Washed at the Mines in 1909." Min. Res. 1909, II, 53-4.

(48) *Savage, T. E.* "The Geology and Coal Resources of the Herrin, Illinois Quadrangle." Ill. State Geol. Sur. Bull. *16*, 284. Makes brief reference to coal washing.

1911

(49) *Fernald, R. H. and Smith, C. D.* "Resume' of Producer Gas Investigations, October 1, 1904-June 30, 1910." U. S. Bur. Mines Bull. *13*. Includes producer tests on 3 Illinois washed coals.

(50) *Link-Belt Co.* "Tipples, Coal Washeries and Conveying Machinery for Coal Mines." Cover has title, "The Handling and Preparation of Coal at the Mine." Book No. *111*. Contains views of 3 Illinois washeries.

(51) *Parker, E. W.* "Coal-Washing Operations." Min. Res. 1910, II, 54-5. Gives quantity of coal washed at Illinois mines.

(52) *Roberts & Schaefer Co.* "Modern Coal Washing Plants." Bull. *24*. Briefly describes 5 Illinois washers and gives views of 10.

1912

(53) *Allen & Garcia Co.* "Coal Washery at Panama, Illinois, for Shoal Creek Coal Co." In business announcement. Gives description and views.

(54) *Bement, A.* "The Screening Problem in Illinois." Coal Age *1*, 1912, 1105, 3 figs. Notes growth of washed coal output and increased demand for No. 4 and No. 5.

(55) *Breckenridge, L. P.; Kreisinger, H.; & Ray, W. T.*
"Steaming Tests of Coals and Related Investigations. September 1, 1904, to December 31, 1908." U. S. B. of M. Bull. *23*. Includes 17 analyses and tests of Ill. washed coals.

(56) *Garcia, J. A.*
"Fireproof Coal Washery, Panama, Ill." Coal Age *1*, 1912, 964-966, 7 figs.

(57) *Parker, E. W.*
"Coal Washing Operations," in Min. Res. 1911, II, 63-64. Gives quantity of coal washed at Illinois mines.

1913

(58) *Bement, A.*
"The Illinois Coal Field." Coal Age *3*, 558-562. Gives sizes of Illinois washed coals.

APPENDIX B.

Chronology of Coal Washing in Illinois.

1870

Osterspey jig washery, 10 tons per hour, under construction by Illinois Patent Coke Co., with Mr. L. Schantl in charge of erection, at East St. Louis to wash coal from the Belleville district for coking in 24 Belgian ovens. This was the first washery in Illinois and the second bituminous coal washery in America. (See 1871).

1871

Piston jig washery erected by St. Louis capitalists at East St. Louis under supervision of Mr. L. Schantl to wash coal for coking in 30 ovens. The plant was a failure and no coke was produced.

The Osterspey jig washery begun at East St. Louis in 1870 was completed and put in operation. (See 1873.)

1872

Piston jig washery erected by Mr. J. J. Endres at Joliet to wash coal for coke.

Piston jig washery erected for Gallatin Coal and Coke Co. at Equality by Mr. J. J. Endres to wash coal for coke. (See 1897.)

1873

The Osterspey jig washery completed at East St. Louis in 1870 was remodeled by Messrs. E. D. and J. W. Meier.

1880

Piston jig washery, 20 tons per hour, erected by Illinois Central Coal and Salt Co. at St. Johns under the supervision of Mr. R. Thomas to wash coal for coking in 18 ovens. (See 1888.)

1882

Washery erected at Brussels to wash coal to supply 50 beehive ovens. Shut down soon because mine was closed.

Prior to 1883

Washery erected at Brookside by Brookside Coal & Coke Co. to wash coal for coking in 6 ovens.

Washery erected at Harrison, Jackson Co., to wash coal from the Big Muddy bed at Carterville, Williamson Co. (See 1886).

1883

Piston jig washery, 35 tons per hour, erected by Luther and Tyler Coal and Coke Co. under supervision of Mr. J. J. Endres on Otter Creek, north of Streator, to wash coal for fuel and for coke making. This was the first bituminous fuel washer. (See 1888.)

1886

The property of the Brookside Coal and Coke Co. was taken over by Consolidated Coal Co. of St. Louis and the washery was not operated again.

1888

The piston jig washery built north of Streator for the Luther and Tyler Coal and Coke Co. in 1883 was moved to the Vermilion River, south of Streator, to obtain better water supply, and its capacity increased to 40 tons per hour. (See 1890.)

The piston jig washery built at St. Johns in 1880 by the Illinois Central Coal and Salt Co. was destroyed by fire on May 30. (For new washery, see 1895.)

1890

Piston jig washery, 27 tons per hour, built for Wilmington Washed Coal Co., successors to Luther & Tyler Coal & Coke Co., on the bank of the Kankakee River at Wilmington to wash coal from the Braidwood district for fuel. (See 1891.)

The piston jig washery built for the Luther and Tyler Coal and Coke Co. south of Streator in 1888 was shut down because it ceased to pay. (For new washery, see above.)

1891

Howe washery, 30 tons per hour, installed by Mr. L. D. Howe, replacing the piston jigs in the washery erected by the Wilmington Washed Coal Co. at Wilmington in 1890. (See 1896.)

1894

Luhrig washery, 60 tons per hour, built for U. S. Coal Washing Co. at the Brush Shaft, Carterville, under direction of Mr. A. Cuninghame with machinery furnished by Link-Belt Co. This was the first washer to wash coal for fuel in the Southern coal field, and the second Luhrig washery in America. (See 1903.)

Forrester washery erected by Illinois Coal Washing Co. at Mine No. 1 of Spring Valley Coal Co., Spring Valley. (See 1903.)

Forrester washery, 50 tons per hour, erected by Himrod Coal Co. at its Pawnee Mine, near Westville. (See 1905.)

1895

Forrester washery, 75 tons per hour, erected by Illinois Coal Washing Co. at Mine No. 1 of the Chicago, Wilmington & Vermillion Coal Co., at Heenanville near Streator, to wash coal for fuel. (See 1905.)

Campbell bumping table washery, 20 tons per hour, erected by Illinois Central Coal & Salt Co. at St. Johns to wash coal for use in making coke. (See 1899.)

1896

Campbell bumping table washery, 40 tons per hour, erected by Big Muddy Coal & Iron Co. at its No. 7 Mine, Herrin, under supervision of Mr. Hiram Willson to wash coal for fuel.

Luhrig washery, 30 tons per hour, erected for St. Louis and Big Muddy Coal Co., at DeSoto under contract with Cuninghame & Co. with machinery furnished by Link-Belt Co. to wash coal for fuel. Acquired later by Inland Steel Co. (See 1908.)

Howe washery, 75 tons per hour built by the Wilmington Washed Coal Co. at Central City in May, the materials and machinery being in part derived from the Howe washery built at Wilmington in 1891.

1897

The piston jig washery erected at Equality in 1872 for the Gallatin Coal and Coke Co. was torn down because worn out. (For new washery see 1901.)

1898

Stewart washery, 75 tons per hour, erected for Big Muddy Coal & Iron Co. at its Harrison Mine, near Murphysboro by E. A. Stewart. This was the first Stewart washery.

Luhrig washery, 60 tons per hour, built for New Ohio Washed Coal Co., Carterville, by Link-Belt Co. (See 1906.)

Luhrig washery, 50 tons per hour, built for Donk Bros. Coal & Coke Co. at its Mine No. 1, Donkville, near Collinsville, by Link-Belt Co. (See 1900.)

Scaife trough washery, 10 tons per hour, erected by Maltby Coal Co. under supervision of J. E. Richmond at Braidwood. (See 1902.)

1899

Stewart washery erected by Chicago & Carbondale Coal Co. at Ward. Later acquired by Mississippi Valley Fuel Co. Abandoned about 1908.

The Campbell bumping table washery, erected in 1895 at St. Johns by the Illinois Central Coal & Salt Co. was shut down because the mine was abandoned.

1900

Howe washery, 50 tons per hour, erected by Acme Coal Co. at Streator in the fall with machinery furnished by Jeffrey Mfg. Co. under supervision of J. R. Holliday. (See 1904.)

Scaife trough washery, 10 tons per hour erected by Gardiner-Wilmington Coal Co., Clarke City.

The Luhrig washery, erected in 1898 for Donk Bros. Coal & Coke Co. at Donkville, near Collinsville, was destroyed by fire. (For new washer, see 1902.)

1901

Stewart washery, 160 tons per hour, built for Chicago & Carterville Coal Co. at its Mine at Herrin by E. A. Stewart.

Piston jig washery, 125 tons per hour, designed by Mr. W. W. Keifer and built by Himrod Coal Co. at its Himrod Mine, Westville. (See 1905.)

Stewart washery, 125 tons per hour, erected by Mr. A. T. Stewart for Southern Illinois Coal Mining & Washing Co. at Marion in April. (See 1911.)

Forrester washery, 100 tons per hour, erected at mine of Chicago, Wilmington & Vermillion Coal Co. at Thayer by and for Illinois Coal Washing Co. in the fall. (See 1911.)

Campbell bumping table washery, 20 tons per hour, erected by and for Gallatin Coal and Coke Co. at Equality. (See 1910.)

1902

Stewart washery, 125 tons per hour, built for Bessemer Washed Coal Co. at East St. Louis by Link-Belt Co. (See 1907.)

Stewart washery, 125 tons per hour, built for Spring Valley Coal Co. at its No. 5 Mine, Dalzell, near Spring Valley by Link-Belt Co.

Stewart washery, 80 tons per hour erected by Mr. E. A. Stewart for Donk Bros. Coal & Coke Co. at its No. 1 Mine, Donkville, near Collinsville.

Stewart washery, 50 tons per hour, built for Illinois Third Vein Coal Co. at its mine at Ladd by Link-Belt Co.

Robinson washery, 50 tons per hour, erected by Wilmington Star Mining Co., with machinery furnished by Link-Belt Co. at No. 5 Mine, Coal City. (See 1911).

The Scaife washery erected at Braidwood in 1898 by the Maltby Coal Co. was burned. (See 1904.)

The Howe washery moved to Central City in 1896 by the Wilmington Coal Washing Co. was sold to T. N. Koehler & Co. (See 1904.)

1903

The Luhrig washery, erected in 1894 at Carterville for U. S. Coal Washing Co. was sold to St. Louis & Big Muddy Coal Co. (See 1911.)

The Forrester washery, erected at Spring Valley No. 1 Mine of the Spring Valley Coal Co. in 1894 was dismantled, because coal from this mine was washed at the Stewart washer erected at Dalzell in 1902.

Prior to 1909

Howe washery, 50 tons per hour, erected by Streator Fuel Co. at Streator. (See 1909.)

Howe washery, 40 tons per hour, erected by Harrison Coal Co. at their mine at Streator under supervision of J. R. Holliday.

1904

Luhrig washery, 125 tons per hour, built for Sunnyside Coal Co. at its Sunnyside Mine, near Herrin, under the supervision of T. G. Warden by Sunnyside Coal Co.

Luhrig washery, 120 tons per hour, built for Consolidated Coal Co. of St. Louis, at its Mine No. 14 near Staunton, by Roberts & Schaefer Co.

American Washery, 100 tons per hour, built for Devlin Coal Co. at Porterfield, near Toluca, by American Coal Washer Co. (See 1909.)

Stewart washery, 60 tons per hour, built for Muddy Valley Mining & Mfg. Co. at Hallidayboro by Roberts & Schaefer Co. (See 1910.)

Luhrig washery, 60 tons per hour, built for Western Coal & Mining Co. at Bush, by Roberts & Schaefer Co.

The Howe washery, 50 tons per hour erected by Acme Coal Co. at Streator in 1900 was burned in the summer and replaced in October.

The machinery from the Scaife washery at Braidwood, burned in 1902, was moved to Central City and erected at the washery of T. N. Koehler & Co. to rewash refuse, but only operated a few days because the refuse contained little coal. (See 1905.)

1905

Luhrig washery, 200 tons per hour, built for Wilson Coal Washery Co. at mine of Carterville Colliery Co., East Carterville, by Roberts & Schaefer Coal Co. (See 1909.)

American washery, 140 tons per hour, built for Lumaghi Coal Co. at Cantine, near Collinsville, by American Coal Washer Co. (See 1911.)

Luhrig washery, 125 tons per hour, built for Consolidated Coal Co. of St. Louis at its Mine No. 15, Mt. Olive, by Link-Belt Co. (See 1907.)

The Forrester washery, erected at Heenanville in 1895 was dismantled and the machinery moved to Mine No. 2 of the Chicago, Wilmington and Vermillion Coal Co. where a new washer was built.

The Forrester washery, built at the Pawnee Mine, Westville, in 1894 was transferred with the mine to the Kelly Coal Co. (See 1907.)

The piston jig washer erected at the Himrod Mine, Westville, in 1901 was transferred with the mine to the Kelly Coal Co. (See 1906.)

The Howe washery at Central City acquired by T. N. Koehler & Co. in 1902 was changed to a Robinson washery with a capacity of 75 tons per hour by the Wilmington Foundry & Machine Co., because the capacity of the Howe washer was too small. Shut down recently because of limited supply of raw screenings.

1906

American washery, 140 tons per hour, built for Madison Coal Corportation at Glen Carbon by American Coal Washer Co.

Luhrig washery, 125 tons per hour, built for Big Muddy Coal & Iron Co. at its No. 8 Mine, Clifford, by Link-Belt Co.

Robinson washery, 100 tons per hour, built for B. F. Berry Coal Co. at Mark, near Granville, by Wilmington Foundry & Machine Co. (See 1908.)

Stewart washery, 90 tons per hour, erected at mine of Royal Collieries Co., near Virden under contract for erection and operation with Carbon Washery Co., who employed Roberts & Schaefer to build the plant. (See 1910.)

Stewart washery, 60 tons per hour, erected at mine of Cardiff Coal Co., Cardiff, under contract for erection and operation with Producers Coal

Co. The washery was built by Roberts & Schaefer Co., being completed in January. (See 1907.)

Stewart washery, 40 tons per hour, erected by Jupiter Collieries Co. at Mine No. 1, DuQuoin. Upon abandonment of Mine No. 1, it was moved to Mine No. 5 and partially erected there.

The Luhrig washery, erected in 1898 at Carterville for New Ohio Washed Coal Co. was shut down because the mine was abandoned.

The washery erected in 1901 at Westville was remodeled into a New Century washery, 60 tons per hour, in August by Link-Belt Co. (See 1907.)

1907

American washery, 280 tons per hour, built for Superior Coal Co. at Gillespie by American Coal Washer Co. Burned July, 1907. (For new washery see 1908.)

Robinson washery, 150 tons per hour, built for Southern Coal & Mining Co. at Lake Station, East St. Louis, by Wilmington Foundry & Machine Co.

American washery, 140 tons per hour, built for Consolidated Coal Co. of St. Louis at its Mine No. 15, Mt. Olive, by American Coal Washer Co. (See 1912.)

Shannon washery, 125 tons per hour, built for Hafer Washed Coal Co. at its mine near Carterville by Link-Belt Co.

Stewart washery, 120 tons per hour, built for Bituminous Coal Washing Co. at mine of Central Washed Coal Co., Pana, by Roberts & Schaefer Co. and turned over to mine owner in August.

Luhrig washery, 100 tons per hour, built for Consolidated Coal Co. of St. Louis at its Mine No. 17, in St. Clair County, near Collinsville, by Link-Belt Co.

American washery, 70 tons per hour, built for Shoal Creek Coal Co., at its Mine No. 1, Panama, by American Coal Washer Co. (See 1911.)

Stewart washery, 60 tons per hour, built for the Standard Washed Coal Co. at Bissell, under contract for erection and operation with Producers Coal Co. Roberts & Schaefer Co. built the plant and washing began in February. (See 1910 and 1911.)

Stewart washery, 50 tons per hour, erected at the mine of the De-Camp Coal Co., De-Camp, near Staunton, under contract for erection and operation with Midland Washery Co. Roberts & Schaefer Co. built the plant and it began operating in August. (See 1910 and 1911.)

Stewart washery, erected by Robert Dick Coal Co. at Cambria. (See 1912.)

The Luhrig washery erected in 1905 at Mine No. 15 of the Consolidated Coal Co. of St. Louis at Mt. Olive was burned. For new washer see above.

A Shannon jig was installed at the Stewart washery of the Bessemer Washed Coal Co., erected in 1902 as an experiment. (See 1912.)

The Stewart washery erected in 1906 at Cardiff for the Producers Coal Co. was purchased by the Cardiff Coal Co. in January. (See 1910.)

The Forrester washery erected at the Pawnee Mine, Westville, acquired by the Kelly Coal Co. in 1905, was transferred to the Dering Coal Co., and shut down December 7th, because it was worn out and never had paid.

The New Century washery remodeled in 1906 was again remodeled by Link-Belt Co. practically into a Luhrig washery. (See 1908.)

1908

American washery, 280 tons per hour, built for Superior Coal Co. at Gillespie by American Coal Washer Co. to replace American washery burned in 1907.

Stewart washery, 50 tons per hour, built for Carterville & Big Muddy Coal Co. at its Cambria mine, under contract with Midland Washery Co. It was built by Roberts & Schaefer Co. and turned over to the mine owner in August. (See 1912.)

The Robinson washery, erected in 1906 for B. F. Berry Coal Co. at Mark, near Granville, by Wilmington Foundry & Machine Co. was burned. It was rebuilt by the same company with capacity reduced to 50 tons per hour.

The Luhrig washery erected in 1896 for the Big Muddy Coal Washing Co., DeSoto, was shut down because the mine was abandoned.

The Luhrig washery remodeled from the New Century washery in 1907 for the Dering Coal Co., Westville, was shut down in March because the mine was abandoned.

1909

Shannon and Foust washery, 125 tons per hour, built for Illinois Hocking Washed Coal Co. at White Ash, near Marion, by Link-Belt Co.

Stewart washery, 100 tons per hour, built for Western Washed Coal Co. at mine of Smith-Lohr Coal Mining Co., Pana, by Roberts & Schaefer Co. and put in operation in June. (See 1910.)

Pittsburg washery, 90 tons per hour, built for the Standard Collieries Co. at White Ash by the Pittsburg Coal Washer Co. This washery was of steel and concrete construction and the first fireproof washery in Illinois.

The Howe washery erected prior to 1909 by the Streator Fuel Co. was shut down because the mine did not pay.

The American washery erected at Porterfield, near Toluca, in 1904 for Devlin Coal Co. passed into the hands of Toluca Coal Co.

The Luhrig washery built for the Wilson Coal Washing Co. at Carterville in 1905 passed into the hands of the Taylor Coal Co.

1910

American washery, 80 tons per hour, built by and for Chicago Coal Washer Co. at Tower Hill with jigs supplied by American Coal Washer Co. and machinery by Stephens-Adamson Mfg. Co. This was the first washery in Illinois to use electric power.

New Century washery, experimental, 30 tons per hour, built for Illinois Steel Co. at their coke plant, Joliet, by American Concentrator Co., and put in operation in April to wash coal for coke. (See 1912.)

New Century washery, experimental, 10 tons per hour, installed for L. L. Summers & Co. at Dalton by American Concentrator Co., October 14th, to wash coal for coke.

Bituminous Coal Washing Co., organized on May 14th by consolidation of Producers Coal Co., Carbon Washery Co., Midland Washery Co. and Western Washed Coal Co. (See 1911 and 1912.)

The Stewart washery, erected at Hallidayboro in 1904 for the Muddy Valley Mining & Mfg. Co. passed into the hands of the Muddy Valley Co.

The Stewart washery acquired by the Cardiff Coal Co. at Cardiff in 1907, was shut down because the mine was closed.

The Campbell bumping table washery erected at Equality in 1901 by the Gallatin Coal and Coke Co. was shut down on account of the strike and remained closed because of financial stringency.

1911

Stewart washery, 150 tons per hour, built by and for the Bituminous Coal Washing Co. at the mine of the Chicago, Wilmington & Vermillion Coal Co., Thayer, started operating December 18th.

Shannon washery, 125 tons per hour, erected by Link-Belt Co. at Mine No. 1, Shoal Creek Coal Co., Panama. The Allen & Garcia Co. were engineers for the operators. Fireproof construction.

Stewart washery, 100 tons per hour, built for the Bituminous Coal Washing Co. at Mine No. 3 of the Chicago, Wilmington & Vermillion Coal Co., So. Wilmington, by Roberts & Schaefer Co. in June and sold to the C. W. & V. Co. August 1st.

The Luhrig washery, erected in 1894 at the Brush shaft, Carterville, and later acquired by the Madison Coal Corporation, was shut down in April.

The American washery, erected at Panama in 1907 for Shoal Creek Coal Co. burned down. (For new washer see above.)

The Stewart washery erected at Bissell in 1907 for the Standard Washed Coal Co. and acquired by the Bituminous Coal Washing Co. in 1910, was moved to the No. 7 Mine of the Wilmington Star Mining Co. at Coal City and remodeled.

The Robinson washery erected at Coal City in 1902 for the Wilmington Star Mining Co. at its No. 5 Mine was shut down because the mine was closed and a new washer was erected at No. 7 Mine. (See above.)

The Stewart washery erected at Marion in 1901 for the Peabody Coal Co. was destroyed by fire on June 19th.

The Forrester washery of the Illinois Coal Washing Co. erected at the Thayer Mine of the Chicago, Wilmington & Vermillion Coal Co. in 1901 was dismantled in December. (For new washer see above.)

Electric power installed in the American washery built at Cantine in 1905 by the Lumaghi Coal Co.

1912

Shannon and Foust washery, 150 tons per hour, under construction for Consolidated Coal Co. of St. Louis at its mine No. 7, Staunton, by Link-Belt Co.

Robinson washery, 120 tons per hour, under construction by Southern Coal Co. under supervision of J. E. Richmond on site of old Stewart washer of Bessemer Washed Coal Co. at East St. Louis. Electric power.

Stewart washery, 60 tons per hour, under construction for Geo. B. Pope & Co. at the Big Muddy Fuel Co. mine, Johnston City, by Roberts & Schaefer Co.

Pyrite cleaning plant, 20 tons per hour, built for Mission Field Coal Co. at Oakwood by American Concentrator Co. and put in operation on October 7th.

Experimental washery, 5 tons per hour, equipped with small-size Stewart jig, New Century jigs, Richards pulsator jig and Robinson tub under construction for the Mining Department of the University of Illinois at Urbana by E. M. Burr & Co.

A Richards-Janney classifier was installed by the Allis-Chalmers Co. in the experimental coal washing plant of the Illinois Steel Co. at Joliet, erected in 1910.

A 50-ton Foust rewashing plant to be installed at the Stewart washery erected at Mt. Olive in 1907 was contracted for by the Consolidated Coal Co. of St. Louis with the Link-Belt Co.

The Stewart washery erected at Cambria in 1908 for the Carterville and Big Muddy Coal Co. was shut down in March, because the mine was closed.

The washery of the Robert Dick Coal Co. erected at Cambria in 1907 was purchased, together with the mine, by the Madison Coal Corporation and shut down.

The Stewart washery erected at DeCamp near Staunton in 1907 and acquired by the Bituminous Coal Washing Co. in 1910 was dismantled in October and moved to Pana for storage on account of a disagreement with the mine owner.

The Stewart washery of the Bessemer Washed Coal Co. at East St. Louis to which a Shannon jig was added in 1907, was dismantled.

The Robinson washery erected at Lake Station, East St. Louis for the Southern Coal & Mining Co. in 1907 was destroyed by fire on October 16th. (For new washer see above.)

APPENDIX C.

Illinois Washeries Operating in the Fall of 1912.

No.	Field	County	Town	Company	Offices and Management	Date of Erection
1	Northern	Bureau	Ladd	Illinois Third Vein Coal Co.	Principal Office: Old Colony Bldg., Chicago, S. M. Dalzell, Pres. Local Office: Ladd. F. D. Chadwick, Supt	1902
2	Northern	Bureau	Dalzell	Spring Valley Coal Co	Principal Office: Old Colony Bldg., Chicago, S. M. Dalzell, Gen. Mgr. Local Office: Spring Valley, R. M. Hill, Supt.	1902
3	Northern	Putnam	Granville	B. F. Berry Coal Co.	Office: Granville, J. T. Cherry, Supt.	1908
4	Northern	Marshall	Porterfield	Toluca Coal Company	Main Office: Wichita, Kas., L. C. Jackson, Pres. Operating Office: Toluca, G. B. Gallon, Gen. Mgr.	1909
5	Northern	LaSalle	Streator	Chicago, Wilmington & Vermillion Coal Co.	General Office: McCormick Bldg., Chicago, T. A. Lemmon, Pres. Local Office: 416 N. Vermillion St., Streator, C. A. Herbert, Gen. Supt.	1905
6	Northern	LaSalle	Streator	Acme Coal Co.	Office: 424 Main St., Streator, T. Fairbairn, Pres; E. L. Atkinson, Supt.	1904
7	Northern	LaSalle	Streator	Harrison Coal Co.	Masonic Temple, Streator, R. Gardener, Gen. Mgr.; J R. Holliday, Supt.	1908
8	Northern	Grundy	Coal City	Wilmington Star Mining Co.	General Sales Office: McCormick Bldg., Chicago, Supt's Office: Coal City, W. Campbell, Supt.	1911
9	Northern	Grundy	S. Wilmington, Ill.	Chicago, Wilmington & Vermillion Coal Co.	General Office: McCormick Bldg., Chi- T. A. Lemmon, Pres. Local Office: S. Wilmington, J. Louis, Supt.	1911
10	Northern	Will	Joliet	Illinois Steel Company	Main Office: Joliet Works, D. R. Mathias, Gen Supt. Local Office: Coke Works, C. Wendell, Washer Supt.	1910
11	Northern	Cook	Dalton	Continuous Process Coal Co.	Office: First Nat'l Bank, Chicago, L. L. Summers, Pres.	1910
12	Central	Sangamon	Thayer	Bituminous CoalWashing Co.	Office: McCormick Bldg., Chicago, D. W. Buchanan, Pres. Supt's. Office: Pana, G. N. St. Clair, Supt.	1911

13	Central	Christian	Pana	Bituminous Coal Washing Co.	Office: McCormick Bldg., Chicago, D. W. Buchanan, Pres. Supt's. Office: Pana, G. N. St. Clair, Supt.	1909
14	Central	Christian	Pana	Central Washed Coal Co.	Office: Fisher Bldg., Chicago, W. E. Zoller, Pres.; R. H. Zoller, Gen. Mgr.	1907
15	Central	Shelby	Tower Hill	Chicago Coal Washing Co.	Office: Fisher Bldg., Chicago, W. J. Shedd, Pres.; F. Gascoigne, Gen. Mgr. Local Office, Tower Hill, F. L. Anderson, Supt.	1910
16	Central	Macoupin	Virden	Royal Colliery Co.	Home Office: Land Title Bldg., Phila., Pa., D. B. Wentz, Pres. Sales Office: Monadnock Bldg., Chicago, A. J. Maloney, V. P. Local Office: At Washer, J. B. Falcetti, Supt.	1906
17	Central	Macoupin	Gillespie	Superior Coal Co.	Main Office: J. P. Reese, Gen. Supt. Local Office: Gillespie, J. H. Ross, Supt.	1908
18	Central	Macoupin	Mt. Olive	Consolidated Coal Co. of St. Louis.	General Office: Syndicate Trust Bldg., St. Louis, Mo., W. L. Schmick, Vice Pres. and Gen. Mgr. Local Office: Staunton, Ill., F. E. Weissenborn, Supt.	1907
19	Central	Macoupin	Staunton	Consolidated Coal Co. of St. Louis.	General Office: Syndicate Trust Bldg., St. Louis, Mo., W. L. Schmick, Vice Pres. and Gen. Mgr. Local Office: Staunton, Ill., F. E. Weissenborn, Supt.	1904
20	Central	Madison	Glen Carbon	Madison Coal Corporation	General Offices: Central Nat'l Bank, St. Louis, Mo., A. J. Moorshead, Pres. and Gen. Mgr. Local Office: Glen Carbon, A. Daenzer, Dist. Supt.	1906
21	Central	Madison	Donkville	Donk Bros. Coal & Coke Co.	314 N. 4th St., St. Louis	1902
22	Central	Madison	Cantine	Lumaghi Coal Co.	Office: Equitable Bldg., St. Louis, Mo., L. F. Lumaghi, Pres.	1905
23	Central	Bond	Panama	Shoal Creek Coal Co.	General Office: Fisher Bldg., Chicago, F. P. Blair, Pres. Local Office: Panama, E. H. Ross, Gen. Mgr.; N. Shannon, Supt.	1911
24	Central	St. Clair	Caseyville Township, near Collinsville	Consolidated Coal Co. of St. Louis.	General Office: Syndicate Trust Bldg., St. Louis, Mo., W. L. Schmick, Vice Pres. and Gen. Mgr. Local Office: Collinsville, P. Grieve, Jr., Supt.; H. P. Altman, Washery Supt.	1907

25	Central......	St. Clair ...	Lake Station E.St. Louis	Southern Coal & Mining Co.	Office: Security Bldg., St. Louis, Mo., W. K. Kavanaugh, Pres.; J. E. Richmond, Supt...	1907
26	Southern.....	Jackson ...	Harrison ...	Big Muddy Coal & Iron Co. .	General Office: Wainwright Bldg., St. Louis, O. L. Garrison, Pres. Dist. Office: Herrin, H. Willson, Mgr. Local Office: E. Walnut St., Murphysboro, J. Forester, Supt.	1898
27	Southern	Jackson ...	Halliday-boro	Muddy Valley Mining & Mfg. Co.	Sales Agt: Carterville Washed Coal Co., Fisher Bldg., Chicago, J. C. Smith, Pres. Local Office: Hollidayboro, J. Forester, Supt.	1904
28	Southern...	Williamson	Bush	Western Coal & Mining Co.	Main Office: Syndicate Trust Bldg., St. Louis, W. J. Jenkins, Pres. Local Office: Bush, E. W Hogan, Supt.	1904
29	Southern.....	Williamson	Clifford	Big Muddy Coal & Iron Co...	General Office: Wainwright Bldg., St. Louis, O. L. Garrison, Pres. District Office: Herrin, H. Willson, Mgr.	1906
30	Southern.....	Williamson	Herrin	Big Muddy Coal & Iron Co...	General Office: Wainwright Bldg., St. Louis, O. L. Garrison, Pres. District Office: Herrin, H. Willson, Mgr.	1896
31	Southern	Williamson	Herrin	Chicago & Carterville Coal Co. .	Sales Office: Old Colony Bldg., Chicago, J. Pease, Pres. General Office: Herrin, J. D. Peters, Vice Pres. and Gen. Mgr..	1901
32	Southern.....	Williamson	Herrin	Sunnyside Coal Co. .	Main Office: Fisher Bldg., Chicago, T. G. Warden, Pres. Local Office: Herrin, J. Rollo, Supt.	1904
33	Southern	Williamson	Carterville .	HaferWashed Coal Co.	Main Office: McCormick Bldg., Chicago. Local Office: Carterville, J. McGonigal, Supt.....	1907
34	Southern	Williamson	Carterville .	Taylor Coal Co.	Main Office: Old Colony Bldg., Chicago, H. H. Taylor, Pres. Local Office: Carterville, P. H. Carroll, Supt.	1905
35	Southern....	Williamson	White Ash .	Illinois Hocking Washed Coal Co. .	Main Office: Fisher Bldg., Chicago, S. M. Goodall, Pres, Local Office: Marion, G. B. Calhoun, Supt.	1909

APPENDIX D.

ANALYSES OF ILLINOIS WASHED COALS AND RELATED PRODUCTS.

Analyses performed by the Chemical Department of the University of Illinois, under the direction of Prof. S. W. Parr.

DRY BASIS.

No.	Bed	Field	Coal	Vol. Matter	Fixed C	Ash	Sulphur	BTU	Unit Coal BTU
1	2	Northern	Raw Screenings	36.64	34.49	28.87	4.64	9870	14542
2	2	“	Washed Nut	43.10	48.15	8.75	3 59		
3	2	“	“ Slack	45 04	45 00	9.96	3.56		
4	2	“	Refuse	27.48	22.47	50.05	9.35		
5	2	“	Raw Screenings	35.51	34.53	29.96	5.63	9645	14506
6	2	“	Washed Screenings	45.02	46.62	8.36	3.88		
7	2	“	Refuse	21 37	9.50	69.13	22.01		
8	2	“	Suspended Matter	24 81	23.21	51.98	5.94		
9	2	“	Raw Screenings	36.79	38.74	24.47	4.95	10666	14704
10	2	“	No. 1 Washed Coal	43 68	48.58	7.74	3.24		
11	2	“	No. 2 “ “	45.18	47.45	7.37	3.74		
12	2	“	No. 3 “ “	43.00	49 09	7.91	3.30		
13	2	“	Refuse	24 83	16.38	58.79	8.06		
14	2	“	Raw Screenings	37 61	33.81	28 58	3.93	10103	14789
15	2	“	Washed Nut	45.34	44.92	9.74	3.72		
16	2	“	“ Slack	43.98	44.59	11.43	3.77		
17	2	“	Refuse	19.84	6.98	73.18	4.57		
18	2	“	Suspended Matter	31.77	29.46	38.77	3.79		
19	2	“	No. 1 Washed Coal	44.69	48.54	6 77	2.72		
20	2	“	No. 2 “ “	46.43	48.64	4.93	2 55		
21	2	“	No. 3 “ “	45.26	49.61	5.13	2.78		
22	2	“	No. 4 & No. 5 W. Coal	43.87	44.30	11.83	3.18		
23	2	“	Refuse	24.94	16.38	58 68	5.27		
24	2	“	Suspended Matter	31.77	39.31	28 92	2.72*	9653	14147
25	2	“	No. 4 & No. 5 W. Coal	32.35	60.55	7.10	3.05		
26	2	“	Refuse	25.55	10 86	63.59	6.77		
27	2	“	Raw Screenings	39.12	43.46	17.42	3.31	11586	14389
28	2	“	No. 1 Washed Coal	45.21	48.93	5.86	2.64		
29	2	“	No. 2 “ “	44.50	48.90	6.60	2.80		
30	2	“	No. 3 “ “	44 63	49.08	6.29	2.81		
31	2	“	No. 4 “ “	43.66	49.49	6.85	2.78		
32	2	“	No. 5 “ “	43 60	49.33	7.07	2.74		
33	2	“	Refuse	24.29	12.32	63.39	4.71		
34	6	Central	Pyrite Crystal	25.97	7.24	66.79			
35	6	“	No. 5 Washed Coal	38 98	42.71	18.31	4 77*		
36	6	“	No. 5 “ “	44.63	43.05	12.32	4.15*	12089	14070
37	6	“	No. 5 “ “	41.65	39.06	19.29	4.83*	11132	14235
38	6	“	Refuse	23.54	12.23	64.23	16.55*		
39	6	“	No. 5 Washed Coal	41.60	43 57	14.83	3 4[illegible]*	12004	14416
40	6	“	No. 5 “ “	40.22	40 37	19.41	4.33*		
41	6	“	Refuse	22.55	8.56	68.89	12.57*		
42	6	“	Suspended Matter	25.04	21.76	53.20	2 67*		
43	6	“	Tank Washings	38.46	38 36	23.18	3.53*		
44	6	“	No. 2 Washed Coal	43.71	46.66	9.63	4.26*	12649	14251
45	6	“	No. 5 “ “	43.39	42.66	13.95	4.19*		
46	6	“	Refuse	25.49	14.37	60.14	22.45*		
47	6	“	Suspended Matter	26.76	26.98	46.26	3 28*		
48	6	Southern	Washed Egg Coal	37.26	53.11	9.63	1.82*		
49	6	“	No. 1 Washed Coal	38.01	52.96	9.03	1.18*		
50	6	“	No. 2 “ “	35.25	56.04	8.71	1.66*	13171	14593
51	6	“	No. 3 “ “	37.22	54.68	8.10	1.73*		
52	6	“	No. 4 “ “	36.63	55.84	7.53	1.48*		
53	6	“	No. 5 “ “	33.79	54 44	11.77	2.26*		
54	6	“	Refuse	20.24	22.32	57.44	10.42*		
55	6	“	Hand Picked Refuse	17 08	13.76	69.16	3.54*		
56	6	“	Suspended Matter	26.53	59.49	13 98	7.19*		
57	6	“	Shale	7.19	.93	91.88			
58	6	“	Raw Screenings	36.77	50.80	12.43	1.87	12651	14679
59	6	“	No. 1 Washed Coal	35.67	55.21	9.12	1.40		
60	6	“	No. 2 “ “	36.38	54.62	9.00	1.55		
61	6	“	No. 3 “ “	35.34	55.38	9.28	1.52		
62	6	“	No. 4 “ “	34.89	54.35	10.76	1.57		
63	6	“	No. 5 “ “	33.[illegible]8	54.66	11.46	1.38		
64	6	“	Refuse	25.26	31.87	42.87	4.34		
65	6	“	Suspended Matter	29.73	51.25	19.02	1.53		

*Sulphur determinations marked with a star were not made in duplicate.

No.	Bed	Field	Coal	Vol. Matter	Fixed C.	Ash	Sul-phur	BTU	Unit Coal BTU
66	6	Southern	Raw Screenings	35.65	50.38	13.97	3.35	12191	14475
67	6	"	No. 1 Extra Washed Coal	37.29	54.84	7.87	2.98		
68	6	"	Washed Stove Coal	37.29	54.02	8.69	2.53		
69	6	"	" Chestnut Coal	37.21	53.58	9.21	2.54		
70	6	"	No. 2 Washed Coal	37.19	53.74	9.07	2.52		
71	6	"	No. 4 " "	37.44	54.90	7.66	2.40		
72	6	"	No 5 " "	36.31	52.75	10.94	2.47		
73	6	"	Refuse	25.09	27.61	47 30	13.67		
74	6	"	Fine Refuse	27 01	29 57	43.42	6.33		
75	6	"	Raw Screenings	33.68	53.06	13.26	2.60	12514	14699
76	6	"	No. 1 Washed Coal	35.91	55.13	8.96	2.09		
77	6	"	No. 2 " "	36 27	55 21	8.52	2.00		
78	6	"	No. 3 " "	34.99	57 12	7.89	1.99		
79	6	"	No. 4 " "	36.29	56.29	7.42	1.82		
80	6	"	No. 5 " "	33.48	51.09	15.43	2.88		
81	6	"	Refuse	21.93	24.59	53.48	10.03		
82	6	"	Raw Screenings	35.88	50 65	13.47	2.61	12397	14598
83	6	"	No. 1 Washed Coal	37.56	53 35	9.09	2.60		
84	6	"	No. 2 " "	37.38	54.46	8.16	1.93		
85	6	"	No. 3 " "	38 05	53.17	8.78	2.64		
86	6	"	No. 4 " "	39.98	50 56	9 46	2.31		
87	6	"	No. 5 " "	36 55	51 85	11.60	2.72		
88	6	"	Coarse Refuse	23.49	20 49	56.02	10.25		
89	6	"	Fine Refuse	18.35	7.52	74.13	10.03		
90	6	"	Suspended Matter	28.12	55.81	16.07	1.93		
91	6	"	No. 1 Cleaned Coal	36.78	53.97	9 25	1.04		
92	6	"	No. 2 " "	35 84	55.73	8 43	.91		
93	6	"	No. 3 " "	35.52	51.29	13.19	1.62		
94	6	"	No. 4 " "	35.30	51.33	13.37	1.11		
95	6	"	No. 5 Raw Coal	34.49	51.96	13.55	1.39		
96	6	"	"Bone" from Cleaner	32.20	47.45	20.35	1.92		
97	6	"	"Slate" " "	22.03	31 16	46.81	3.13		
98	6	"	Rescreened No. 1 Coal	35.21	52.96	11.83	2 87	12688	14646
99	6	"	" No. 2 "	35 52	53 09	11.39	2 64		
100	6	"	" No. 3 "	35.47	53.25	11.28	2.53		
101	6	"	" No. 4 & 5 Coal	35.55	54.35	10.10	2.46		
102	6	"	Hand Picked Refuse	20.76	22 45	56 79	4.84		

APPENDIX E.

Screens in Illinois Washeries.

I. FIXED SCREENS

(a) SIZING RAW COAL

Washery No.	Screen No.	Type	Length (Feet)	Width (Feet)	Slope (Degrees)	Travel of Drag Feet per Minute	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
30	1a	Drag	24	2	Level	180	3/8	Round	8	3 3/4–0	40	....	0.2
	b						1 3/4	Slot	16	3 3/4–3/8	...	....	...
(b) RESIZING WASHED COAL SIZED PRIOR TO WASHING													
20	2	Drag	8	3	10	...	1/16	Round	8	3/4–0	25	(Rinsings only)	1.0
	3	Gravity	4	1	36	...	3/4	"	4	1 3/4–1 1/8	19	(Supplementing Revolving Screen No. 25).....	0.2
	4	"	3	1	31	...	1 1/8	"	3	3–1 3/4	20	(Supplementing Revolving Screen No. 26).....	0.2
24	5	"	.....	.....	....	.....	3/8	"	.....	1 3/4–1	18	(Supplementing Revolving Screens Nos. 27 and 28)................	...
32	6	"	8	1 1/2	....	...	1/4	"	8	3/4–0	54	(Relieving Revolving Screens No. 48).......	0.2
	7	Drag	10	3	11	60	3/16	"	10	3/8 & 1/4–0	23	(Rinsings only)	1.3
34	8	"	16 & 22	4	Level	40	3/16	"	16 & 22	3/8–0	...	" "	...
35	9	Gravity	4	4	....	...	3/4	"	4	3–0	65	(Relieving Revolving Screens No. 57......	0.2
	10	Drag	15	3	10	60	1/4	"	15	3/4–0	43	23	1.0
(c) SIZING WASHED COAL													
3	11a	Drag	20	4	40	100	3/32	Round	12	7/8–0	...	(Rinsings only)	...
	b						1/16	"	8	7/8–3/32	...	" "	...
7	12	"	9	2 1/2	....	60	1/16	"	9	1 1/2 slot–0	...	" "	...
14	13	Gravity	..	..	....	...	1 1/2x2 3/8	Elliptical	..	3 1/2–1 3/4 sq.	...	....	...
	14	"	..	..	....	...	1 3/8	Round	..	1 3/4 sq.–1 3/8 r. & 1 sq.	...	....	...
	15	"	..	..	....	...	..	Elliptical	..	1 3/8 r. & 1 sq. –7/8	...	....	...
	16	"	..	..	....	...	1/4	Round	..	7/8–1/4	...	....	...
18	17	"	3	1	....	...	3/4	"	3	1 1/8–3/4	...	(Supplementing Shaker Screen No. 27c)......	...
21	18	Drag	12	2	4 1/2	...	3/16	"	12	3/8–0	...	....	...
22	19	Gravity	6	1 1/2	31	...	3/4	"	6	3–1 3/4	1	(Supplementing Revolving Screen No. 76a)....	9.0

I. (c) SIZING WASHED COAL (Continued)

Wash-ery No.	Screen No.	Type	Length (Feet)	Width (Feet)	Slope (Degrees)	Travel of Drag Feet per Minute	Openings		Length of Screen-ing Surface (Feet)	Feed		Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
							Diam. (Ins.)	Shape		Diameter (Inches)	Tons per Hour		
22	20	Gravity	..	..	....	...	¾	Round	..	1¾–¾	43	(Supplementing Revolving Screen No. 76a)....	...
25	21	Drag	16	3½	Inclined	60	¼	"	16	3–0	131	28	0.4
26	22	"	13	4	7½	240	¼	"	13	1¼–0	...	....	...
27	23	Gravity	..	..	....	...	⅛	"	..	....	...	(Rinsings only)	...
31	24a	Drag	28	3	16	120	$\frac{7}{16}$	"	7	3–0	76	25	1.1
	b						⅜	"	21	3–$\frac{7}{16}$			
	25a	"	28	3	16	120	$\frac{7}{16}$	"	7	3–0	76	25	1.1
	b						⅜	"	21	3–$\frac{7}{16}$			
	26	Double-decked Drag	12	3½	Level	90	$\frac{5}{16}$	"	12	$\frac{7}{16}$ & ⅜–0	38	(Rinsings only)	1.1
33	27	Gravity	1¼	3	22	...	$\frac{5}{16}$	"	1¼	3½–1¾	25	(Supplementing Revolving Screen No. 90a)....	0.2
	28	"	4	2	21	...	$\frac{6}{16}$	"	4	3½–1¾	25	(Supplementing Revolving Screen No. 90a)....	0.3
	29	"	3	2	38	...	¾	Slot	3	1¾–1	22	(Supplementing Revolving Screen No. 90b)...	0.3
	30	"	$1\frac{1}{6}$	2	31	...	½	Square	2	1–¾	23	(Supplementing Fixed Screen No. 29)........	0.1
	31	"	$1\frac{5}{6}$	3	32	...	½	"	3	¾–0	42	(Supplementing Revolving Screen No. 90c)....	0.1

II. SHAKING SCREENS

(a) SIZING RAW COAL

Washery No.	Screen No.	Type	Length (Feet)	Width (Feet)	Slope (Degrees)	Shakes: No. per Minute	Shakes: Length (Inches)	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
32	1	Rod suspension	7	6	10	132	5	3/4	Round	7	3½–0	62½	52	0.7
	2	" "	7	6	10	132	5	3/4	"	7	3½–0	62½	52	0.7

(b) RESIZING WASHED COAL SIZED PRIOR TO WASHING

Washery No.	Screen No.	Type	Length (Feet)	Width (Feet)	Slope (Degrees)	Shakes: No. per Minute	Shakes: Length (Inches)	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
2	3a	Rod suspension	13	8	9	180	2½	3/8	Round	13	3/4–0	71	....	1.5
	b	double-deck, baffle plates on upper deck			7			3/16	"	13	3/8–0	..	(Rinsings only)	...
29	4a	Rod suspension, double	8	4	8	140	4½	2¾	"	8	3–1¾	..	....	...
	b	deck......						1¾	"	8	2¾–1¾	..	(Breakage only)	...
	5	Rod suspension	8	4	8	140	4½	1¾	"	8	3–1¾	..	" "	...
	6	" "	8	4	8	140	4½	1	"	8	1¾–1	..	" "	...
	7	" "	8	4	8	140	4½	1	"	8	1¾–1	..	" "	...
	8	" "	8	4	8	140	4½	5/8	"	8	1–3/4	..	" "	...
	9	" "	8	4	8	140	4½	5/8	"	8	1–3/4	..	" "	...
	10a	" "	12	4	8	140	4½	3/8	"	4	3/4–0	..	....	...
	b							3/16	"	8	3/8–0	..	....	...
	11a	" "	12	4	8	140	4½	3/8	"	4	3/4–0	..	....	...
	b							3/16	"	8	3/8–0	..	....	...
30	12	" "	10½	4½	10	144	4½	1/4	"	10½	3¾–0	40	....	1.2
	13	"	7	3½	15	176	4½	1/4	"	7	3/4–1/4	..	(Breakage only)	...
	14	"	7	3½	15	176	4½	1/4	"	7	3/4–1/4	..	" "	...
	15	"	7	3½	15	176	4½	3/4	"	7	1–3/4	..	" "	...
	16	" "	7	3½	15	176	4½	1	"	7	1¾–1	..	" "	...
	17a	Rod suspension, double	7	3½	15	176	4½	1¾	"	7	3¾–1¾	..	" "	...
	b	deck.....						3	"	7	3¾–1¾	..	....	...
Average					10½	155	4½							1.35

II. (c) SIZING WASHED COAL

Washery No.	Screen No.	Type	Length (Feet)	Width (Feet)	Slope (Degrees)	Shakes: No. per Minute	Shakes: Length (Inches)	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
1	18	Rod suspension	9	3	8	160	2½	⅛	Round	9	¾–0	31	(Rinsings only)	0.9
3	19	Supported Parrish...	10	4	4	132	6	15/16	"	10	⅞ slot–0	34	(Just installed)	1.2
6	20	Roller support...	14	4	18	40	5	1/16	"	14	2¼ & 2½–0	38	(Rinsings only)	1.5
8	21a	Supported Parrish double deck	24	4	4	120	6	¾	"	12	2x3½ ell.–0	44	57	1.1
	b							⅜	"	12	¾–0	..	....	...
	c							1¾	"	12	2x3½ ell.–¾	25	81	1.9
	d							1¼	"	12	1¾–¾	20	66	2.4
9	22a	Rod suspension, tandem screens double decked	20	6	6	148	6	1½	"	16	3–5/16	55	87	1.7
	b							2¼	"	12	3–1½	7	66	10.3
	c		20		5			5/16	"	12	1½–5/16	48	(Supplementing Rev. Screen No. 63)	
	d							1/16	"	8	5/16–0	26	(Rinsings only)	1.5
	e							¾	"	12	1½–5/16	48	43	1.8
12	23a	Supported Parrish, tandem screens, fastened together	22	5	5	124	6	¾	"	22	3–5/16	..	....	1.5
	b							5/16	"	22	¾–5/16	..	(Supplementing Rev. Screen No. 64)	...
	c							1/16	"	22	5/16–0	..	(Rinsings only)	...
	d							1¼	"	10	3–¾	..	....	...
	e		20	4	7			1¾	"	10	3–1¼	..	....	...
13	24a	Hung Parrish, pair, one above other	19	5	5	126	6	2⅞	"	18½	3½–5/16	..	....	...
	b			6				2¼	"	18½	2⅞–5/16	..	....	...
	c			7				1¾	"	18½	2¼–5/16	..	....	...
	d		24	6				¾	"	16½	1¾–5/16	..	....	.....
	e			6				1¼	"	7½	1¾–¾	..	....	...
	f			7				5/16	"	18½	¾–5/16	..	(Supplementing Rev. Screen No. 65)	...
15	25a	" "	12	6	4	130	4	2½	"	12	3½–0	..	....	..
	b			6				1¼	"	12	2½–0	..	....	...
	c			7				⅞	"	12	1¼–0	..	....	...
	d			7				⅝	"	4	⅞–0	..	....	...
16	26a	" "	12	5	5	144	6	1⅞	"	12	3–5/16	..	....	...
	b			5	5			1¼	"	12	1⅞–5/16	..	....	...
	c			6	14 & 5			⅞	"	12	1¼–5/16	..	....	...
	d			6	2½			¼	"	9	⅞–5/16	..	....	...
	e			7	2½			1/16	"	8	5/16 & ¼–0	..	....	...
18	27a	Hung Parrish	13	6	4½	150	4	1¾	"	13	3–0	109	86	0.7
	b							1⅛	"	13	1¾–0	94	79	0.8

II. (c) SIZING WASHED COAL—(Continued)

Washery No.	Screen No.	Type	Length (Feet)	Width (Feet)	Slope (Degrees)	Shakes: No. per Minute	Shakes: Length (Inches)	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
	c	Hung Parrish						¾	Round	13	1⅛–0	74	82	1.1
21	28a	Rod suspension, pair, one above other	40	4¾	16	78	8	¼	"	6	2 & 1¼–0	..	....	...
	b				16			1	"	16	2 & 1¼–¼	..	....	...
	c				16			1½	"	16	2 & 1¼–1½	..	....	...
	d				16			⅜	"	16	1–¼	..	....	...
	e				16			¼	"	12	⅜–¼	..	(Supplementing a)	...
	f				14			⅜	"	16	1–⅜	..	(Supplementing d)	...
23	29a	Hung Parrish, pair, one above other	10½	6	4	128	6	1¾	"	10½	3–0	113	17	0.6
	b			6	4			1¼	"	10½	1¾–0	94	15	0.7
	c		12	7	5			¾	"	12	1¼–0	80	29	1.1
25	30a	Rod suspension, pair, one above other, each double decked	16	4	21	158	4	2	"	16	3–¼	94	4	0.7
	b							1⅛	"	16	2–¼	90	25	0.7
	c							11/16	"	16	1⅛–¼	67	41	1.0
	d							¼	"	16	11/16–¼	40	(Supplementing Fixed Screen No. 21)	1.6
	31	Rod suspension	6	3	...	156	..	1	"	6	2–1⅛	23	(Supplementing Screen No. 30b)	0.8
	32	" "	6	3	..	156	..	1½	"	6	3–2	4	(Supplementing Screen No. 30a)	4.5
27	33	" "	5	4	..	160	3	2	"	5	3–2	..	(Supplementing Rev. Screen No. 87a)	...
	34	" "	5	4	..	160	3	1	"	5	2–1	..	(Supplementing Rev. Screen No. 87b)	...
	35	" "	5	4	..	160	3	¾	"	5	1–¾	..	(Supplementing Rev. Screen No. 87c)	...
	36	" "	4	4	..	200	3	¼	"	4	¾–0	..	....	...
31	37a	Rod suspension, double decked	10	3	15	130	..	11/16	"	10	3–1¾	..	(Supplementing Rev. Screen No. 88a)	...
	b			3½	20			¼	"	6	1¾–1¼	..	(Supplementing Rev. Screen No. 88b)	....
Average						138	5							1.7

III. REVOLVING SCREENS

(a) SIZING RAW COAL

Washery No.	Screen No.	Type	Length (Feet)	Dia. (Feet)	Slope (Degrees)	R. P. M.	Peripheral Speed Feet per Minute	Openings		Length of Screening Surface (Feet)	Feed		Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
								Diam. (Ins.)	Shape		Diameter (Inches)	Tons per Hour		
2	1	Cylindrical	9	5	7	18	283	¾	Square	9	1⅛–0	125	89	1.1
19	2a	Triple-jack-	14	6	5	12	200	1¾	Round	14	3–0	120	90	2.2
	b	eted, cylin-		8			301	1	"	14	1¾–0	108	74	3.3
	c	drical		10			377	¾	"	14	1–0	80	65	5.5
20	3	Cylindrical	14	5	3	30	471	9/16	Square	14	1¼–0	140	......	1.5
24	4a	Triple-jack-	10½	3 & 5	Axis level	12	151	1¾	Round	10½	3–0	100	83	1.3
	b	eted, conical	10	5 & 7			200	1	"	10	1¾–0	83	89	2.3
	c		9½	7 & 9			301	¾	"	9½	1–0	74	77	3.2
28	5a	Triple-jack-	9	4½	3	16	226	2	"	9	3–0	60	90	2.1
	b	eted, cylin-		5¾			289	1	"	9	2–0	54	69	3.0
	c	drical		7			352	¾	"	9	1–0	37	86	5.3
29	6a		15	4	4	10	126	1¾	"	15	3–0	62½	......	11.3
	b	" "		5½			173	1	"	15	1¾–0	..	......	...
	c			7			220	¾	"	15	1–0	..	......	...
	7a		15	4	4	10	126	1¾	"	15	3–0	62½	......	11.3
	b	" "		5½			173	1	"	15	1¾–0	..	......	....
	c			7			220	¾	"	15	1–0	..	......	...
32	8a	Roller	16	5	6½	7	110	1¾	"	10	3½–¾	33	53	4.8
	b	cylindrical					110	2¾	"	6	3½–1¾	15	64	6.3
	9a	" "	16	5	6½	7	110	1¾	"	10	3½–¾	33	53	4.8
	b						110	2¾	"	6	3½–1¾	15	64	6.3
34	10a	Triple-jack-	16	5½ & 10	Axis level	8	195	1¾	"	16	3½–0	200	......	1.9
	b	eted, conical	15	7¾ & 12¼			251	1⅛	"	15	1¾–0	..	......	...
	c		14	10 & 14½			308	¾	"	14	1–0	..	......	...
	11a	Double-jack-	5½	2½	4½	18	141	3	"	5½	3½–1¾	..	......	...
	b	eted, cylindri-	5	3½			198	1¾	"	5	3–1¾	..	(Supplementing	...
		cal											Screen 10a)	...
35	12	Conical	7	5 & 6	Axis level	11	190	¾	"	7	3–0	125	36	1.0
Average						13	219							4.2

(b) RESIZING WASHED COAL SIZED PRIOR TO WASHING

Washery No.	Screen No.	Type	Length (Feet)	Dia. (Feet)	Slope (Degrees)	R. P. M.	Peripheral Speed Feet per Minute	Diam. (Ins.)	Shape	Length of Screening Surface (Feet)	Diameter (Inches)	Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
2	13	Cylindrical	8	4	3	20	251	¾	Square	8	1⅛–¾	9	(Breakage only)	11.1
19	14	"	7	3	3	15	141	1¾	Round	7	3–1¾	6	" "	11.0
	15	"	7	3	3	15	141	1¾	"	7	3–1¾	6	" "	11.0
	16	"	7	3	3	15	141	1	"	7	1¾–1	14	" "	4.7
	17	"	7	3	3	15	141	1	"	7	1¾–1	14	" "	4.7

III. (b) RESIZING WASHED COAL SIZED PRIOR TO WASHING—Continued

Washery No.	Screen No.	Type	Length (Feet)	Dia. (Feet)	Slope (Degrees)	R. P. M.	Peripheral Speed Feet per Minute	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
19	18	Cylindrical	7	3	3	15	141	½	Round	7	1–¾	28	(Breakage only)	2.4
	19	"	7	2½	3	15	118	¾	"	7	...	..	" "	...
	20	"	8	4	4	18	226	¼	"	8	¾ & ½–0	52	50	1.9
20	21a	"	19½	6	4½	12	226	¾	"	6	3–9/16 sq.	53	(Very little)	2.6
	b							1⅛	"	8	3–¾	53	29	2.8
	c							1¾	"	5½	3–1⅛	39	49	2.7
	22a	Roller cylindrical	16	5	4	16	251	¼	"	12	¾ & 9/16 sq.–0	58	56	4.3
	b							⅜	"	4	¾ & 9/16 sq.–¼			
	23	Cylindrical	8	3	2½	36	339	¾	"	8	1⅛–¾	16	(Supplementing Screen No. 21a)	4.7
	24	"	8	3	2½	36	339	1⅛	"	8	1¾–1⅛	20	(Supplementing Screen No. 21b)	3.8
	25	"	8	3	2½	36	339	1¾	"	8	3–1¾	19	(Supplementing Screen No. 21c)	4.0
	26	"	8	4	2½	36	452	¼	"	8	¾–¼ & ⅜	32	(Supplementing Screen No. 22a)	3.1
24	27	"	8½	3	5	24	226	1¾	"	8½	3–1¾	9	(Breakage only)	9.0
	28	"	8½	3	5	24	226	1¾	"	8½	3–1¾	9	" "	9.0
	29	"	8½	3	5	24	226	1¾	"	8½	1¾–1	9	" "	9.0
	30	"	8½	3	5	24	226	1¾	"	8½	1¾–1	9	" "	9.0
	31	"	8½	3	5	24	226	¾	"	8½	1–¾	15	" "	5.3
	32	"	6½	3	5	24	226	1	"	6½	1¾–1	18	(Supplementing Nos. 27 & 28)	3.5
	33	"	10½	4	3½	30	377	¼	"	10½	¾–0	50	36	2.6
28	34a	Double-jacketed, cylindrical	5½	2	2½	15	94	1	"	5½	3–2	6	(Breakage only)	5.8
	b			3			141	¾	"	5½			" "	
	35	Cylindrical	7	2	5	15	94	¾	"	7	2–1	5	" "	8.8
	36a	Double-jacketed, cylindrical	5½	2	3	15	94	1	"	5½	2–1	12	" "	2.9
	b			3			141	¾	"	5½			" "	
	37	Cylindrical	7	2	5	15	94	¾	"	7	1–¾	5	" "	8.8
	38	"	8	4	5	24	301	¼	"	8	¾–0	32	36	3.1
30	39a	Double-jacketed, cylindrical	..	..	....	15	...	¾	"	8	3¾–¼	..	......	...
	b							1	"	..	3¾–¾	..	......	...
	c							1¾	"	..	3¾–1	..	......	...
32	40a	Double-jacketed, cylindrical	7	3	6	14	132	2½	"	7	3½–2¾		(Breakage only)	
	b			2			88	¾	"	7		5	" "	8.8
	41a	" "	7	3	6	14	132	2½	"	7	3½–2¾		" "	8.8
	b			2			88	¾	"	7		5	" "	8.8
	42a	" "	7	3	7	14	132	1¾	"	7	2¾–1¾		" "	

III. (b) RESIZING WASHED COAL SIZED PRIOR TO WASHING (Continued)

Washery No.	Screen No.	Type	Length (Feet)	Dia. (Feet)	Slope (Degrees)	R. P. M.	Peripheral Speed Feet per Minute	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
32	b	Double-jacketed, cylindrical		2			88	¾	Round	7		6	(Breakage only)	7.4
	43a		7	3	7	14	132	1¾	“	7	2¾–1¾		“ “	
	b			2			88	¾	“	7		6	“ “	7.4
	44a	“ “	7	3	7	14	132	1¾	“	7	2¾–1¾		“ “	
	b			2			88	¾	“	7		6	“ “	7.4
	45a	“ “	7	3	7½	14	132	1⅜	“	7	1¾–¾		“ “	
	b			2			88	⅞	“	7		9	“ “	4.9
	46a	“ “	7	3	7½	14	132	1⅜	“	7	1¾–¾		“ “	
	b			2			88	⅞	“	7		9	“ “	4.9
	47a	“ “	7	3	7½	14	132	1⅜	“	7	1¾–¾	9	“ “	
	b			2			88	⅞	“	7		9	“ “	4.9
	48	Conical	10	4 & 5	Axis level	15	212	¼	“	10	¾–0 }	54	42	3.8
	49	“	6	3 & 4	“ “	24	264	⅜	“	6	¾–0 }			
34	50a	Triple-jacketed, cylindrical	7	2½	3½	24	188	2¼	“	7	3½–3	..	(Breakage only)	...
	b		6	3⅙			264	1½	“	6				
	c		5	4½			339	¾	“	5				
	51a	“ “	7	3½	3½	24	264	2¼	“	7	3–1¾	..	“ “	...
	b		6	4½			339	1½	“	6				
	c		5	5½			414	¾	“	5				
	52a	Double-jacketed, cylindrical	7	2¼	3½	24	170	1 $\frac{3}{16}$	“	7	1¾–1⅛	..	“ “	...
	b		6	3⅙			239	¾	“	6				
	53a		7	2¼	3½	24	170	1 $\frac{3}{16}$	“	7	1¾–1⅛	..	“ “	...
	b	“	6	3⅙			239	¾	“	6				
	54	Cylindrical	7	3⅙	3½	24	239	¾	“	7	1⅛–¾	..	“ “	...
	55	Cylindrical	7	3⅙	3½	24	239	¾	“	7	1⅛–¾	..	“ “	...
	56		8	5	1	12	188	$\frac{5}{16}$	“	8	¾–0	..	......	...
35	57a	Double-jacketed, conical	20	5 & 8	Axis level	14	252	1	“	9	3–0	119	27	1.4
	b						319	{ 1¼	“	5½	3–1 }	87	25	2.9
								{ 2	“	5½	3–1¼ }	87	25	2.9
	c			7 & 8½		340				9	1–0	65	34	3.4
Average						20	202	......	......	......	......	......	......	5.66

(c) SIZING WASHED COAL

Washery No.	Screen No.	Type	Length (Feet)	Dia. (Feet)	Slope (Degrees)	R. P. M.	Peripheral Speed Feet per Minute	Openings: Diam. (Ins.)	Openings: Shape	Length of Screening Surface (Feet)	Feed: Diameter (Inches)	Feed: Tons per Hour	Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
1	58	Cylindrical	10	4½	5	14	196	¾	Square	10	1⅛–0	34	90	4.2
4	59a	“	23½	4	5	22	182	$\frac{3}{32}$	Round	10	1½–0	67	(Rinsings only)	1.9
	b	“						⅞	Square	8	1½–$\frac{3}{32}$	67	76	1.5
	c	“						1⅜	“	5½	1½–⅞	16	90	4.3
5	60	Conical	10	2½ & 3	Axis 2½	48	396	2	Round	5	2½–0	19	21	2.3
6	61	Cylindrical	12	3	3½	20	188	1¼	“	12	2½ & 2¼–$\frac{1}{16}$	38	80	3.0
7	62	“	12	3	3½	20	188	1¼	“	12	1½ slot–$\frac{1}{16}$	30	......	3.8
9	63	“	10	5	4	21	330	$\frac{5}{16}$	“	10	3–0	81	32	1.9
12	64	“	14	5	2½	15	236	$\frac{5}{16}$	“	14	3–0	..	....	...

III. (c) SIZING WASHED COAL (Continued)

Washery No.	Screen No.	Type	Length (Feet)	Dia. (Feet)	Slope (Degrees)	R. P. M.	Peripheral Speed Feet per Minute	Openings		Length of Screening Surface (Feet)	Feed		Per Cent Through Screen	Sq. Ft. Screen Per Ton per Hour
								Diam. (Ins.)	Shape		Diameter (Inches)	Tons per Hour		
13	65	Cylindrical	9⅓	4⅔	5½	40	588	5/16	Round	9⅓	3½–0	..	......	...
14	66	"	10	5	4	14	220	¼	"	10	3½–0	108	......	1.5
	67a	Double-jacketed, conical	21	5 & 8	Axis level	15	267	1⅜	"	9	3½–¼	..	......	...
	b						315	1	Square	6	3½–1⅜	..	......	...
	c						356	1¾	"	6	3½–1 sq.	..	......	...
	d		9	7 & 8⅓			362	⅞	Round	9	1⅜–¼	..	......	...
15	68	Cylindrical	9⅓	4	5	20	251	⅜	"	9⅓	⅝–0	..	......	...
16	69	Conical	6½	4 & 5	Axis level	25	353	5/16	"	6½	3–0	..	......	...
17	70	Roller	16	6	5	12	226	⅜	"	16	1¼–0	125	39	2.4
	71	cylindrical	16	6	5	12	226	⅜	"	16	1¼–0	125	39	2.4
18	72	Cylindrical	12	4	5	20	251	¼	"	12	¾–0	61	48	
	73	"	7	3	7	11½	108	1½	"	7	3–1¾	109	(Supplementing Shaker Screen No. 27)	0.6
	74	"	7	3	7	11½	108	1	"	7	1¾–1⅛	94	" "	0.7
	75	"	7	3	7	11½	108	⅜	"	7	1⅛–¾	74	" "	0.9
22	76a	"	18	5	4	18	283	¾	"	12	3–0	113	61	1.7
	b							1¾	"	6	3–¾	44	3	2.2
	77	"	12	5	3	18	283	⅜	"	12	¾–0	69	62	2.7
	78	"	12	4	2	21	265	1/16	"	12	⅜–0	43	(Rinsings only)	3.5
	79	"	8	4	5	15	180	1¾	"	8	3–1¾	1	(Supplementing Screen No. 76b)	100.
	80	"	7	3	7	15	141	1⅛	"	7	1¾–¾	22	50	3.0
	81	"	7	4	7	15	188	1⅛	"	7	1¾–¾	22	50	3.0
	82	"	8	4	4	18	226	¾	"	4	..	..	(Supplementing Screen No. 76c)	...
								1	"	4				
23	83	"	12	5	5	16	251	⅜	"	12	¾–0	28	36	6.7
	84	"	12	5	5	16	251	⅜	"	12	¾–0	28	36	6.7
26	85a	"	16	4	5	18	226	2	"	8	2½–0	..	......	...
	b							1¼	"	8	2–0	..	......	...
27	86	"	8	4	4½	20	251	¾	"	8	3–0	..	......	...
	87a	"	12	4	5	18	226	2	"	6	3–¾	..	......	...
	b							1¾	"	3	2–¾	..	......	...
	c							1	"	3	1¾–¾	..	......	...
31	88a	Triple-jacketed, conical	15	4 & 7½	Axis level	12	216	1¾	Round	15	3–7/16 & ⅜	114	69	2.4
	b		16	6½ & 10			311	1¼	"	16	1¾–7/16 & ⅜	78	51	5.3
	c		17	8 & 12			377	¾	"	17	1¼–7/16 & ⅜	50	67	10.7
	89	Cylindrical	11	3	8	20	188	¾	"	11	1¼–¾	16	(Supplementing Screen No. 88c)	6.5
33	90a	Triple-jacketed, conical	14	4 & 6	Axis level	8½	133	1¾	"	14	3½–0	106	24	2.1
	b			6 & 8			188	1	"	14	1¾–0	81	28	3.8
	c			7½ & 9½			227	¾	"	14	1–0	58	27	6.4
	91	Cylindrical	8	4	4	16	201	5/16 & ¼	"	8	¾–0	21	45	4.8
	92	"	8	4	4	16	201	5/16 & ¼	"	8	¾–0	21	45	4.8
Average						18	245							6.5

APPENDIX F

FLOW DIAGRAMS OF DIFFERENT TYPES OF ILLINOIS WASHERIES

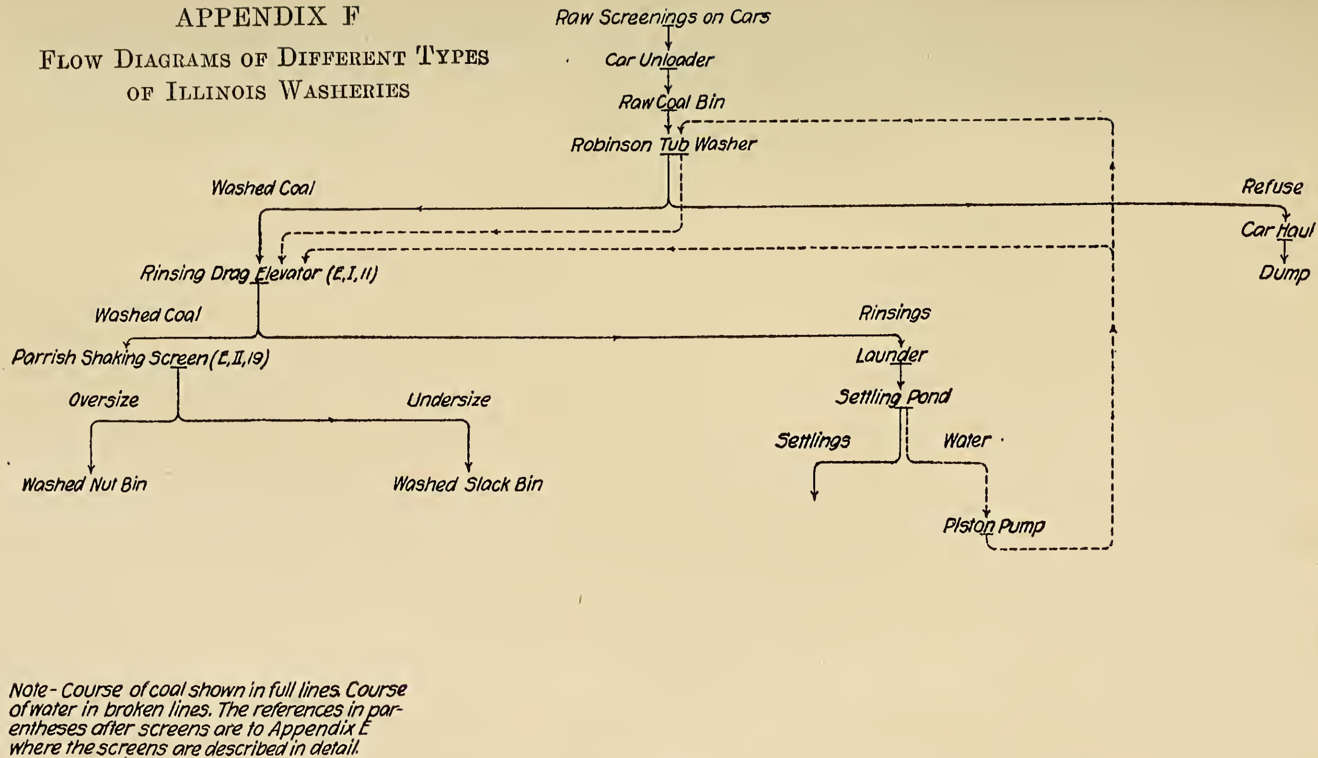
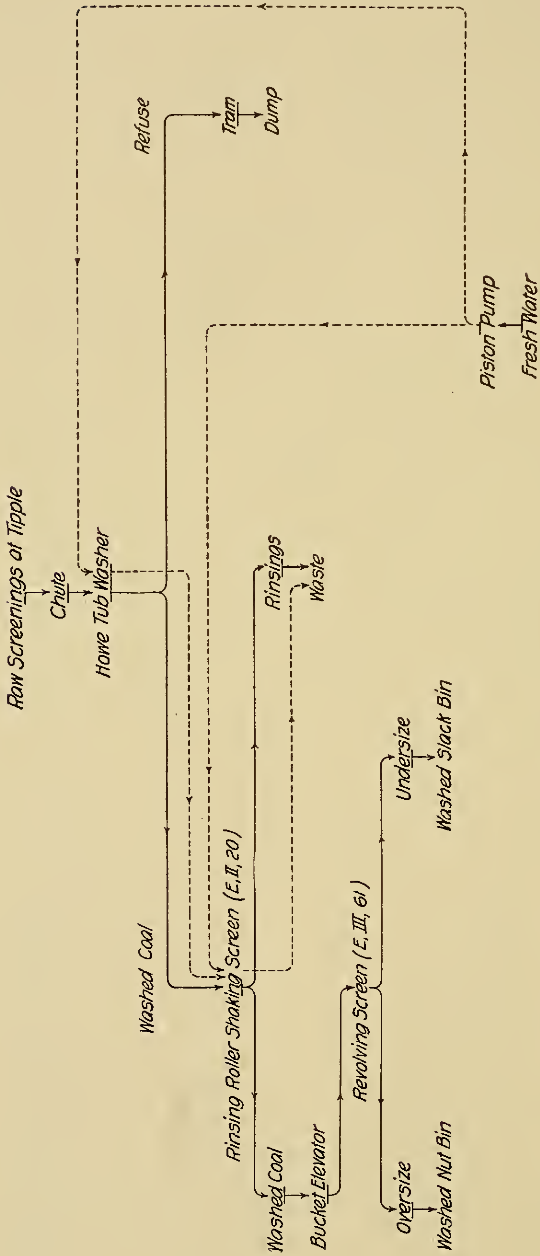


FIG. 18. FLOW SHEET OF A ROBINSON TUB WASHERY (No. 3)



Note—Course of coal shown in full lines. Course of water in broken lines. The references in parentheses after screens are to Appendix E where the screens are described in detail.

FIG. 19. FLOW SHEET OF A HOWE TUB WASHERY (NO. 6)

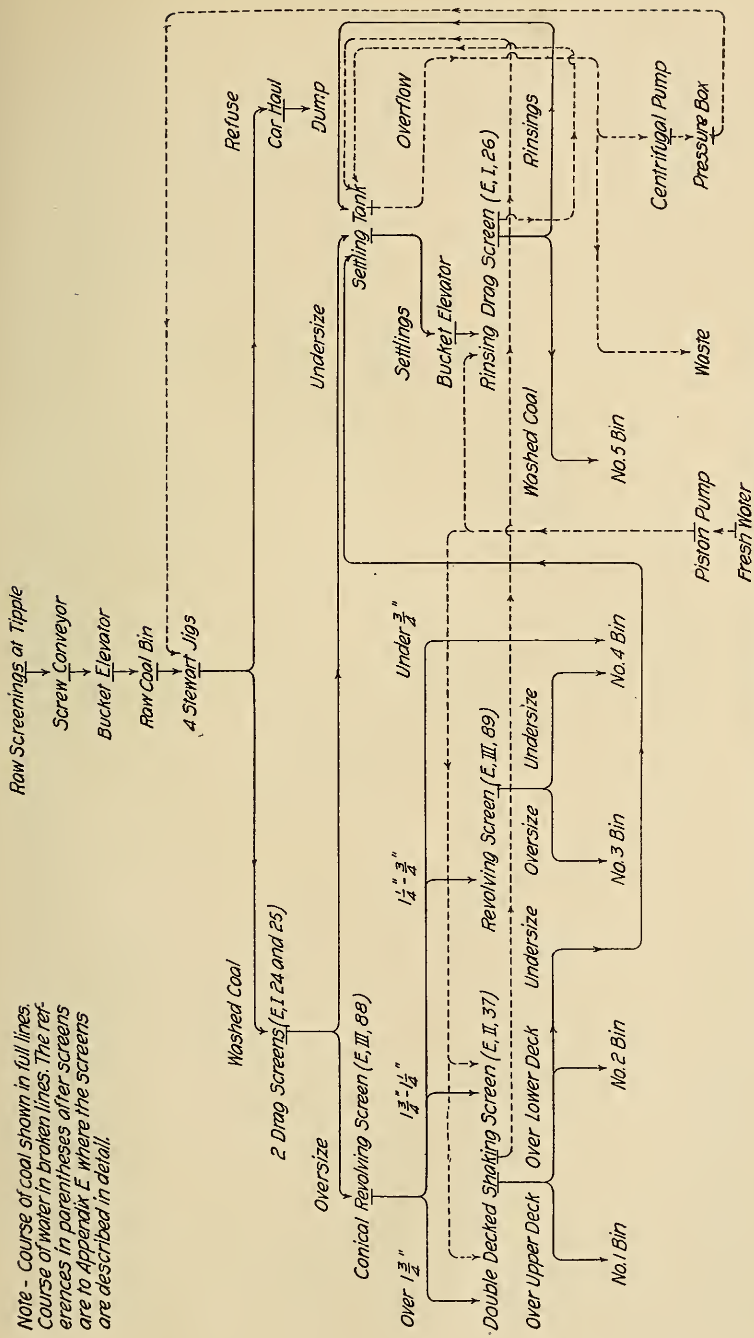
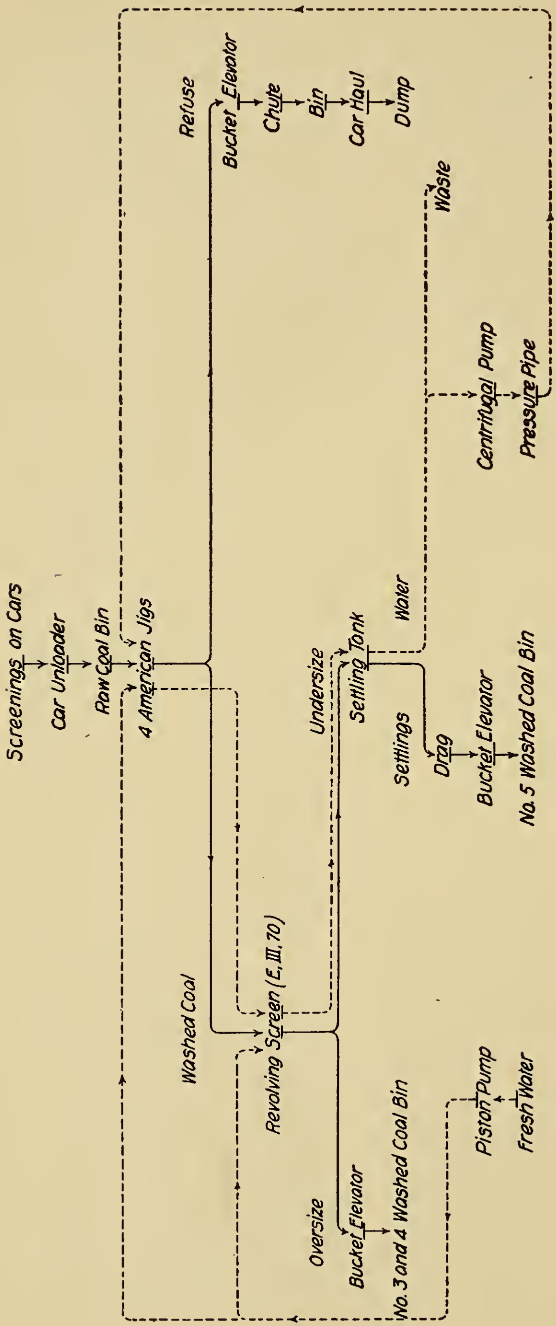


Fig. 20. Flow Sheet of a Stewart Pan Jig Washery (No. 31)



Note—Course of coal shown in full lines. Course of water in broken lines. The references in parentheses after screens are to Appendix E where the screens are described in detail.

Fig. 21. Flow Sheet of an American Pan Jig Washery

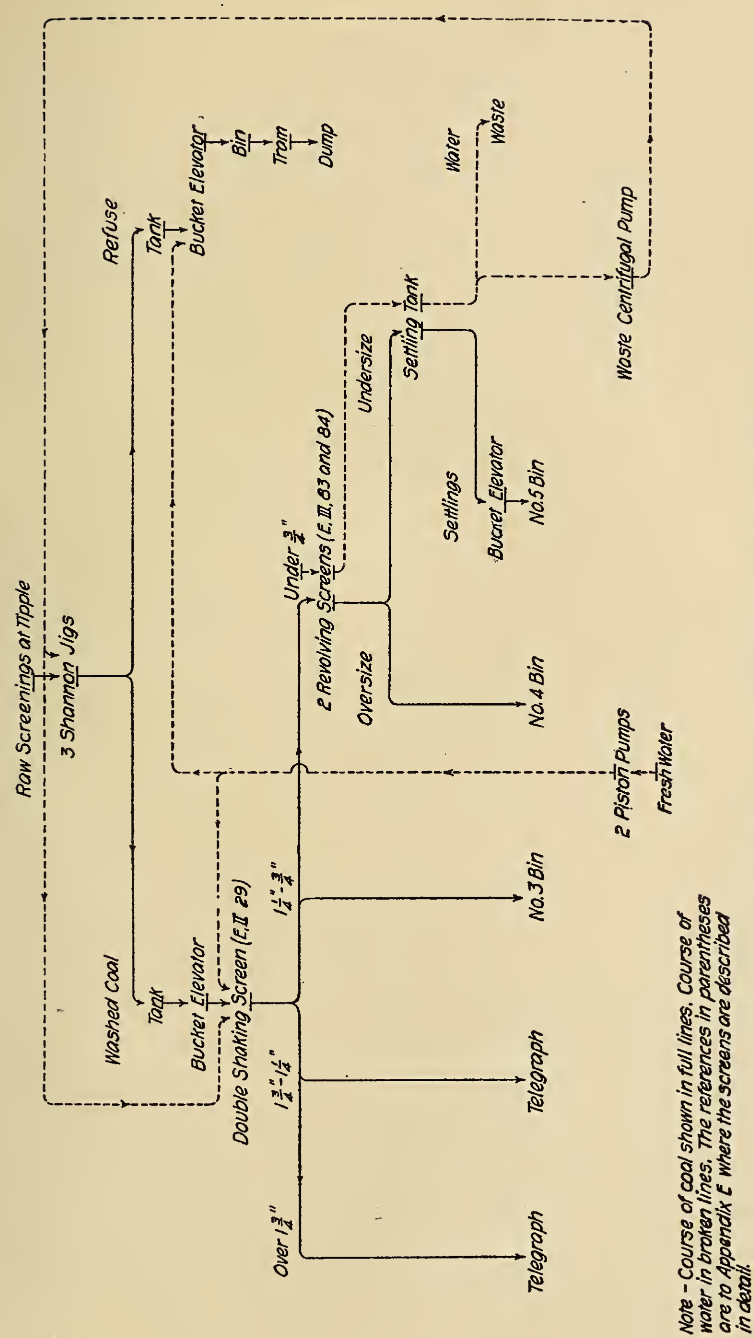


FIG. 22. FLOW SHEET OF A SHANNON PAN JIG WASHERY (No. 23)

Note - Course of coal shown in full lines. Course of water in broken lines. The references in parentheses after screens are to Appendix E where the screens are described in detail.

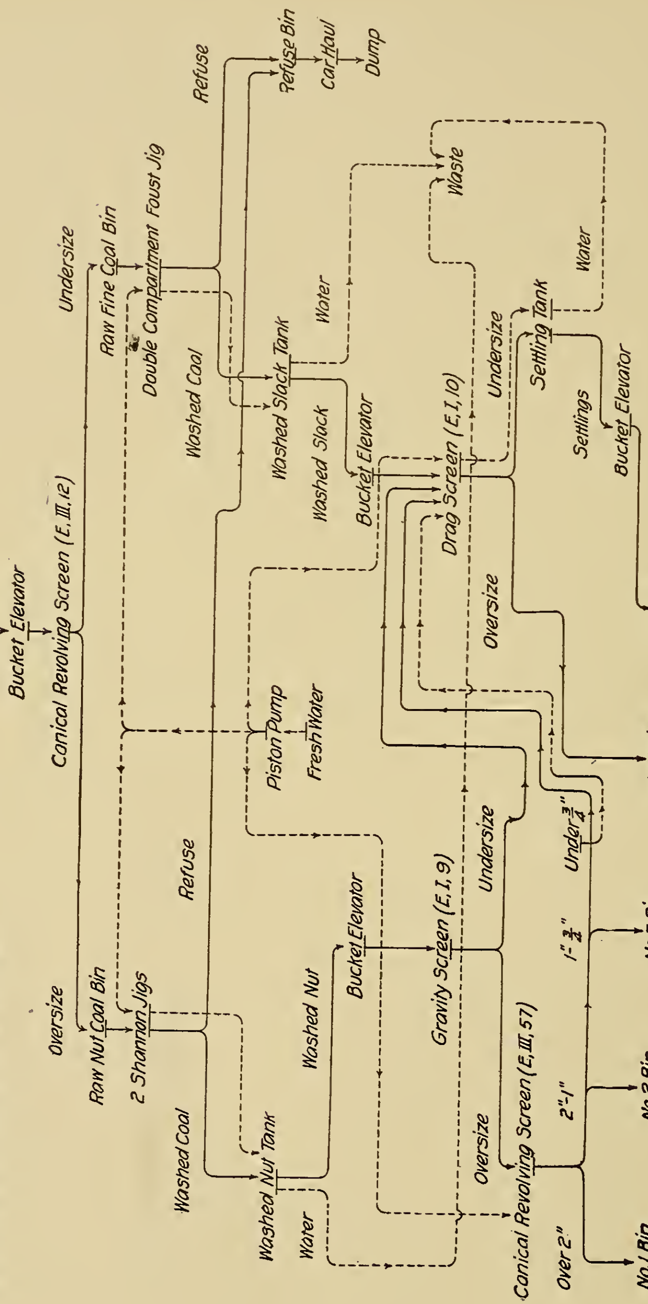


FIG. 23. FLOW SHEET OF A SHANNON AND FOUST JIG WASHERY (NO. 35)

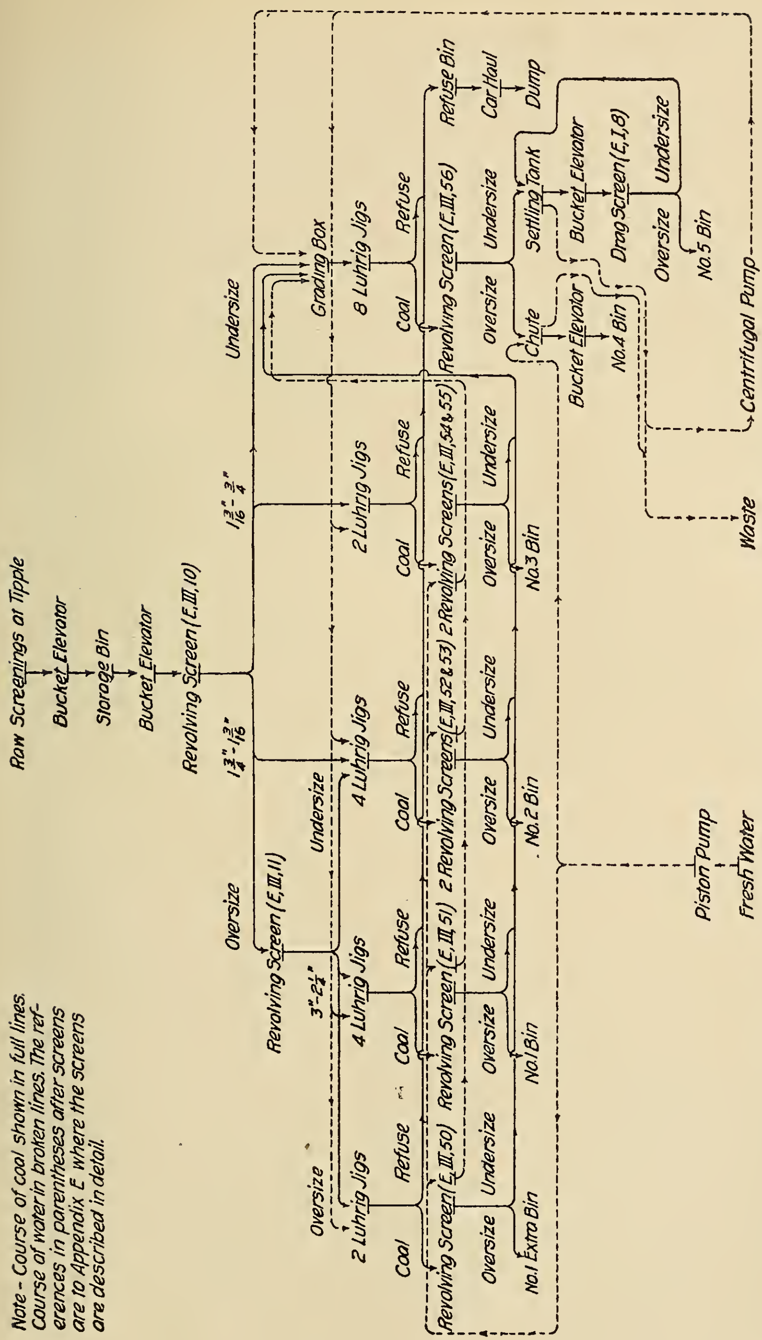


FIG. 24. FLOW SHEET OF A LUHRIG PISTON JIG WASHERY (No. 34)

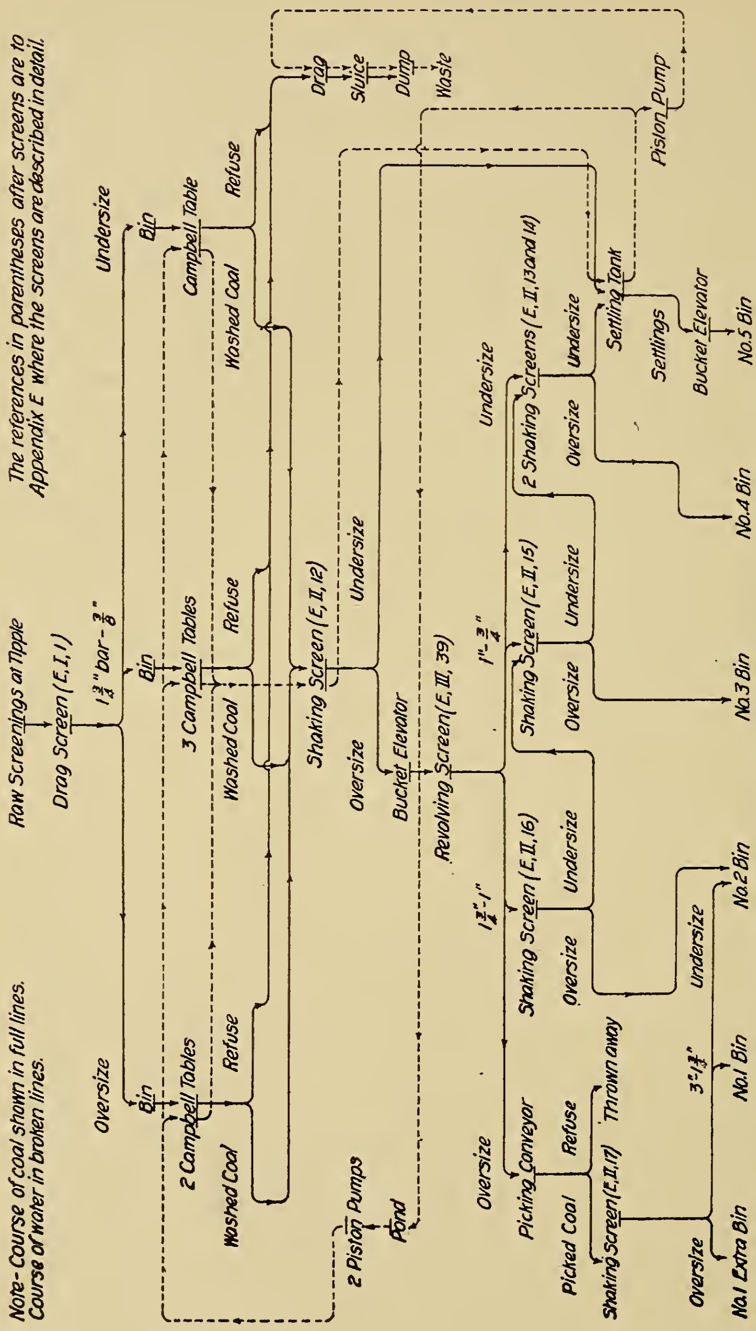


FIG. 25. FLOW SHEET OF A CAMPBELL BUMPING TABLE WASHERY (No. 30)

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge, 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal. (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907, by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *Fifteen Cents.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised edition, 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *Thirty-five cents.*

Bulletin No. 23. Voids, Settlement and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

Bulletin No. 24. The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Free upon request.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Free upon request.*

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Bulletin No. 27. Tests of Brick Columns and Terra Cotta Block Columns, by Arthur N. Talbot and Duff A. Abrams. 1909. *Thirty Cents.*

Bulletin No. 28. A Test of Three Large Reinforced Concrete Beams, by Arthur N. Talbot. 1909. *Free upon request.*

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Bulletin No. 30. On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Free upon request.*

Bulletin No. 31. Fuel Tests with House-heating Boilers, by J. M. Snodgrass. 1909. *Free upon request.*

Bulletin No. 32. The Occluded Gases in Coal, by S. W. Parr and Perry Barker. 1909. *Fifteen cents.*

Bulletin No. 33. Tests of Tungsten Lamps, by T. H. Amrine and A. Guell. 1909. *Twenty cents.*

Bulletin No. 34. Tests of Two Types of Tile Roof Furnaces under a Water-tube Boiler, by J. M. Snodgrass. 1909. *Free upon request.*

Bulletin No. 35. A Study of Base and Bearing Plates for Columns and Beams, by N. Clifford Ricker. 1909. *Twenty cents.*

Bulletin No. 36. The Thermal Conductivity of Fire-Clay at High Temperatures, by J. K. Clement and W. L. Egy. 1909. *Free upon request.*

Bulletin No. 37. Unit Coal and the Composition of Coal Ash, by S. W. Parr and W. F. Wheeler. 1909. *Thirty-five cents.*

Bulletin No. 38. The Weathering of Coal, by S. W. Parr and W. F. Wheeler. 1909. *Free upon request.*

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Bulletin No. 40. A Study in Heat Transmission, by J. K. Clement and C. M. Garland. 1910. *Free upon request.*

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Bulletin No. 42. The Effect of Keyways on the Strength of Shafts, by Herbert F. Moore. 1910. *Free upon request.*

Bulletin No. 43. Freight Train Resistance, by Edward C. Schmidt. 1910. *Seventy-five cents.*

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Bulletin No. 48. Resistance to Flow through Locomotive Water Columns, by Arthur N. Talbot and Melvin L. Enger. 1911. *Free upon request.*

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Bulletin No. 50. Tests of a Suction Gas Producer, by C. M. Garland and A. P. Kratz. 1912. *Free upon request.*

Bulletin No. 51. Street Lighting, by J. M. Bryant and H. G. Hake, 1912. *Free upon request.*

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THE MORTAR-MAKING QUALITIES OF ILLINOIS SANDS

BY

C. C. WILEY

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 70 DECEMBER, 1913

THE MORTAR-MAKING QUALITIES OF ILLINOIS SANDS

By C. C. Wiley, Associate in Civil Engineering

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LIST OF TABLES

FIGURES

THE MORTAR-MAKING QUALITIES OF ILLINOIS SANDS.

I. Introduction.

1. *Preliminary.*—Mortar is defined by the American Railway Engineering Association* as "a mixture of fine aggregate, cement or lime, and water used to bind together the materials in concrete, stone or brick masonry, or to form a covering for the same". The fine aggregate is further defined as "sand or crushed stone screenings".

It would be very difficult to obtain an accurate estimate of the total amount of sand annually used in Illinois for building purposes, but it is doubtless considerably in excess of a million cubic yards.† This enormous quantity of sand is obtained from many deposits, which differ widely in geological conditions, and consequently the sands from different localities may differ greatly in character. It is, therefore, only reasonable to expect that the mortar made of these different sands may vary greatly in quality. Since the mortar is an important element in masonry construction, it would be expected that the sand would be carefully selected and tested in order that the mortar may be of the highest quality, and yet this is not usually done. As a rule, the brick or other materials are selected with reasonable care and the cement is carefully inspected and tested, while the sand usually receives little more than a brief "clean and sharp" in the specifications and a casual glance at the sand pile by way of inspection. As a result, inferior sands producing mortars deficient in both durability and strength are often used when excellent sands are almost as easily obtainable. The existence of such a condition can be attributed only to a failure to realize that sands differ greatly in quality and that the quality of the mortar depends largely on the character of the sand. On some important work elaborate tests of the sand have been made, and the results of these tests, together with those from tests on concrete aggregates, have attracted considerable attention, which has resulted in an increased appreciation of the value of good sands. There is a growing demand for more information concerning the general characteristics of sands, as well as for reliable data on the mortar-making qualities of typical sands.

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*Bulletin No. 130, p. 173-174.

†Circular No. 5 of the Illinois State Geological Survey gives the total amount of building sand produced in the State in 1908 as 1,082,507 cubic yards. Since this report fails to take into account sand used by private parties from private deposits, and since considerable quantities are shipped in from adjoining States, probably balancing that shipped out, it is safe to say that these figures are conservative.

2. *Purpose.*—In view of the conditions mentioned above, a series of tests to determine the mortar-making qualities of a number of representative sands in common use in the cities of Illinois was undertaken with the belief that such data would be of assistance to the engineers and builders of the State in making an intelligent and economical use of the available sands. It is not the function of this bulletin to give a treatise on the use of sands, and therefore only such theoretical or general points are discussed as are essential to the explanation of the tests or to the interpretation of the results. The data obtained from these tests are directly applicable to mortar for stone or brick masonry; and in all probability apply equally well to concrete, since concrete is essentially a mass of rock fragments bound together by mortar, and in no other kind of masonry is the ultimate strength as dependent on the strength of the mortar as in concrete.

3. *Acknowledgment.*—The tests were made in the Cement Laboratory of the Department of Civil Engineering of the University of Illinois under the immediate supervision of the writer. Acknowledgment is here made of the assistance in making the tests of Messrs. J. W. McManus, E. B. Adams, F. T. Heyle, G. A. Barth, and W. Koestner, senior civil engineering students; and also of the assistance of Mr. B. L. Bowling, Assistant in Charge of the Laboratory, both in making the tests and in comparing, arranging, and checking the data.

The work was made possible in a large measure by the courtesy of a number of city engineers of the State in furnishing samples of sand; and acknowledgment is here made of their co-operation. The following list gives the cities whose engineers furnished samples of sand for these tests, together with the reference numbers used to designate the various samples. In several instances the locality from which the sand was obtained is not the same as the city furnishing the sample, and the name given any particular sand in the following tables and discussion is that of the location of the deposit from which the sample was taken.

Aurora	17
Beardstown	27
Bloomington	7
Cairo	25
Champaign	12
Chicago	1, 2, 3, 4
Decatur	14
East St. Louis	21
Elgin	5, 6
Freeport	15, 16
Galesburg	13
Jacksonville	26
Joliet	18
Moline	19
Mt. Carmel	28
Paris	23
Rockford	8
Springfield	9, 10, 11
Taylorville	24
Waukegan	22

Samples No. 20, 29, 30, 31, and 32 were collected personally by the students mentioned above.

II. Description of Tests.

4. *Preparation of Samples.*—The samples of sand were shipped to the laboratory in bags or in tight, paper-lined boxes. As soon as a sample was received it was given a "sample number" for identification. It was then screened to pass a wire sieve having square openings of 0.20 in., thoroughly dried over a steam coil, and then stored in wooden lockers in a room maintained at about 70° F. It will be noted that a maximum size of grain of 0.20 in. was used throughout these tests. This size was adopted at the beginning of the series of tests which was before the national engineering societies generally adopted 0.25 in. as the maximum size*; and in the later tests it was thought unwise to make a change as it was impracticable to repeat some of the earlier tests. Fortunately the difference arising from using 0.20 instead of 0.25 is very small, since none of the sands contained an appreciable amount of material larger than 0.20 in.

Each sand was tested for cleanness, gradation of size of grains (sieve analysis), specific gravity, voids, and weight. The approximate mineralogical composition and comparative sharpness were also determined. Tests for tensile strength were made on mortars made of each of the sands. In making all of these tests especial care was taken to eliminate the personal factor from the results.

5. *Tensile Strength.*—Tensile tests were made on mortars composed of portland cement and the various sands mixed in the proportions of 1:3 by weight. The cement used was a standard brand bought on the open market. Two bags of cement purchased at the same time were used. Tests of the neat cement from each bag gave the same results, within the limits of observation, as follows: Percentage passing the No. 100 and 200 sieves, 96 and 80 respectively; percentage of water for normal plasticity 21.0; tensile strength 642, 772, and 785 lb. per sq. in. at seven, twenty-eight, and ninety days, respectively. The sand mortars were mixed with 9 per cent of water, corresponding to the 21 per cent above, as required by the Specifications for the Uniform Tests of Cements with standard sand. This amount of water was sufficient for the clean, hard sands but with dirty or soft sands the mortars were quite dry. However, as the utmost care was taken in molding the briquettes, it is not believed that this has any considerable effect on the results, especially at the ages of twenty-eight and ninety days. The mortars were mixed in batches sufficient for six briquettes. Eighteen briquettes were made from each

*Recommended to the societies in 1909 by their joint committee and adopted by the societies in 1910 and 1911.

Table 1.—Tensile Strength of 1:3 Mortars.

Sand. No.	Sand. Name.	Age, Days	Tensile Strength—Pounds per Sq. In. Individual Briquettes.						Average
00	Neat cement	7	615	645	640	690	620	...	642
		28	780	730	770	760	720	...	772
		90	755	820	785	780	...	...	785
0	Ottawa standard, 20-30	7	220	200	220	230	230	...	225
		28	310	300	315	340	315	295	313
		90	375	365	375	370	360	365	368
1	Chicago K. I. Co.	7	115	110	108	113	108	...	111
		28	205	195	190	200	165	195	192
		90	210	210	210	190	205	...	205
2	Chicago Z. R. Co.	7	150	175	175	145	150	...	159
		28	300	260	275	290	250	270	274
		90	315	310	330	290	...	...	311
3	Chicago K. I. Co.	7	200	220	205	170	180	...	195
		28	265	295	260	270	305	290	281
		90	320	310	320	305	315	...	314
4	Joliet limestone screenings	7	220	240	200	210	220	...	218
		28	295	335	365	350	330	...	335
		90	450	530	505	460	...	...	485
5	Elgin, Hammond pit	7	245	270	240	220	250	...	245
		28	380	375	395	425	410	380	394
		90	465	455	450	445	490	480	464
6	Elgin, Stimpson pit	7	190	180	210	225	210	200	203
		28	340	340	360	350	390	...	356
		90	415	420	405	400	...	...	410
7	Bloomington	7	225	230	225	235	210	225	225
		28	225	265	250	240	235	260	246
		90	295	305	275	285	270	...	286
8	Rockford	7	250	240	260	230	245	265	248
		28	335	345	350	335	330	340	339
		90	385	380	410	360	395	385	386
9	Lincoln	7	235	220	230	210	200	205	217
		28	285	270	275	285	275	290	279
		90	310	310	290	300	300	305	303
10	Alton	7	185	180	175	185	150	175	175
		28	245	230	250	255	220	260	243
		90	275	270	260	270	270	270	269
11	Alton	7	180	195	210	195	190	...	194
		28	245	235	250	245	240	...	243
		90	270	285	295	280	270	280	280
12	Covington, Ind	7	205	185	200	200	205	195	198
		28	265	280	250	280	285	245	267
		90	315	315	310	305	315	300	310
13	Gladstone	7	190	180	180	170	170	180	178
		28	230	220	205	210	225	220	218
		90	250	235	240	230	240	250	241
14	Decatur	7	270	310	285	285	305	265	287
		28	380	375	355	355	360	...	365
		90	425	435	420	430	430	...	428
15	Freeport	7	245	205	200	220	190	225	214
		28	275	285	270	260	265	265	270
		90	335	330	320	315	310	295	317

Table 1.—Tensile Strength of 1:3 Mortars—Continued.

Sand. No.	Sand. Name.	Age, Days	Tensile Strength—Pounds per Sq. In. Individual Briquettes.						Average
16	Freeport Sandstone screenings	7	180	180	195	220	170	195	190
		28	220	250	240	210	250	235	234
		90	235	260	270	270	300	280	269
17	Aurora	7	270	290	260	270	270	...	272
		28	390	415	400	435	410	...	410
		90	470	460	450	450	440	...	454
18	Joliet	7	120	130	98	100	100	100	108
		28	100	105	125	115	140	160	124
		90	145	170	155	150	150	175	159
18	Joliet (washed) .	..	...	...	...	...	...	...	...
		28	150	170	190	175	185	185	176
		..	...	...	...	...	...	...	...
19	Moline	7	160	180	175	170	160	180	171
		28	250	225	235	245	...	...	239
		90	275	260	260	245	240	265	257
20	Urbana	7	190	185	190	200	200	195	193
		28	240	240	230	220	245	265	240
		90	365	320	315	310	340	330	330
21	E. St. Louis	7	170	185	170	160	170	175	172
		28	195	210	205	190	210	230	207
		90	285	275	230	245	245	265	258
22	Waukegan	7	140	145	160	175	155	140	158
		28	235	230	225	225	230	220	228
		90	300	270	280	310	305	310	296
23	Paris	7	125	130	120	130	120	115	123
		28	200	200	200	205	190	190	198
		90	260	245	285	265	245	255	259
24	Taylorville	7	110	100	100	115	130	135	113
		28	140	155	150	155	135	160	149
		90	210	220	195	220	200	220	211
25	Cairo	7	155	155	140	155	175	140	153
		28	260	210	235	220	245	240	234
		90	300	295	300	325	280	285	298
26	Hannibal,	7	160	150	150	175	175	180	165
		28	220	200	205	225	180	220	209
		90	290	270	290	275	275	275	279
27	Beardstown	7	95	80	90	90	90	70	86
		28	115	105	100	105	115	100	107
		90	145	130	150	135	135	145	140
28	Mt. Carmel	7	100	100	85	100	100	105	99
		28	140	135	130	140	125	140	135
		90	185	175	175	165	160	160	170
29	La Salle	7	130	120	130	110	100	110	117
		28	200	180	175	190	200	170	186
		90	175	200	185	190	210	200	194
30	Peoria	7	195	170	180	185	170	170	178
		28	200	205	200	205	210	210	205
		90	235	260	265	260	280	270	262
31	Peoria	7	165	140	175	185	150	185	167
		28	210	220	230	200	235	235	221
		90	315	320	295	285	320	300	306
32	Peoria	7	120	110	105	105	115	125	114
		28	155	155	145	145	160	160	153
		90	185	170	190	180	165	180	178

sand. Two briquettes from each batch, or six in all from each sand, were tested at the ages of seven, twenty-eight, and ninety days. The entire series of briquettes were molded as nearly at the same time as practicable, and all other precautions were taken to maintain uniform conditions throughout the mixing and molding. After being molded the briquettes were kept in moist air for about twenty-four hours and then stored in water maintained at about 70° F. in tanks so arranged that the water was gradually changed. Upon reaching the required age the briquettes were removed from the water and immediately broken in a Riehle automatic cement testing machine, applying the load at the rate of 600 lb. per minute.

The detailed results of these tests are given in Table 1. It will be noted that these results are quite uniform, *i. e.,* the variation in the strength of the briquettes of the same sand at the same age is small. The sands were ranked in the order of the tensile strength of the mortars at ninety days. This ranking is shown in Table 4.

6. *Cleanness.*—The sands were tested for cleanness as follows: 1,000 grams of the sand were thoroughly agitated in about one gallon of water. The mixture was then allowed to settle for about one minute, experience showing that this allowed sufficient time for the finest sand to settle. The dirty water was then siphoned off with a ¼ in. rubber tube, care being taken that none of the sand was carried over with the water. This washing process was repeated until the water showed no discoloration. It was usually found necessary to "scrub" the dirtier sands between the hands to entirely remove the coating of clay from the grains. The sand was then transferred to a pan and as much water as possible drawn off by jarring the sand until the water flushed to the surface, and then removing the water with a pipette. The sand was then re-dried over the steam coil and weighed. The loss in weight due to the washing was taken as the amount of suspended matter.

The time required to accomplish this washing varied from about fifteen minutes for a fairly clean sand to about two and one-half hours for the dirtiest, the average being about one hour. The results of these tests are given in Tables 2 and 4.

7. *Sieve Analysis.*—The following standard sieves were used: Nos. 5, 8, 10, 16, 20, 30, 40, 60, 74, 100, 150, and 200. In some of the earlier tests sieves Nos. 5, 10, 15, 150, and 200 were omitted. The sieves were nested in order of size, the largest at the top. The washed sand was then placed on the top sieve and the whole shaken for forty minutes on a Per Se Sieve Agitator driven by power at 100 r. p. m. After shaking,

TABLE 2.

SIEVE ANALYSIS.

Sand.		Suspended Matter.	Percent by Weight Passing Sieve No.												
No.	Name.		200	150	100	74	60	40	30	20	16	10	8	5	0.2 in.
1	Chicago, K. I. Co.	0.3	...	...	4.2	48.6	69.5	96.4	99.1	99.7	99.8	....	100.0	....	100.0
2	Chicago, Z.-R. Co.	0.3	...	...	2.5	20.0	25.9	77.3	87.5	93.7	94.7	....	99.1	....	100.0
3	Chicago, K. I. Co.	0.3	...	...	2.4	9.9	15.9	49.1	69.0	78.3	79.8	....	89.1	....	100.0
4	Limestone Scr...	0.0	...	...	8.1	9.3	10.7	14.9	15.5	19.9	21.2	....	52.3	....	100.0
5	Elgin	1.0	...	...	2.8	5.0	7.4	20.9	36.4	51.7	55.0	....	84.0	....	100.0
6	Elgin............	0.9	...	...	2.1	5.2	9.0	23.6	39.2	60.1	65.5	....	97.3	....	100.0
7	Bloomington	8.0	9.0	9.7	11.4	19.8	21.7	43.0	56.5	67.2	69.9	85.8	92.1	96.6	100.0
8	Rockford	0.6	...	...	1.7	5.5	10.8	25.1	34.3	60.0	68.2	....	98.0	....	100.0
9	Lincoln	1.1	...	...	2.4	4.1	6.5	26.7	46.4	61.7	65.8	....	95.4	....	100.0
10	Alton	0.3	...	...	1.2	5.2	10.5	45.6	70.2	87.8	90.3	....	98.2	....	100.0
11	Alton............	0.2	...	...	2.2	5.3	8.9	30.1	53.8	76.2	80.0	....	97.0	....	100.0
12	Covington, Ind..	1.5	...	...	3.6	5.7	7.6	19.8	31.6	47.9	53.3	....	93.5	....	100.0
13	Gladstone	0.2	...	...	1.1	6.8	12.9	63.8	88.8	97.0	97.9	....	99.8	....	100.0
14	Decatur	2.5	2.9	3.1	3.9	6.0	7.8	21.0	35.8	45.9	48.7	71.6	81.5	93.7	100.0
15	Freeport	1.3	3.0	3.7	6.6	14.4	18.5	43.9	63.6	79.2	82.5	97.2	99.2	99.9	100.0
16	Sandstone Scr...	0.0	...	...	0.2	1.4	4.2	30.9	63.2	82.9	86.4	....	98.3	....	100.0
17	Aurora	0.5	...	...	0.9	1.1	1.6	12.3	35.0	55.7	61.3	....	97.8	....	100.0
18	Joliet	18.3	19.3	19.5	20.2	21.1	22.2	32.5	63.7	94.8	96.9	99.5	99.7	99.9	100.0
19	Moline	0.0	0.0	0.1	0.2	1.6	4.3	35.2	67.1	85.8	88.6	96.9	98.3	99.4	100.0
20	Urbana	3.5	6.7	8.6	17.0	39.1	52.4	74.4	79.6	83.5	84.5	92.3	95.3	98.3	100.0
21	E. St. Louis.....	trace	0.4	0.7	1.8	5.9	9.9	39.5	55.1	79.4	82.6	95.9	97.8	99.4	100.0
22	Waukegan	0.0	0.1	0.2	0.8	8.8	12.1	38.4	61.7	82.0	85.8	98.0	99.0	100.0	100.0
23	Paris	1.2	1.7	2.1	4.3	12.9	16.9	64.4	91.4	97.6	98.1	99.1	99.4	99.8	100.0
24	Taylorville	4.0	7.4	9.4	22.1	55.6	75.4	99.5	99.8	99.9	99.9	99.9	100.0	100.0	100.0
25	Cairo	0.3	0.4	0.5	0.8	2.5	3.9	24.9	62.9	81.9	84.8	95.7	98.1	99.8	100.0
26	Hannibal, Mo....	0.2	0.3	0.4	0.9	3.9	7.4	33.2	58.5	76.9	80.4	93.3	96.1	99.3	100.0
27	Beardstown	4.4	6.6	7.6	13.2	35.3	47.4	97.0	99.6	99.9	99.9	99.9	100.0	100.0	100.0
28	Mt. Carmel......	2.4	2.6	2.7	3.9	18.8	34.2	98.7	99.8	99.9	99.9	99.9	100.0	100.0	100.0
29	La Salle.........	3.2	4.2	4.5	6.8	16.5	26.0	87.6	97.7	99.1	99.3	99.7	99.8	99.9	100.0
30	Peoria	2.7	3.1	3.5	6.0	14.5	17.8	47.8	74.9	88.2	90.3	96.7	98.0	99.0	100.0
31	Peoria...........	1.9	2.3	2.4	2.6	3.0	4.0	20.0	59.5	83.6	86.5	96.2	97.9	99.3	100.0
32	Peoria...........	0.6	0.8	0.9	2.0	9.2	13.3	68.7	89.1	96.0	97.1	99.0	99.4	99.7	100.0

the sand retained on each sieve was weighed and from these weights the percentage passing each sieve was calculated. In making these calculations the amount of suspended matter, as determined by the cleanness test, was included with the material passing the finest sieve.

The results of these tests are given in Table 2 and are shown graphically in Figs. 1 to 4. The "size of grains" used in platting these curves is the nominal clear opening of the sieve meshes. The use of these curves is more fully discussed under Size and Gradation of Grains on page 29.

8. *Specific Gravity.*—The specific gravity of the sands was determined by means of a Schumann specific gravity flask. This flask consists of a bulb or bottle into the neck of which a graduated stem fits with a ground joint. The bulb was filled with water and the height of the column of water in the stem read from the graduations. Fifty grams of sand were admitted, care being taken to permit the escape of the air, and

TABLE 3.

SPECIFIC GRAVITY, VOIDS, AND WEIGHT.

Sand. No.	Sand. Name.	D, Water Displaced by 100 G. Sand Cu. Cm.	Specific Gravity, $\frac{100}{D}$	S, 500xSp. Gr.	W, Weight of 500 c.c. of Sand, Grams.	Voids, $\frac{S-W}{S}$ Percent.	Weight Per Cu. Ft. Pounds.
0	Standard	37.57	2.660	1330.0	870.0	34.6	108.7
1	Chicago, K. I. Co.	37.62	2.655	1327.5	847.0	36.2	105.8
2	Chicago, Z.-R. Co.	37.65	2.655	1327.5	871.0	34.4	108.8
3	Chicago, K. I. Co.	37.13	2.695	1347.5	922.0	31.6	115.1
4	Limestone screenings	36.40	2.750	1375.0	983.0	28.5	122.8
5	Elgin	36.69	2.720	1360.0	926.0	31.9	115.7
6	Elgin	37.27	2.680	1340.0	916.0	31.6	114.4
7	Bloomington	38.10	2.625	1312.5	911.5	30.5	113.8
8	Rockford	37.51	2.665	1332.5	905.0	32.0	113.0
9	Lincoln	37.72	2.650	1325.0	895.5	32.4	111.9
10	Alton	38.03	2.630	1315.0	901.0	31.5	112.5
11	Alton	37.57	2.660	1330.0	917.0	31.0	114.5
12	Covington, Ind.	37.60	2.660	1330.0	884.5	33.5	110.4
13	Gladstone	37.65	2.655	1327.5	852.0	35.8	106.3
14	Decatur	37.60	2.660	1330.0	927.5	30.3	115.8
15	Freeport	36.80	2.720	1360.0	912.0	33.0	113.9
16	Sandstone screenings	36.97	2.705	1352.5	899.0	33.0	112.2
17	Aurora	36.75	2.720	1360.0	894.0	34.2	111.7
18	Joliet	37.60	2.660	1330.0	802.0	39.7	100.2
19	Moline	37.77	2.650	1325.0	897.0	32.3	112.0
20	Urbana	37.75	2.650	1325.0	861.0	35.0	107.5
21	East St. Louis	37.73	2.650	1325.0	927.0	30.1	115.8
22	Waukegan	37.25	2.680	1340.0	901.0	32.7	112.5
23	Paris	37.52	2.665	1332.5	867.0	35.2	108.3
24	Taylorville	37.88	2.685	1317.5	798.5	39.4	99.7
25	Cairo	37.79	2.645	1322.5	863.0	34.7	107.8
26	Hannibal, Mo.	37.33	2.680	1340.0	921.0	31.3	115.0
27	Beardstown	38.40	2.605	1302.5	809.0	37.9	101.0
28	Mt. Carmel	37.90	2.635	1317.5	835.5	36.5	104.4
29	La Salle	37.70	2.650	1325.0	854.0	35.5	106.7
30	Peoria	37.50	2.665	1332.5	896.5	32.9	111.9
31	Peoria	37.80	2.640	1320.0	832.0	36.9	103.9
32	Peoria	37.61	2.660	1330.0	870.0	34.6	108.7

the new height of the water column read. The difference in the two readings gave the volume of water displaced by the fifty grams of sand. A second fifty grams of sand was immediately admitted in order to secure a double determination as well as a check on the observations. From these data the specific gravity of the sand was computed. The results of these tests are shown in Tables 3 and 4.

9. *Voids.*—The large variation in the percentage of voids in sands as determined by different observers is doubtless due to errors of observation, particularly to a failure to compact all sands to the same extent. For the present series of tests some method of operation was desired which would compact all sands alike, and hence yield reliable results; and, further, a method was desired which would be easy to perform and which could make use of a limited quantity of sand. After a number of trials the following method was adopted:

A graduated cylinder about 2 in. in diameter and of 500 c.c. capacity was used. About 20 c.c. of sand was placed in this cylinder and compacted by striking lightly with the cylinder on a pad composed of eight thicknesses of heavy cotton flannel. Twelve blows were given at the rate of about two per second with a fall of about one inch, care being taken that the blow was not hard enough to cause the sand to bounce and also that the cylinder struck squarely so that the sand was not thrown from side to side. Successive increments of the sand were added in this way until the cylinder was filled. The difference between the weight of the empty cylinder and the cylinder full of sand gave the weight of 500 c.c. of dry compacted sand. Then, knowing the specific gravity of the sand, the percentage of voids was computed from the equation

$$V = 100 \frac{S - W}{S}$$

in which V is the voids expressed in percents of the total volume, W is the weight of the sand, and S is the product of the volume of the sand, its specific gravity, and the weight of a unit volume of water; *i. e.,* S is the weight of an equal volume of sand containing no voids. In the present case S equals 500 multiplied by the specific gravity.

The results obtained by this method were quite uniform. The maximum variation from the mean of five determinations of the weight of 500 c.c. of the same sand made by the same observer was only 7.5 grams or about 0.8 per cent. The maximum variation from the mean of four sets of determinations on the same sands by four different observers was only 1.5 per cent. The results of these tests are given in Tables 3 and 4,

10. *Weight.*—The weight per cubic foot of each sand was computed from the weight of 500 c.c. and is given in Tables 3 and 4. These results average about 3 per cent higher than the results obtained by other observers on similar sands; but it is believed that this is due to a difference in the method of compacting the sands, and also that the results given here are more nearly correct for the dry sands after being transported some distance in wagons or cars.

TABLE 4.
SUMMARY OF DATA.

Sand.			Tensile Strength of 1:3 Mortar at 90 Days.			Rank According to Tensile Strength	Voids, Per Cent.	Weight per Cu. Ft., Pounds.	Suspended Matter, Per Cent.	Specific Gravity.
No.	Name.	Kind.	Pounds per Sq. In.	In Per Cent of the Strength of Standard Sand Mortar.	In Per Cent of the Strength of Neat Cement					
00	Neat cement...	Portland	785	213.4	100.0	..	...	....	...	....
0	Ottawa standard	20-30	368	100.0	46.9	7	34.6	108.7	0.0	2.660
1	Chicago K. I. Cc	Lake	205	55.7	26.1	27	36.2	105.8	0.3	2.655
2	Chicago Z-R Co	Lake	311	84.5	39.6	11	34.4	108.8	0.3	2.655
3	Chicago K. I. Cc	Lake	314	85.4	40.0	10	31.6	115.1	0.3	2.695
4	Limestone scr..	Joliet	485	131.8	61.8	1	28.5	122.8	0.0	2.750
5	Elgin, Hammond pit	Bank	464	126.0	59.1	2	31.9	115.7	1.0	2.720
6	Elgin, Stimpson pit	Bank	410	111.5	52.2	5	31.6	114.4	0.9	2.680
7	Bloomington ..	Bank	286	77.8	36.4	17	30.5	113.8	8.0	2.625
8	Rockford	Bank	386	104.9	49.2	6	32.0	113.0	0.6	2.665
9	Lincoln	Bank	303	82.4	38.6	14	32.4	111.9	1.1	2.650
10	Alton	River	269	73.2	34.3	20	31.5	112.5	0.3	2.630
11	Alton	River	280	76.2	35.7	18	31.0	114.5	0.2	2.660
12	Covington, Ind.	Bank	310	84.4	39.5	12	33.5	110.4	1.5	2.660
13	Gladstone	Bank.	241	65.5	30.7	25	35.8	106.3	0.2	2.655
14	Decatur	Bank	428	116.4	54.5	4	30.3	115.8	2.5	2.660
15	Freeport	Bank	317	86.2	40.4	9	33.0	113.9	1.3	2.720
16	Sandstone sci...	Freeport	269	73.2	34.3	20	33.0	112.2	0.0	2.705
17	Aurora	Bank	454	123.3	57.8	3	34.2	111.7	0.5	2.720
18	Joliet	Bank	159	43.3	20.3	31	39.7	100.2	18.3	2.660
19	Moline	River	257	69.8	32.7	24	32.3	112.0	0.0	2.650
20	Urbana	Bank	330	89.7	42.0	8	35.0	107.5	3.5	2.650
21	E. St. Louis....	River	258	70.2	32.8	23	30.1	115.8	Trace	2.650
22	Waukegan	Lake	296	80.5	37.7	16	32.7	112.5	0.0	2.680
23	Paris	Bank	259	70.4	33.0	22	35.2	108.3	1.2	2.665
24	Taylorville.....	Bank	211	57.4	26.9	26	39.4	99.7	4.0	2.635
25	Cairo	River	298	81.0	38.0	15	34.7	107.8	0.3	2.645
26	Hannibal, Mo..	River	279	75.8	35.5	19	31.3	115.0	0.2	2.680
27	Beardstown	Bank	140	38.1	17.9	32	37.9	101.0	4.4	2.605
28	Mt. Carmel.....	River	170	46.2	21.7	30	36.5	104.4	2.4	2.635
29	La Salle........	Bank	194	52.7	24.7	28	35.5	106.7	3.2	2.650
30	Peoria	Bank	262	71.2	33.4	21	32.9	111.9	2.7	2.665
31	Peoria	Bank	306	83.2	39.0	13	36.9	103.9	1.9	2.640
32	Peoria	River	178	48.4	22.7	29	34.6	108.7	0.6	2.660

11. *Mineralogical Composition and Sharpness.*—The sands were examined under a magnifying glass having a power of about five diameters to determine the approximate mineralogical composition, the sharpness or irregularity in shape of the grains, and the roughness of the surfaces. The results are therefore only relative and hence are not tabulated but are given in connection with the description of the sands.

III. Description of Sands.

The following description of the sands is based on the results of the tests and on general information furnished by the parties who sent the sands. The general appearance of the sands is well shown by Figs. 6 to 11, inclusive, which are reproduced from photographs. In making these photographs care was taken to secure a representative surface of the sand, and as the illustrations are full size the relative appearance and fineness of the different sands are well indicated.

Sample No. 0 (See Fig. 6).—This is the standard sand used in the comparative tests of cements. It is pure quartz and comes from the St. Peter formation near Ottawa, Ill. The grains are spherical, almost transparent with dull surfaces, and are nearly of the same size, since the sand is screened to pass a No. 20 sieve and be retained on a No. 30. The specific gravity is 2.66, the percentage of voids 34.6, and the weight per cu. ft. 108.7 lb. The tensile strength of 1:3 mortar at 90 days was 368 lb. per sq. in., or 46.9 per cent of that of the neat cement. This sand ranked seventh in strength, *i. e.*, only six other sands produced mortars of greater strength.

Sample No. 1 (See Fig. 6).—This sand, supplied by the Knickerbocker Ice Co., Chicago, was taken from banks along the shore of Lake Michigan. Its color is light yellow, and the grains are angular and consist principally of quartz. This sand is very fine, 99.8 per cent passing the No. 16, and 69.5 per cent the No. 60 sieve (see Fig. 1). There is 3 per cent of suspended matter, and the specific gravity is 2.655, the percentage of voids 36.2, and the weight per cu. ft. 105.8 lb. The tensile strength of 1:3 mortar at 90 days was 205 lb. per sq. in., 55.7 per cent of the strength of the Ottawa standard sand mortar. It ranks twenty-seventh in strength.

Sample No. 2 (See Fig. 6).—This is also a bank sand from the shores of Lake Michigan and was supplied by the Zander-Reum Company, Plasterers, Chicago. It is light gray in color and is composed principally of quartz with a small proportion of granite, flint, and limestone. It is somewhat better graded than the preceding sample (see Fig. 1), 94.7 per cent passing the No. 16 sieve and 25.9 per cent the No. 60. It contains 3 per cent of suspended matter, the specific gravity is 2.655, the weight per cu. ft. is 109.8 lb., and the percentage of voids is 34.4. The strength of 1:3 mortar at 90 days was 311 lb. per sq. in., 84.5 per cent of that of the Ottawa standard sand mortar. It ranks eleventh in strength.

Sample No. 3 (See Fig. 6.)—This is another lake sand furnished by the Knickerbocker Ice Co., Chicago. The smaller grains are principally quartz, while the larger grains are composed of granite, flint, limestone, and chert, which give the sand its light gray color. It is better graded than either of the preceding samples (see Fig. 1), and the percentage of voids is somewhat lower than would be expected of a sand of this grading, being only 31.6. It contains 3 per cent of suspended matter, the specific gravity is 2.695, and the weight per cu. ft. is 115.1 lb. The tensile strength of 1:3 mortar at 90 days was 314 lb. per sq. in., 85.4 per cent of that of the Ottawa standard sand mortar. It ranks tenth in strength.

Sample No. 4 (See Fig. 6).—This is a sample of Joliet limestone screenings which is being used in Chicago to a considerable extent. The stone is very close and even-grained and of a pale blue tint. In crushing some of it passes into very fine dust, but there is no suspended matter. The specific gravity is 2.75, the weight per cu. ft. is 122.8 lb., and the percentage of voids is 28.5. The sieve analysis curve (see Fig. 1) shows that this sample is almost uniformly graded. The tensile strength of a 1:3 mortar at 90 days was 485 lb. per sq. in., 131.8 per cent of the Ottawa standard sand mortar, and was the highest tensile strength of mortar recorded in these tests. From this one sample it would appear that limestone screenings give a much stronger mortar than natural sand; but it will be noted that this sample is particularly well graded, has very little fine dust, and the stone itself is uniform and hard, hence this sample should be regarded as exceptional and not as typical.

Sample No. 5 (See Fig. 6).—This sample of sand comes from the Hammond pit of the Chicago Gravel Company near Elgin. It is yellowish gray in color and contains less quartz than the average Illinois sand. The other materials present are principally limestone, flint, and chert. This is one of the coarsest sands tested, only 55 per cent passing the No. 16 sieve and 7.4 per cent the No. 60. (See Fig. 1.) Although this sand was reported as a "washed sand", it contains 1 per cent of suspended matter, a trifle more than Sample No. 6 which is a "screened" sand from the same locality. The specific gravity is 2.72, the weight per cu. ft. 115.7 lb., and the percentage of voids 31.9. This sand made the strongest mortar of the natural sands, being second only to the limestone screenings. The strength of a 1:3 mortar at 90 days was 464 lb. per sq. in., 126 per cent of that of the Ottawa standard sand mortar.

Sample No. 6 (See Fig. 7).—This is a screened sand from the

Stimpson pit, near Elgin. It is grayish in color and contains a small proportion of flint, granite, and chert, the majority of the material being quartz. Note that in this respect it differs quite a little from the preceding sample from the same locality. This sand is somewhat exceptional in that the fine grains are the more angular, the large grains being well rounded. The sieve analysis (see Fig. 1) shows this sand to be somewhat finer than sample No. 5. The specific gravity is 2.68, the weight per cu. ft. 114.4 lb., and the percentage of voids 31.6. A 1:3 mortar gave a strength of 410 lb. per sq. in. at 90 days, 111.5 per cent of that of the Ottawa standard sand mortar. This sand ranks fifth in tensile strength.

Sample No. 7 (See Fig. 7).—This is a bank sand obtained along Sugar Creek near Bloomington. It is quite uniform in quality, and has been used a great deal for pavement cushions and in common brick work, but has not been used to any great extent in concrete, as it was thought to be too dirty. It contains 8 per cent of suspended matter in the form of finely divided clay which is distributed loosely throughout the mass of the sand and does not form a coating on the grains. The tensile tests (see Table 1) show that it forms a mortar considerably stronger than several of the cleanest river sands; and since it is fairly well graded (see Fig. 1), and its mineralogical composition is satisfactory, it could doubtless be safely used in nearly all kinds of work. The tensile strength was not improved by washing the sand. The specific gravity is 2.625, the weight per cu. ft. 113.8 lb., and the percentage of voids 30.5. The tensile strength of a 1:3 mortar at 90 days was 286 lb .per sq. in., 67.7 per cent of that of the Ottawa standard sand mortar. It ranks seventeenth in tensile strength.

Sample No. 8 (See Fig. 7).—This is a bank sand from near Rockford. The deposits are large and very uniform in quality. The sand is light gray in color and contains considerable chert with some limestone and flint. The sieve analysis (see Fig. 1) shows it to be fairly well graded. The amount of suspended matter is 6 per cent, the specific gravity 2.665, the weight per cu. ft. 113 lb., and the percentage of voids 32.0. This sand ranks sixth in tensile strength, a 1:3 mortar giving a strength of 386 lb. per sq. in. at 90 days, 104.9 per cent of the strength of the Ottawa standard sand mortar.

Sample No. 9 (See Fig. 7).—This is a sample of the sand used in the construction of the Illinois Supreme Court building at Springfield. It is a bank sand from near Lincoln, and is dark gray in color due to nearly half of its mass being composed of dark colored flints and granites,

the remainder being principally quartz with some limestone. It is fairly well graded (see Fig. 2), 65.8 per cent passing the No. 16 sieve and 6.5 per cent the No. 60. It contains 1.1 per cent of clay which adheres to the grains and which possibly accounts for the lower tensile strength than appears to be indicated by the other characteristics of the sand; but this can not be stated definitely as no test was made on the washed sand. It ranks fourteenth in tensile strength, a 1:3 mortar showing a strength of 303 lb. per sq. in. at 90 days, 82.4 per cent of the strength of the Ottawa standard sand mortar. The specific gravity is 2.65, the weight per cu. ft. 111.9 lb., and the percentage of voids 32.4.

Sample No. 10 (See Fig. 7).—This is a bar sand from the Mississippi River near Alton. It is dark gray in color, and contains some limestone and flint. There is 3 per cent of suspended matter present. The sieve analysis (see Fig. 2) shows it to be very fine, 90.3 per cent passing the No. 16 sieve. The specific gravity is 2.63, the weight per cu. ft. 112.5 lb., and the percentage of voids 31.5. The tensile strength of 1:3 mortar at 90 days was 269 lb. per sq. in., 73.2 per cent of that of the Ottawa standard sand mortar. It ties with sample No. 16 for twentieth place in the order of tensile strength.

Sample No. 11 (See Fig. 7).—This is also a bar sand from near Alton. It is brownish gray in color and contains some flint and limestone. There is also a small amount of coal and cinders present, probably as the result of transportation by rail. The finer grains are well rounded but the coarser ones are quite angular. This sand is quite fine although not quite as fine as sample No. 10 (see Fig. 2). The specific gravity is 2.66, the weight per cu. ft. 114.5 lb., the percentage of voids 31.0, and the suspended matter amounts to 2 per cent. It ranks eighteenth in tensile strength, a 1:3 mortar having a strength of 280 lb. per sq. in. at 90 days, 76.2 per cent of that of the Ottawa standard sand mortar.

Sample No. 12 (See Fig. 8).—This sand, used in large quantities in and about the cities of Champaign and Urbana, is a bank sand from along the Wabash River near Covington, Ind. It is yellowish gray in color and contains some limestone, flint, and shale. The grains are all well rounded. This sand is fairly well graded but has an excess of material between the No. 8 and No. 16 sieves (see Fig. 2). It contains 1.5 per cent of suspended matter in the form of loose clay. The specific gravity is 2.66, the weight per cu. ft. 110.4 lb., and the percentage of voids 33.5. The tensile strength of 1:3 mortar at 90 days was 310 lb. per sq. in., 84.4 per cent of that of the Ottawa standard sand mortar. This sand ranks twelfth in tensile strength.

Sample No. 13 (See Fig. 8).—This is a bank sand from near Gladstone. It is gray in color and contains some flint and granite, the grains being moderately angular. It is quite fine, 97.9 per cent passing the No. 16 sieve and 12.9 per cent the No. 60 (see Fig. 2). This is the cleanest of the bank sands, there being but 0.2 per cent of suspended matter present. The specific gravity is 2.655, the weight per cu. ft. 106.3 lb., and the percentage of voids 35.8. It ranks twenty-fifth in tensile strength, a 1:3 mortar showing a strength at 90 days of 241 lb. per sq. in., 65.5 per cent of that of the Ottawa standard sand mortar.

Sample No. 14 (See Fig. 8).—This sand is screened from bank gravel found along the Sangamon River near Decatur. It is brownish gray in color and contains considerable flint, granite, limestone, and shale, with some softer rocks containing iron oxide. As shown by Fig. 2, this is the coarsest sample of natural sand tested. It might therefore be expected that this sand would yield a mortar next in strength to the limestone screenings, and it probably would but for the fact that many of the grains are so soft that they were ruptured in the tensile tests, showing that they were weaker than the adhesion of the cement to them. In spite of this it makes a very strong mortar, ranking fourth in this respect, a 1:3 mixture having a strength at 90 days of 428 lb. per sq. in., 116.4 per cent of that of the Ottawa standard sand mortar. The specific gravity is 2.66, the weight per cu. ft. 115.8 lb., and the precentage of voids 30.3.

Sample No. 15 (See Fig. 8).—This is a bank sand from near Freeport. It is yellow in color due to 1.3 per cent of clay which adheres very tightly to the sand grains. It is composed principally of quartz with some flint, limestone, and granite. It is moderately fine, 82.5 per cent passing the No. 16 sieve (see Fig. 2). The specific gravity is high, being 2.72; and the grains are quite hard and well rounded. This sand would probably be improved by washing, provided it could be done without washing out the fine sand, since the suspended matter forms a coating on the grains. The tensile strength of 1:3 mortar was 317 lb. per sq. in. at 90 days, 86.2 per cent of that of the Ottawa standard sand mortar. It ranks ninth in tensile strength. The weight per cu. ft. is 113.9 lb. and the percentage of voids 33.0.

Sample No. 16 (See Fig. 8).—This is a sample of Freeport sandstone screenings. The stone is rather soft which probably accounts for the comparatively low tensile strength, the fragments themselves being frequently ruptured in the tests. This sample is deficient in fine material (see Fig. 2), only 4.2 per cent passing the No. 60 sieve. This is due

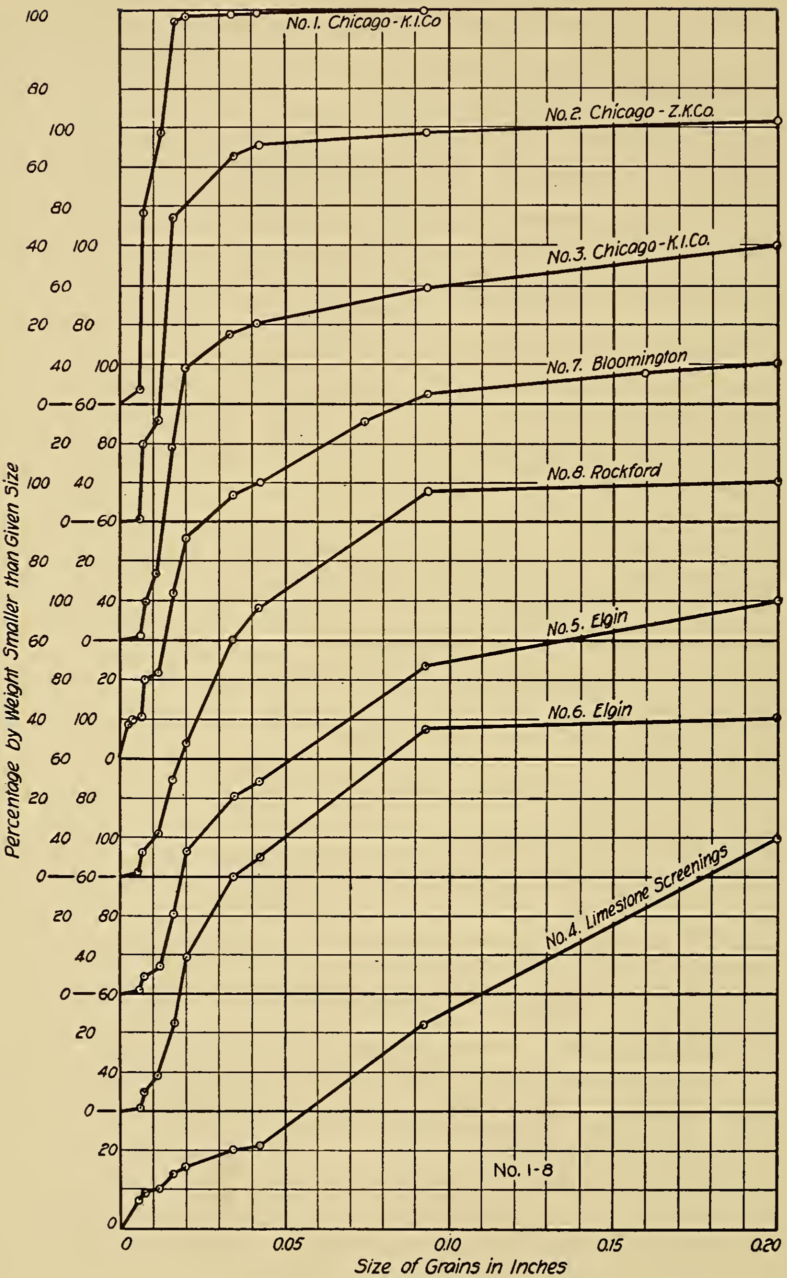


FIG. 1. SIEVE ANALYSIS CURVES FOR SAMPLES 1 TO 8.

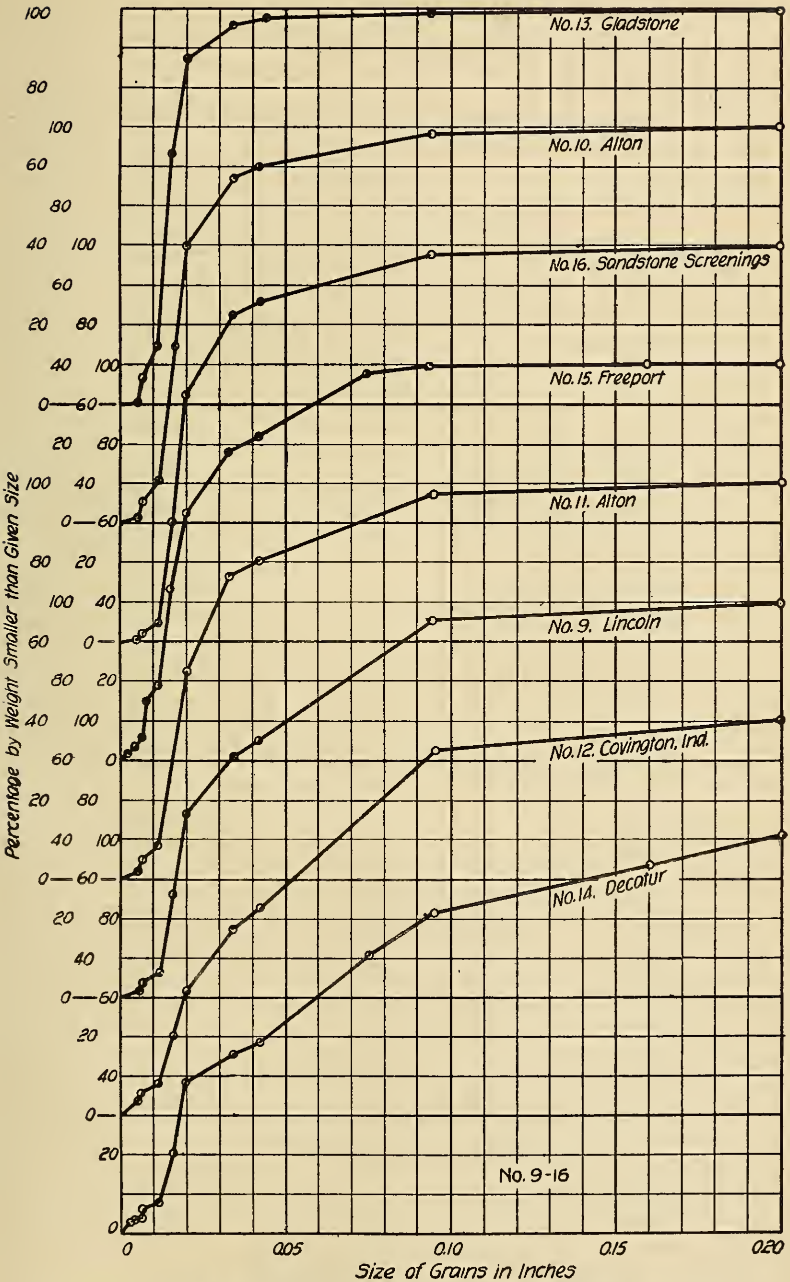


FIG. 2. SIEVE ANALYSIS CURVES FOR SAMPLES 9 TO 16.

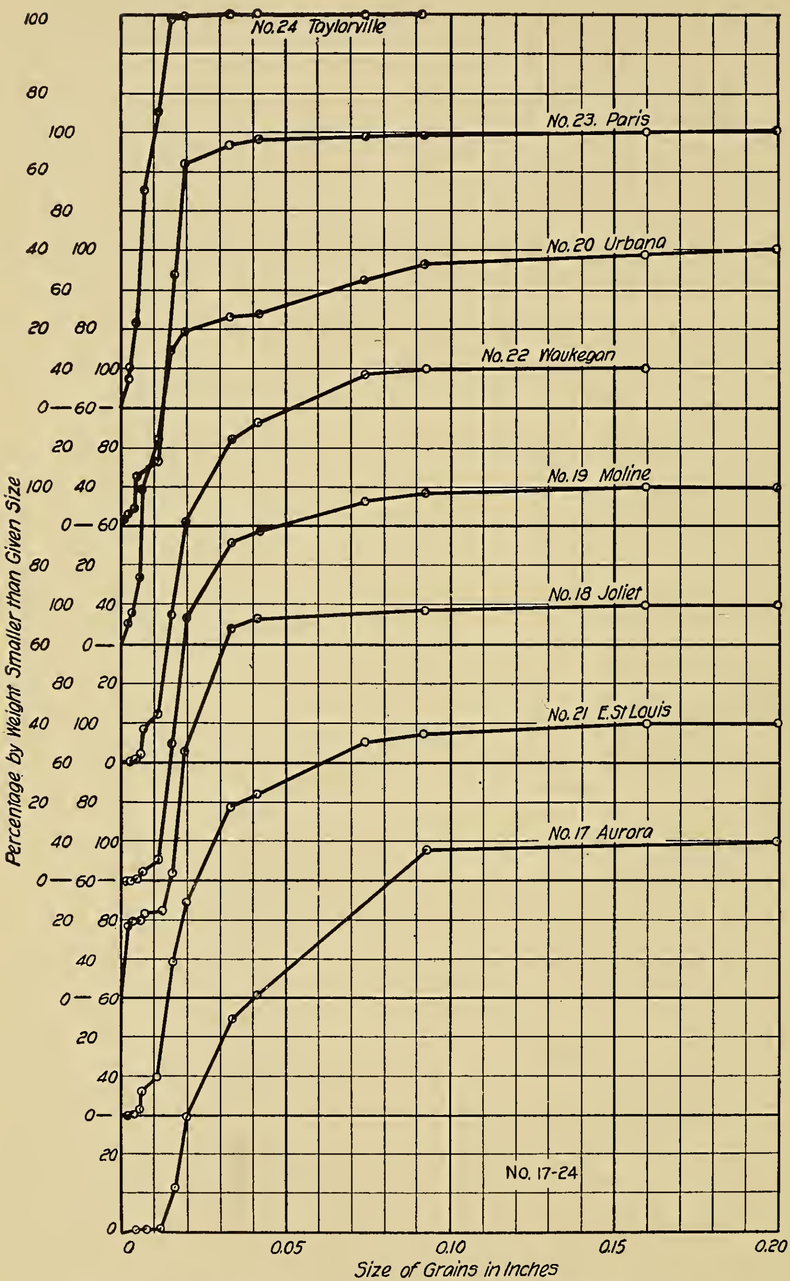


FIG. 3. SIEVE ANALYSIS CURVES FOR SAMPLES 17 TO 24.

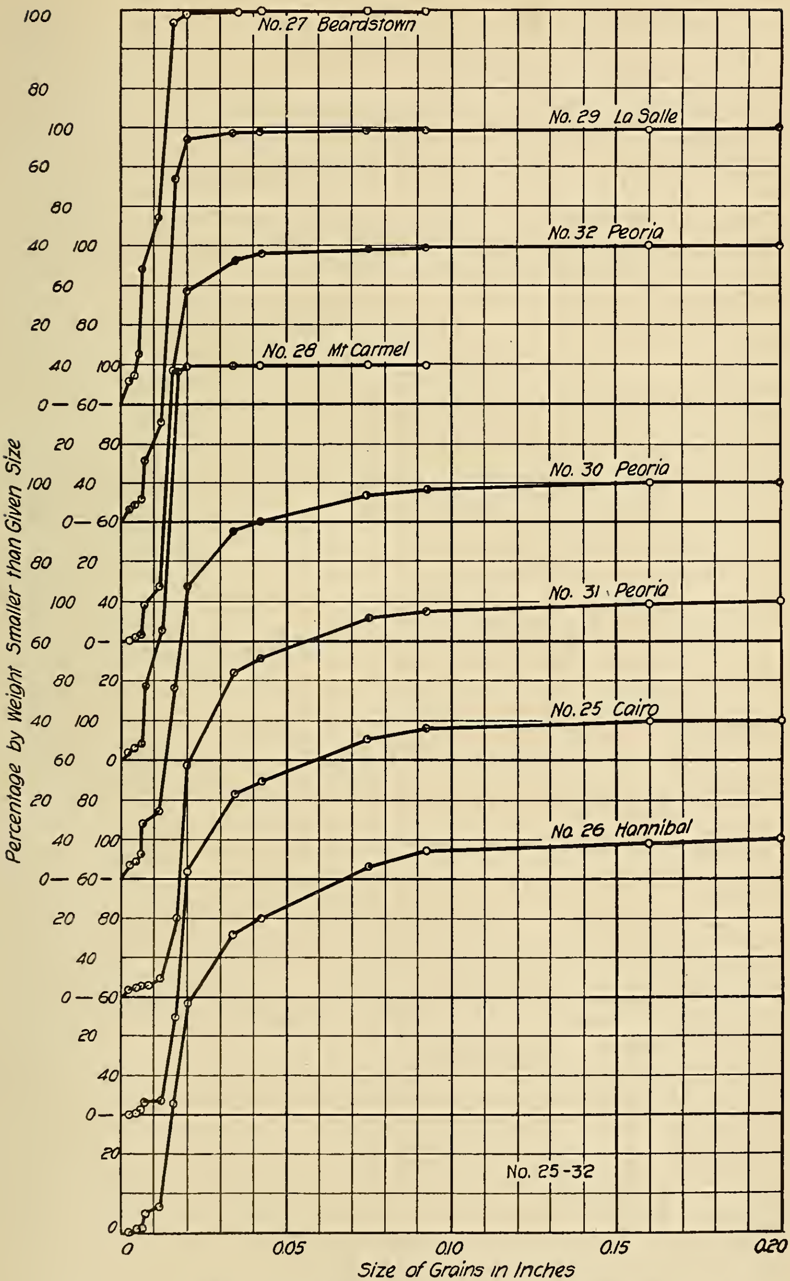


FIG. 4. SIEVE ANALYSIS CURVES FOR SAMPLES 25 TO 32.

to the fact that the stone is composed of quartz sand cemented together and in the crushing the sand grains are simply separated without being broken up. There is no suspended matter, the specific gravity is 2.705, the weight per cu. ft. 112.2 lb., and the percentage of voids 33.0. The tensile strength of 1:3 mortar at 90 days was 269 lb. per sq. in., 73.2 per cent of that of the Ottawa standard sand mortar. This sample ties with sample No. 10 for twentieth place in the order of strength. It compares quite favorably with the average natural sand but it is not the equal of the best natural sands in this series.

Sample No. 17 (See Fig. 8).—This sand is screened from a bank gravel found near Aurora. It is yellowish in color and is composed almost entirely of quartz with some shale. The grains are rounded and the suspended matter, of which there is 0.5 per cent, adheres to them but is easily removed by wetting and therefore does not affect the tensile strength of the mortar. The sieve analysis indicates that while not well graded this sand is comparatively coarse, 61.3 per cent passing the No. 16 sieve (see Fig. 3). The specific gravity is 2.72, the weight per cu. ft. 111.7 lb., and the percentage of voids 34.2. Although not as well graded as sample No. 14, the tensile strength is higher since the grains are all hard and strong. It ranks third in tensile strength, a 1:3 mortar showing a strength at 90 days of 454 lb. per sq. in., 123.3 per cent of that of the Ottawa standard sand mortar.

Sample No. 18 (See Fig. 9).—This sand, which comes from the Fuller pit near Joliet, presents some rather unusual characteristics. The color is reddish yellow, due to the large amount of clay present. The grains appear quite uniform in size and rounded in shape and the sieve analysis shows that the greater part of the material lies between the No. 20 and No. 60 sieves and also that there is a very large amount of fine material, including the suspended matter (see Fig. 3). The suspended matter amounts to 18.3 per cent by weight of the total sand, and consists almost entirely of clay which adheres very tightly to the sand grains, each grain being entirely coated to such an extent that it is impossible to recognize the materials of which the grains are formed without first washing the sand. On wetting, this clay becomes gummy making it impossible to wash it from the sand without resorting to a thorough "scrubbing", and hence it is doubtful if this sand could be economically washed on a commercial scale. After a thorough washing the sands present quite a marked change in appearance. The color is gray, and the grains are quite angular, this sample being the sharpest sand in the series with the exception of sample No. 21. The composition is shown

to be about half quartz, the other materials being limestone and granite with large proportions of shale and other soft rocks. The sand as a whole is of very poor quality, due to the large amount of soft materials, the poor grading, and the abnormal amount of suspended matter and the way in which it coats the sand grains. The unwashed sand ranked thirty-first (next to lowest) in tensile strength, a 1:3 mortar having a strength at 90 days of only 159 lb. per sq. in., only 43.3 per cent of that of the Ottawa standard sand mortar. The tensile strength at 28 days (not tested at 90 days) was increased about 35 per cent by thoroughly washing the sand. The specific gravity is 2.66, the weight per cu. ft. 100.2 lb., and the percentage of voids 39.7.

Sample No. 19 (See Fig. 9).—This is a bar sand from the Mississippi River a few miles above Moline. The bars are constantly shifting and therefore the sand is more or less variable in character. It is gray in color and is composed principally of quartz with some flint, limestone, and granite. It is quite fine, 88.6 per cent passing the No. 16 sieve (see Fig. 3). There is a decided deficiency of fine material as it contains no suspended matter and only 0.2 per cent passes the No. 100 sieve. As a result, the tensile strength is less than would be expected from the other characteristics of the sand, and it would probably improve the mortar to add some fine material to it. It ranks twenty-fourth in tensile strength, the strength of 1:3 mortar at 90 days being 257 lb. per sq. in., 69.8 per cent of that of the Ottawa standard sand mortar. The specific gravity is 2.65, the weight per cu. ft. 112 lb., and the percentage of voids 32.3.

Sample No. 20 (See Fig. 9).—This sand, obtained from banks of limited extent near Urbana, is quite variable in quality but the sample used in these tests was fairly representative. It is yellowish in color and contains some granite, flint, shale, and chert. The grains are rounded and are coated to a considerable extent with clay of which there is 3.5 per cent. It is not well graded, there being a large amount of fine material and a fair proportion of coarse material, but a marked deficiency in grains of intermediate size (see Fig. 3). When wet the sand behaves considerably like quick-sand, and on drying shows a decided tendency to form lumps. The use of this sand has been practically abandoned on account of its variable character and the limited amounts available, and because a good sand of uniform quality (sample No. 12) can be shipped in at a price sufficiently low to compete with it. The specific gravity is 2.65, the weight per cu. ft. 107.5 lb., and the percentage of voids 35.0. The tensile strength of 1:3 mortar at 90 days was 330 lb. per sq. in., 89.7 per cent of that of the Ottawa standard sand mortar. This sand ranks eighth in tensile strength.

Sample No. 21 (See Fig. 9).—This sand is dredged from the bed of the Mississippi River at East St. Louis and varies but little in quality. It is light gray in color and contains some granite and flint, but the bulk of the material is pure quartz and is almost transparent. This is the sharpest of the natural sands in this series, the grains being very irregular in shape with the edges only slightly rounded. The faces of the grains are all very smooth, those of quartz having the appearance of broken glass. The tensile strength of the mortar was doubtless reduced by this condition, since an examination of the broken briquettes showed that the cement did not adhere well to the grains. This sand is not well graded (see Fig. 3); but the percentage of voids is the lowest of the natural sands, being 30.1 per cent. This is probably due to the extreme "sharpness" of the sand. The specific gravity is 2.65, and the weight per cu. ft. 115.8 lb. The tensile strength of 1:3 mortar at 90 days was 258 lb. per sq. in., 70.2 per cent. of that of the Ottawa standard sand mortar. It ranks twenty-third in strength.

Sample No. 22 (See Fig. 9).—This sand is taken from the shore of Lake Michigan near Waukegan and varies in quality from time to time, as the result of storms. This sample is dark gray in color. The principal components are quartz, flint, limestone, granite, sandstone, and shale. There is also a considerable amount of coal and cinders present, the quantities of each indicating that their presence can not be attributed to transportation. This sand is quite fine, 85.8 per cent passing the No. 16 sieve (see Fig. 3). The specific gravity is 2.68, the weight per cu. ft. 112.5 lb., and the percentage of voids 32.7. It ranks sixteenth in strength, 1:3 mortar showing a tensile strength at 90 days of 296 lb. per sq. in., 80.5 per cent of that of the Ottawa standard sand mortar.

Sample No. 23 (See Fig. 9).—This is a bank sand from limited but uniform deposits near Paris, Ill. It is very fine, 98.1 per cent passing the No. 16 sieve, and there is only an occasional large grain (see Fig. 3). The grains are all well rounded and are composed principally of quartz with some limestone and chert. The color is pale yellow. The suspended matter amounts to 1.2 per cent, the specific gravity is 2.665, the weight per cu. ft. 108.3 lb., and the percentage of voids 35.2. It ranks twenty-second in tensile strength, 1:3 mortar having a strength at 90 days of 259 lb. per sq. in., 70.4 per cent of that of the Ottawa standard sand mortar. This sand is too fine to be desirable for ordinary purposes, but if the large grains were screened out it would form an excellent grout sand.

Sample No. 24 (See Fig. 10).—This is a bank sand from limited

but very uniform deposits near Taylorville. The mortar made of this sand is reported as working best when the proportion of cement is small, and considerable quantities of it have been used this way, but as would be expected it has not proved satisfactory. The sand is light yellow in color and the grains are well rounded. The principal components are quartz, limestone, and shale. This sand is the finest in this series, 100 per cent passing the No. 16 sieve, 99.5 per cent the No. 40, and 75.4 per cent the No. 60 (see Fig. 3). It is also the lightest sand, its weight per cu. ft. being only 99.7 lb. There is 4 per cent of suspended matter, and the sand shows a decided tendency to form lumps on drying. The specific gravity is 2.635 and the percentage of voids 39.4. The tensile strength of 1:3 mortar at 90 days was 211 lb. per sq. in., 57.4 per cent of that of the Ottawa standard sand mortar. It ranks twenty-sixth in tensile strength. Like sample No. 23 this sand is too fine for ordinary purposes but would be quite satisfactory as a grout sand.

Sample No. 25 (See Fig. 10).—This sand, furnished by the Halliday Sand Company, Cairo, is pumped from the bed of the Ohio River above that city. It is gray in color and contains some granite and limestone, the grains all being somewhat rounded. It is moderately fine (see Fig. 4), and is deficient in both coarse and very fine grains. The suspended matter amounts to 0.3 per cent. The specific gravity is 2.645, the weight per cu. ft. 107.8 lb., and the percentage of voids 34.7. The tensile strength of 1:3 mortar at 90 days was 298 lb. per sq. in., 81 per cent of that of the Ottawa standard sand mortar. This sand ranks fifteenth in strength.

Sample No. 26 (See Fig. 10).—This sand is taken from the bed of the Mississippi River near Hannibal, Mo. It is bluish gray in color and contains considerable flint, granite, and chert, all of the grains being moderately round. The sieve analysis shows it to be deficient in both coarse and very fine grains (see Fig. 4). There is 0.2 per cent of suspended matter present. The specific gravity is 2.68, the weight per cu. ft. 115 lb., and the percentage of voids 31.3. It ranks nineteenth in tensile strength, 1:3 mortar showing a strength at 90 days of 279 lb. per sq. in., 75.8 per cent of that of the Ottawa standard sand mortar.

Sample No. 27 (See Fig. 10).—This is a bank sand from deposits of more or less variable character near Beardstown. It is reddish yellow in color due to the 4.4 per cent of reddish clay which forms a coating around the grains. It contains some granite, limestone, flint, and considerable amounts of soft materials, and shows a tendency to form lumps. The sieve analysis (see Fig. 4) shows that this sand is next in fineness to

sample No. 24, practically all passing the No. 16 sieve and 47.4 per cent the No. 60. This sand has been used a great deal for pavement foundations, common brick work, etc., and has given fairly satisfactory service although it is entirely too fine and too deficient in strength for good results. It yielded the weakest mortar tested, the strength of a 1:3 mixture at 90 days being only 140 lb. per sq. in., 38.1 per cent of that of the Ottawa standard sand mortar. This low strength is due both to the coating of clay on the sand grains and to the fineness and softness of the grains themselves. The tensile strength is so low that it would be advisable, and perhaps more economical, to abandon the use of this sand entirely. The specific gravity is 2.605, the weight per cu. ft. 101.0 lb., and the percentage of voids 37.9.

Sample No. 28 (See Fig. 10).—This is a bar sand from the Wabash River near Mt. Carmel. It is yellowish in color and contains some flint, granite, and cinders. It is nearly as fine as sample No. 27 (see Fig. 4). It is the finest and also the dirtiest of the river sands, containing 2.4 per cent of suspended matter. The grains are well rounded. In common with the other fine sands, it forms a mortar of low strength, a 1:3 mixture showing a strength of only 170 lb. per sq. in. at 90 days, 46.2 per cent of that of the Ottawa standard sand mortar. The specific gravity is 2.635, the weight per cu. ft. 104.4 lb., and the percentage of voids 36.5. It ranks thirtieth in strength.

Sample No. 29 (See Fig 10).—This is a bank sand from along the Little Vermillion River above La Salle. It is gray in color and is composed almost entirely of quartz and the grains are well rounded. It is very fine, 99.3 per cent passing the No. 16 sieve and 26 per cent the No. 60 (see Fig. 4). There is 3.2 per cent of suspended matter present. The specific gravity is 2.65, the weight per cu. ft. 106.7 lb., and the percentage of voids 35.5. It ranks twenty-eighth in tensile strength, 1:3 mortar showing a strength at 90 days of 194 lb. per sq. in., 52.7 per cent of that of the Ottawa standard sand mortar.

Sample No. 30 (See Fig. 11).—This sand is from the Crescent Gravel pit near Peoria. It is yellowish in color and contains some shale, flint, and granite. The grains are well rounded and quite fine, 90.3 per cent passing the No. 16 sieve and 17.8 per cent the No. 60 (see Fig. 4). It contains 2.7 per cent of suspended matter. The specific gravity is 2.665, the weight per cu. ft. 111.9 lb., and the percentage of voids 32.9. The tensile strength of 1:3 mortar at 90 days was 262 lb. per sq. in., 71.2 per cent of that of the Ottawa standard sand mortar. It ranks twenty-first in strength.

Sample No. 31 (See Fig. 11).—This sand was taken from a large sewer trench near the center of the city of Peoria and was being used (1908) in the construction of the sewer. This sand is yellowish gray in color and contains some flint, granite, limestone, and chert. The medium-sized grains are more angular than either the coarser or the finer. It contains 1.9 per cent of suspended matter. It is quite fine, 86.5 per cent passing the No. 16 sieve (see Fig 4). This sand formed a stronger mortar than the other sands from the vicinity of Peoria. The tensile strength of 1:3 mortar at 90 days was 306 lb. per sq. in., 83.2 per cent of that of the Ottawa standard sand mortar. It ranks thirteenth in strength. The specific gravity is 2.64, the weight per cu. ft. is 103.9 lb., and the percentage of voids is 36.9.

Sample No. 32 (See Fig. 11).—This is a bar sand from the Illinois River above Peoria. It is brownish gray in color and only about half of the grains are quartz, the remainder being flint, limestone, granite, chert, and cinders. The suspended matter amounts to 0.6 per cent. This sand is very fine (see Fig. 4), 97.1 per cent passing the No. 16 sieve. The specific gravity is 2.66, the weight per cu. ft. is 108.7 lb., and the percentage of voids is 34.6. The tensile strength of 1:3 mortar at 90 days was only 178 lb. per sq. in., 48.4 per cent of that of the Ottawa standard sand mortar. It ranks twenty-ninth in strength.

IV. Discussion of Tests.

Although this series of tests includes only a small proportion of the different sands used in Illinois, it is believed that a sufficient number of samples, well distributed geographically, have been tested to serve as an index to the mortar-making qualities of the sands available in various sections of the State, and at the same time afford some information relative to the mortar-making qualities of sands in general. Following is a brief discussion of the results of this series of tests, together with the conclusions which may be drawn.

12. *Mineralogical Composition.*—The mineralogical composition of a sand is the fundamental factor in its mortar-making qualities, since not only its durability and hence the durability of the mortar but the size and gradation of the grains, the nature of the grain surfaces, the strength of the grains themselves, and all the other factors which affect the strength of the mortar are more or less directly dependent on the nature of the component materials of the sand.

The samples tested indicate that the majority of Illinois sands are

composed of satisfactory materials. Quartz is the principal component, forming the bulk of the medium-sized grains, and in most cases more than half of the entire volume of the sand. Limestone is the next in importance, while the other rocks form comparatively a small proportion of the whole.

13. *Specific Gravity.*—The specific gravity of a sand affords but little information relative to its mortar-making qualities, its principal value being as a factor in certain computations. Quartz has a specific gravity of about 2.65; and the nearer the specific gravity of a sand approaches this value the greater is the content of silicious material. A higher value indicates considerable quantities of materials other than quartz, which are likely to be hard and durable; while a lower value usually indicates the presence of soft, unsatisfactory material, or of considerable quantities of clay and loam or other foreign matter. The specific gravities of the samples tested ranged from 2.60 to 2.75.

14. *Sharpness*.*—Angularity or irregularity of the sand grains appears to exert no effect on the tensile strength of the mortar. In compression, the sharp sands may show a slight advantage due to the interlocking of the angular grains; but evidently such action is insignificant as compared with the resistance to displacement of the grains afforded by the bond between them due to the adhesion of the cement to their surfaces, hence the strongest mortars are invariably those in which the cement most readily adheres to the sand grains. Crystalline rocks when freshly fractured generally show surfaces of great smoothness to which the cement does not adhere well; but when these grains have been worn down the surfaces become roughened and the cement adheres much more readily. This is particularly true of quartz as is evidenced by the fact that rounded silicious sands usually form mortars of greater strength than similar sharp sands. For example, note that sample No. 21 with its very sharp, glass-like grains gives only 70.2 per cent as much strength as the Ottawa standard sand with its spherical dull-surfaced grains, although the former is the better graded and the two are practically identical in composition and cleanness. With sands composed of rocks which naturally show a rough granular fracture this advantage of round grains is lost, but in any case mortars made of round-grained sands will compact more readily than those of sharp sands; hence such mortars in place are likely to be more compact and dense, which conduces to greater strength. The usual requirements of specifications that sands for mor-

*Sharpness is here used in its common meaning of angularity or irregularity in shape.

tar and concrete shall be sharp is not only useless but may even be detrimental and should therefore be omitted. Further, since the condition of the grain surfaces does materially affect the strength of the mortar, the specifications should fully cover this point.

15. *Voids.*—The percentage of voids in dry sand is a function of its compactness and the gradation of sizes in the sand grains, hence the effect of the voids on the strength of the mortar is included with that of the gradation of the sizes of the grains. The percentage of voids is valuable in determining the amount of cement necessary to give the densest mortar. The sands tested show voids varying from 28.5 to 39.4 per cent. The average sands contained 32 to 34 per cent when dry and well compacted.

16. *Size and Gradation of Grains.*—It has been demonstrated both by experiment and practice, that coarse sands will yield denser mortars than fine sands, and that the maximum density is obtained when the various sizes of grains are present in the proper proportions. Just what the proper proportions are, or in other words, what the ideal form of the sieve analysis curve is, has not as yet been determined. Experiments with materials for concrete indicate that for a mixture of cement, sand, and stone the sieve analysis curve should approximate a parabola, and by analogy it would seem that the same should be true of a mixture of cement and sand as well. Assuming that the parabola is the ideal line for the mixture, and remembering that the cement is very fine*, it follows that the ideal line for the sand considered alone must lie *below* the parabola, and that it would be different for each proportion of cement used. But since the cement is very fine and forms a relatively large proportion in most mortars, the consequent variation in the ideal gradation for the sand considered alone will not be great; and hence it may be said that for the common proportions of cement and sand (1:1 to 1:4) the *flatter* the sieve analysis curve of the sand the denser will be the mortar made from it.

It has also been demonstrated that, other things being equal, the denser a mortar the greater is its strength; and hence the size and gradaion of the grains is indirectly a factor in the strength of the mortar. Unfortunately, the fact that "other things must be equal" has been frequently overlooked with disappointing and sometimes disastrous results. It is only when the various sands are identical in mineralogical composition, condition of the grain surfaces, cleanness, etc., that the size and

*Standard specifications require that at least 92 per cent must pass the No. 100 sieve.

gradation of the grains becomes the controlling factor in the strength of the mortar, and even then it is only relative. Thus if two sands are exactly alike in all respects except size and gradation of the grains, the better graded sand will yield the mortar of both greater density and greater strength; but this strength may be decidedly less than that of a mortar from a third sand which may be less perfectly graded than either of the two given sands but is superior in other characteristics. Consequently we may have two sands which have identical sieve analyses and yield mortars of the same density, and yet these mortars may differ greatly in strength; or, two sands which differ greatly in their sieve analyses may form mortars of the same strength, although different in density. For example, note that samples Nos. 2 and 12 give mortars of practically the same strength, although their sieve analyses, as shown by Figs. 1 and 2, are decidedly different; while samples Nos. 13 and 32 are almost identical in sieve analysis (see Figs. 2 and 4), but their mortars show a wide difference in strength.

Where strength is of the first importance in a mortar the only criterion is a direct test for strength; but after the absolute strength of the mortar from any one sand is determined, the sieve analysis may indicate whether the strength can be somewhat increased by improving the gradation of the sand. In general this improvement must be accomplished by judicious screening rather than by the addition of other material, since usually the added material will not be of the same general character as the given sand. Where a dense or impervious mortar is required, the size and gradation of the grains is of more importance; and if strength is secondary, may control the choice of the sand. In this case the sieve analysis curves may be of considerable assistance in selecting the proper sand, or in indicating whether the desired results can be obtained either by combining several sands or by screening. In no case, however, should the direct test for strength be omitted, for a certain amount of strength is always required, and a good result for the sieve analysis is not sufficient evidence that the mortar will have sufficient strength.

The maximum size of grain permissible depends on the work for which the mortar is intended. For concrete the maximum size is now taken at ¼ inch, but certain kinds of masonry, etc., require a much smaller maximum size. The maximum size of grain fixes the limits of sieve analysis, and consequently two sands may differ greatly in average fineness, and yet each one may be more perfectly graded and hence more suitable for some particular work than the other.

In general, the sands tested were quite fine and several of them

were extremely so. The bank sands showed the greatest difference in gradation. The river sands showed less difference in gradation than the bank sands, and although the latter also showed the greatest difference in strength, this is probably due more to a greater difference in the other characteristics than to a difference in gradation. In nearly every case there was a large amount of material caught between the No. 30 and the No. 60 sieves, and it is of interest to note that this material is nearly all silicious. It is also worthy of note that practically no quartz grains were retained on the No. 16 sieve. It is therefore to be expected that silicious sands will be comparatively fine and that the coarser sands will contain considerable quantities of less brittle rocks.

17. *Cleanness.*—Foreign material in a sand may affect the strength of the mortar by retarding or preventing the hardening of the cement, by preventing the adhesion of the cement to the sand grains, and if present in sufficient quantities by simple "dilution" of the cement. Organic matter is the most common source of trouble, but inert clay and loam may prove deleterious under certain conditions*. Experiments indicate that finely divided inert clay or loam may be present in an average sand to the extent of 10 to 15 per cent without appreciably affecting the strength of the mortar, provided the clay or loam does not adhere to the grains, while a very small quantity may seriously impair the strength of the mortar if it forms a coating around the sand grains.

As would be expected, the river sands are the cleanest, showing an average of only 0.3 per cent. of suspended matter while the bank sands averaged 2.2 per cent even after omitting sample No. 18. This sand was the dirtiest one in the series. It contained 18.3 per cent of clay which adhered tightly to the grains and decreased the strength of the mortar both by its large quantity and by preventing a perfect bond between the cement and the sand grains. Sample No. 7 was the next dirtiest, containing 8 per cent of clay, but in contrast with No. 18 it did not coat the grains, and a test of the washed sand showed no increase in strength of the mortar over that of the unwashed sand, showing that even this amount of clay, when loose, had no appreciable effect on the strength of the mortar. In every case the suspended matter was inert clay or loam. It may be expected that in general Illinois sands will be free from injurious silts. It must not be forgotten, however, that appearances are deceiving and that a test is the only sure method of determining the quality of the sand or the effect of foreign matter.

*For a very interesting discussion of the effect of organic silt on mortar, see Transactions of the American Society of Civil Engineers, Vol. 65, pp. 250-273.

18. *Tensile Strength.*—The tensile strength of the natural sand mortars varied from 140 to 464 lbs. per sq. in. at the age of 90 days. By comparing the tensile strengths with the sieve analysis curves, it will be seen that the coarser sands give the mortars of greater strength, and it will be noted that several of the sands show almost identical curves but that their tensile strengths are quite different, while still others giving practically the same strength differ greatly in their sieve analysis curves. An examination of the sands themselves shows that this variation is due to some of the other characteristics of the sands. Only five of the natural sands gave a tensile strength greater than the Ottawa standard sand; and it will be noted that all of these are bank sands with rounded grains, comparatively well graded and containing considerable material other than quartz. The river sands showed less variation in strength than the bank sands but this is to be expected since they differ less in composition, grading, and cleanness. It may, therefore, be expected that river sands will form mortars of only moderate strength, and that there is likely to be little variation between different sands; and that bank sands will furnish the mortars of greatest strength, but that the strength from different sands will vary through a wide range.

In an experimental investigation there is often a question as to how many individual results are necessary for the accurate determination of the result. It will be found that in any extended series of observed values of a single quantity one value is considerably less than the average, another considerably greater than the average, while the remaining values fill in the interval between these extremes more or less completely. If the values are platted as ordinates in the order of their magnitude with uniform horizontal spacing, the resulting curve will be found to have the form of an ogee or reversed curve, and the average value will lie at the point of reversal of the ogee. If the values are too few in number, the curve will be discontinuous, and the average value cannot be said to be accurately determined. As the number of values is increased the curve will become smoother and more complete; and it is only when the typical ogee form is distinctly and continuously outlined that the average value of the results can be considered as accurately determined.

Fig. 5 shows the results of the tensile tests of the mortars platted in the above manner, the ordinates being the strengths of the individual mortars in terms of the strength of standard Ottawa sand mortar. It will be observed that this figure shows the characterstic ogee curve quite distinctly, and therefore it may be considered that a sufficient number of

samples have been tested to yield a fairly representative average value of the strength which may be expected of mortars made from Illinois sands. This figure also shows graphically that the extreme values range from less than one-half of that of the standard Ottawa sand mortar to nearly one-third greater, and that the average value (at the point of reversal of the ogee) is about 80 per cent of the strength of the standard Ottawa sand mortar.

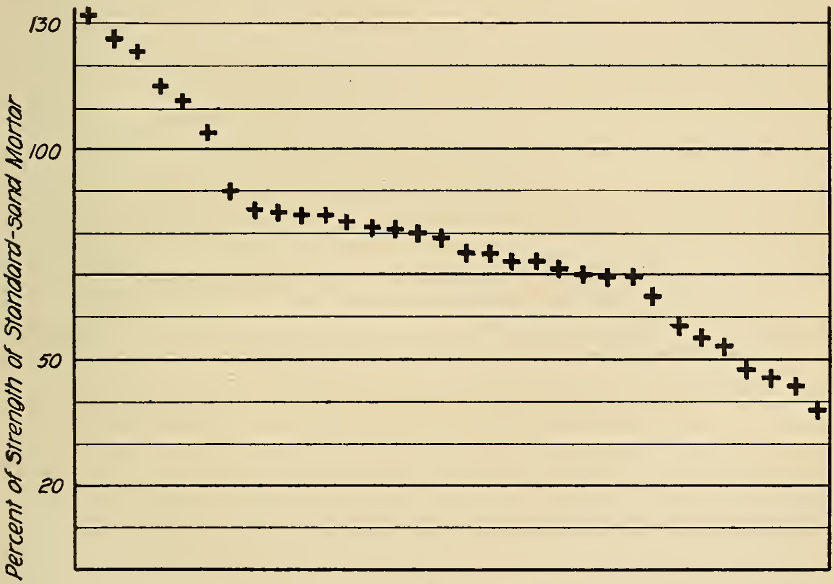


FIG. 5. STRENGTH OF BRIQUETTES MADE OF ILLINOIS SANDS IN TERMS OF THE STRENGTH OF STANDARD OTTAWA MORTAR.

It will be noted that in making these comparisons the tensile strength at 90 days is used. This age was chosen in that the results most nearly represent the ultimate strength of the mortars. A study of Table 1 will show that the mortars differ widely in the time required to attain approximately their maximum strength, and that the weak mortars invariably require less time than the strong mortars. This is explained by the fact that the strength of a mortar is largely dependent on the bond between the cement and sand. With poor sands the maximum bond is reached long before the cement has reached its full strength, while with the better sands the maximum bond is not attained until the cement has more completely hardened, and hence more time is required. Seven, fourteen, and possibly twenty-eight-day tests are there-

fore not absolutely reliable for comparing the ultimate strength of different sand mortars. For such comparisons the longest time available should be used. The short time tests are, however, of value in determining how soon it is permissible to remove forms or to subject the material to limited use.

19. *Crushed Stone Screenings.*—Although the limestone screenings (sample No. 4) formed the strongest mortar, it must not be concluded that all screenings are superior to natural sands. This sample of screenings was of exceptional quality in every respect, while the natural sands tested were the average quality met in practice. The sandstone screenings (sample No. 16) gave only medium results. The results for the limestone and the sandstone screenings indicate that there may be as great a variation in the mortars made from screenings as in those made from natural sands. These two samples of screenings are not sufficient to warrant any general conclusions as to the relative merits of sands and screenings except that under favorable conditions of quality of stone and of crushing, screening, and handling, the screenings may compete favorably with good natural sands.

20. *Proportioning.*—These tests indicate that the variation in the mortar-making qualities of sands may be so great that a stronger mortar may be obtained from a lean mixture of one sand than from a much richer mixture of another sand, and since the cement is the most expensive part of the mortar a substantial saving in the cost of materials may be effected by a judicious choice of the sand. For example, assume that a certain sand requires a 1:1 mixture by weight to give sufficient strength. If the cement cost $1.50 per barrel and the sand $0.40 per cu. yd., the materials for a cubic yard of mortar would be $6.67.* If the same strength could be secured with a 1:3 mixture of another sand which costs $1.50 per cu. yd., the cost of the materials for a cubic yard of mortar would be only $4.71. Thus a saving of $1.96 per cubic yard of mortar could be effected by simply substituting a leaner mixture of a better sand although it is more expensive. It is thus seen that the *cost per unit of strength* of mortar can be materially reduced by a proper choice of materials. The same thing likewise applies to concrete, since the strength of the concrete is governed by the quality of the mortar entering into it and also by the quality of the aggregate. Thus a 1:3:6 mixture using one kind of sand and stone may form a stronger concrete than a 1:2:4 mixture using another kind of sand and stone and the same

*For data on the quantities required, see Baker's Masonry Construction, 10th ed., p. 120.

cement. If the unit cost of the material is the same and strength is the governing factor, it is evident that a considerable saving could be effected by using the leaner mixture of the better materials, and a higher price could often be paid for these materials and still yield a substantial saving.

The selection of the proper proportions for a mortar is essentially a problem in economics, whether the result to be attained is a mortar or concrete of maximum strength, or one of the greatest density and imperviousness, or simply one of sufficient strength for the purpose in hand. The object in any case is to obtain the desired result with the minimum cost. If this fact is kept in mind, it will become clear that it may really be true economy to spend what appears to be a considerable amount of time and money in investigating the available sands and determining the best combinations. Obviously, the saving to be effected varies with the magnitude of the work, and hence the bigger the job the more important it is that the materials be properly selected. If such investigations were generally made it would certainly result in reducing the careless and extravagant use of cement, the most costly material entering into mortar or concrete.

V. Specifications for Sand.

21. *Need for Definite Specifications.*—It is generally acknowledged that the specifications most frequently used for sand are inadequate in that they are too brief or too indefinite to secure the desired results. Recent specifications have overcome these defects in some respects, but most of them are objectionable in that they are too inflexible, *i. e.,* fail to allow variations in the quality of the sand to meet varying conditions or different requirements, or else by placing undue stress on some particular requirement bar from use sands which would prove entirely satisfactory. The following specifications have been prepared with the idea of giving this necessary flexibility, and at the same time of making them sufficiently rigid. It is not intended, however, that these specifications should be used indiscriminately for all purposes but rather that they should serve simply as a guide in preparing the specifications for any particular piece of work. In preparing these specifications both the specifications proposed by the national engineering societies and the results of the test described in this bulletin have been taken as guides.

22. *Definition of Sand and Screenings.*—The term *sand* shall be understood to mean natural sand which will pass, when dry, a screen

having ¼ in. clear openings. Similar material which is the product of artificial crushing shall be known as *screenings*, and shall conform to the specifications for sand.

23. *Suggested Classification of Sands.*—Sands shall be classified as No. 1, No. 2, No. 3, plastering sand, and grout sand, the several grades being suitable for the following classes of work:

No. 1 sand is that required in reinforced concrete and in other work requiring a mortar of maximum strength and density.

No. 2 sand is that required in work not demanding maximum strength or density but still requiring a mortar of high quality.

No. 3 sand is that required where high strength or density are not controlling factors.

Plasterng sand is that for use in ordinary plastering over masonry, concrete, and wood or metal lath. Either No. 3 sand or plastering sand is of high enough quality for use in lime mortars. The latter sand should be used where the thickness of the mortar joint is such as to require grains of small size.

Grout sand is that for use in pavement fillers and other work requiring a thin, smooth, free-running grout.

24. *Suggested Specifications for Sand.*—The author offers the following specifications for the various grades of sand according to the above classification. These specifications are based primarily upon the tests described in this bulletin, but it is hoped that they may be useful in preparing specifications for masonry work in general.

SPECIFICATIONS FOR NO. 1 SAND.

Composition.—No. 1 sand shall consist of grains from hard, tough, durable rocks, and be free from soft, decayed, or friable material.

Cleanness.—The sand must be free from lumps of clay, loam, or other foreign material. It shall not contain more than 2 per cent by weight of finely divided clay, loam, or other suspended matter when tested by washing in such a manner as to remove all such material without removing any of the finest sand; *provided*, that if the strength of the mortar made from the sand is greater than 110 per cent of the strength of a smilar mortar made with standard Ottawa sand, the amount of suspended matter may reach 3 per cent. This suspended matter must not form a coating around the grains to such an extent that such coating is not entirely broken up and removed from the grains by sprinkling with water or in the mixing of the mortar or concrete. The sand shall

be free from oily or greasy matter in any form and must contain no organic silt.

Roughness.—The grains shall have rough, unpolished surfaces to which the cement paste will readily adhere.

Size of Grains.—The grains shall be well graded in size from the finest to the coarsest. For the greatest density not more than 8 per cent by weight, including the suspended matter, shall pass the No. 100 sieve, and not more than 60 per cent the No. 16 sieve. If maximum density is not essential and the mortar yields the required strength, these quantities may be increased to 12 per cent and 75 per cent, respectively.

Voids.—The voids in the dry sand, when well shaken, shall not exceed 33 per cent of the total volume of the sand.

Tensile Strength.—Mortar, in the proportions of 1:3 by weight, when tested at an age of 28 days shall develop a tensile strength at least equal to the strength of a similar mortar made of the same cement and standard Ottawa sand tested at the same age.

SPECIFICATIONS FOR NO. 2 SAND.

General Requirements.—No. 2 sand shall meet the requirements for No. 1 sand in all respects except as follows:

Cleanness.—The suspended matter shall not exceed 6 per cent by weight when tested in the same manner as described for No. 1 sand.

Size of Grains.—Not more than 15 per cent by weight, including the suspended matter, shall pass the No. 100 sieve, and not more than 80 per cent the No. 16 sieve.

Voids.—The voids shall not exceed 35 per cent of the total volume.

Tensile Strength.—The tensile strength shall equal at least 80 per cent of that of the standard Ottawa sand mortar when tested as described for No. 1 sand.

SPECIFICATIONS FOR NO. 3 SAND.

No. 3 sand shall meet the requirements of No. 2 sand, except that the suspended matter may reach 8 per cent and the tensile strength be as low as 65 per cent of that of the standard Ottawa sand mortar.

SPECIFICATIONS FOR PLASTERING SAND.

Plastering sand shall meet the requirements for No. 3 sand in all respects except that for the finishing coat it shall be of the requisite fineness to give the desired finish.

SPECIFICATIONS FOR GROUT SAND.

Grout sand shall meet the requirements for No. 3 sand except as follows:

It shall all pass a No. 16 sieve. The voids shall not exceed 38 per cent of the total volume. The tensile strength shall be at least 40 per cent of that of the standard Ottawa sand mortar.

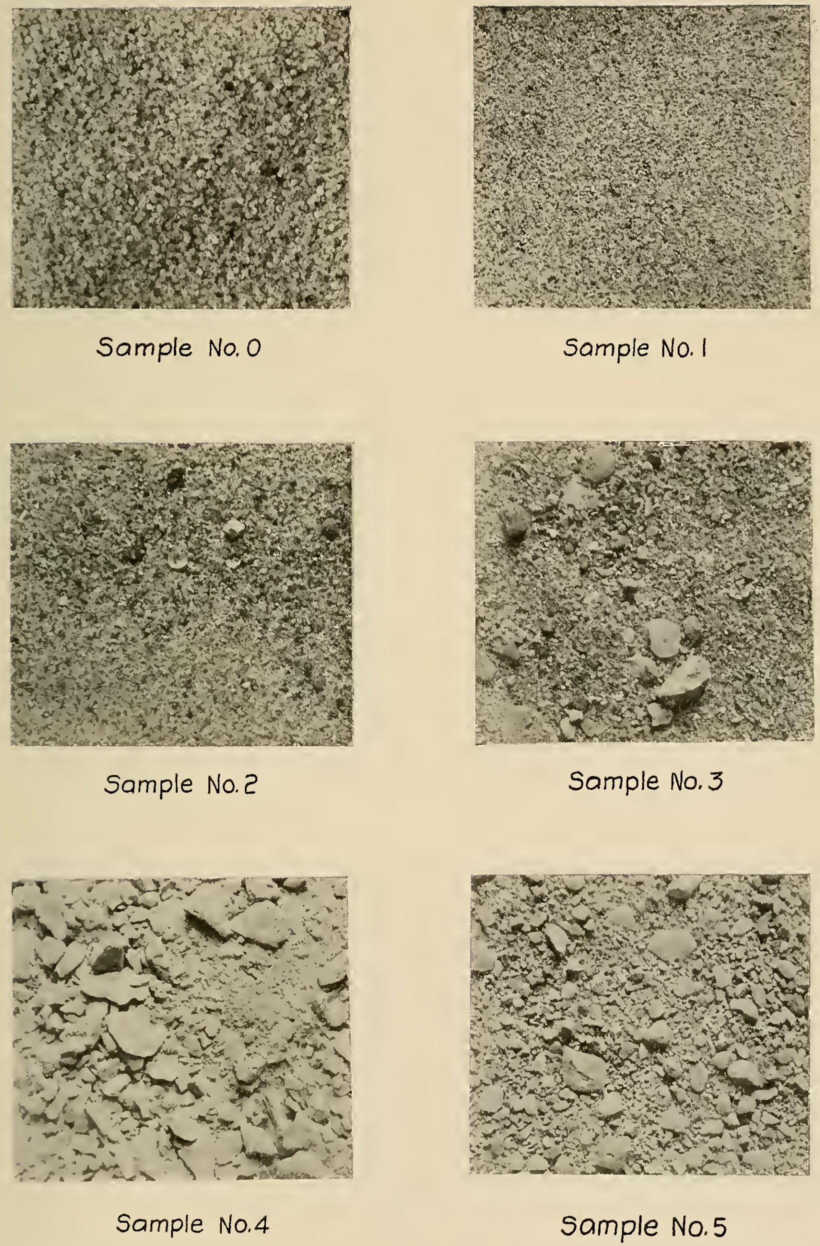
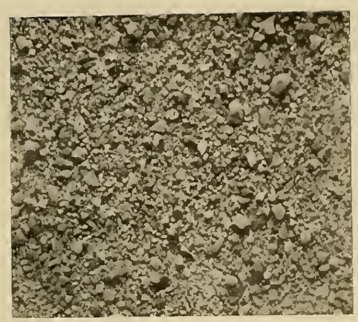


FIG. 6. PHOTOGRAPHS OF SAMPLES NO. 0 TO 5.

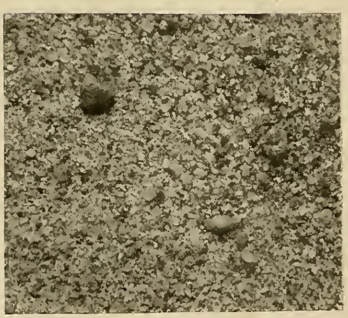


Sample No. 6

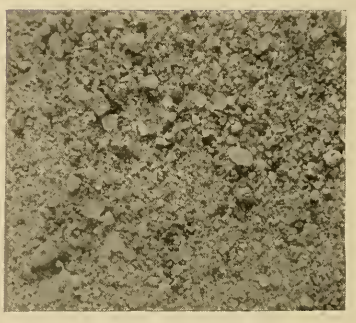
Sample No. 7

Sample No. 8

Sample No. 9



Sample No. 10



Sample No. 11

FIG. 7. PHOTOGRAPHS OF SAMPLES NO. 6 TO 11.

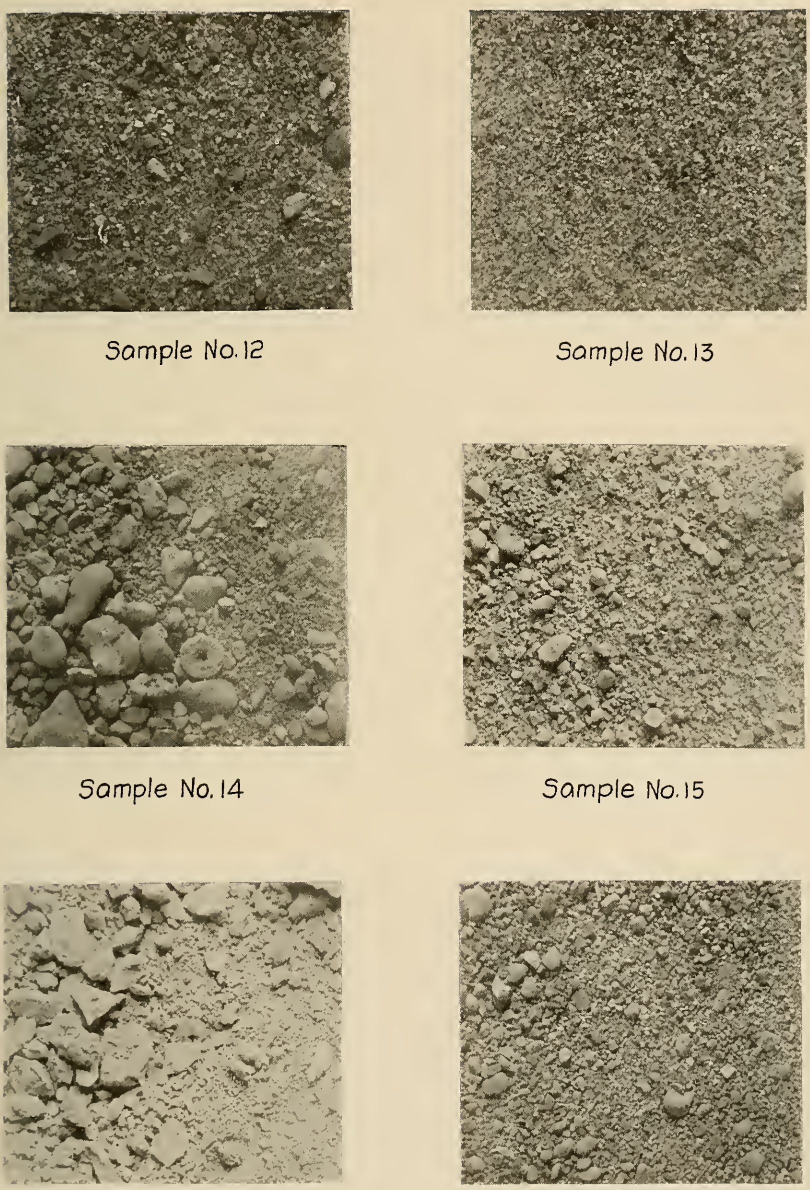
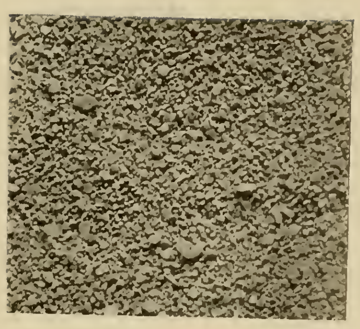


Fig. 8. Photographs of Samples No. 12 to 17.

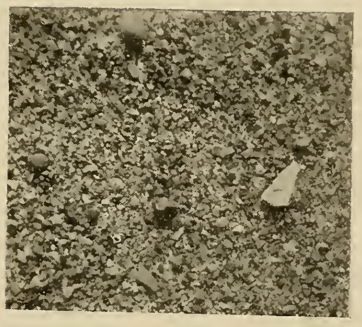


Sample No. 18

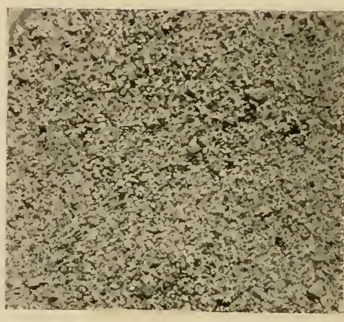
Sample No. 19

Sample No. 20

Sample No. 21

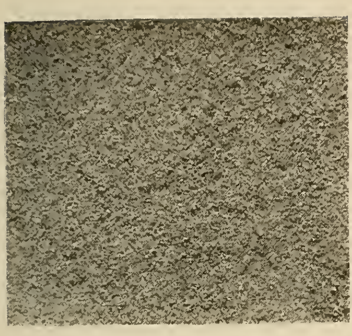


Sample No. 22



Sample No. 23

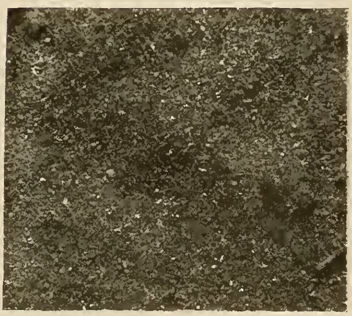
FIG. 9. PHOTOGRAPHS OF SAMPLES NO. 18 TO 23.



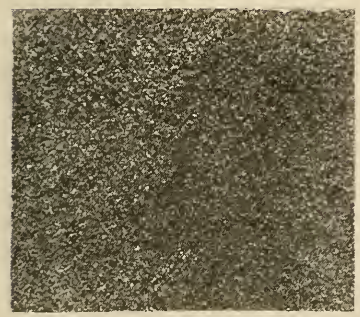
Sample No. 24

Sample No. 25

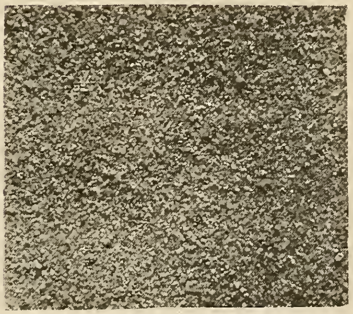
Sample No. 26



Sample No. 27



Sample No. 28



Sample No. 29

FIG. 10. PHOTOGRAPHS OF SAMPLES NO. 24 TO 29.

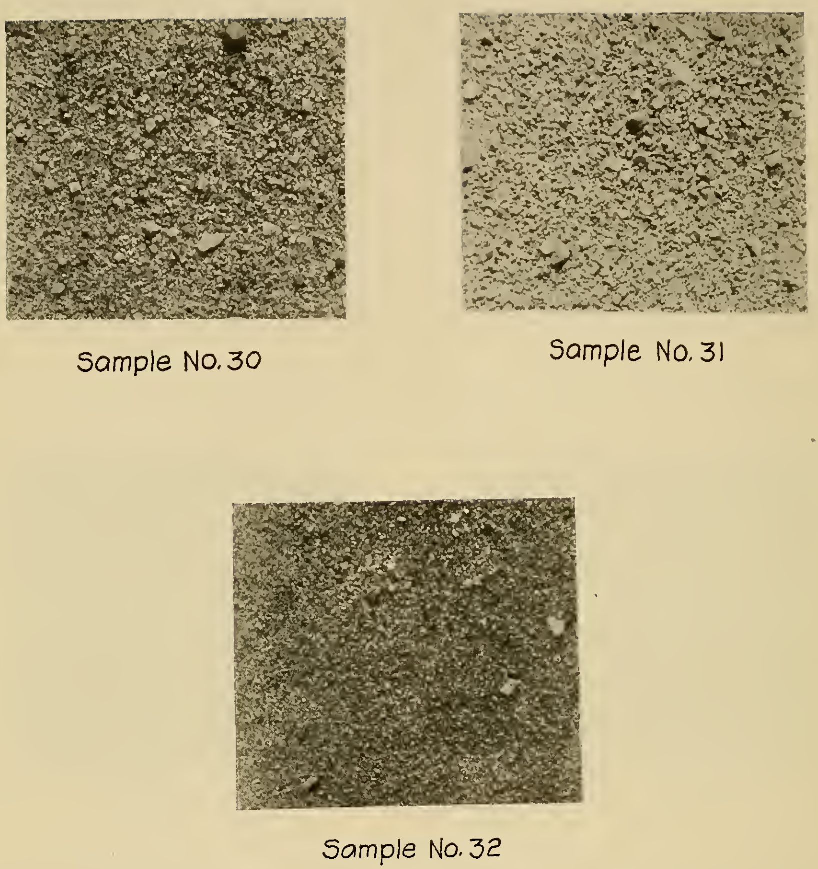


Fig. 11. Photographs of Samples No. 30 to 32.

PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *Fifteen cents.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat Through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

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